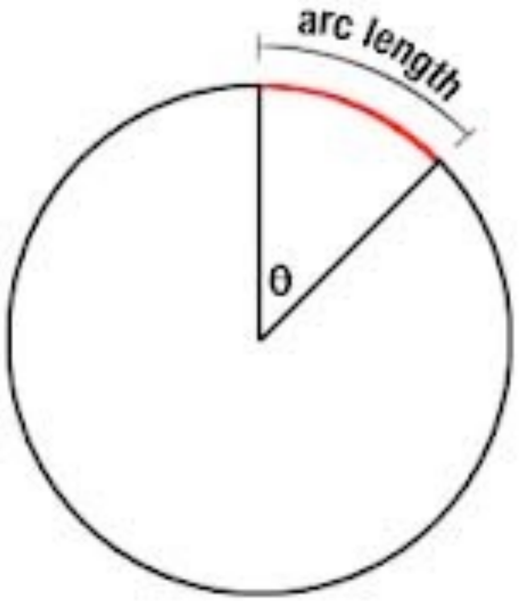
Planets +
Comets

Orbits: The Basics

$$E_{\text{tot}} = \text{K.E.} + V_{\text{pot.}}$$



$$l = r\theta$$

$$\text{K.E.} = p_r^2 + p_\theta^2$$

$$\text{K.E.} = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right)$$

Orbits: The Basics

$$V_{\text{pot.}} = -\frac{GMm}{r}$$

$$L = r \times p_\theta = r^2\dot{\theta}$$

$$F_r = \frac{d}{dt}p_r$$

$$F_r = \frac{d}{dr}\left(\text{K.E.} - V_{\text{pot.}}\right)$$

$$= 2mr\dot{\theta}^2 - \frac{GMm}{r^2}$$

$$m\ddot{r} = 2mr\dot{\theta}^2 - \frac{GMm}{r^2}$$

$$= \frac{2mL^2}{r^3} - \frac{GMm}{r^2}$$

Orbits: The Basics

$$u = \frac{1}{r}$$

$$\begin{aligned}\dot{r} =& \dot{\theta}\frac{dr}{d\theta} \\ =& -r^2\dot{\theta}\frac{d}{d\theta}\frac{1}{r} \\ =& -L\frac{du}{d\theta}\end{aligned}$$

Orbits: The Basics

$$
\begin{aligned}
\ddot{r} &= -\,\dot{\theta}\frac{d}{d\theta}\left(L\frac{du}{d\theta}\right) \\
&= -\,L^2u^2\frac{d^2u}{d\theta^2}
\end{aligned}
$$

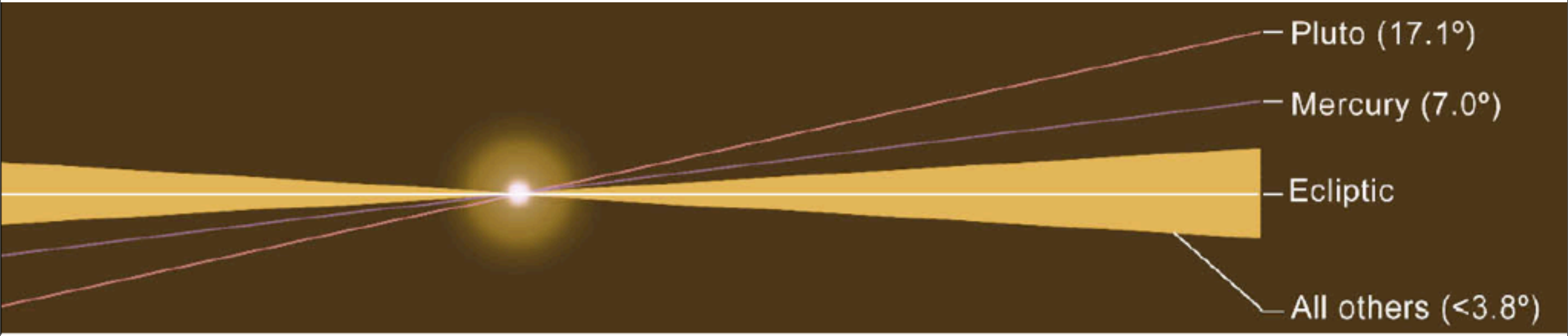
$$m\ddot{r} = \frac{2mL}{r^3} - \frac{GMm}{r^2}$$

$$\frac{d^2u}{d\theta^2} + u = \frac{GM}{L^2} \quad \Longrightarrow \quad u = \frac{GM}{L^2}(1 + e\cos\theta)$$

Orbits: Planets

Planet	Eccentricity
Mercury	0.21
Venus	0.01
Earth	0.02
Mars	0.09
Jupiter	0.05
Saturn	0.06
Uranus	0.05
Neptune	0.01
Pluto	0.25

Orbits: Planets



Planet	Eccentricity
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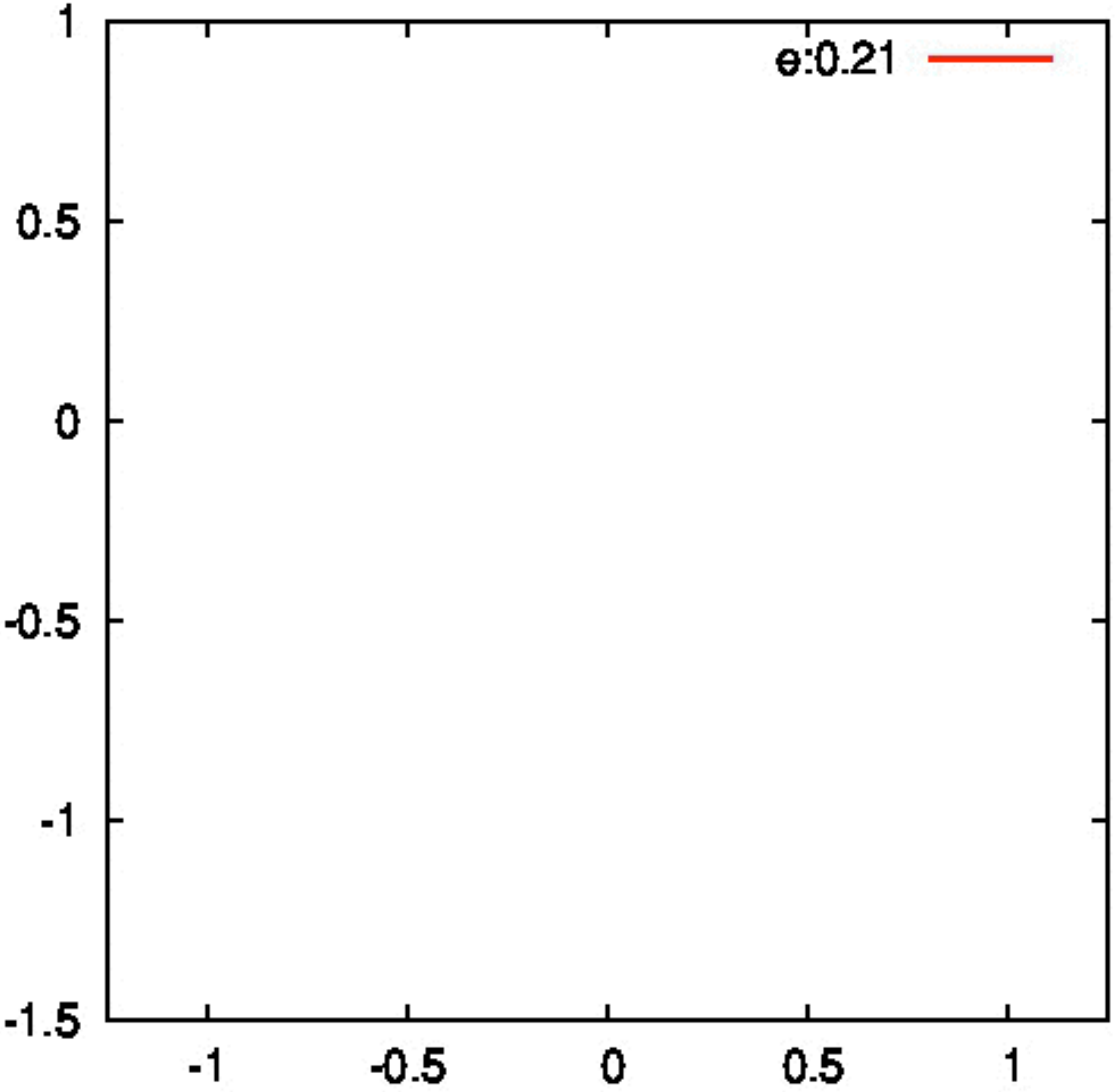
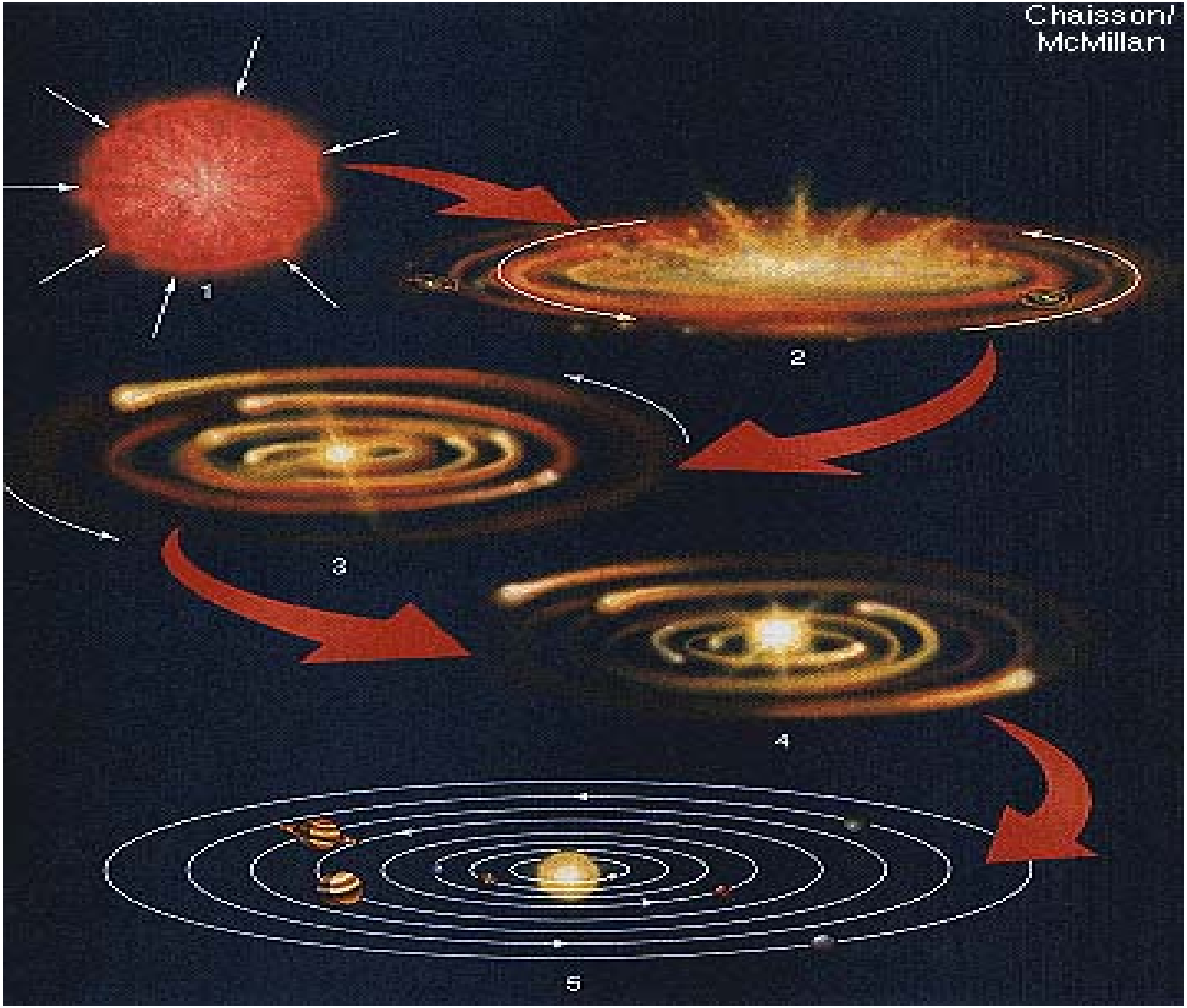
Venus	0.01
Earth	0.02
Mars	0.09
Jupiter	0.05
Saturn	0.06
Uranus	0.05
Neptune	0.01
Pluto	0.25

Orbits: Mercury




Chaisson/
McMillan
1
2
3
4
5

Evidence for Proto-Planetary Disks

Planet Formation

Initially grain size < 1mu to a few mm

Growth of ice mantles and grains to interstellar gravel, max size a few cm's

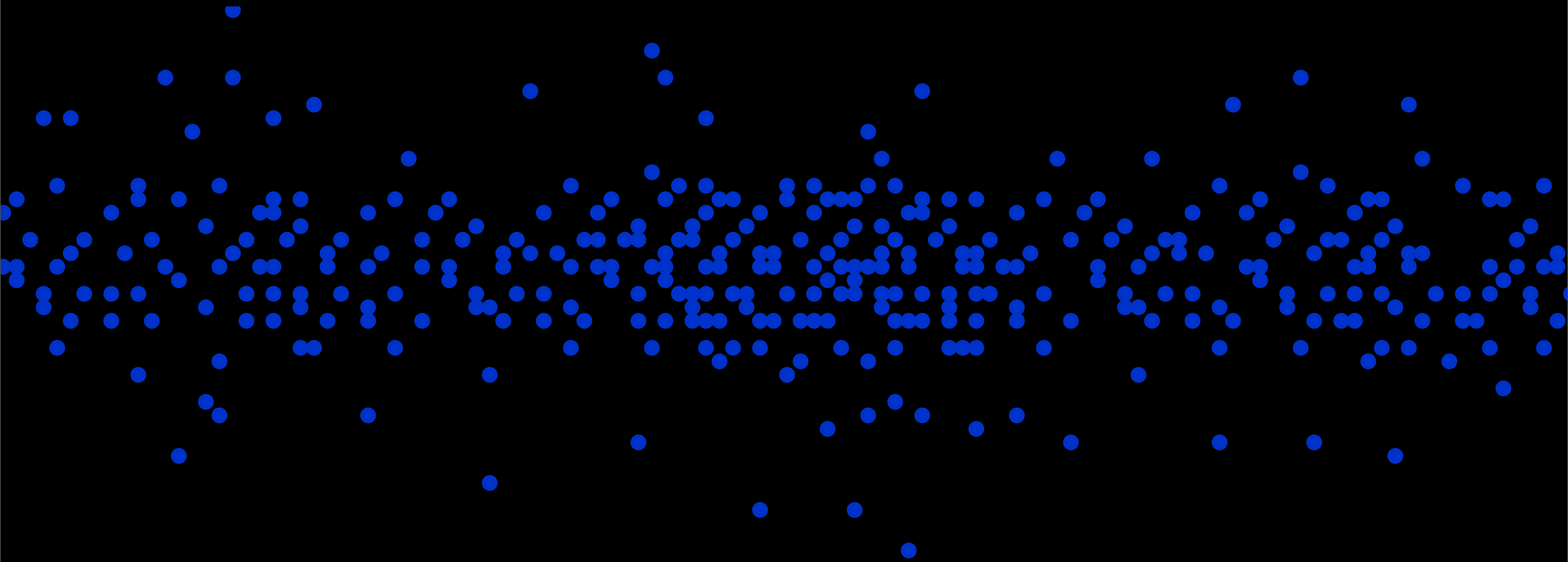
Growth of ice balls, rocks, max size a few m's

Planetisimal Formation km's in size

Planetisimal's accelerated by each others gravity, 10-1000 km's protoplanets formed

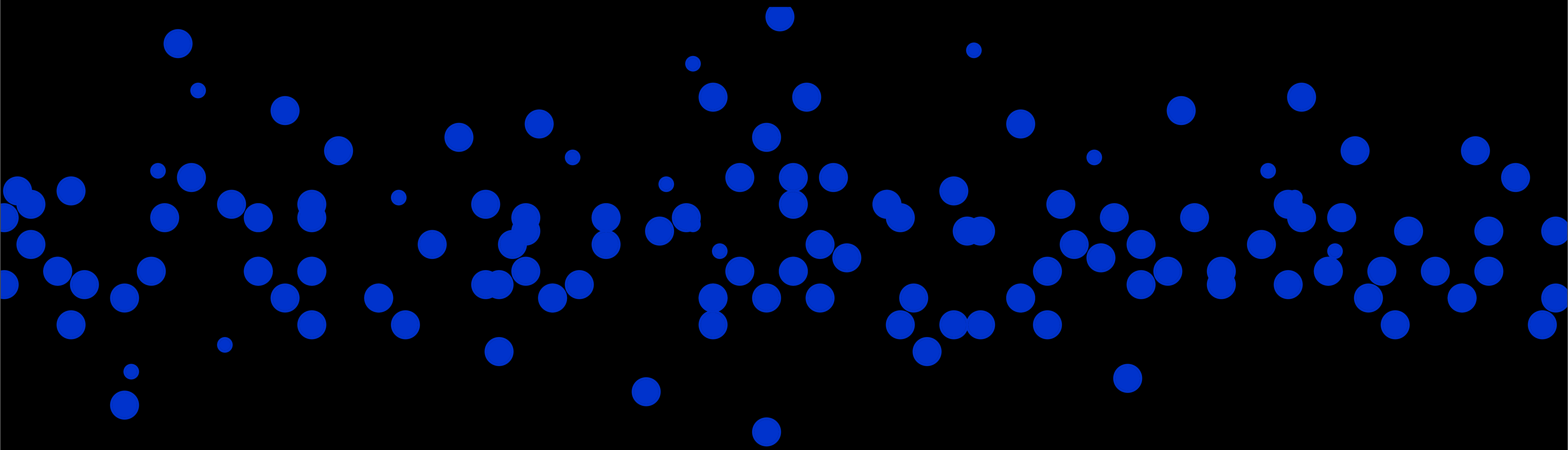
timescale of millions of years

A cross-section through a circumstellar disk



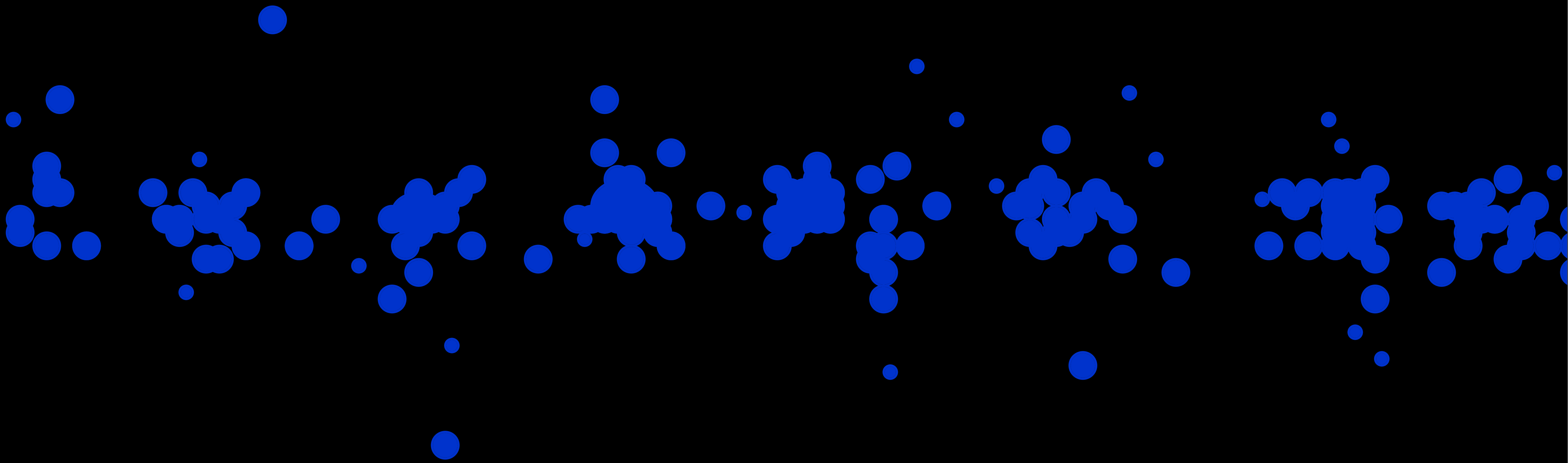
Grain size < 1mu to a few mm

Growth of ice mantles and grains to interstellar gravel



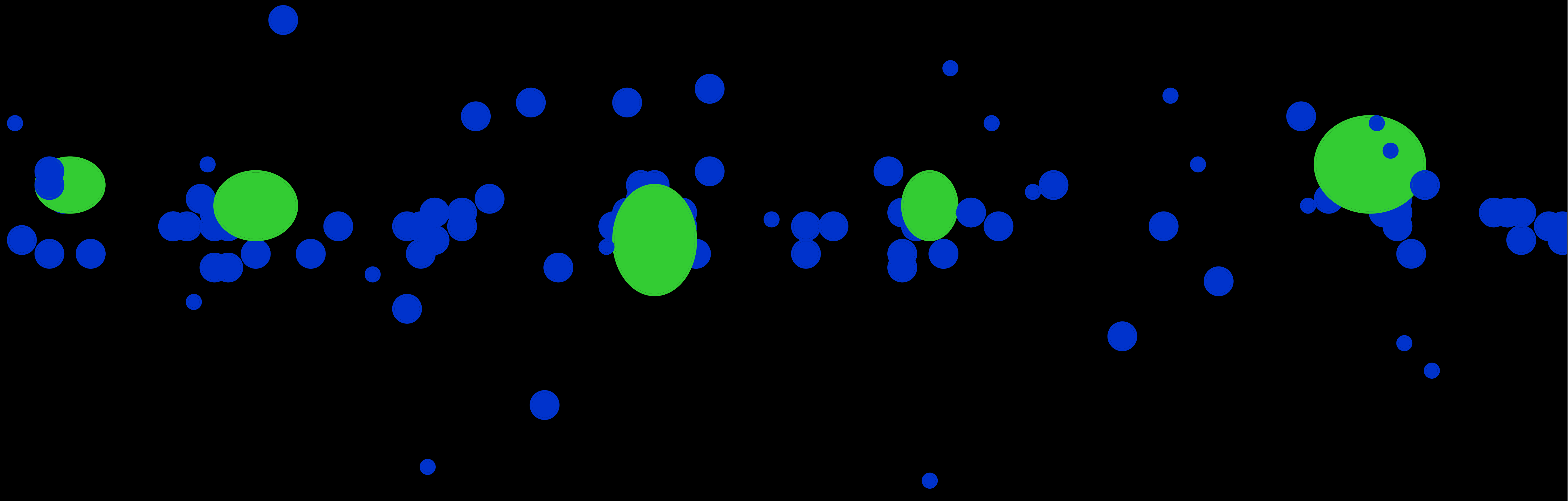
Maximum size: few centimeters

Growth of ice balls, rocks



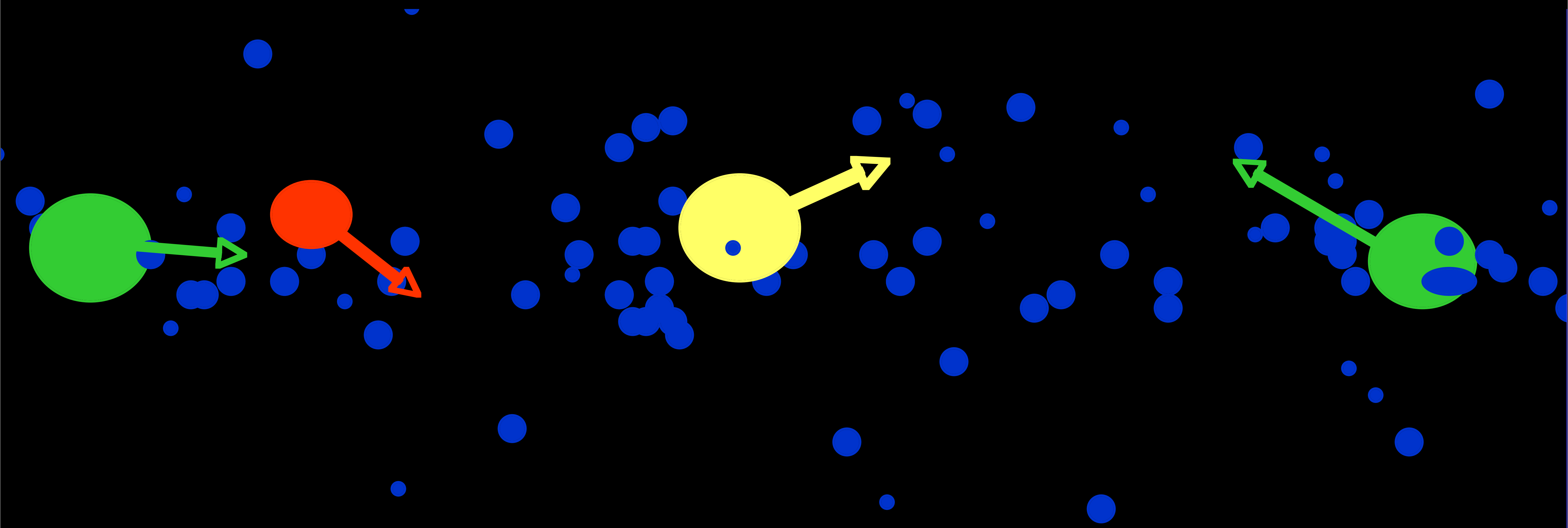
Maximum size: few meters

Planetisimal Formation



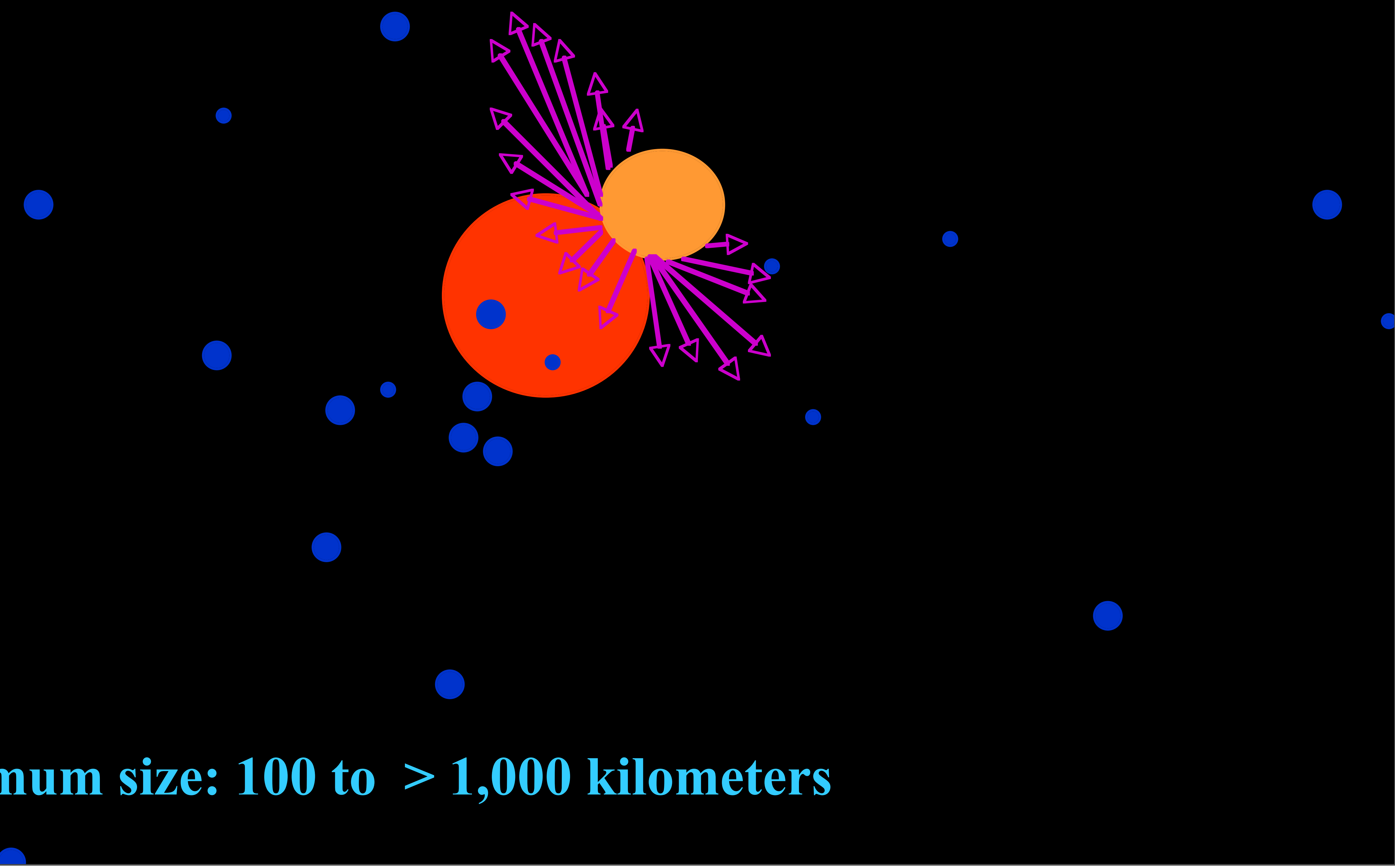
Maximum grain size: few kilometers

Planetisimal's accelerated by each others gravity



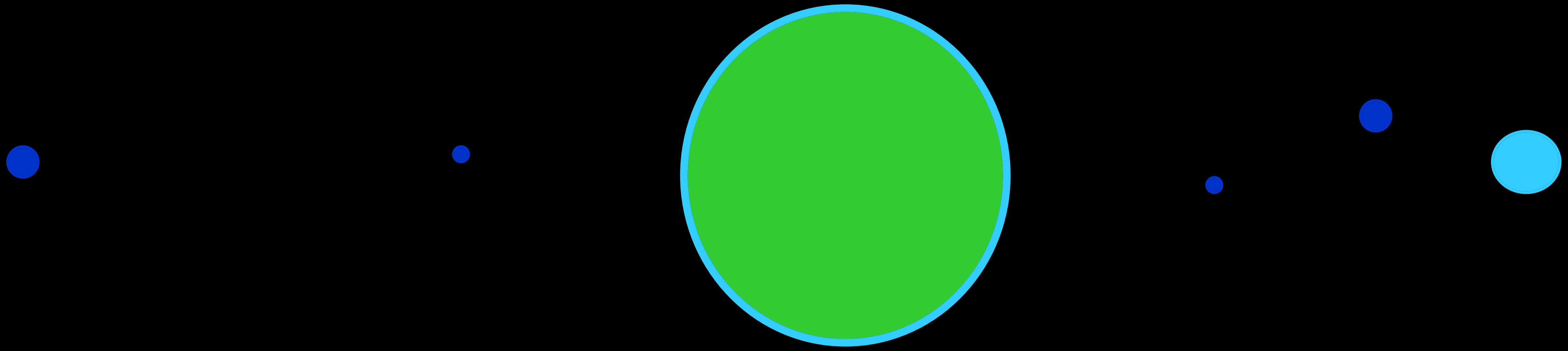
Maximum size: 10 to 1,000 kilometers

Protoplanet collisions



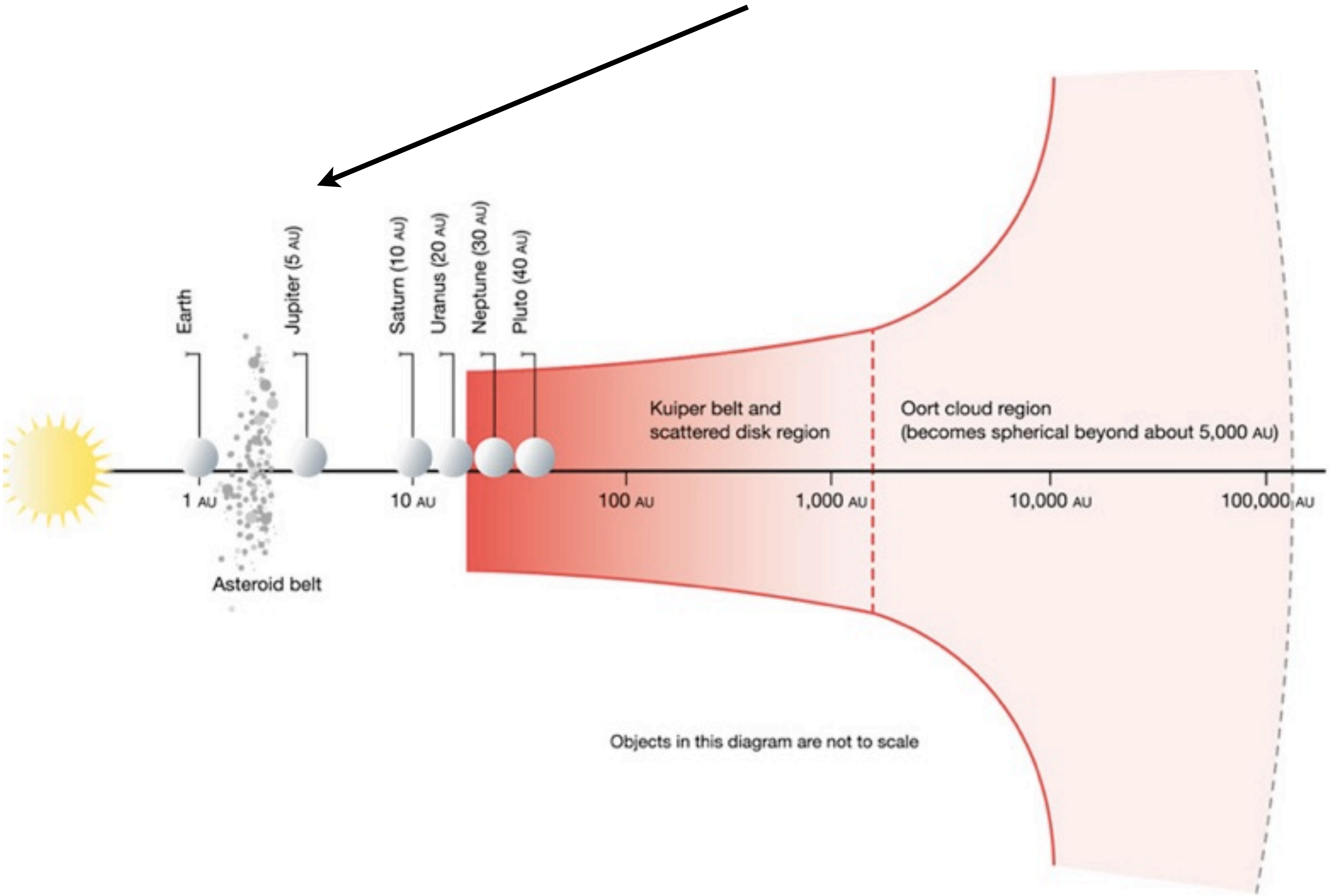
Maximum size: 100 to > 1,000 kilometers

A Young Planet!



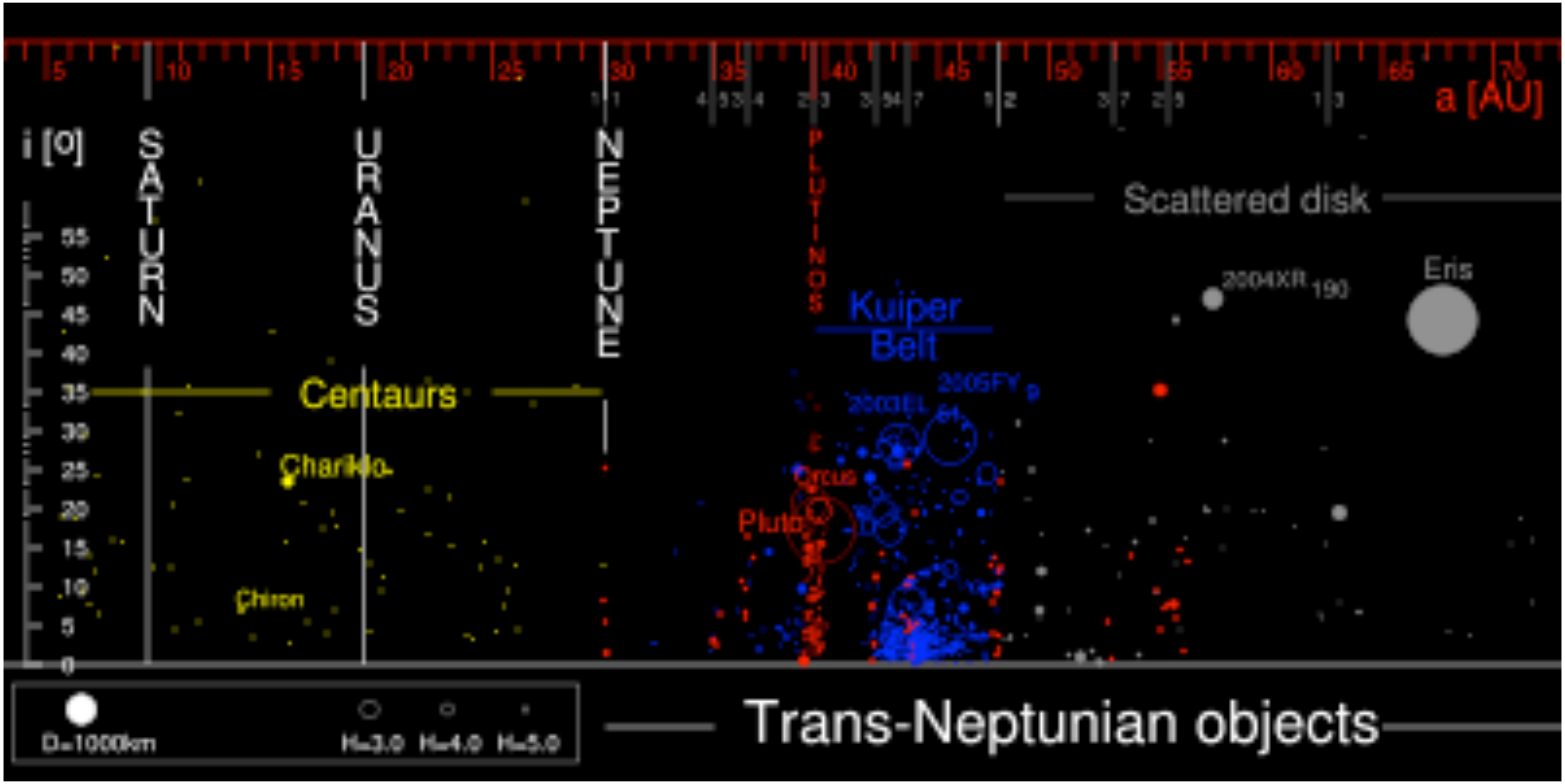
Maximum size: > 10,000 kilometers

The Inner Solar System

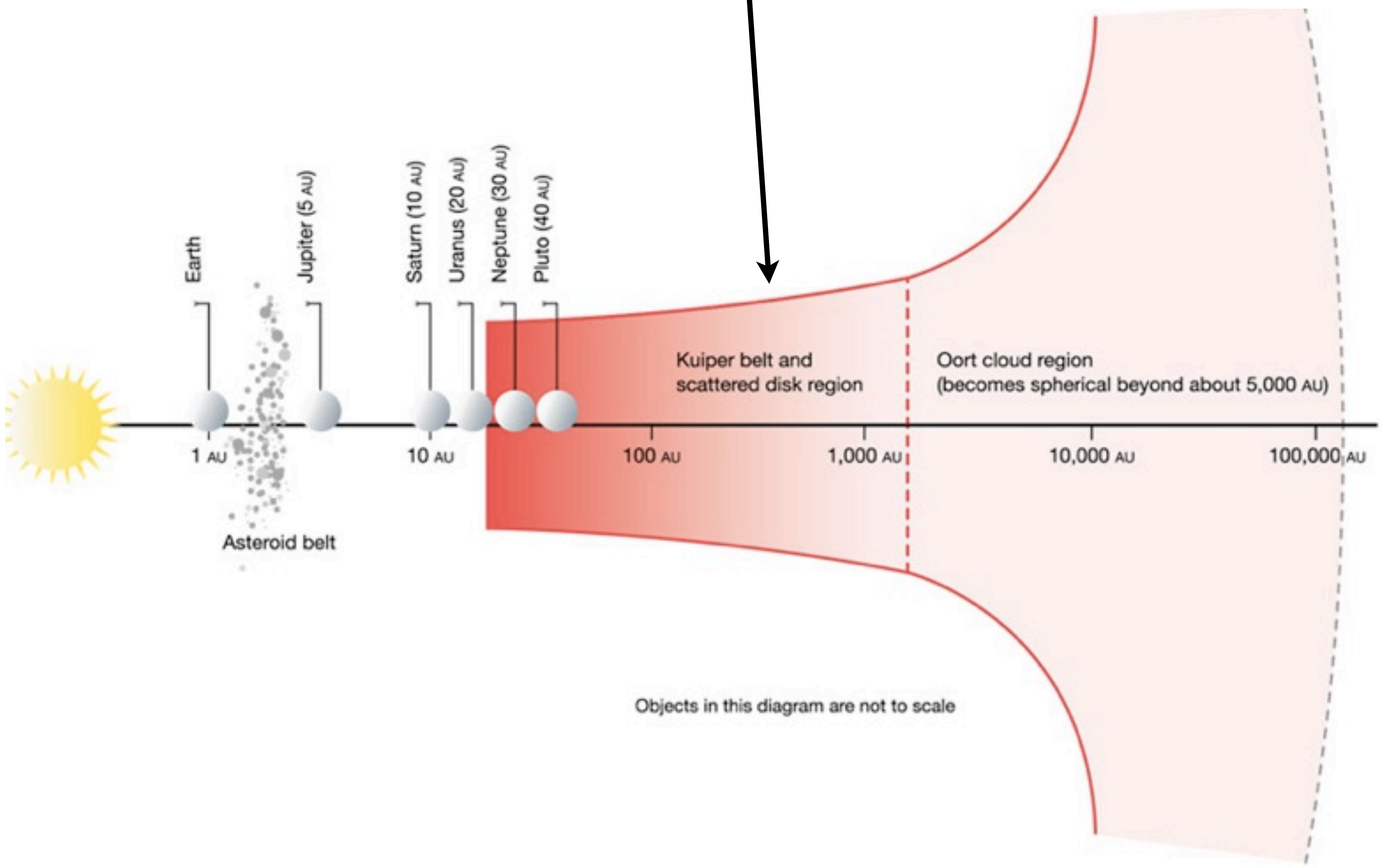


Trans-Neptunian Objects

Trans-Neptunian Objects are the name given collectively to any object in the solar system that orbits at a greater distance on average than Neptune. The Edge-worth Kuiper belt, Scattered disk and Oort cloud are the names for the three divisions of this volume of space.



The Edge-worth Kuiper Belt

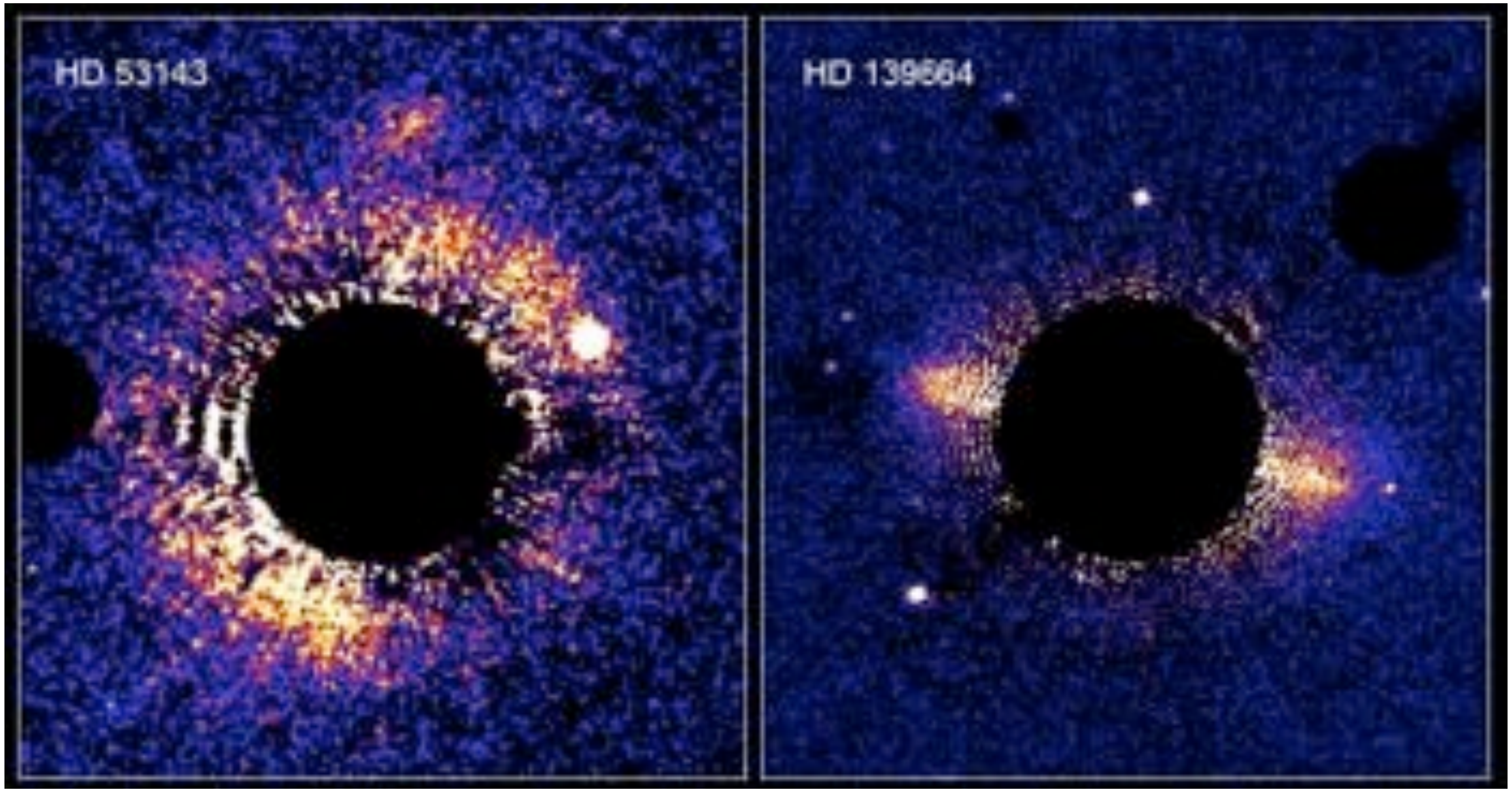


Objects in this diagram are not to scale

1943 Irish astronomer Kenneth E. Edgeworth and Gerard Kuiper 1951 suggested the existence of a belt of objects beyond Neptune that might be source of short period comets. Gerard Kuiper is generally credited with its discovery.

The E-Kuiper belt is an area of the solar system that extends beyond Neptune (at 30 AU) to 50 AU from the Sun. It is a vast resevoir of icy bodies.

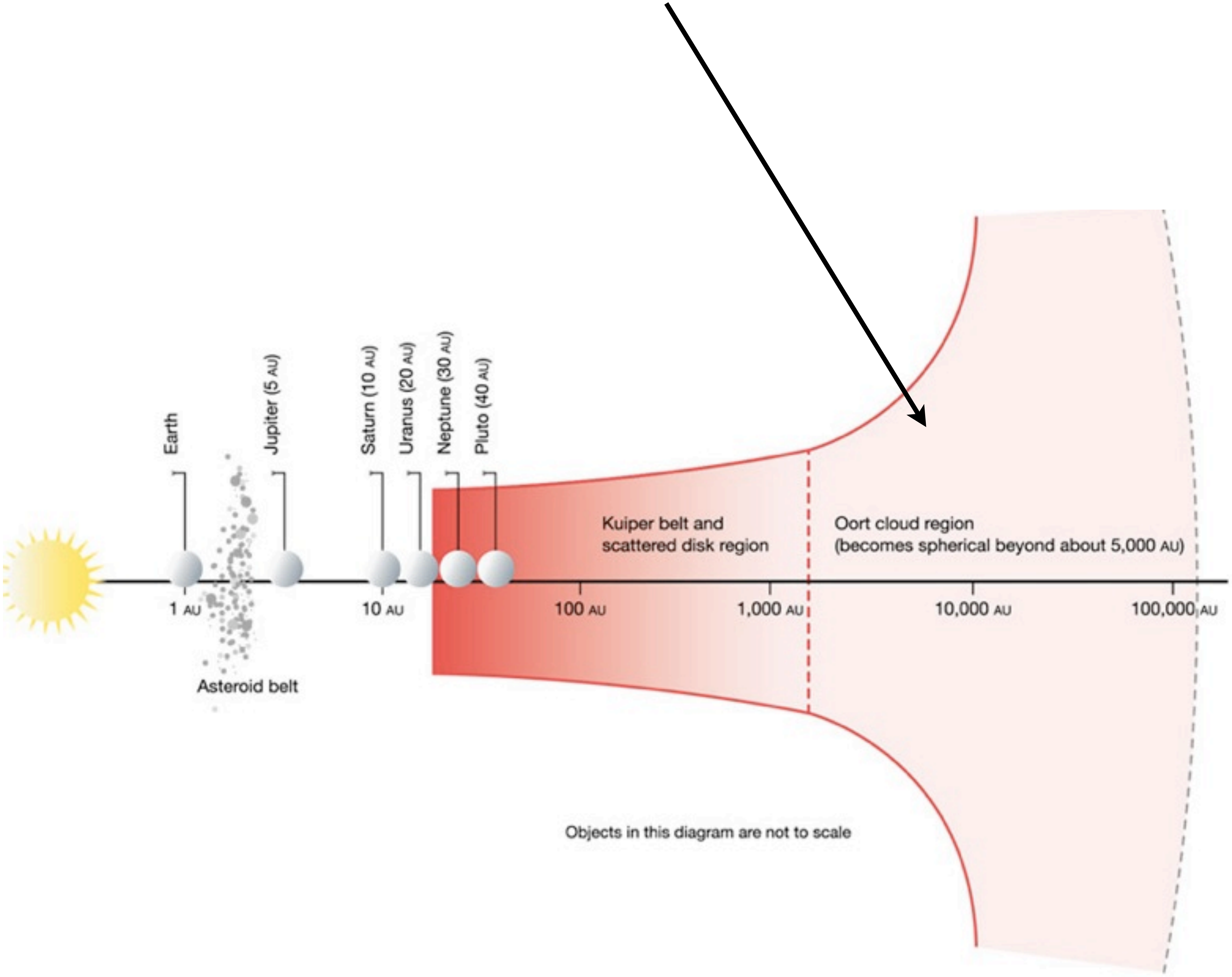
Objects in the E-Kuiper belt are a sub-set of the **trans-Neptunian objects.** Over 800 Kuiper belt objects have been discovered to date. The first was found in 1992

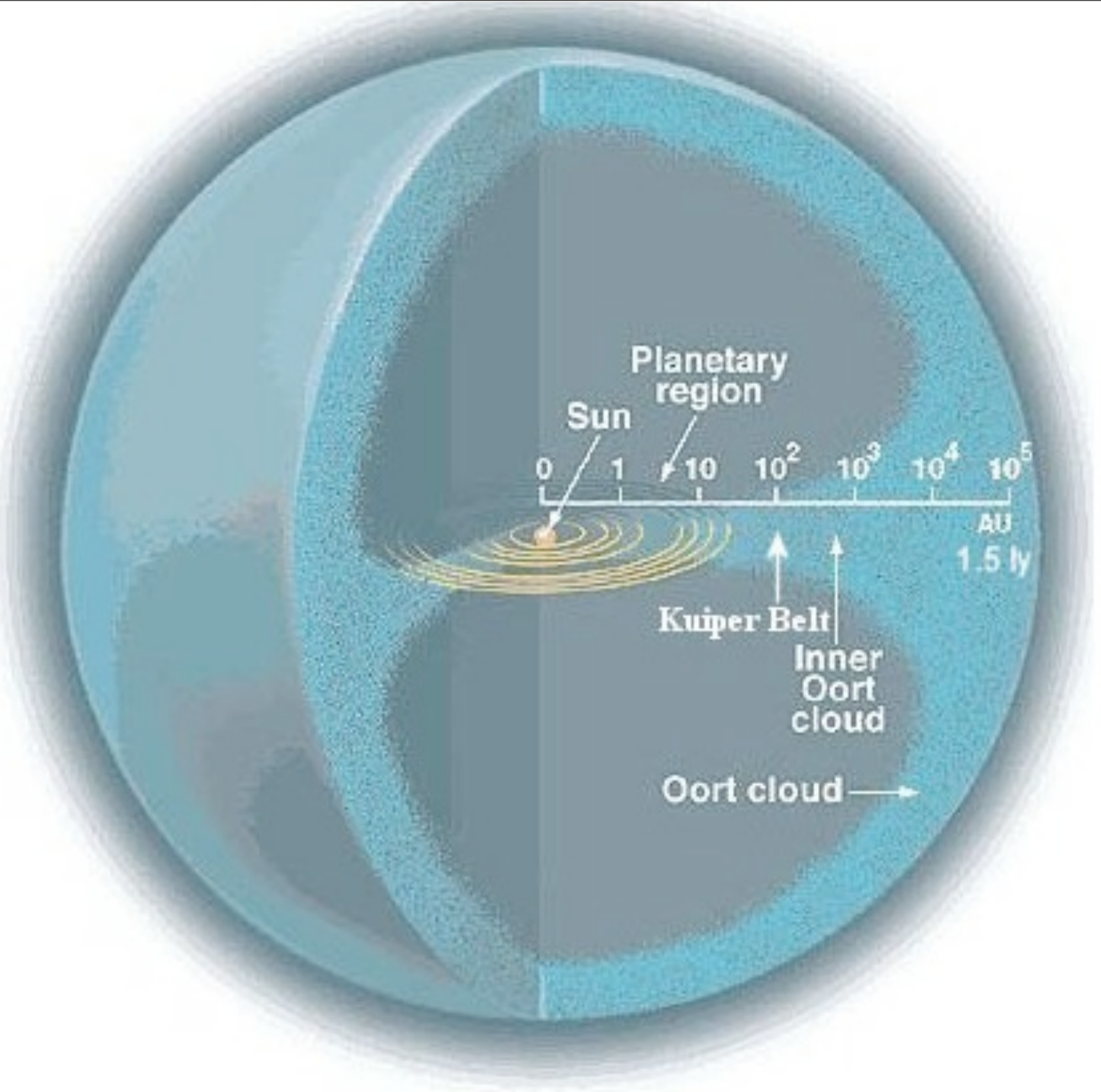


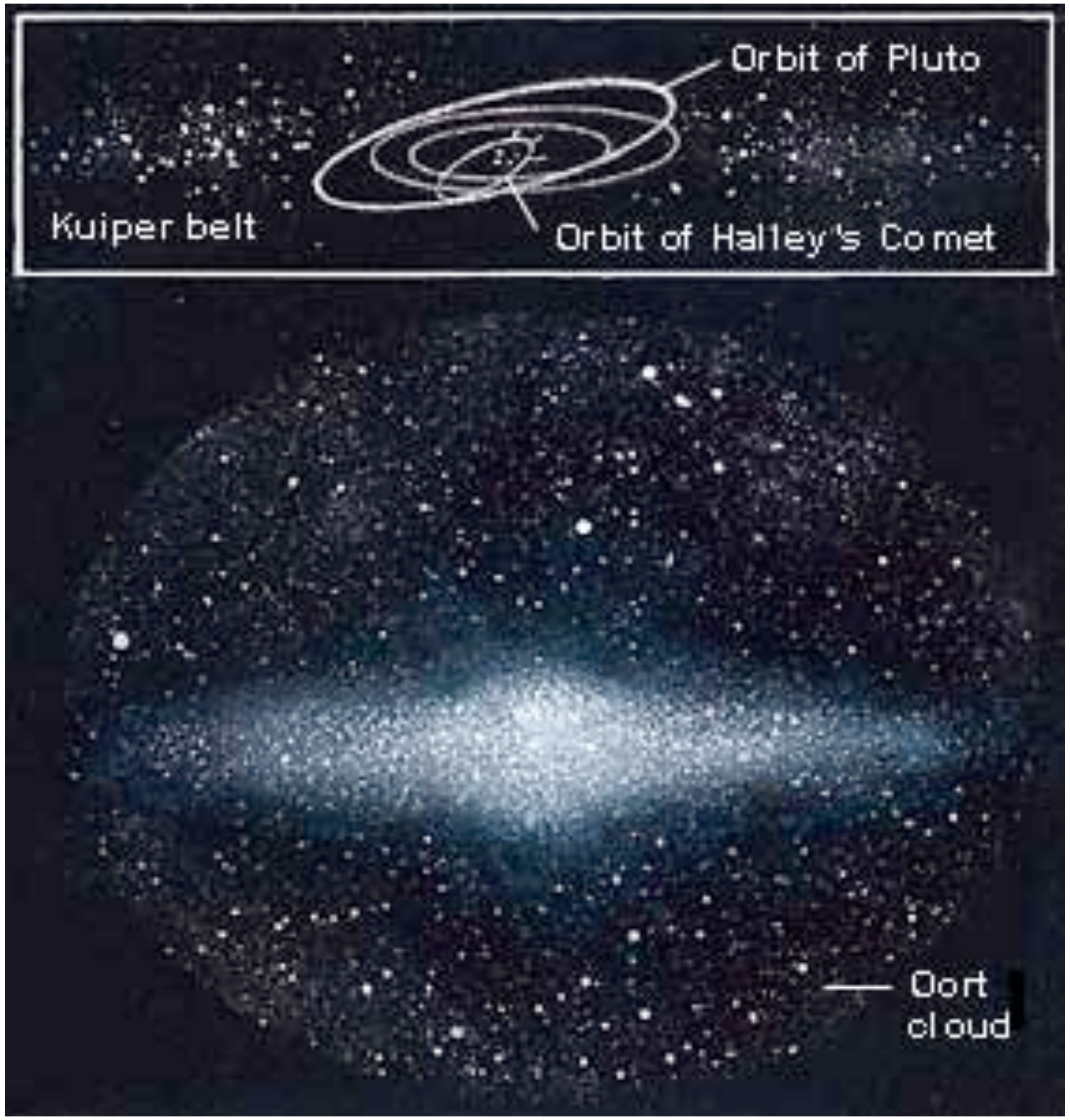
Origin of E-Kuiper Belt

The E-Kuiper belt and scattered disk region represent the edge of the Sun's accretion disk. The disk became much less dense towards the edges and accretion progressed very slowly. Hence planets did not form and a ring of debris material or small objects remained. As these objects were located far beyond the Giant planets they escaped being ejected from the solar system.

The Oort Cloud



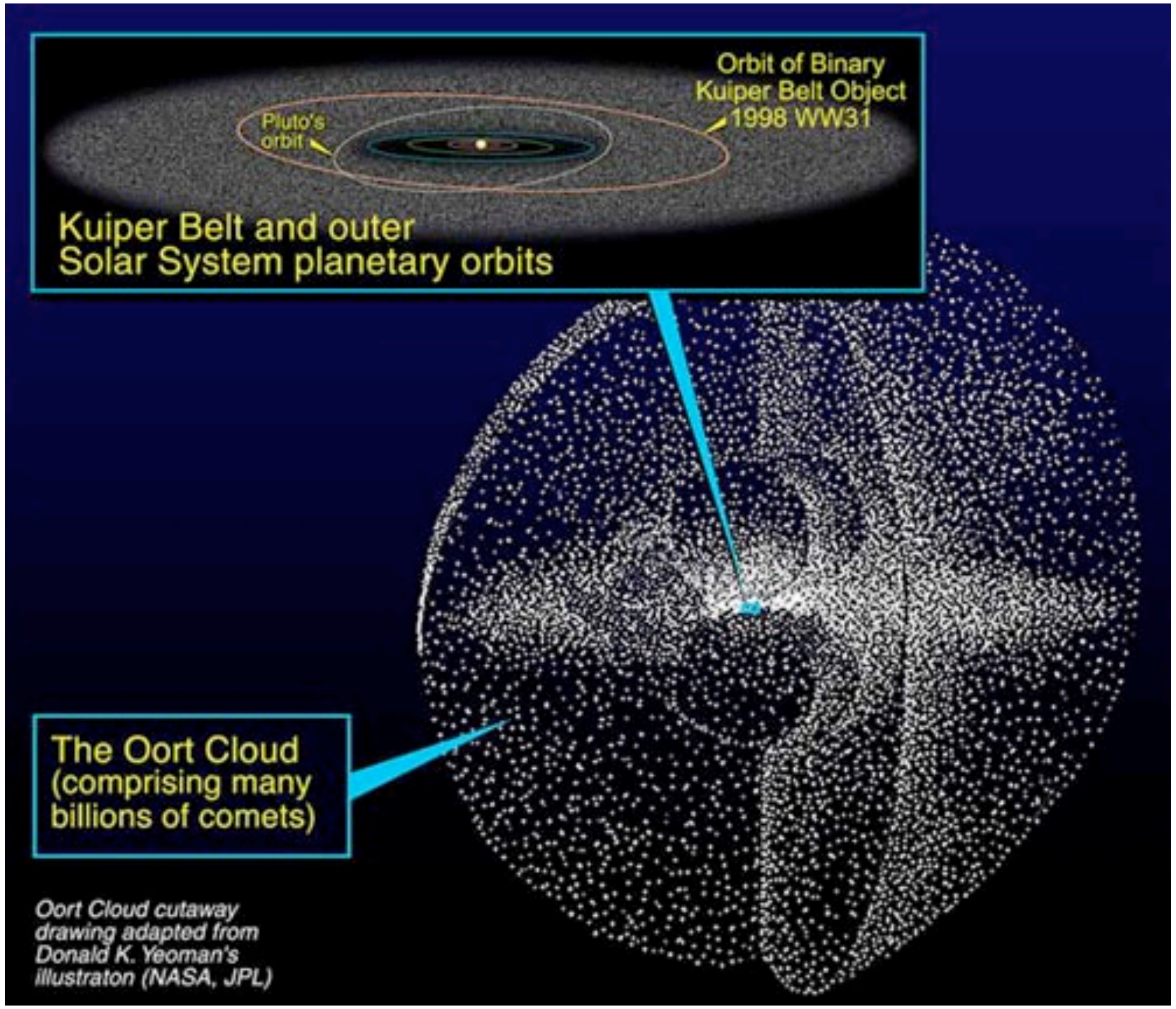

Planetary region
Sun
0
1
10
10^2
10^3
10^4
10^5
AU
1.5 ly
Kuiper Belt
Inner Oort cloud
Oort cloud


Orbit of Pluto
Kuiper belt
Orbit of Halley's Comet
Oort cloud

The Oort Cloud is a spherical cloud of billions of comets at a distance of 50,000 to 100,000 AU from the Sun and with a predicted mass of 5-100 times the mass of the Earth. It is a resevoir for long-period comets

Its existence was first predicted by Jans Oort

The Oort cloud is a remnant of the original molecular core that collapsed to form the solar system. It is made up of left over fossil material. Some of the material formed in situ while some originated closer in but was ejected from the inner solar system and scattered out of the ecliptic plane due to gravitational interactions with the giants gas planets.



Oort Cloud cutaway drawing adapted from Donald K. Yeoman's illustraton (NASA, JPL)

Comets

A comet is a small icy rocky body that has a highly elliptical orbit around the sun. If the orbit of this object is perturbed it can be forced into the inner solar system where it is theorised that the Sun's radiation causes the outer layers to melt and evaporate (Whipples Dirty Snowball Model). The streams of dust and gas thus released form a very large, extremely tenuous atmosphere around the comet called the *coma*, and the force exerted on the coma by the Sun's radiation pressure and solar wind cause an enormous *tail* to form, which points away from the sun.

Comet Hale-Bopp

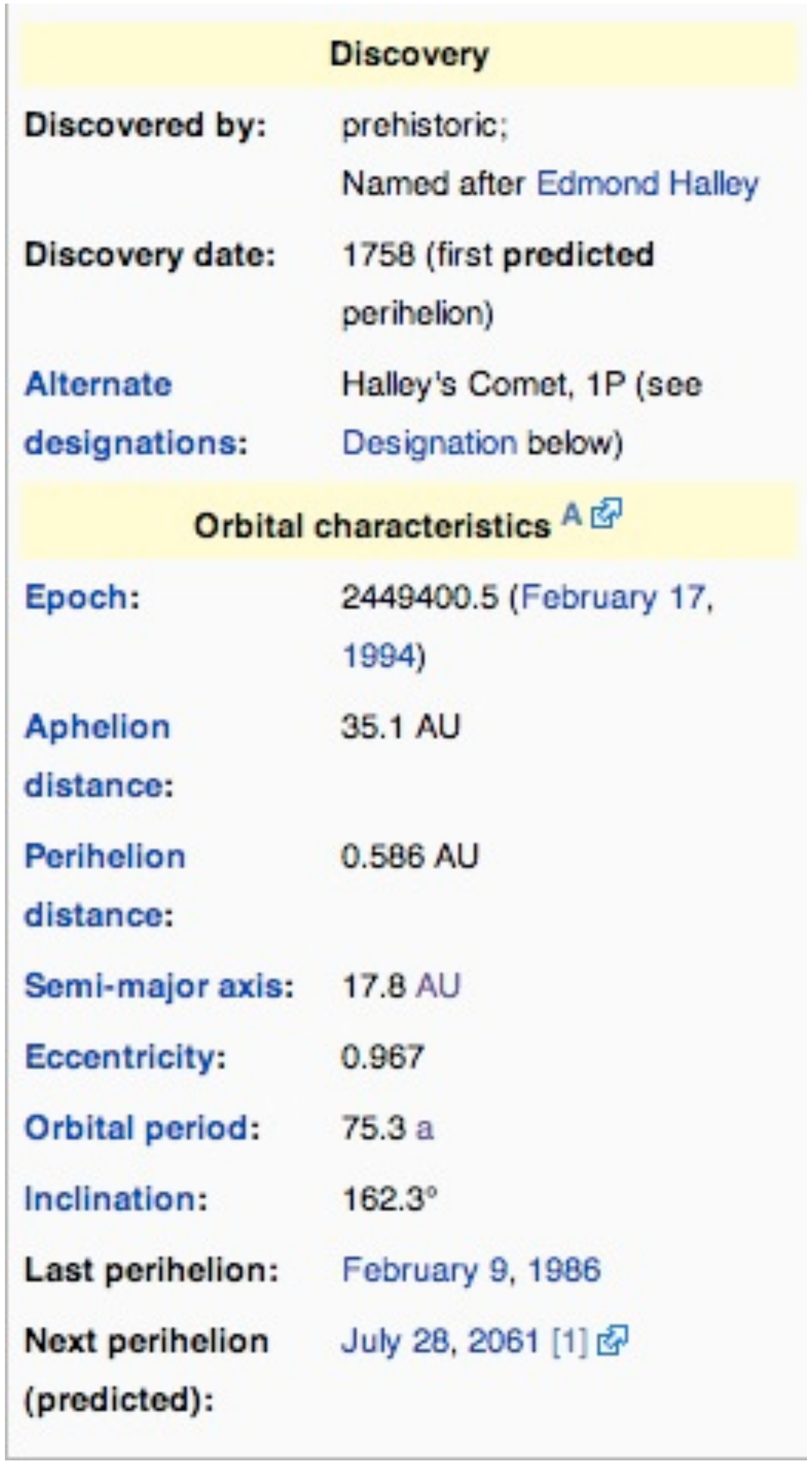
Comet Mc Naught

Orbits: Comets

Comet	Eccentricity
Haley's	0.97
Hale-Bopp	0.99
McNaught	N.A.
Lovejoy	0.99

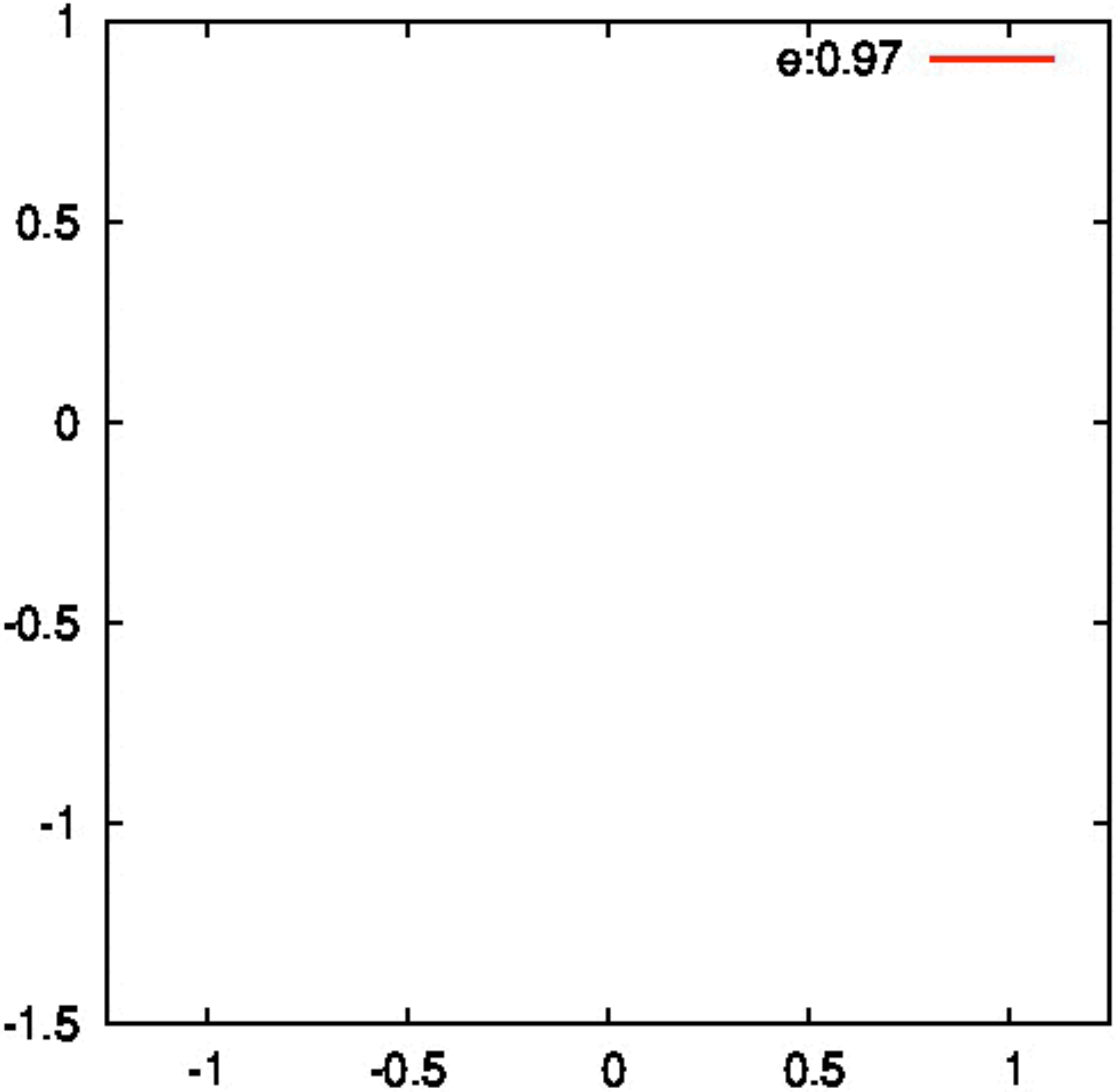
Halley’s Comet

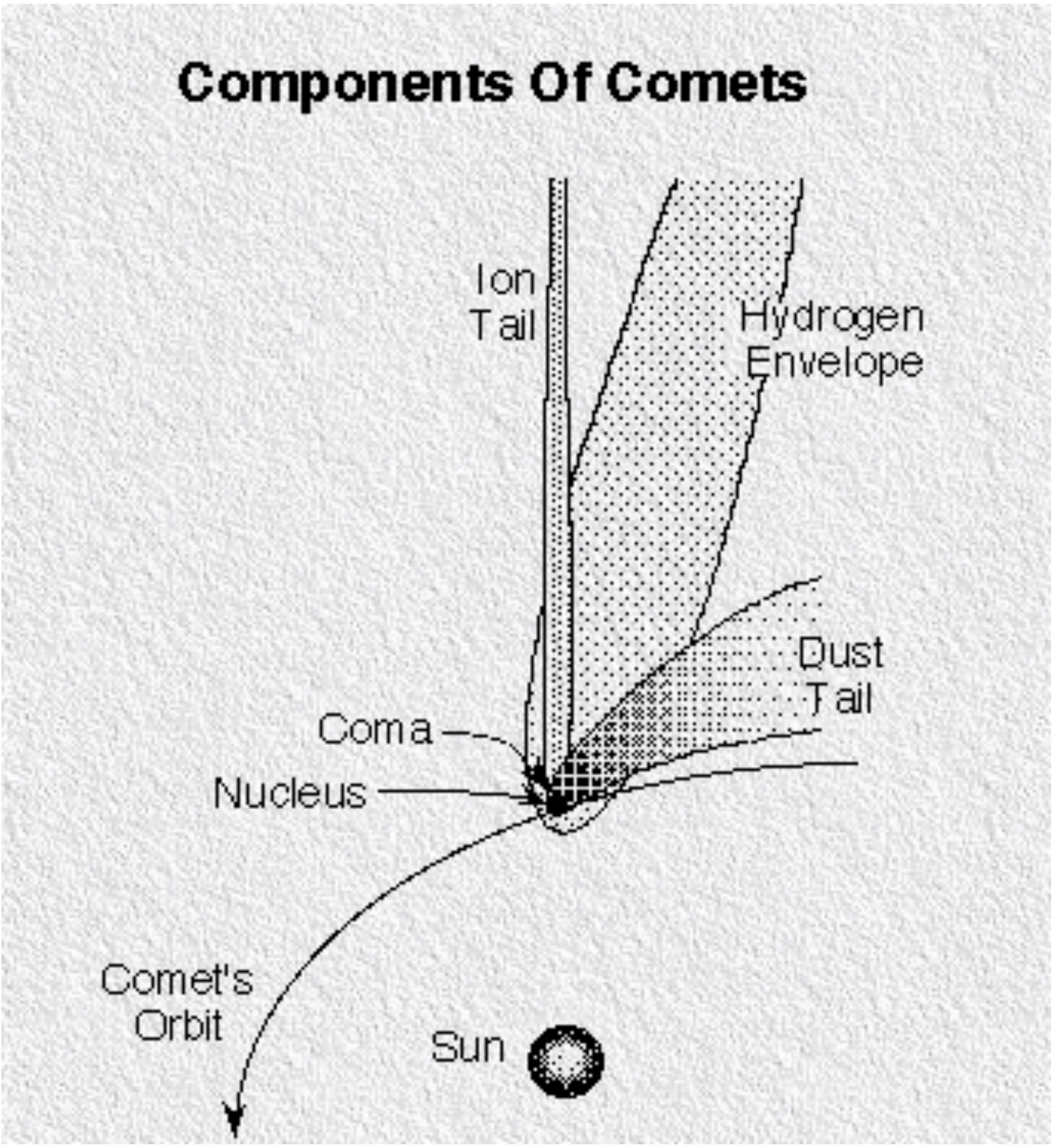
The most famous and the first comet shown to be periodic. Edmund Halley observed the comet of 1682 and recognised thats its characteristics were the same as comets that had been observed in 1531 and 1607. He predicted its return in 1758. Last time it was observed was 1986. The next perihelion is July 2061.

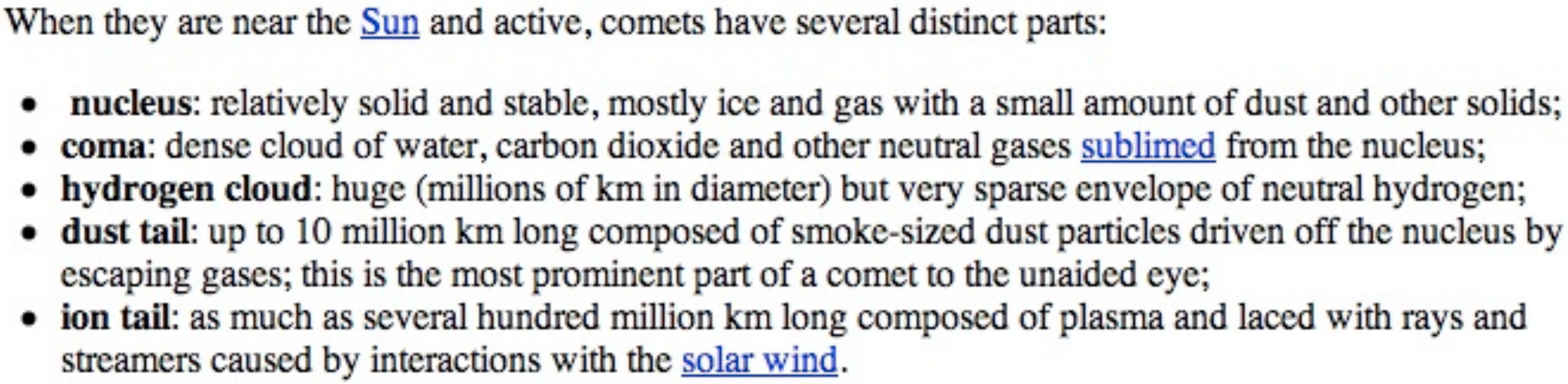


Discovery	
Discovered by:	prehistoric; Named after Edmond Halley
Discovery date:	1758 (first **predicted** perihelion)
Alternate designations:	Halley's Comet, 1P (see Designation below)
Orbital characteristics A	
Epoch:	2449400.5 (February 17, 1994)
Aphelion distance:	35.1 AU
Perihelion distance:	0.586 AU
Semi-major axis:	17.8 AU
Eccentricity:	0.967
Orbital period:	75.3 a
Inclination:	162.3°
Last perihelion:	February 9, 1986
Next perihelion (predicted):	July 28, 2061 [1]

Orbits: Haley's Comet







When they are near the Sun and active, comets have several distinct parts:

- **nucleus**: relatively solid and stable, mostly ice and gas with a small amount of dust and other solids;
- **coma**: dense cloud of water, carbon dioxide and other neutral gases sublimed from the nucleus;
- **hydrogen cloud**: huge (millions of km in diameter) but very sparse envelope of neutral hydrogen;
- **dust tail**: up to 10 million km long composed of smoke-sized dust particles driven off the nucleus by escaping gases; this is the most prominent part of a comet to the unaided eye;
- **ion tail**: as much as several hundred million km long composed of plasma and laced with rays and streamers caused by interactions with the solar wind.

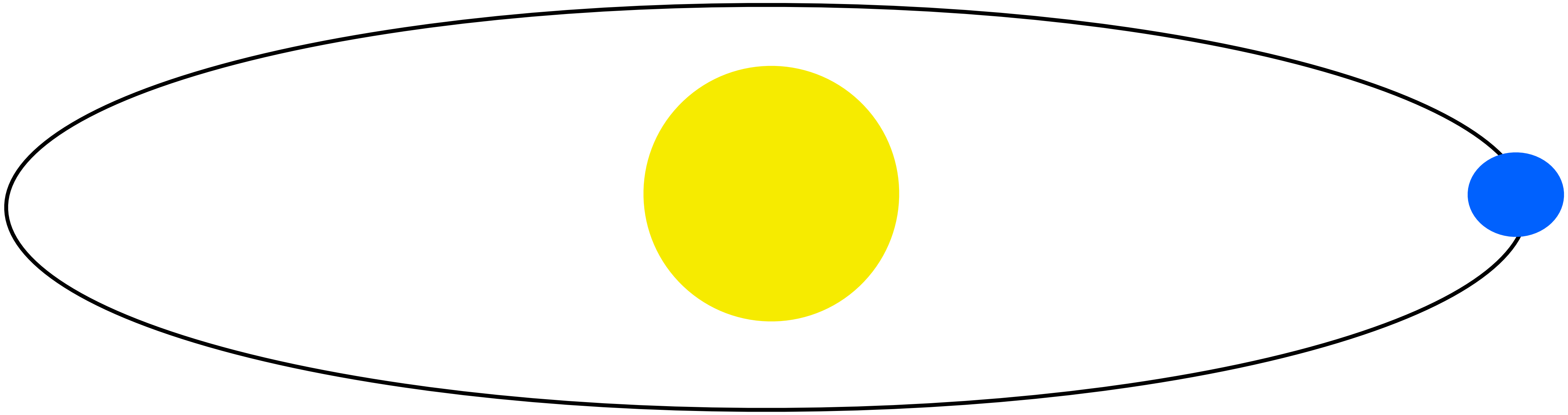
Short Period and Long Period

Comets are classified according to their orbital periods. *Short-period comets*, also called *periodic comets*, have orbits of less than 200 years, while *long-period comets* have longer orbits but remain gravitationally bound to the Sun

Short-period comets are thought to originate in the E-Kuiper belt whereas the source of long-period comets is thought to be the Oort Cloud.

Life on Planets

Determination of Liquid Water Zone



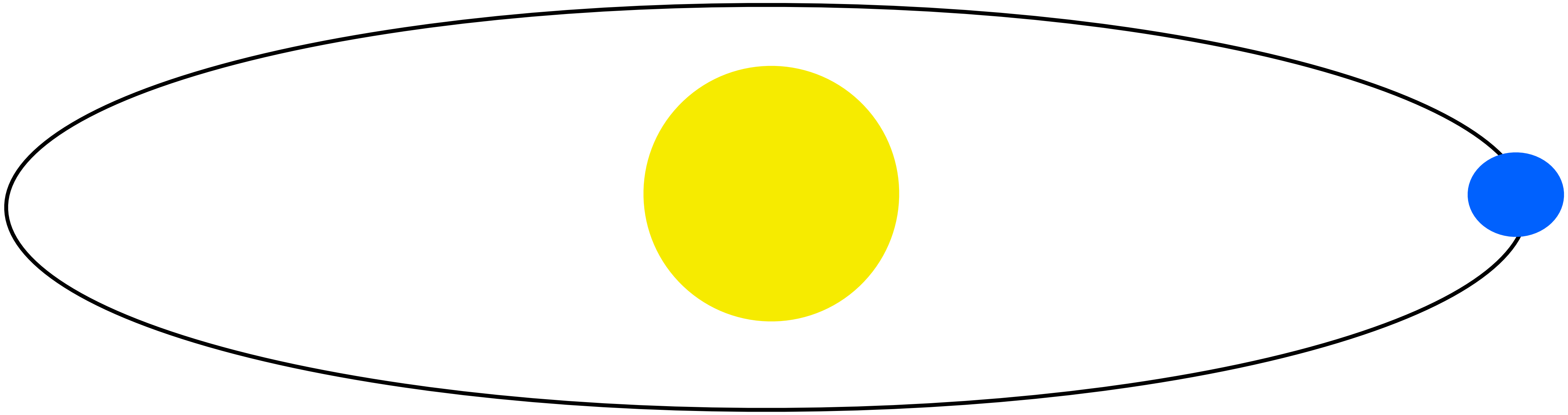
Earth- Power Received:

$$L_{\mathrm{Earth}} = L_{\mathrm{Sun}} \left(\frac{R_{\mathrm{Earth}}}{\mathrm{A.U.}} \right)^2$$

Blackbody Relation:

$$T^4 \propto \frac{L}{R^2}$$

Determination of Liquid Water Zone



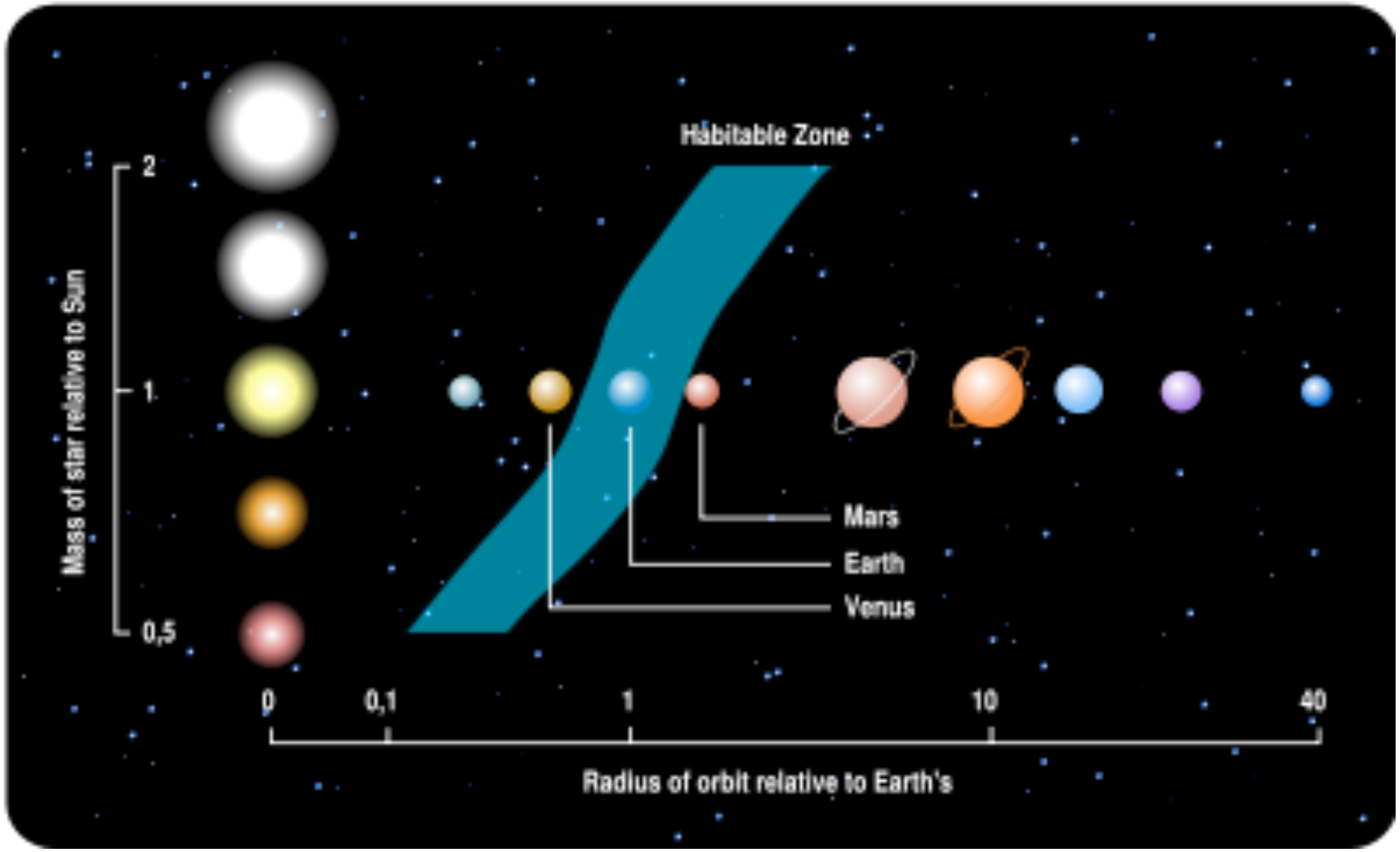
$$T_{\text{Earth}} = T_{\text{Sun}} \left(\frac{L_{\text{Earth}}}{L_{\text{Sun}}}\right)^{1/4} \left(\frac{R_{\text{Sun}}}{R_{\text{Earth}}}\right)^{1/2}$$

$$= T_{\text{Sun}} \left(\frac{R_{\text{Sun}}}{\text{A.U.}}\right)^{1/2}$$

$$= 10^{3.7} \left(\frac{10^{8.8}}{10^{11}}\right)^{1/2} \approx 10^{2.6}\ \text{K}$$

Low mass stars more likely to have life supporting planets

low mass stars (our sun) longer lived (5-10 byrs), solar winds and UV radiation not strong, High mass stars excessive UV radiation, strong winds and short-lived

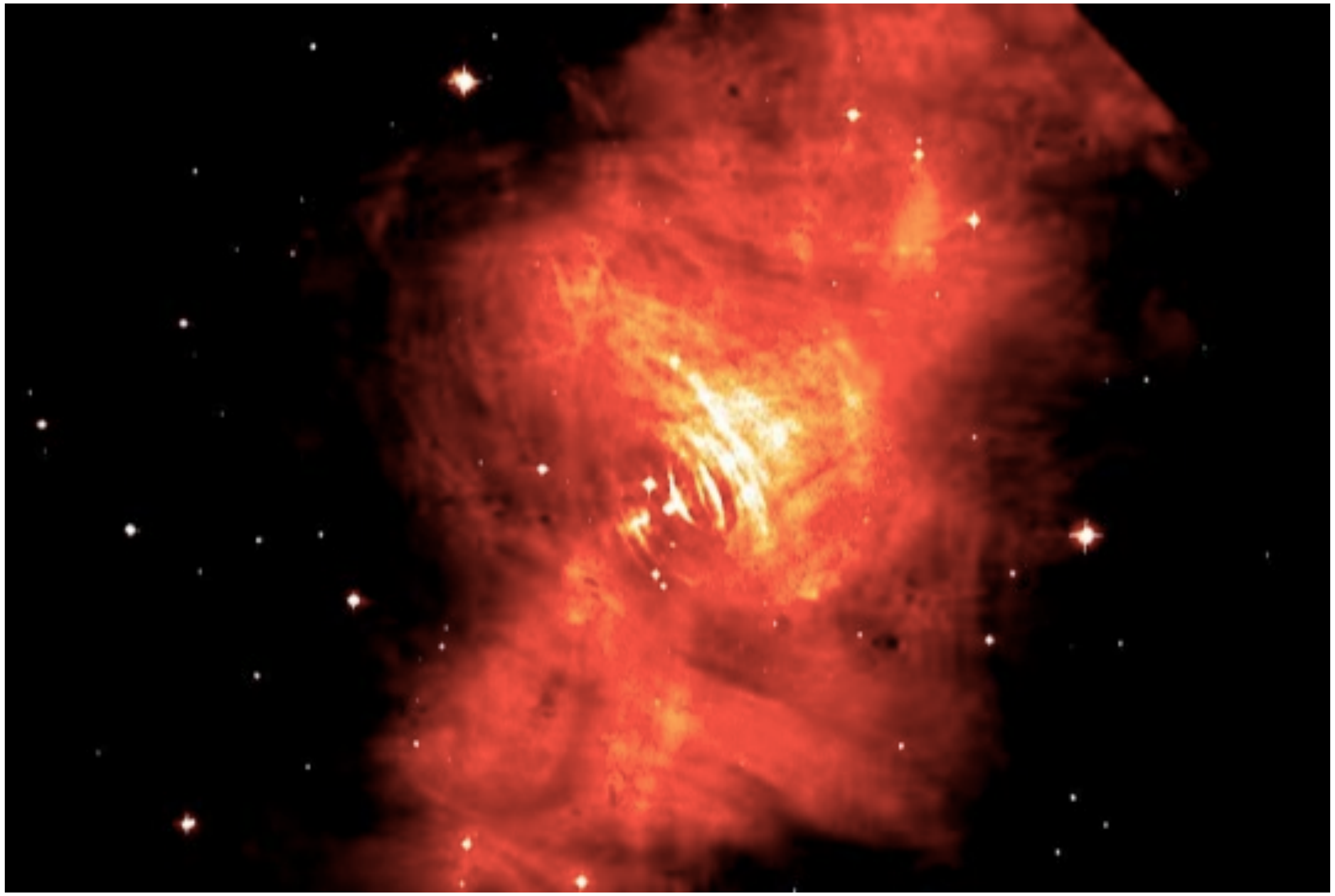


Lecture Problems

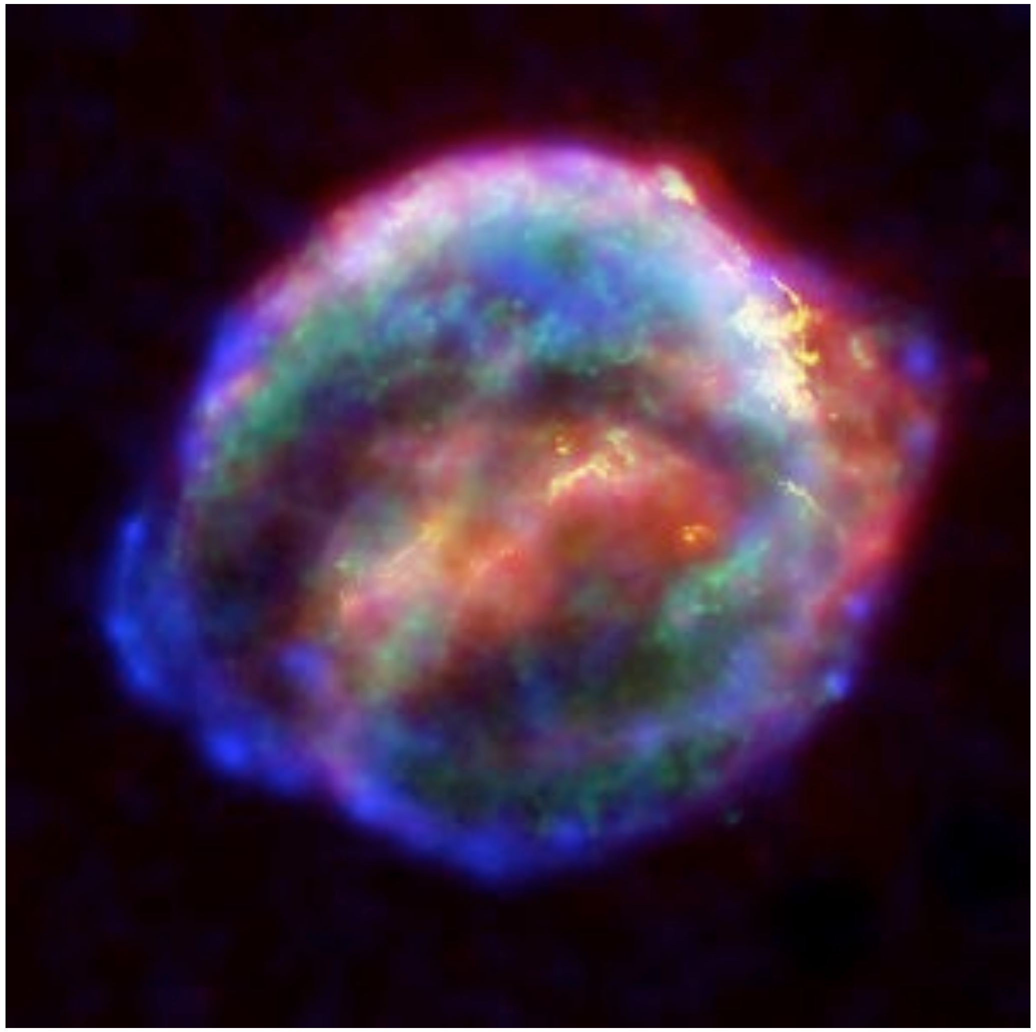
A rocky body in the asteroid belt, at radius R_{AB} (~3 A.U.) from the Sun, has a circular orbit around the sun. Find an expression for its kinetic energy in terms of its radius, R_{AB}, the Gravitational constant, G, and its mass, M_A.

Following a collision in the asteroid belt, the rocky object has1/5 of its original kinetic energy, what is the eccentricity of its subsequent orbit? Will the objects orbit now come within the Earth's orbital radius?

Stellar Death



The Crab Nebula, exploded in 1054



SN1604 discovered
in 1604

Betelgeuse A Red Giant 9th Brightest Star in the Night Sky

White Dwarfs

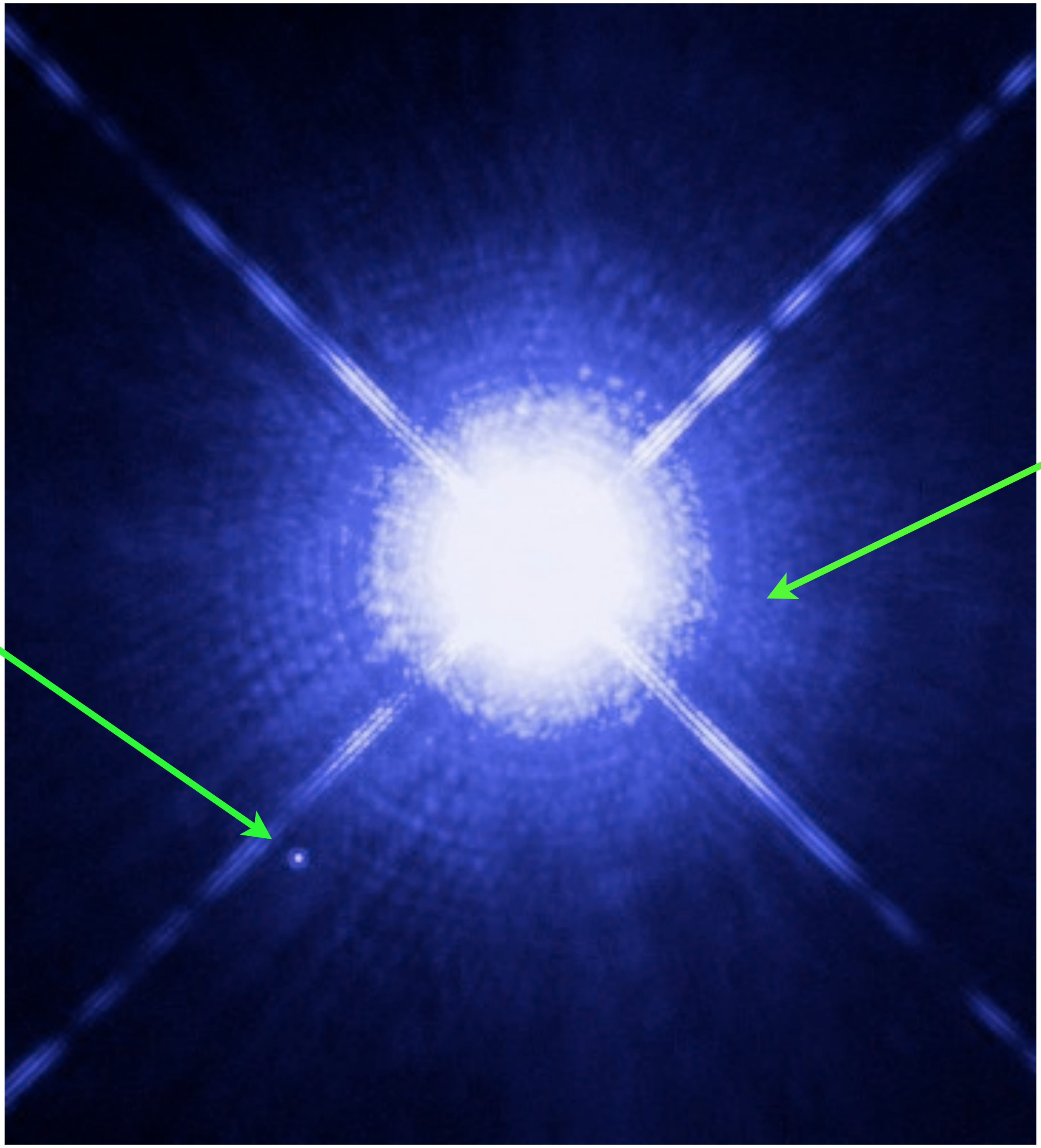
A White Dwarf forms when a star similar in mass to the Sun dies.

These stars are not massive enough to generate the core temperatures required to continue past the red giant phase.

After such a star has become a red giant it will shed its outer layers to form a planetary nebula, leaving behind an inert core.

This core has no further source of energy, and so will gradually radiate away its energy and cool down. The core, no longer supported against gravitational collapse by fusion reactions, becomes extremely dense, with a typical mass of that of the sun contained in a volume about equal to that of the Earth. The core which is now a white dwarf is supported only by electron degeneracy pressure

Examples of White Dwarfs



Sirius A
the brightest star i
the night sky

Sirius B
A White Dwarf

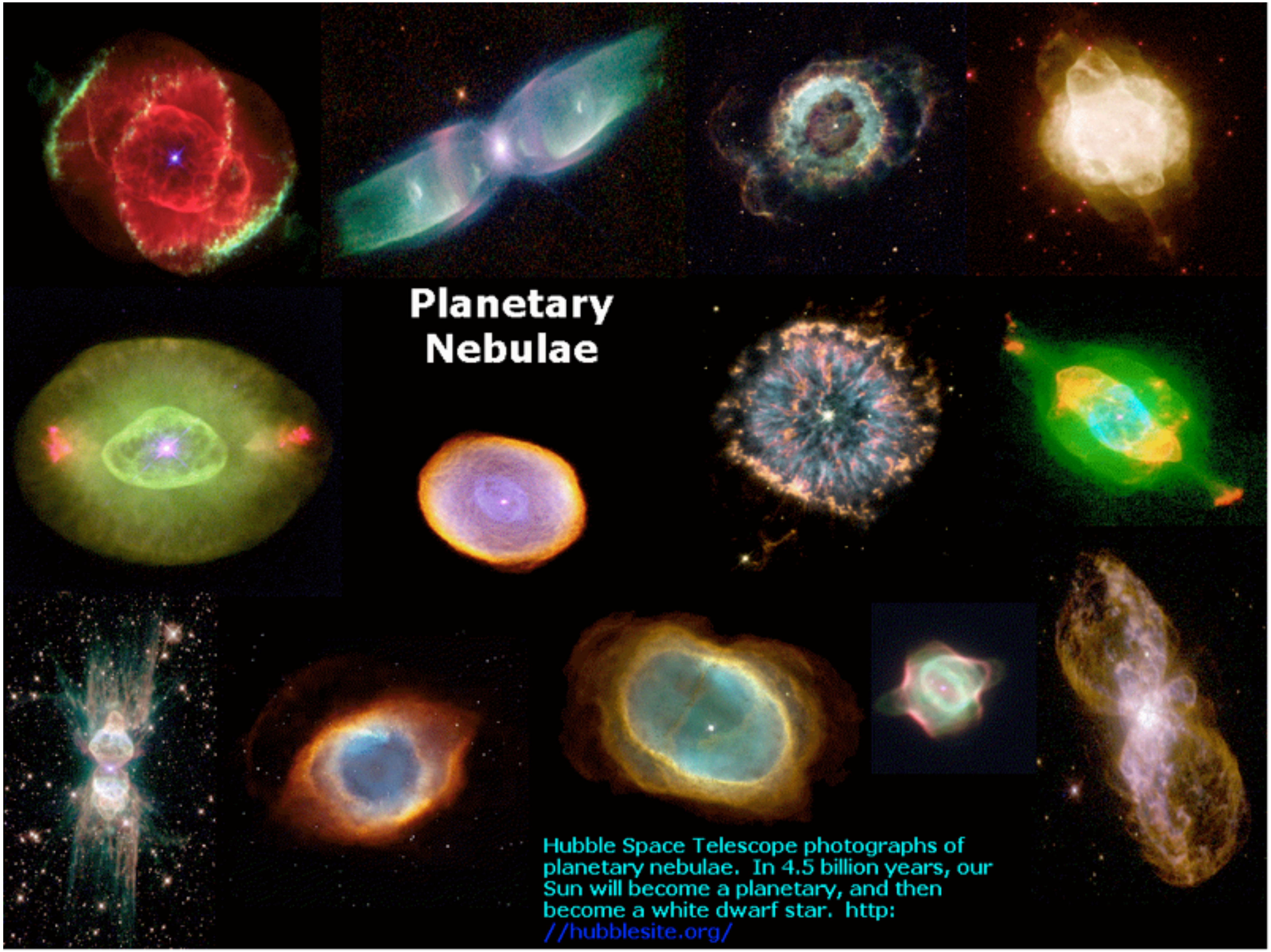
Planetary Nebulae

Before a dying low mass star becomes a White Dwarf it passes through the Planetary Nebula phase. Due to pulsations built up in the star it throws off its outer layers and the UV radiation from the star ionises and lights up the thrown off material.

A **planetary nebula** consists of a glowing shell of gas and plasma. They are a relatively short-lived phenomenon, lasting a few tens of thousands of years, compared to a typical stellar lifetime of several billion years. About 1,500 are known to exist in the Milky Way.

Planetary nebulae are important objects in astronomy because they play a crucial role in the chemical evolution of the galaxy, returning material to the interstellar medium which has been enriched in heavy elements and other products of nucleosynthesis (such as carbon, nitrogen, oxygen and calcium).

Some Planetary Nebulae



Hubble Space Telescope photographs of planetary nebulae. In 4.5 billion years, our Sun will become a planetary, and then become a white dwarf star. http://hubblesite.org/

Electron Degeneracy Pressure

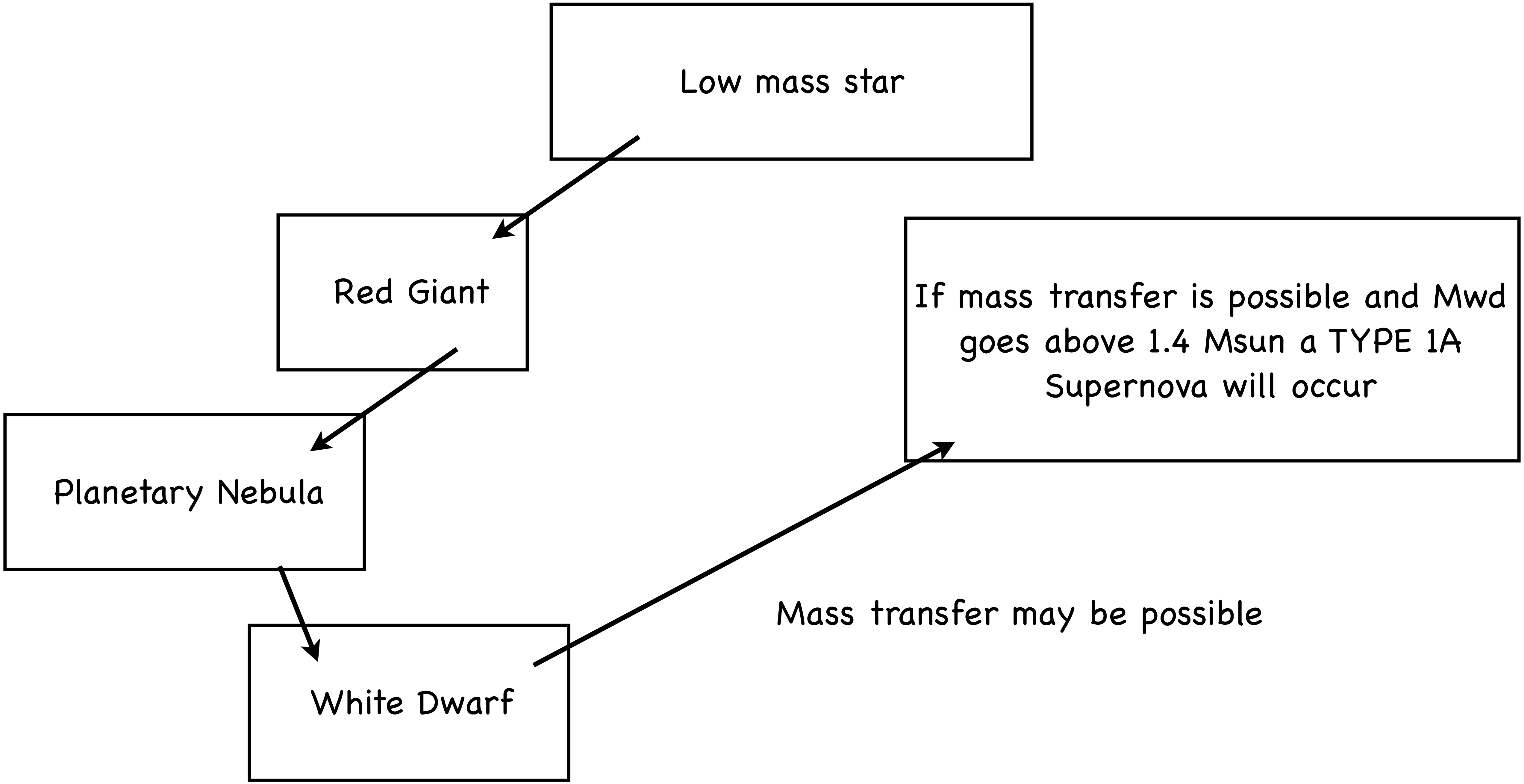
Electron degeneracy pressure is a force caused by the Pauli Exclusion Principle, which states that two electrons cannot occupy the same quantum state at the same time. This force sets a limit to how much matter can be squeezed together. It is an important factor in stellar physics as objects such as white dwarfs and brown dwarfs are supported against collapse by electron degeneracy pressure.

The Chandrasekhar Limit

The Chandrasekhar **limit** is the maximum nonrotating mass which can be supported against gravitational collapse by electron degeneracy pressure. It is commonly given as being about 1.4 solar masses. Anything above this mass limit will end its life in a SUPERNOVA EXPLOSION. Anything below as a WHITE DWARF

Thermonuclear Explosion

If a white dwarf has a companion it may obtain enough mass from its companion via mass transfer for its mass to go above the Chandrasekhar limit. In this case a Type I a Supernova explosion occurs.



Low mass star

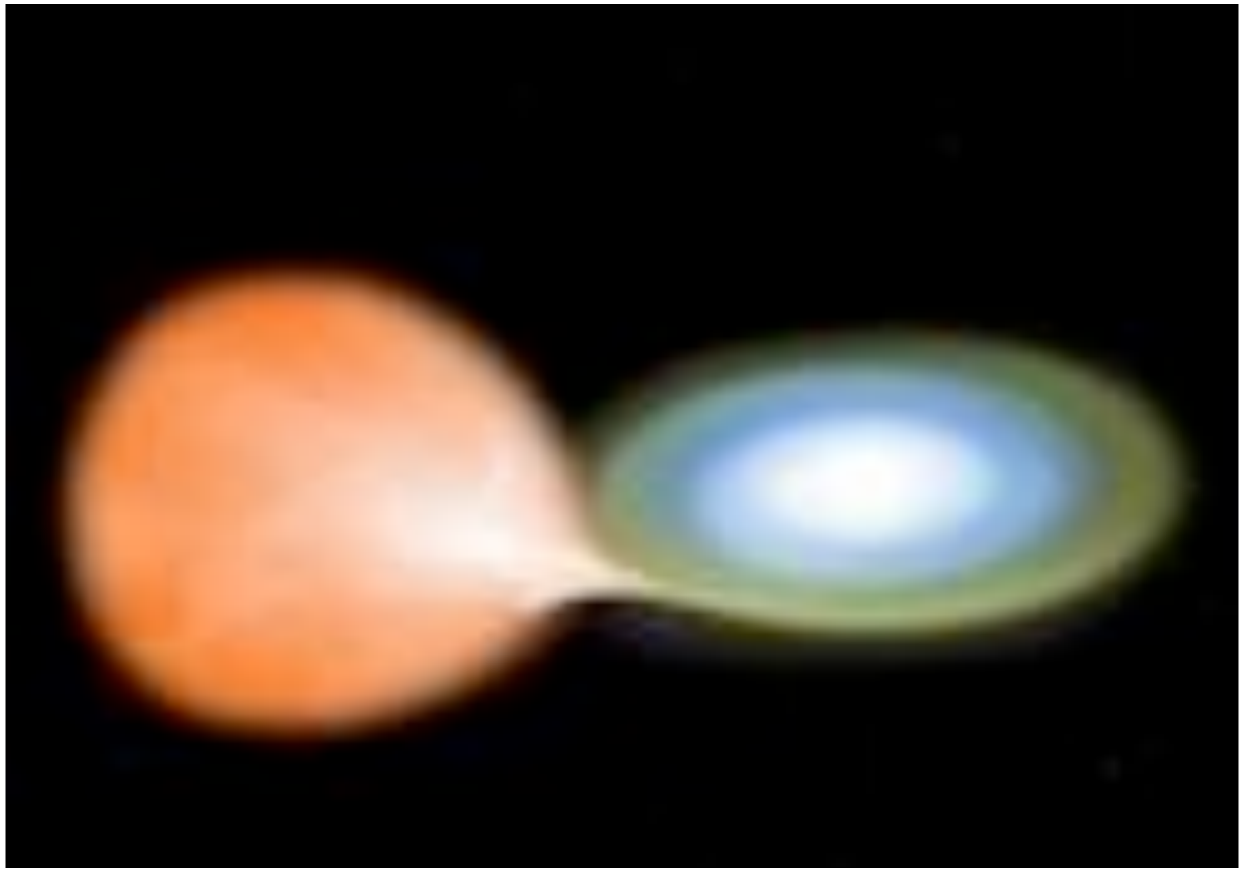
Red Giant

Planetary Nebula

White Dwarf

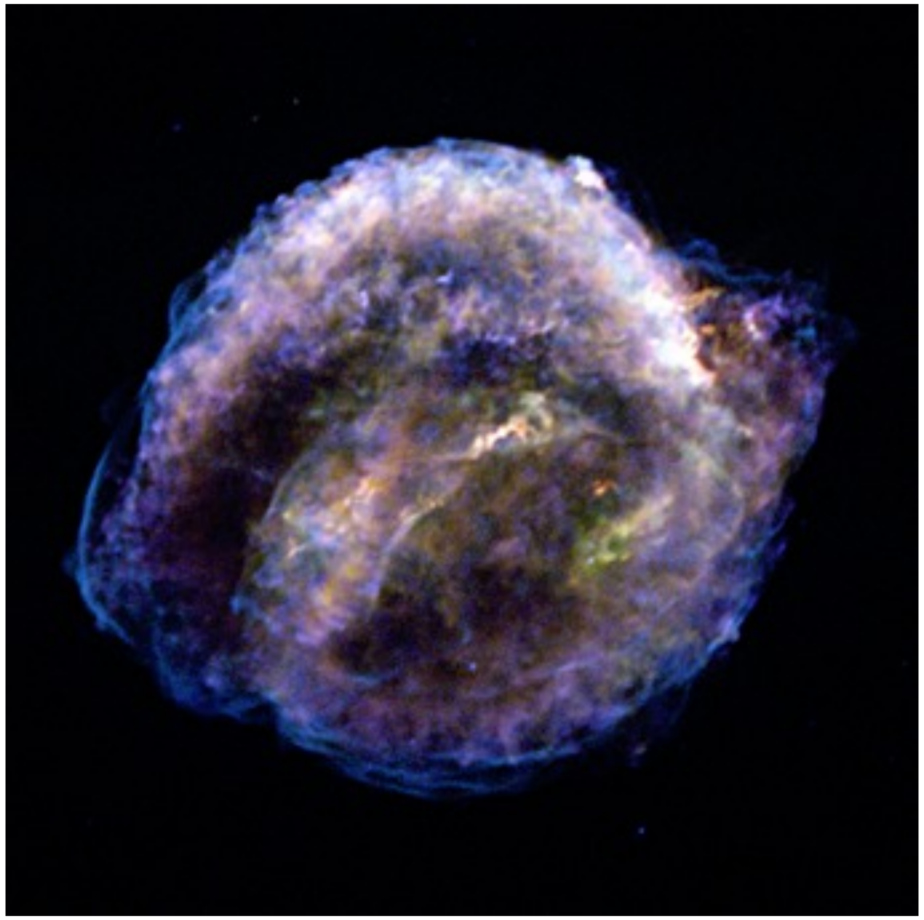
Mass transfer may be possible

If mass transfer is possible and Mwd goes above 1.4 Msun a TYPE 1A Supernova will occur



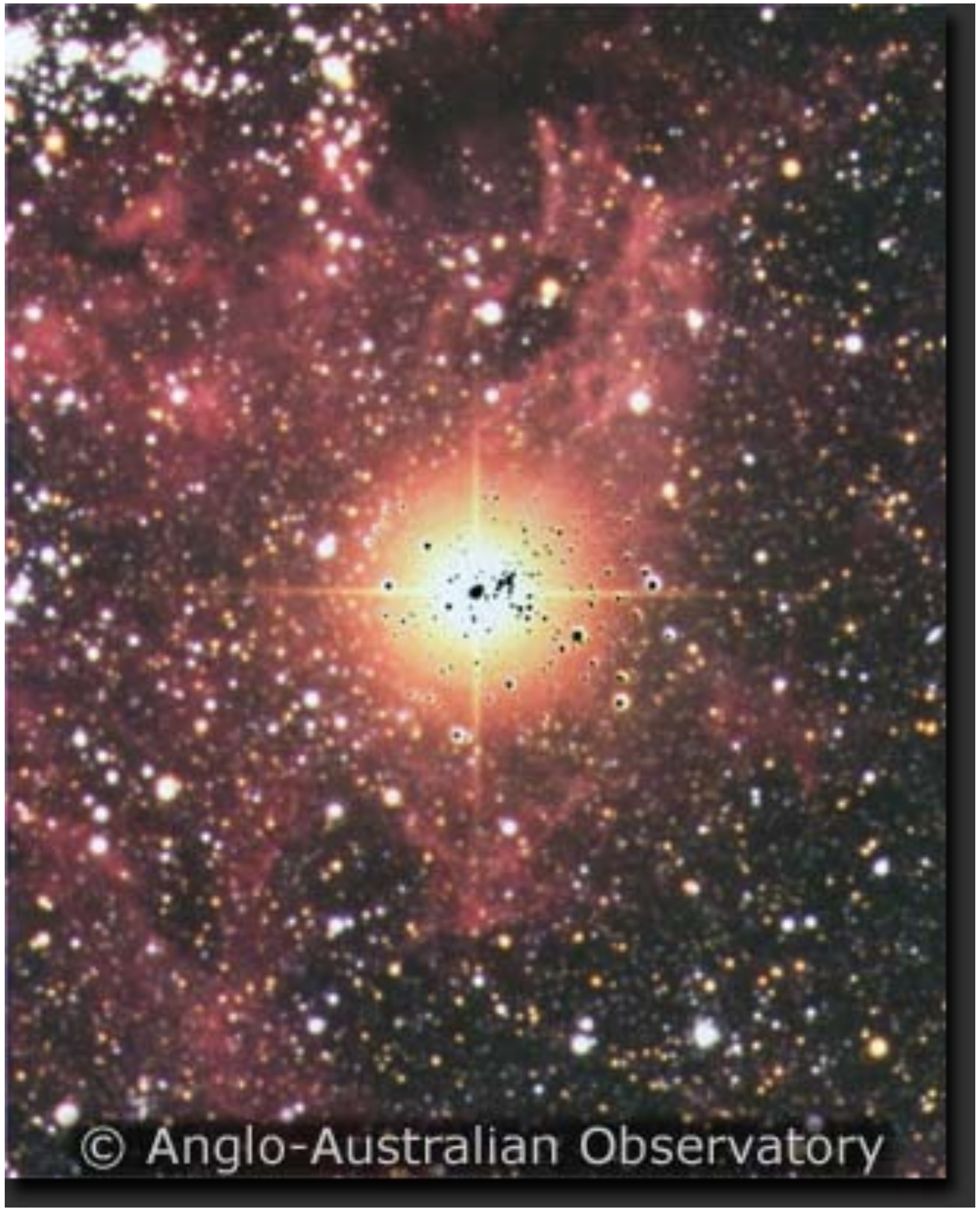
Mass Transfer

Capturing a Type Ia Supernova



A Type Ia
SN1604
(Kepler's SN)

Core Collapse Explosion


© Anglo-Australian Observatory

SN1987a- Supernova explosion in the Large Magellanic Cloud (inspiration for lecture problem!)

Outer parts of star expand to form opaque and relatively cool envelope (red giant phase).

Once Fe production ceases all fuel in the core is exhausted, star collapses

Red Giant - Betelgeuse

massive star fuses material in core
until Fe is produced

↓

not possible to fuse Fe so an
iron core builds up

↓

when core exceeds chandrasekhar
limit collapses ensues

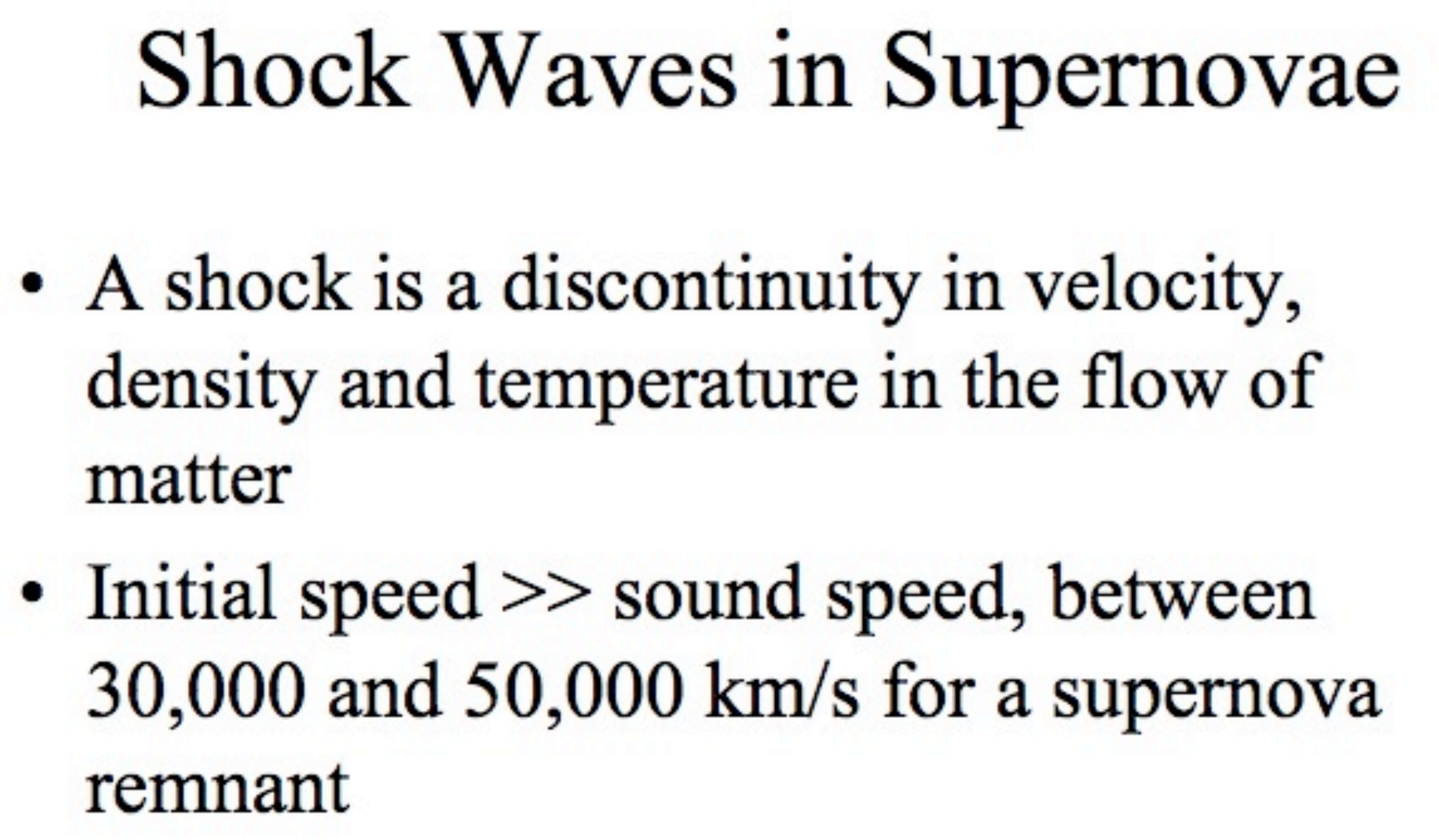
↗

gamma rays and neutrinos
produced. neutron-neutron
interactions halt the collapses

↓

infalling matter rebounds
producing a supernova explosion

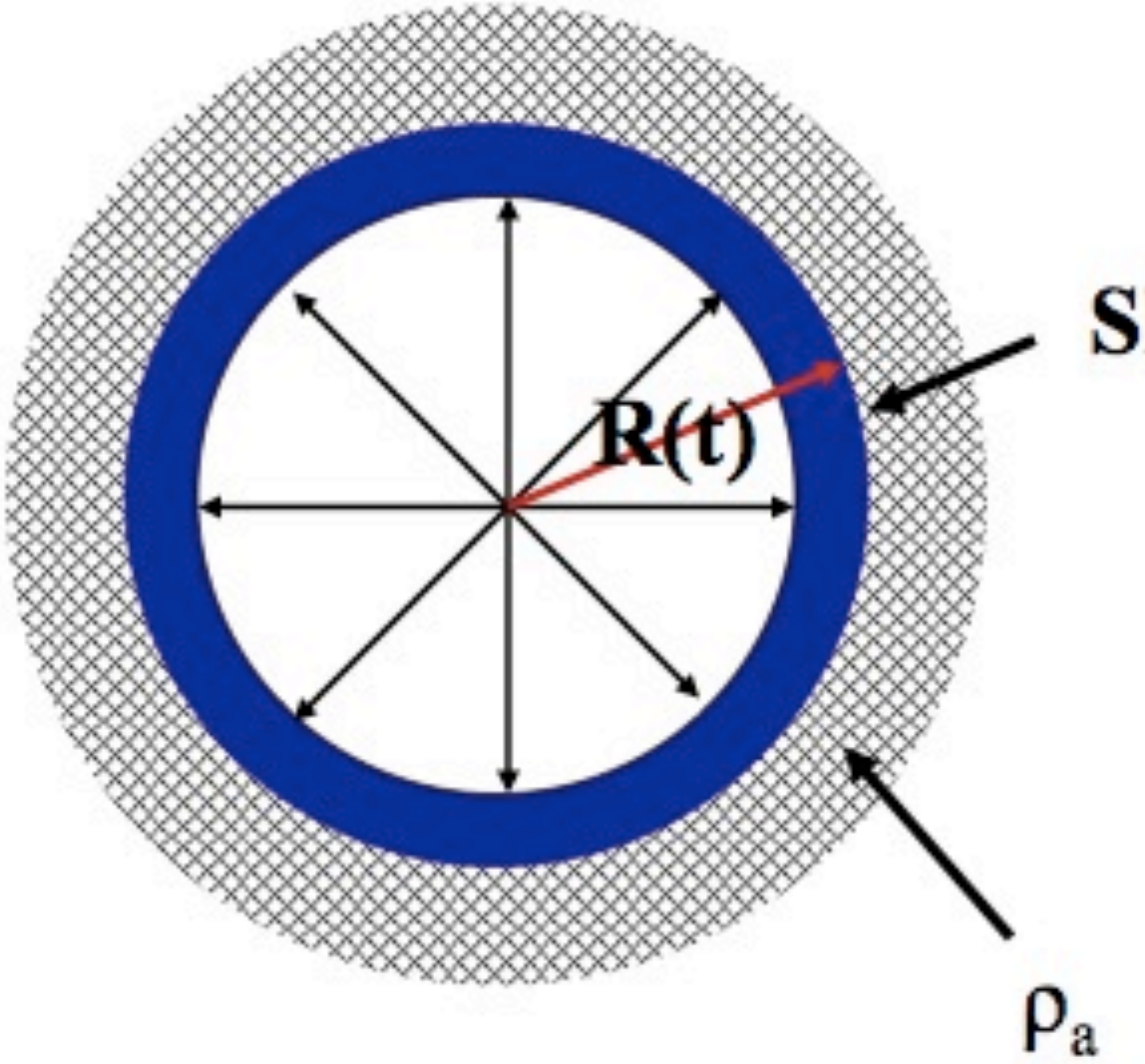
- Ejecta from supernova contain around 10^{44}J of energy but how does the implosion lead to an explosion?
- Once the core density has reached 10^{17} -10^{18} kg m^{-3}, further collapse impeded by nucleons resisting compression
- Shock waves bounces back leading to explosion of outer layers



Shock Waves in Supernovae

- A shock is a discontinuity in velocity, density and temperature in the flow of matter
- Initial speed >> sound speed, between 30,000 and 50,000 km/s for a supernova remnant

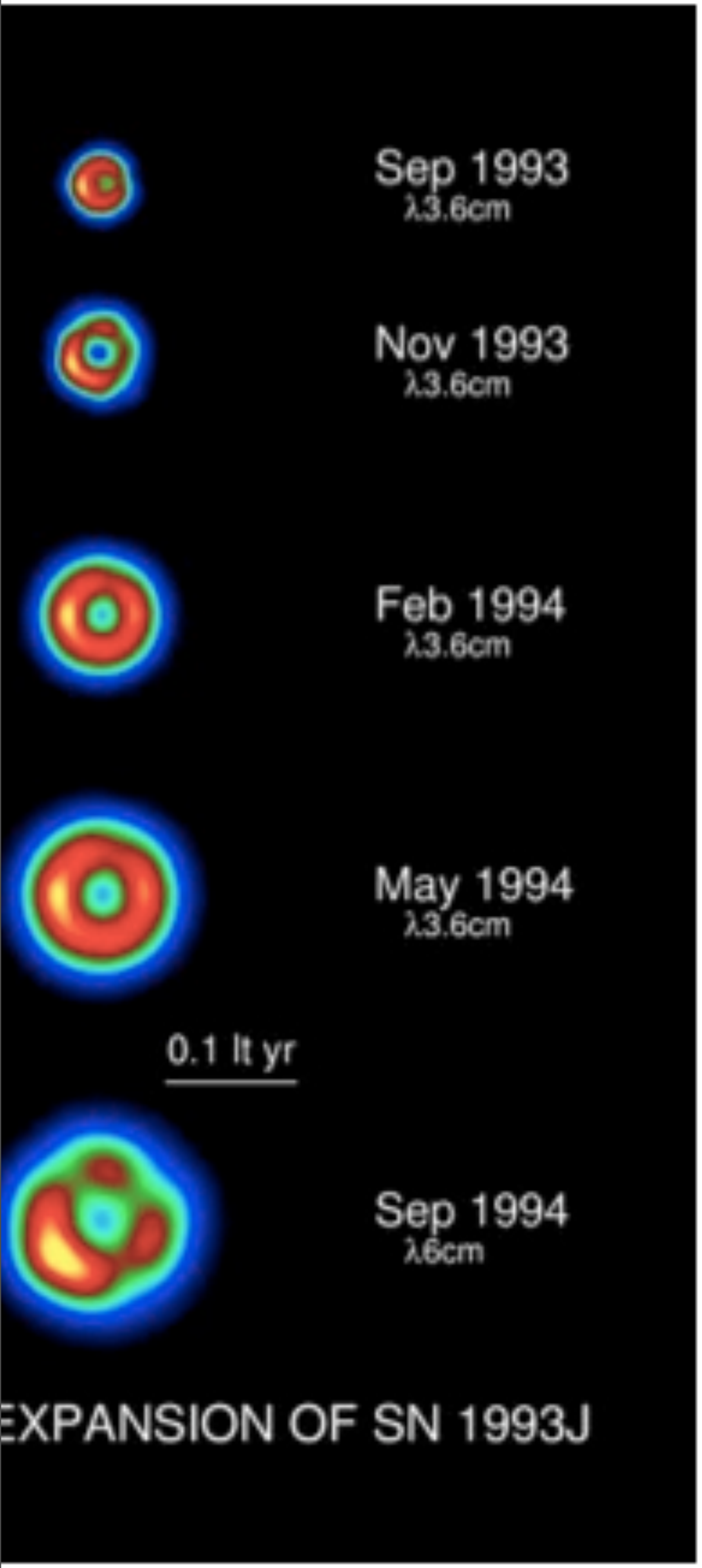
At time $t = 0$, mass m_0 of gas is ejected with velocity v_0 and total energy E_{SN}.



Shell velocity much higher than sound speed in the ambient medium, so shock front of radius R forms.

The Different Phases of a Supernova Explosion

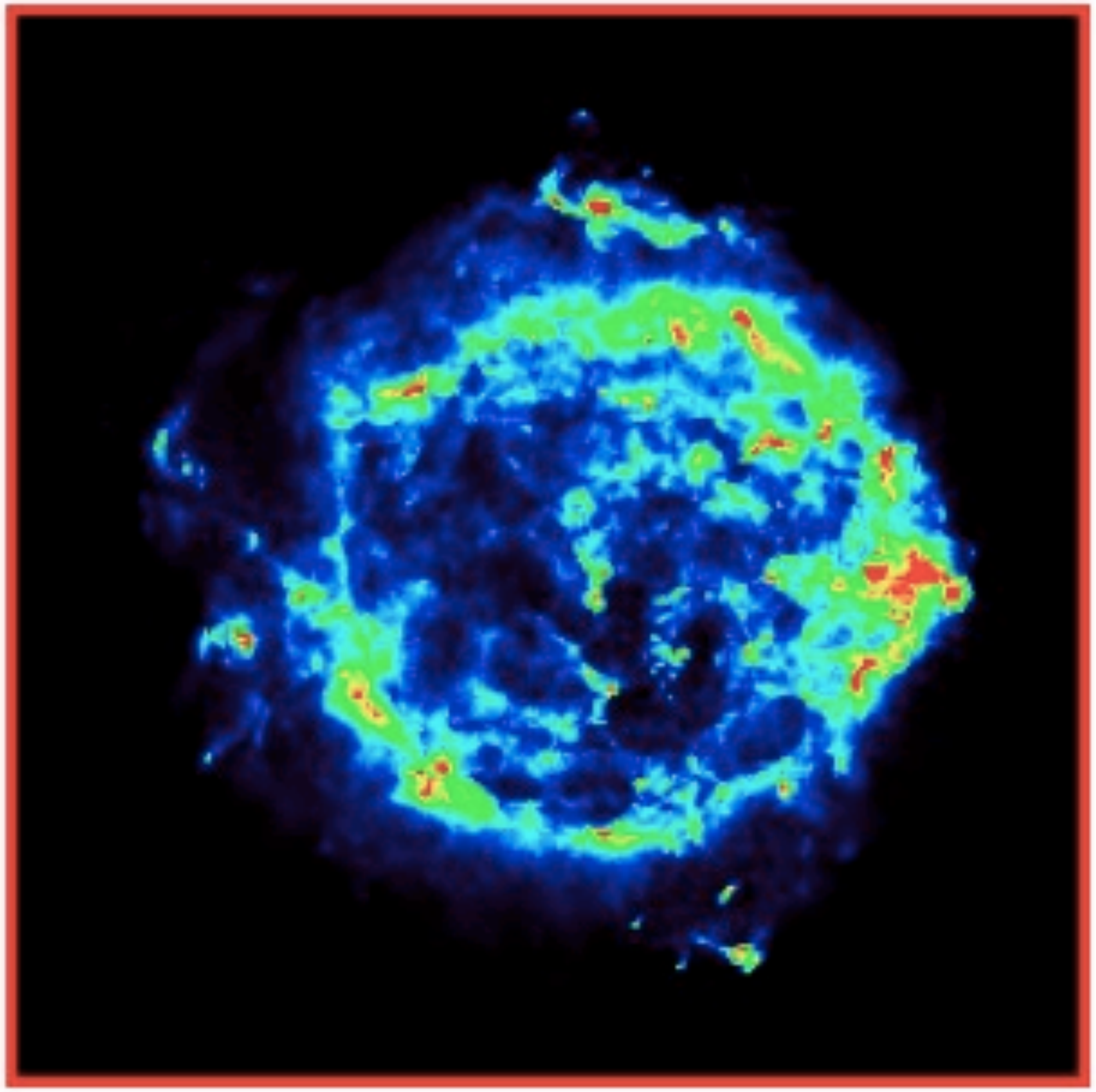
Phase I - The Ballistic Phase



Shell of swept-up material in front of shock is not massive compared to mass of ejecta m_0

$$m_0 >> \frac{4\pi}{3}\rho_a R^3(t) \quad \textbf{(1)}$$

Phase II - Adiabatic (Energy Conserving or Sedov) Expansion



Cassiopeia A

Large amount of ambient medium swept-up:

$$m_0 << \frac{4\pi}{3}\rho_a R^3(t) \quad \textbf{(2)}$$

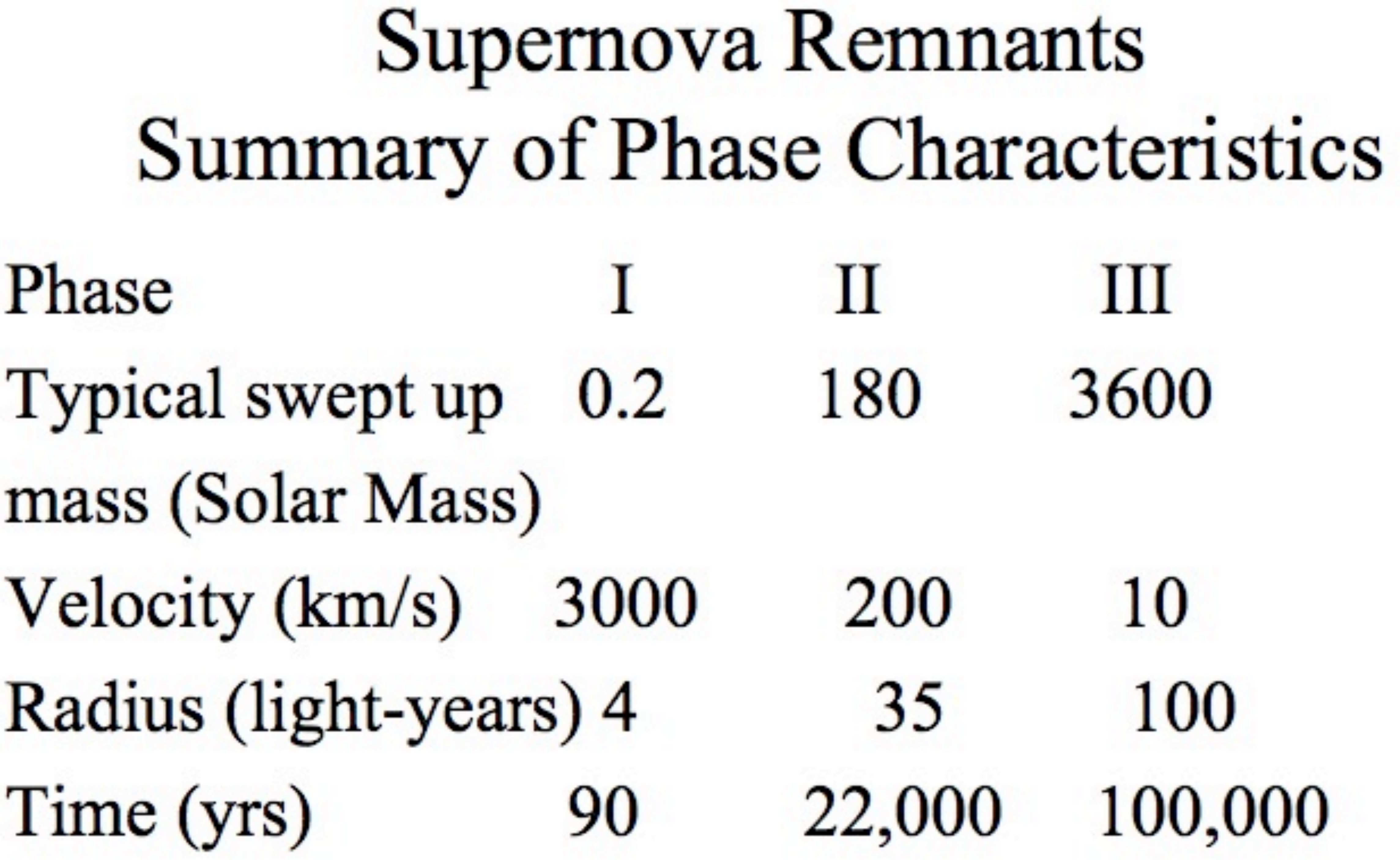
Phase III - Rapid Cooling (Snow-Plough Phase)

- Remnant cools, no high pressure to drive it forward.
- Shock front is coasting i.e. momentum is conserved

$$\frac{4}{3}\pi R^3 \rho_a U_{sh} = \text{constant}$$

Phase IV - Disappearance

- The Interstellar Medium (ISM) has random velocities ~10 km/s.
- When velocity(SNR)=10 km/s, it merges with ISM and is ‘lost’.



Supernova Remnants
Summary of Phase Characteristics

Phase	I	II	III
Typical swept up mass (Solar Mass)	0.2	180	3600
Velocity (km/s)	3000	200	10
Radius (light-years)	4	35	100
Time (yrs)	90	22,000	100,000

Types of Supernovae

Type 1a

The most commonly accepted theory of this type of supernova is that they are the result of a white dwarf accreting matter from a nearby companion star. If the accretion continues long enough, the white dwarf may eventually approach the Chandrasekhar limit of 1.44 solar masses.

Type 2

Type II supernovae occur at the end of the life of a massive star. Massive stars evolve in far more complicated ways than stars of the same mass as our Sun

Lecture Problems

1 solar mass of stellar material collaped down from a solar radius to 10 km, and released its gravitational potential energy- how much energy did it release? (10^{46} J)

This energy was released in neutrinos from the Large Magellanic Cloud. A 50 m x 50 m x 50 m detector on Earth detected neutrinos from this explosion- how many neutrinos in total did it detect? (assume each neutrino was MeV in energy)