Lecture Plan:

1) Cosmic Ray acceleration- accelerated spectrum, efficient accelerators, nuclei friendly

PROBLEMS

2) Cosmic Ray proton + nuclei interaction rates in extragalactic radiation fields

PROBLEMS

3) Cosmic Ray propagation through Galactic and extragalactic magnetic fields

Andrew Taylor

When E^{-2} and when not?

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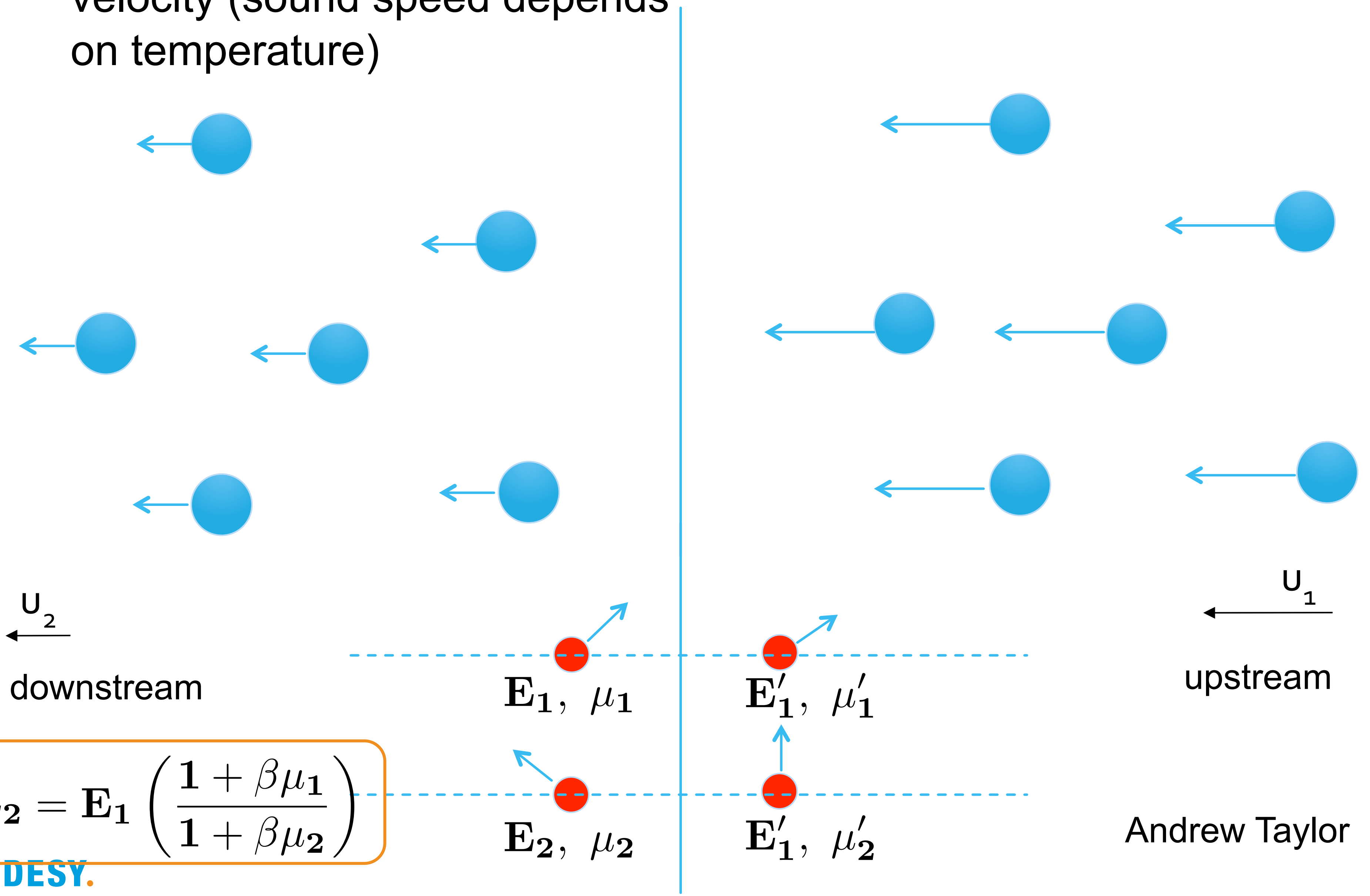
DESY.

Shock Acceleration

Strong shock wave propagating at supersonic velocity (sound speed depends on temperature)



$$\mathbf{E_2} = \mathbf{E_1}\left(\frac{1+\beta\mu_1}{1+\beta\mu_2}\right)$$

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Fermi Acceleration (more)

Energy

$$\frac{\Delta E}{E} = \frac{4v}{3c} = \frac{4}{3}\beta \text{(energy gain)}$$

$$E_1 = \left(1 + \frac{4}{3}\beta\right) E_0$$

$$E_n = \left(1 + \frac{4}{3}\beta\right)^n E_0$$

Number

$$\frac{\Delta N}{N} = -\frac{4v}{3c} = -\frac{4}{3}\beta \text{ (advection downstream)}$$

$$N_1 = \left(1 - \frac{4}{3}\beta\right) N_0$$

$$N_n = \left(1 + \frac{4}{3}\beta\right)^n N_0$$

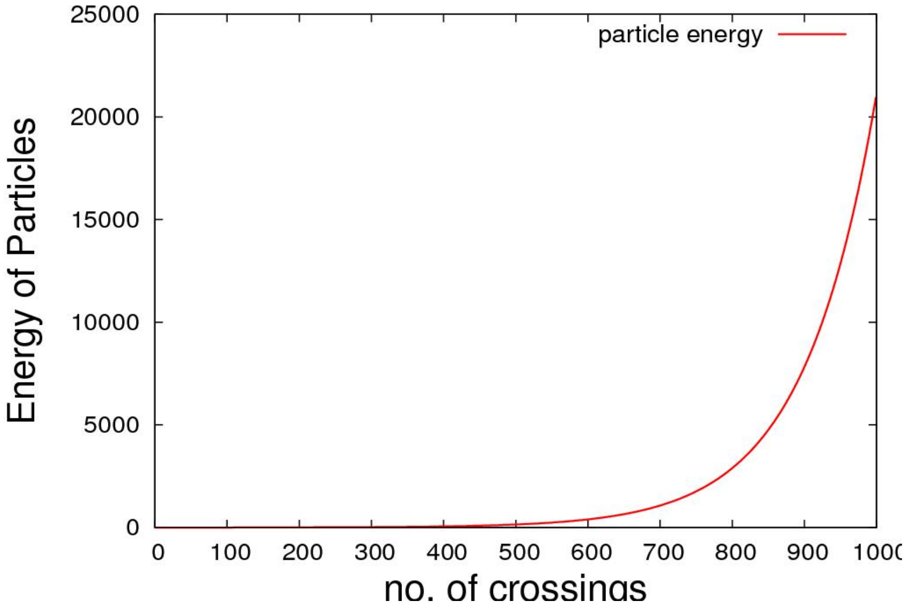
So n~1/β crossings are needed before the particle population is significantly altered → SNRs have $v_{sh} \sim 10^3$ km s^{-1} so $\beta \sim 10^{-2}$

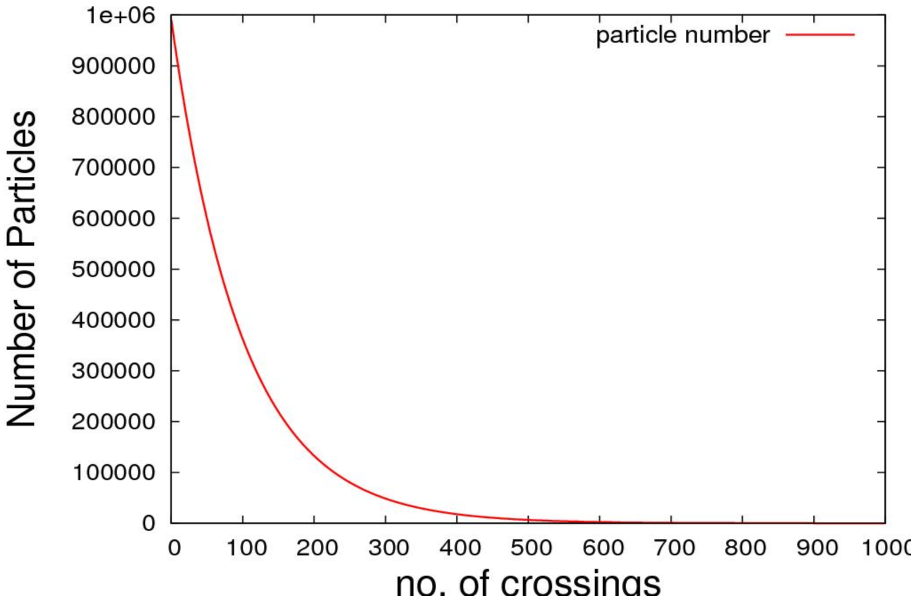
Fermi Acceleration (more)

Energy

Number

$\beta \sim 10^{-2}$





DESY.

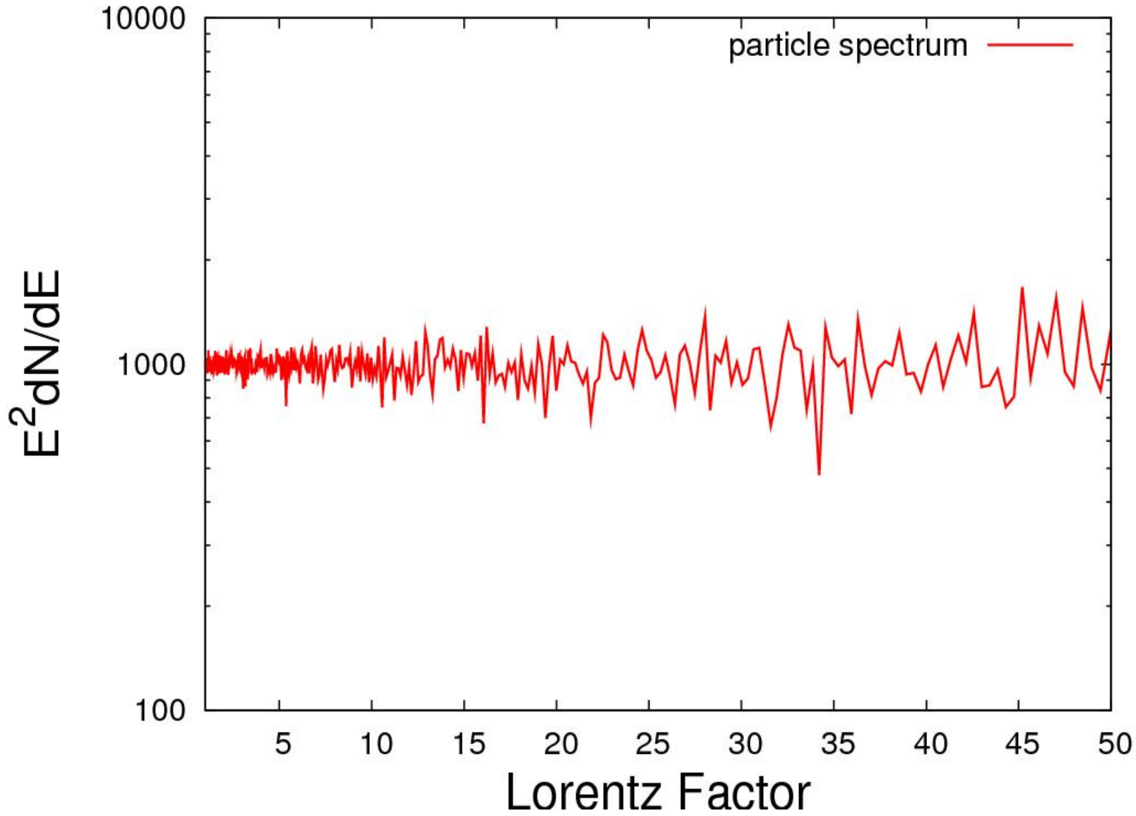
Fermi Acceleration (more)

So,

$$\frac{\Delta N}{\Delta E} = \frac{N_0}{E_0}\left(\frac{1-4\beta/3}{1+4\beta/3}\right)^{n}$$

$$\approx \frac{N_0}{E_0}\left(1+4\beta/3\right)^{-2n}$$

$$\approx N_0 E_0 E^{-2}$$



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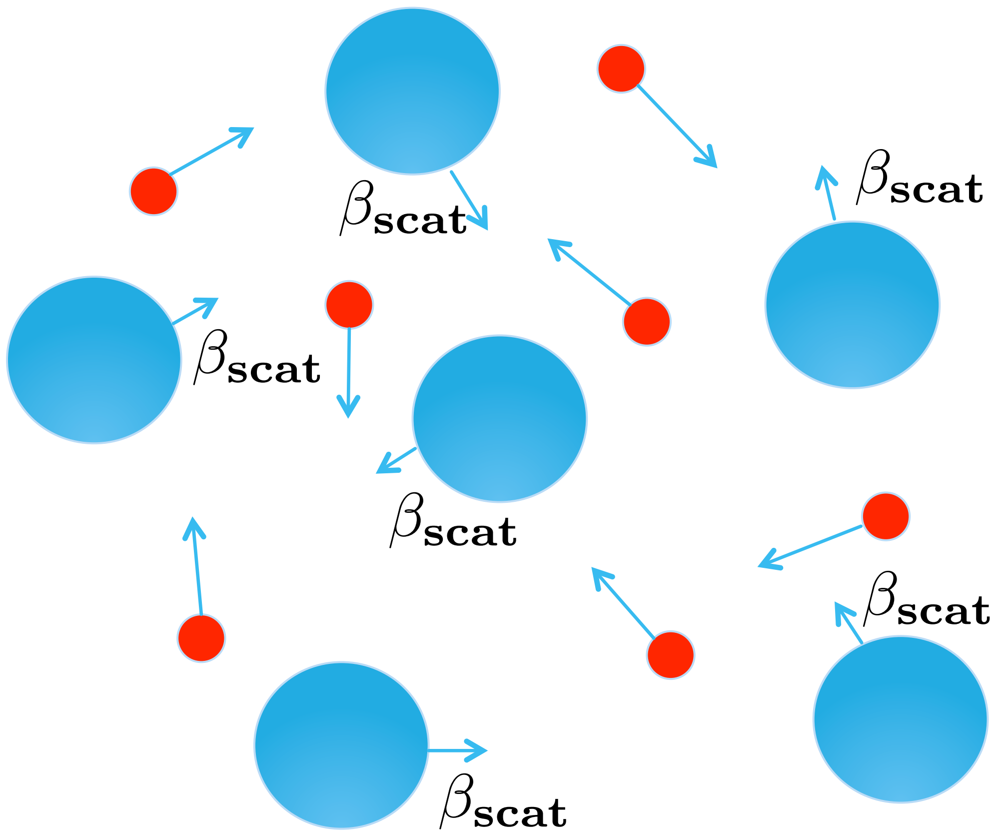
Stochastic Acceleration/Propagation

$$\mathbf{D_{xx}D_{pp}} \approx \beta^{\mathbf{2}}_{\mathbf{scat}}\mathbf{p^2}$$



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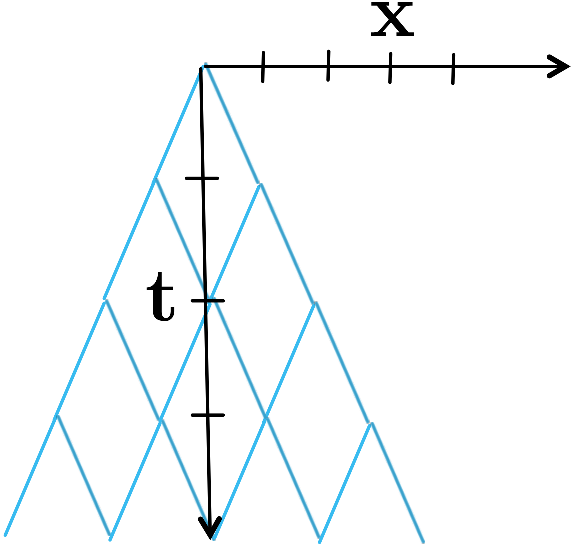
Random Walks



$$\gamma(\mathbf{t}+\mathbf{1}) = \mathbf{t}!$$

$$\gamma(\mathbf{t}+\mathbf{1}) = \int_0^\infty \mathbf{x}^{\mathbf{t}} \mathbf{e}^{-\mathbf{x}} \mathbf{dx}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) = \gamma(\mathbf{t}+\mathbf{1})/[\gamma([\mathbf{t}-\mathbf{x}]/\mathbf{2}+\mathbf{1})\gamma([\mathbf{x}+\mathbf{t}]/\mathbf{2}+\mathbf{1})]/(\mathbf{2}^{\mathbf{t}})$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) \approx \frac{\mathbf{e}^{-\mathbf{x}^2/(\mathbf{2t})}}{[\pi/(\mathbf{t}/\mathbf{2})]^{\mathbf{1}/\mathbf{2}}}$$

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Random Walks

Spatial spread:

$$\frac{\mathbf{dN}}{\mathbf{dx}} \propto \mathbf{e}^{-\mathbf{x^2}/\mathbf{4D_{xx}t}}$$

$$\frac{\mathbf{dN}}{\mathbf{dx}} \propto \mathbf{e}^{-\mathbf{x^2}/\mathbf{4c^2t_{scat}t}}$$

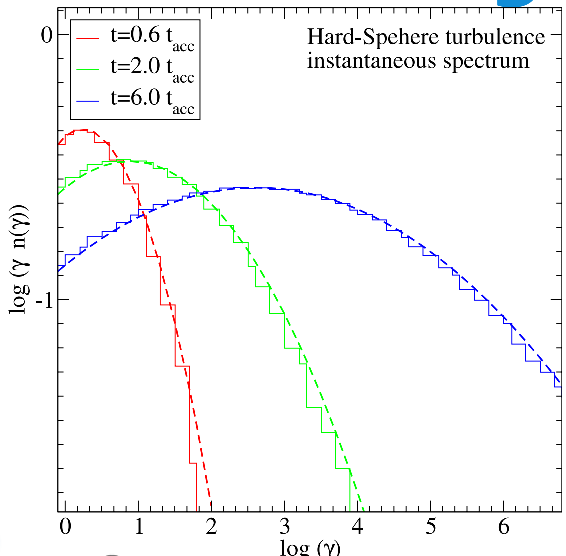
Momentum spread:

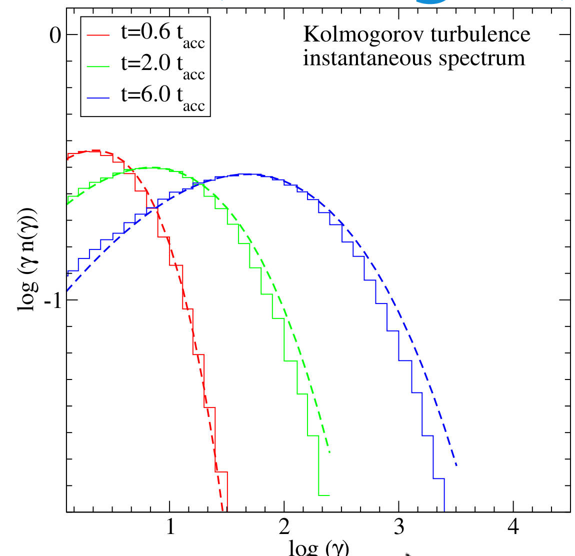
$$\frac{\mathbf{\Delta E}}{\mathbf{E}} \propto \beta$$

$$\frac{\mathbf{dN}}{\mathbf{dp}} \propto \mathbf{e}^{-(\ln \mathbf{p})^\mathbf{2}/\mathbf{4}(\mathbf{D_{pp}}/\mathbf{p^2})\mathbf{t}}$$

$$\frac{\mathbf{dN}}{\mathbf{dp}} \propto \mathbf{e}^{-(\ln \mathbf{p})^\mathbf{2}/\mathbf{4}(\mathbf{t}/\mathbf{t_{acc}})}$$

Gamma-Ray Probes of Particle Acceleration –Flaring Blazars (Mrk 501)





No energy losses

$$\frac{\partial}{\partial \mathbf{t}}\mathbf{n}(\mathbf{p},\mathbf{t}) = \nabla_{\mathbf{p}} \cdot \mathbf{D}_{\mathbf{p}} \nabla_{\mathbf{p}} \mathbf{n}(\mathbf{p},\mathbf{t}) - \frac{\mathbf{n}(\mathbf{p},\mathbf{t})}{\tau_{\mathbf{esc.}}(\mathbf{p})} + \mathbf{Q}(\mathbf{p},\mathbf{t})$$

astro-ph/1107.1879, Tramacere et al.

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DESY.

Fermi (Second Order) Acceleration

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \nabla_{\mathbf{p}} \cdot \left[(\mathbf{D}_{\mathbf{pp}} \nabla_{\mathbf{p}} \mathbf{f}) - \frac{\mathbf{p}}{\tau_{\mathbf{loss}}(\mathbf{p})} \mathbf{f} \right] - \frac{\mathbf{f}}{\tau_{\mathbf{esc}}(\mathbf{p})} + \frac{\mathbf{Q}}{\mathbf{p}^2}$$

Acceleration

Radiative Losses

Escape

Source term

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Stochastic Particle Acceleration- Random Walk Result (Spatial)

$$\nabla \cdot (\mathbf{D_{xx}} \nabla \mathbf{f}) = \delta(\mathbf{r})$$

$$\frac{\partial^2 \mathbf{f}}{\partial \mathbf{r}^2} + \frac{\mathbf{2}}{\mathbf{r}} \frac{\partial \mathbf{f}}{\partial \mathbf{r}} = \delta(\mathbf{r})$$

$$\mathbf{f} = \mathbf{r}^{-\alpha}$$

$$-\alpha(-\alpha - \mathbf{1}) - \mathbf{2}\alpha = \mathbf{0}$$

$$\alpha(\alpha - \mathbf{1}) = \mathbf{0}$$

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Stochastic Particle Acceleration- Random Walk Result (Momentum)

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \nabla_{\mathbf{p}} \cdot \left[(\mathbf{D_{pp}} \nabla_{\mathbf{p}} \mathbf{f}) - \frac{\mathbf{p}}{\tau_{\mathbf{loss}}(\mathbf{p})} \mathbf{f} \right] - \frac{\mathbf{f}}{\tau_{\mathbf{esc}}(\mathbf{p})} + \frac{\mathbf{Q}}{\mathbf{p^2}}$$

Steady state

No losses

Delta injection

$$\mathbf{D_{pp}} \propto \mathbf{p^q}$$

$$\frac{\partial^2 \mathbf{f}}{\partial \mathbf{p^2}} + \frac{(\mathbf{2}+\mathbf{q})}{\mathbf{p}} \frac{\partial \mathbf{f}}{\partial \mathbf{p}} - \frac{\mathbf{4}\tau_{\mathbf{acc}}}{\tau_{\mathbf{esc}}} \frac{\mathbf{f}}{\mathbf{p^2}} = \delta(\mathbf{p})$$

For $\mathbf{f} = \mathbf{p}^{-\alpha}$ and $\mathbf{q} = \mathbf{2}$

$$\alpha^{\mathbf{2}} - \mathbf{3}\alpha - \frac{\mathbf{4}\tau_{\mathbf{acc}}}{\tau_{\mathbf{esc}}} = \mathbf{0}$$

Stochastic Particle Acceleration- Random Walk Result (Momentum)

$$\alpha^{2} - 3\alpha - \frac{4\tau_{\mathrm{acc}}}{\tau_{\mathrm{esc}}} = 0$$

$$\alpha = \frac{3}{2} \pm \left(\frac{4\tau_{\mathrm{acc}}}{\tau_{\mathrm{esc}}} + \frac{9}{4}\right)^{1/2}$$

$$\frac{\tau_{\mathrm{acc}}}{\tau_{\mathrm{esc}}} = 1$$

$$\mathbf{f} = \frac{\mathrm{dN}}{\mathrm{d^{3}p}} = \mathrm{p}^{-4}$$

Fermi (First Order) Acceleration Time

$$\mathbf{t_{acc} = E\frac{\Delta t_{cycle}}{\Delta E_{cycle}}}$$

Transport of particles in each region is dictated by competition between diffusion and advection

downstream | upstream

$$\mathbf{t_{diff} = \frac{R^2}{D_{xx}}} \qquad \mathbf{t_{adv} = \frac{R}{v_{adv}}}$$

Balancing these timescales

$$\mathbf{t_{resid} = \frac{D_{xx}}{(c\beta_{sh})^2}}$$

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Fermi (First Order) Acceleration Time

$$\mathbf{t_{acc} = E\frac{\Delta t_{cycle}}{\Delta E_{cycle}}}$$

$$\mathbf{t_{resid} = \frac{D_{xx}}{(c\beta_{sh})^2}}$$

However, during the time it takes advection to dominate over diffusion, the particle will have crossed the shock $\mathbf{1}/\beta$ times

$$\mathbf{\Delta t_{cycle} = \frac{D_{xx}}{(c^2\beta_{sh})}}$$

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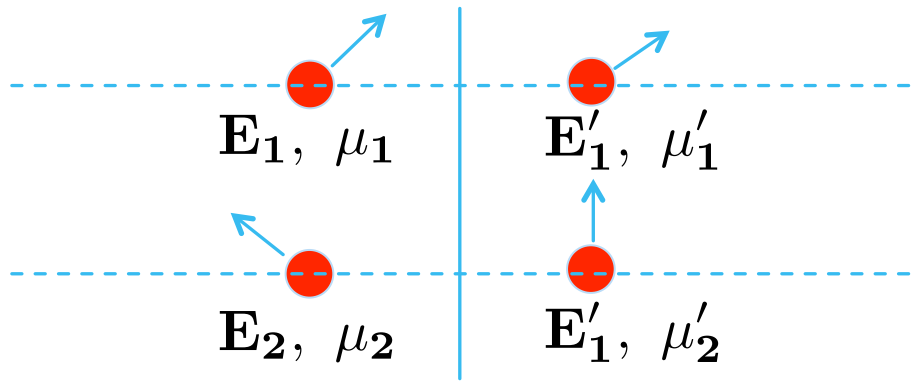
Fermi (First Order) Acceleration Time

$$\mathbf{t_{acc} = E\frac{\Delta t_{cycle}}{\Delta E_{cycle}}}$$

$$\mathbf{\Delta t_{cycle} = \frac{D_{xx}}{(c^2\beta_{sh})}}$$

$$\mathbf{\Delta E_{cycle} = E\beta_{sh}}$$

$$\mathbf{t_{acc} = \frac{D_{xx}}{(c\beta_{sh})^2} = \frac{t_{scat}}{\beta_{sh}^2}}$$



$$\mathbf{E_2 = E_1\left(\frac{1+\beta\mu_1}{1+\beta\mu_2}\right)}$$

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Fermi (Second Order) Acceleration Time

$$\mathbf{t_{acc} = E\frac{\Delta t_{scat}}{\Delta E_{scat}}}$$

$$\mathbf{\Delta E_{scat} = E}\beta^{\mathbf{2}}_{\mathbf{scat}}$$

$$\mathbf{t_{acc}} = \frac{\mathbf{t_{scat}}}{\beta^{\mathbf{2}}_{\mathbf{scat}}}$$

Efficient Accelerators….what means efficient?

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Particle Acceleration in AGN

$$\mathbf{t_{acc}} = \eta \frac{\mathbf{R_{lar}}}{\mathbf{c}\beta^{\mathbf{2}}}$$

$$\mathbf{t_{esc.}} = \frac{\mathbf{R^2}}{\eta \mathbf{c} \mathbf{R_{lar}}}$$

Maximum energy
(Hillas criterion)

$$\mathbf{R_{lar}} = \frac{\beta}{\eta}\mathbf{R}$$

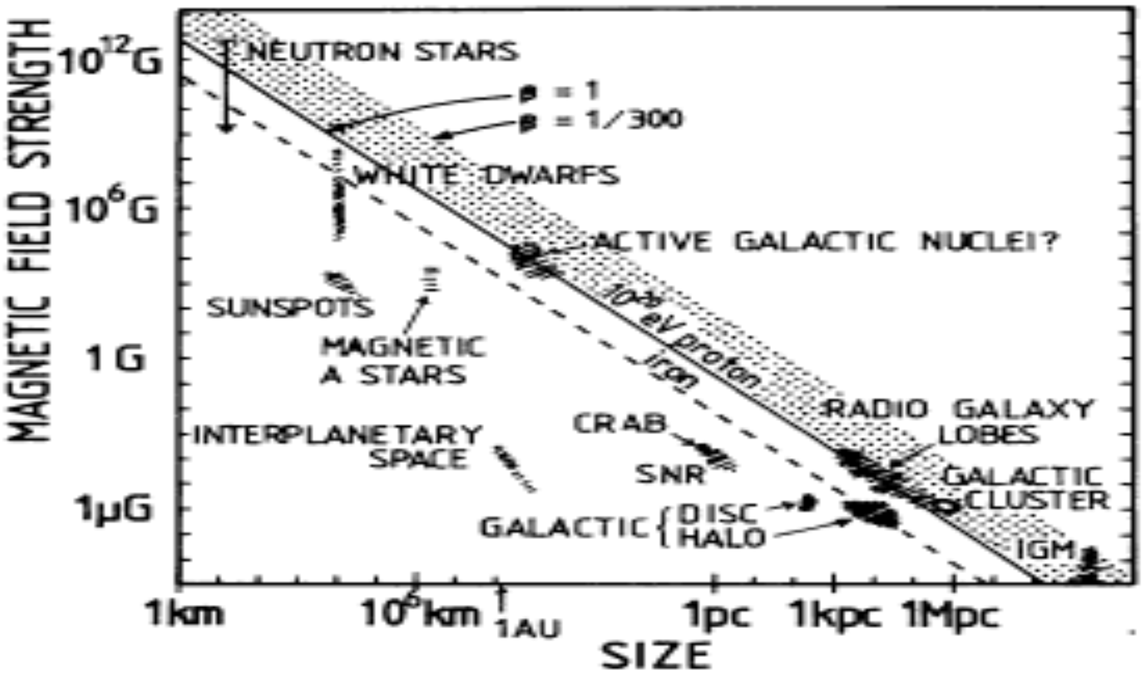
AM Hillas (1984)

$$\mathbf{R_{lar}}(\mathbf{E}, \mathbf{B}) = \left(\frac{\mathbf{E}}{\mathbf{10\ EeV}}\right)\left(\frac{\mathbf{1\ mG}}{\mathbf{B}}\right)\mathbf{10\ pc}$$

DESY.

Compactness of UHECR Sources: Proton/Nuclei Synchrotron Losses

AM Hillas (1984)



$\eta \approx 1$ assumed in above plot

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Particle Acceleration with Cooling

$$\mathbf{t_{acc}} = \eta \frac{\mathbf{R_{lar}}}{\mathbf{c}\beta^2}$$

$$\mathbf{t_{cool}} = \frac{\mathbf{9}}{\mathbf{8}\pi\alpha} \left(\frac{\mathbf{m_e}}{\mathbf{E_\gamma^{sync}}} \right) \mathbf{t_{lar}}$$

$$\mathbf{E_\gamma^{sync}} \approx \frac{\mathbf{9}}{\mathbf{4}} \eta^{-\mathbf{1}} \beta^{\mathbf{2}} \frac{\mathbf{m_e}}{\alpha}$$

Maximum synchrotron energy tells us how efficient accelerator is!

$$\eta < \mathbf{10^3}$$

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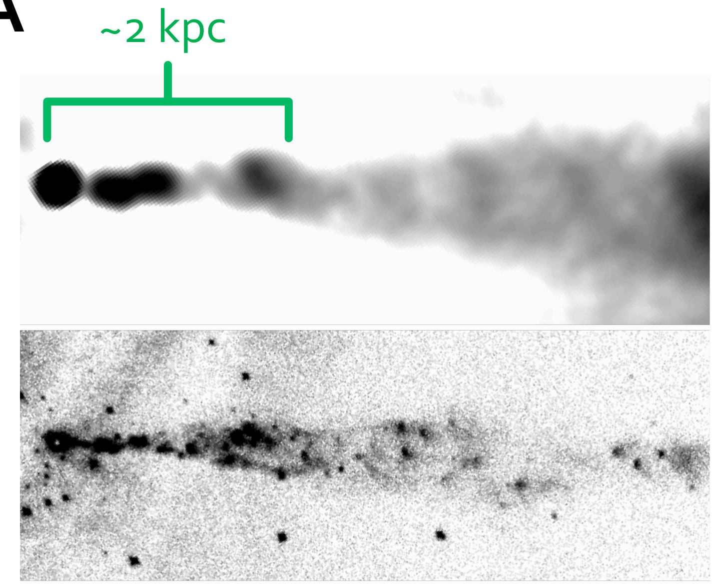
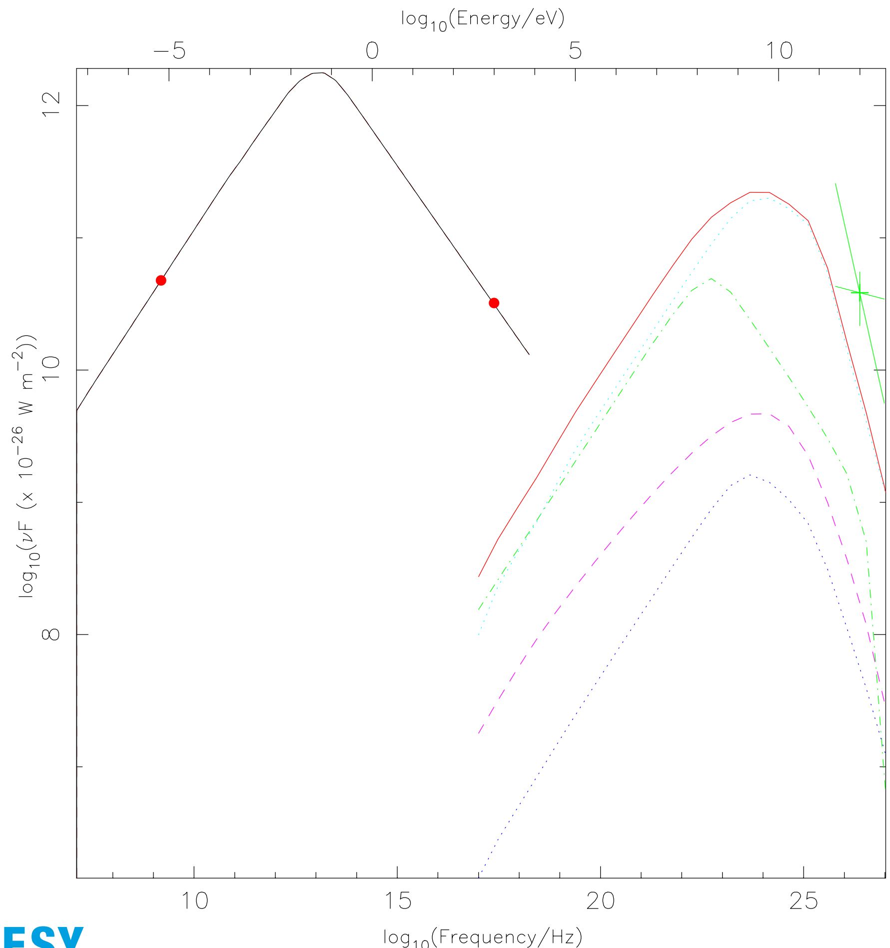
Emission Site?

Cen A

Where are the misaligned (X)HBLs?



Hardcastle et al. (1103.1744)



$$\eta < \mathbf{10^3}$$

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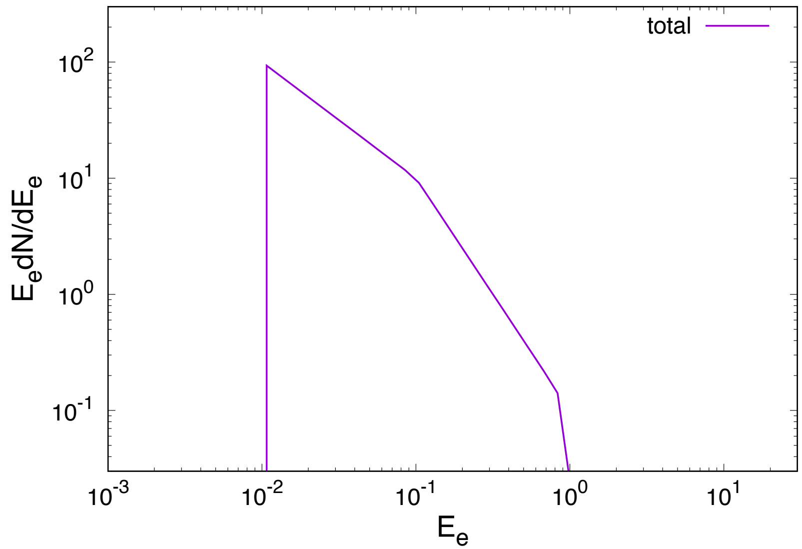
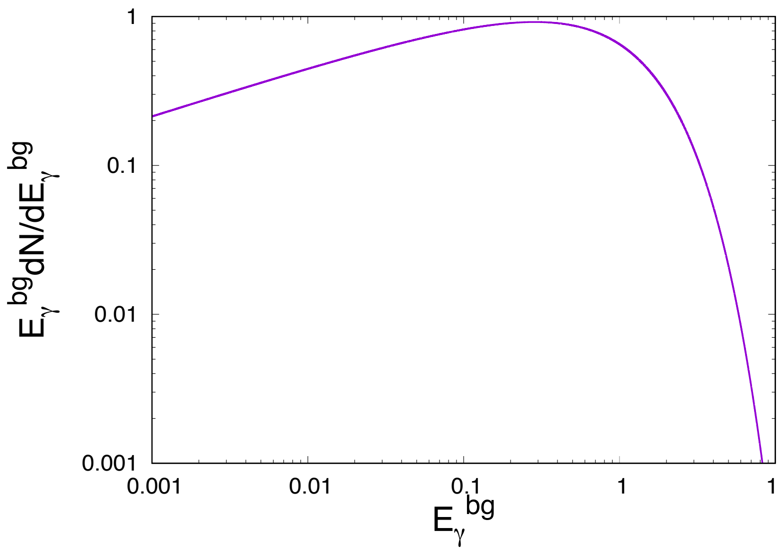
Future Probes- Cutoff Region

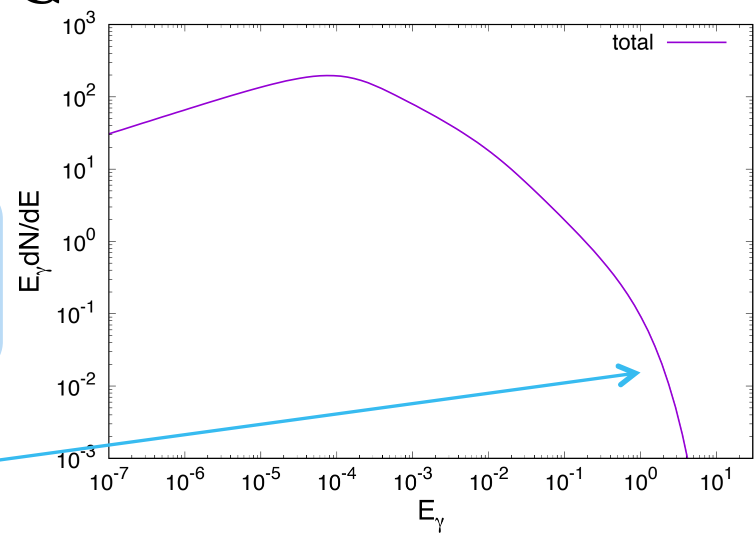




$$\mathbf{E_\gamma^{sync} = \Gamma_e^2 \left(\frac{B}{B_{crit}}\right) m_e}$$

$$\mathbf{B_{crit} = 4 \times 10^{13}\ G}$$

$$E_\gamma \frac{dN}{dE_\gamma}_{tot} = \int \left(\frac{E_\gamma}{E_e^2}\right) \frac{dN}{dE_\gamma} \left(\frac{E_\gamma}{E_e^2}\right) E_e \frac{dN}{dE_e} dE_e$$



Possibility to probe cutoff region

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Nuclei Friendly Accelerators

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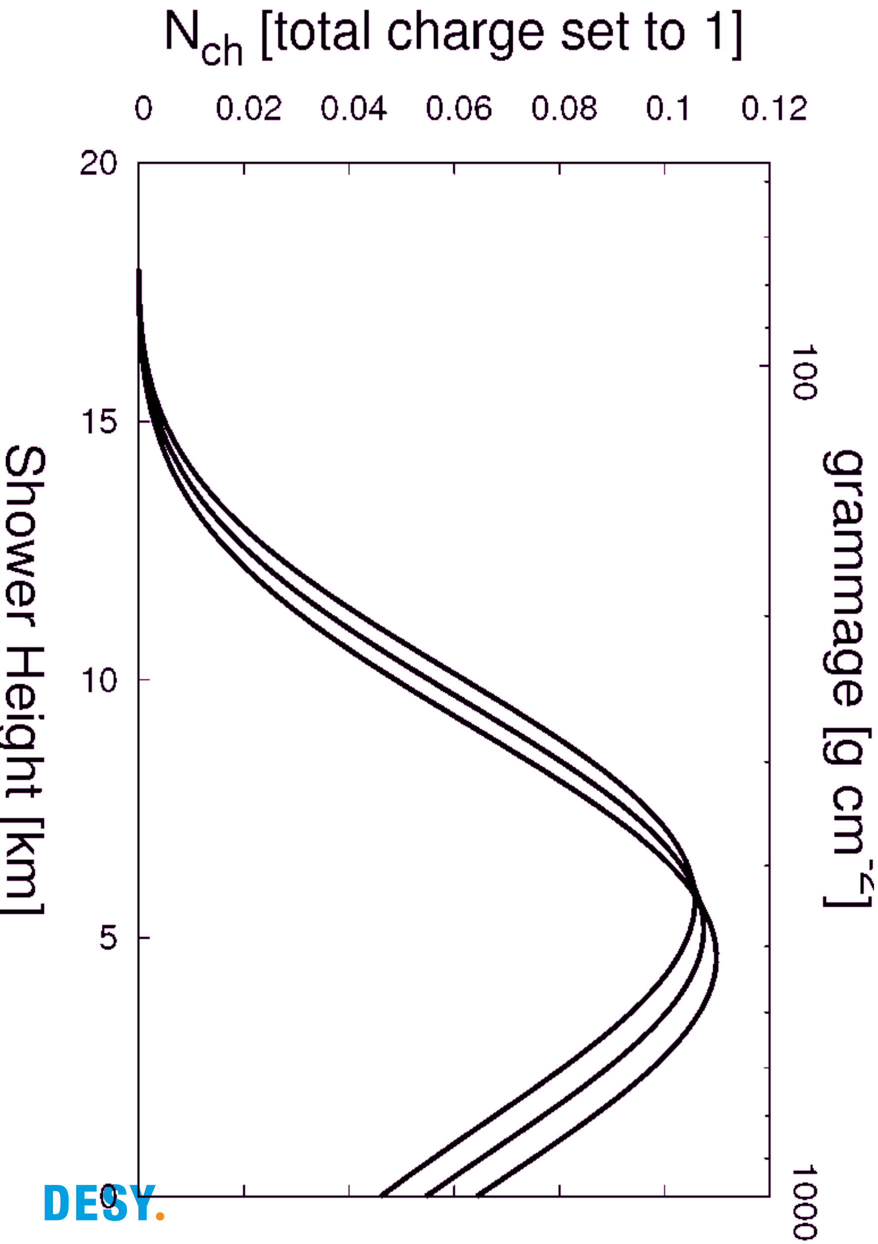
UHECR Air Showers



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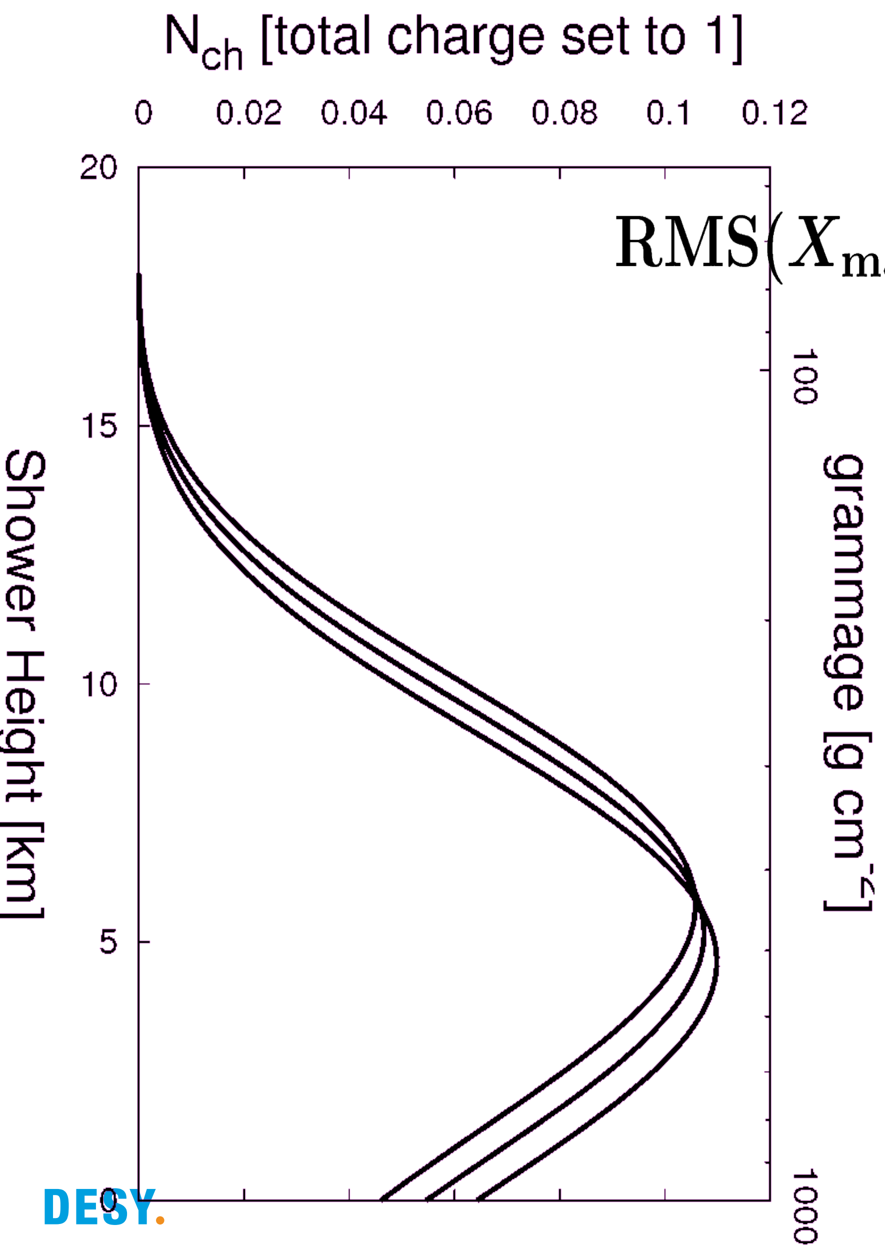
UHECR Air Showers

$$\langle X_{\max} \rangle = \frac{1}{N} \sum_{n=1}^{N} X_{\max,n}$$

$$\mathrm{RMS}(X_{\max}) = \frac{1}{N} \sqrt{\sum_{n=1}^{N} (X_{\max,n} - \langle X_{\max} \rangle)^2}$$



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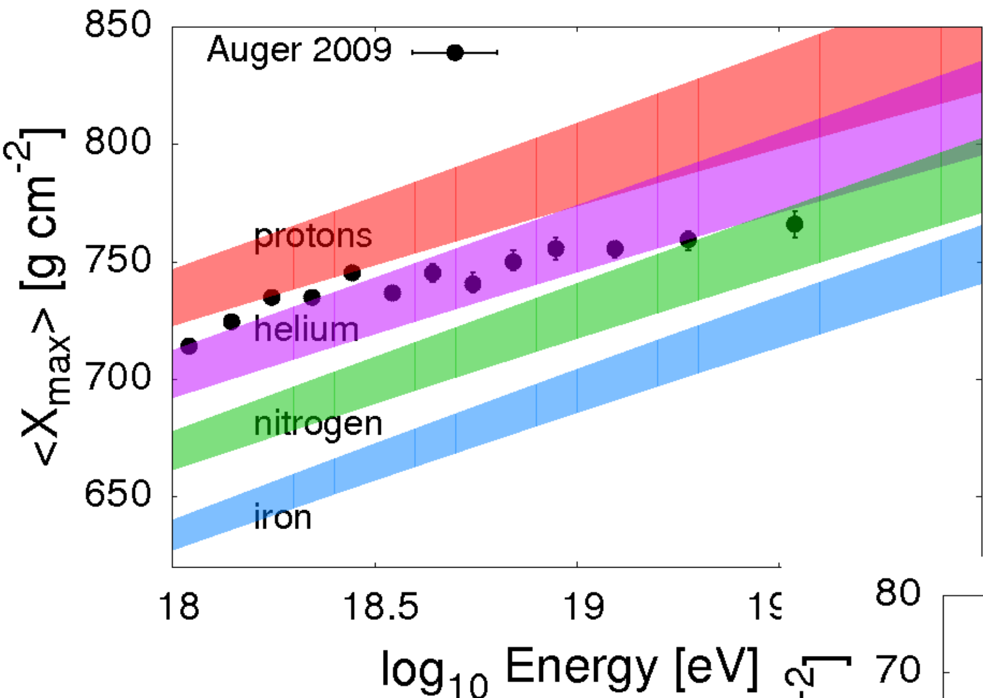
Composition Measurements by the PAO

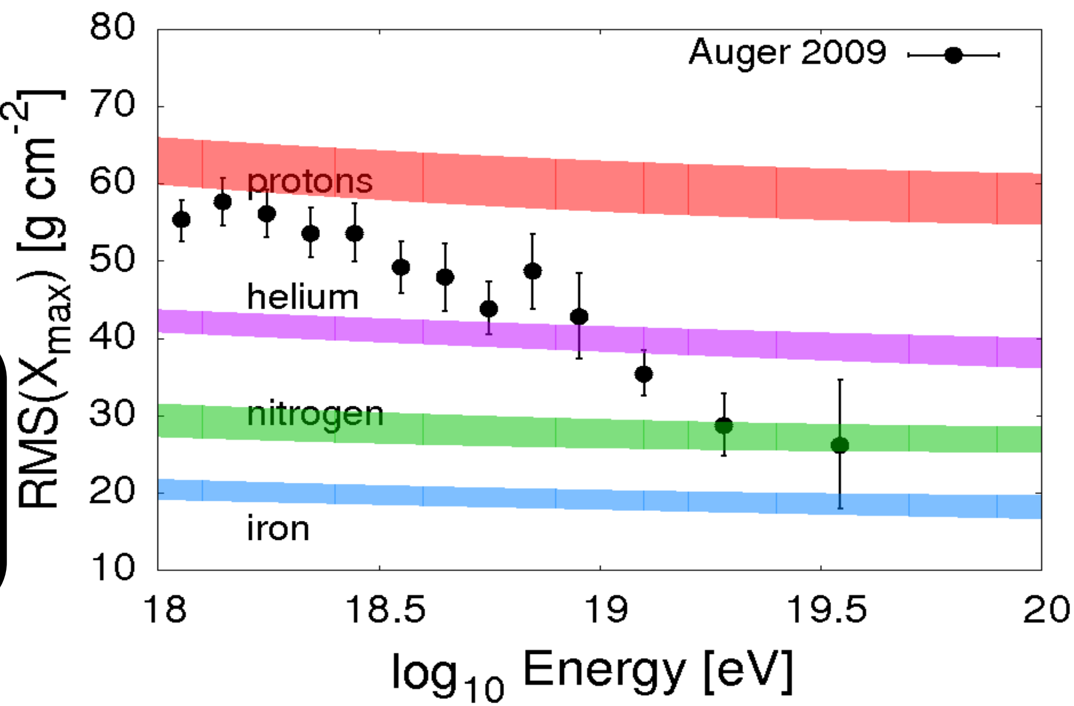


$\leftarrow \langle X_{\mathrm{max}} \rangle$

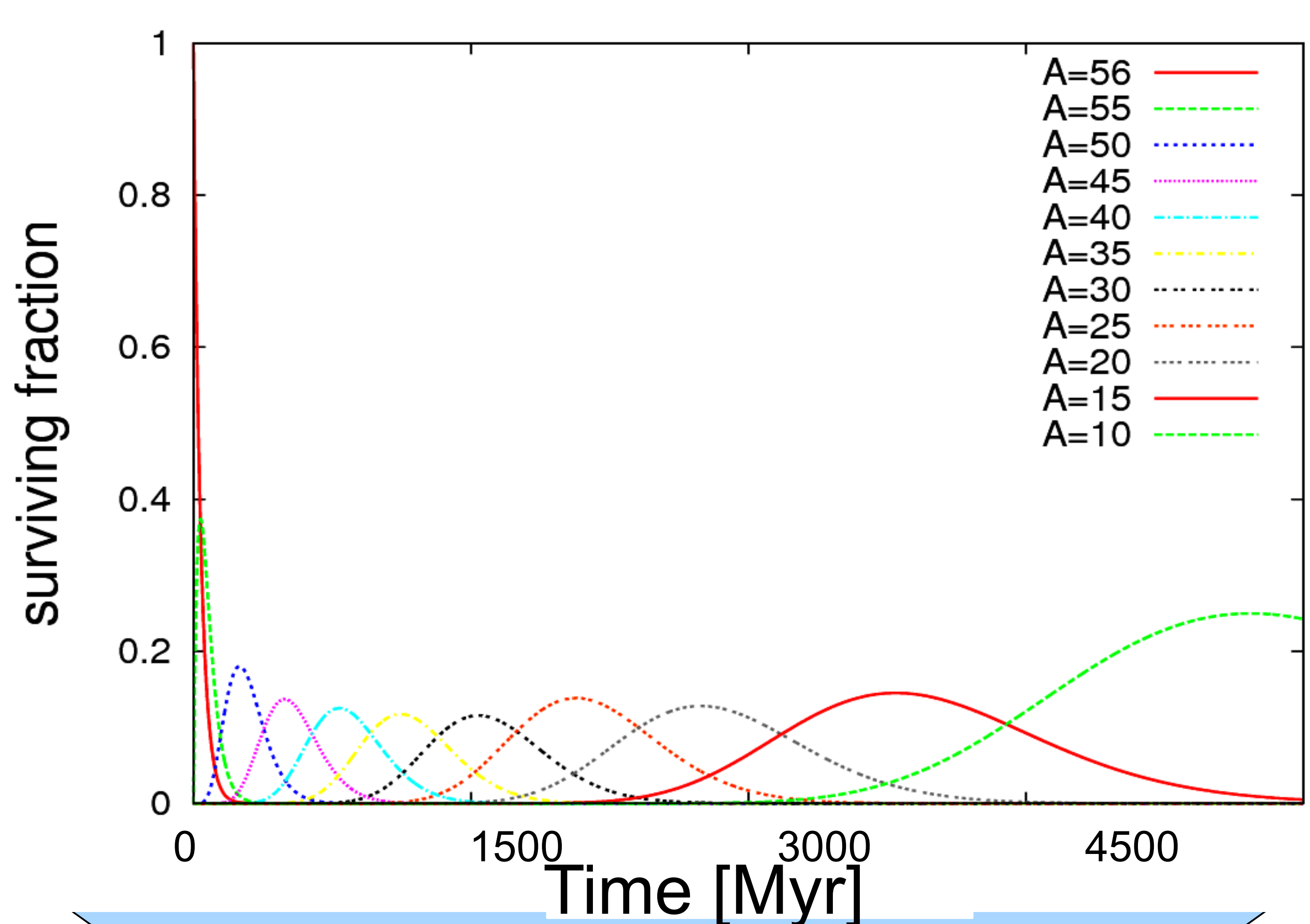
$$\langle X_{\mathrm{max}} \rangle = \sum_i f_i X_{\mathrm{i,max}}$$

$\mathrm{RMS}(X_{\mathrm{max}}) \longrightarrow$

$$\mathrm{RMS}^2_{X_{\mathrm{max}}} = \sum_i f_i \mathrm{RMS}^2_{X_{\mathrm{i,max}}} + \sum_i f_i \left(X_{\mathrm{i,max}} - \langle X_{\mathrm{max}} \rangle \right)^2$$



Nuclei Transmutation Within their Source



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IMPLICATIONS for UHECR Sources

$$f = \frac{t_{\rm trap}}{t_{\rm int.}^{\rm CR\gamma}}$$

$$t_{\rm int.}^{\rm CR\gamma} \approx \frac{1}{n_\gamma \sigma_{\rm CR\gamma} c}$$

$$n_\gamma = \frac{L_\gamma}{c 4\pi R^2 \epsilon_\gamma}$$

$$t_{\rm trap} \approx \frac{R^2}{2D} = \frac{3R^2}{2R_{\rm Larmor}}$$

$$f^{\rm CR\gamma} = \frac{3L_\gamma \sigma_{\rm CR\gamma} Z B}{8\pi \epsilon_\gamma E_{\rm CR}}$$

IMPLICATIONS for UHECR Sources

$$f^{\mathrm{CR}\gamma} = \frac{3L_\gamma \sigma_{\mathrm{CR}\gamma} ZB}{8\pi\epsilon_\gamma E_{\mathrm{CR}}} = \frac{s_1}{s_2}$$

Photo-disintegration threshold:

$2E_{\mathrm{CR}}\epsilon_\gamma > Am_p c^2 E_{\mathrm{bind.}}$, where $m_p c^2 E_{\mathrm{bind.}} = 10^{16}\ \mathrm{eV}^2$

Since,

$$L_\gamma[10^{44}\mathrm{erg\ s}^{-1}] = 2 \times 10^{45}\mathrm{eV\ cm}^{-1}$$

$$\sigma_{\mathrm{CR}\gamma}[\mathrm{A\ mb}] = \mathrm{A} \times 10^{-27}\ \mathrm{cm}^2$$

$$B[10^{-4}\ \mathrm{G}] = 3 \times 10^{-2}\ \mathrm{eV\ cm}^{-1}$$

$\frac{L_\gamma \sigma_{\mathrm{CR}\gamma} B}{A} = 6 \times 10^{16}\ \mathrm{eV}^2$, ergo.... $f^{\mathrm{CR}\gamma} = 50\frac{Z}{26}$

DESY.

A similar expression holds for TeV photon transparency

IMPLICATIONS for UHECR Sources

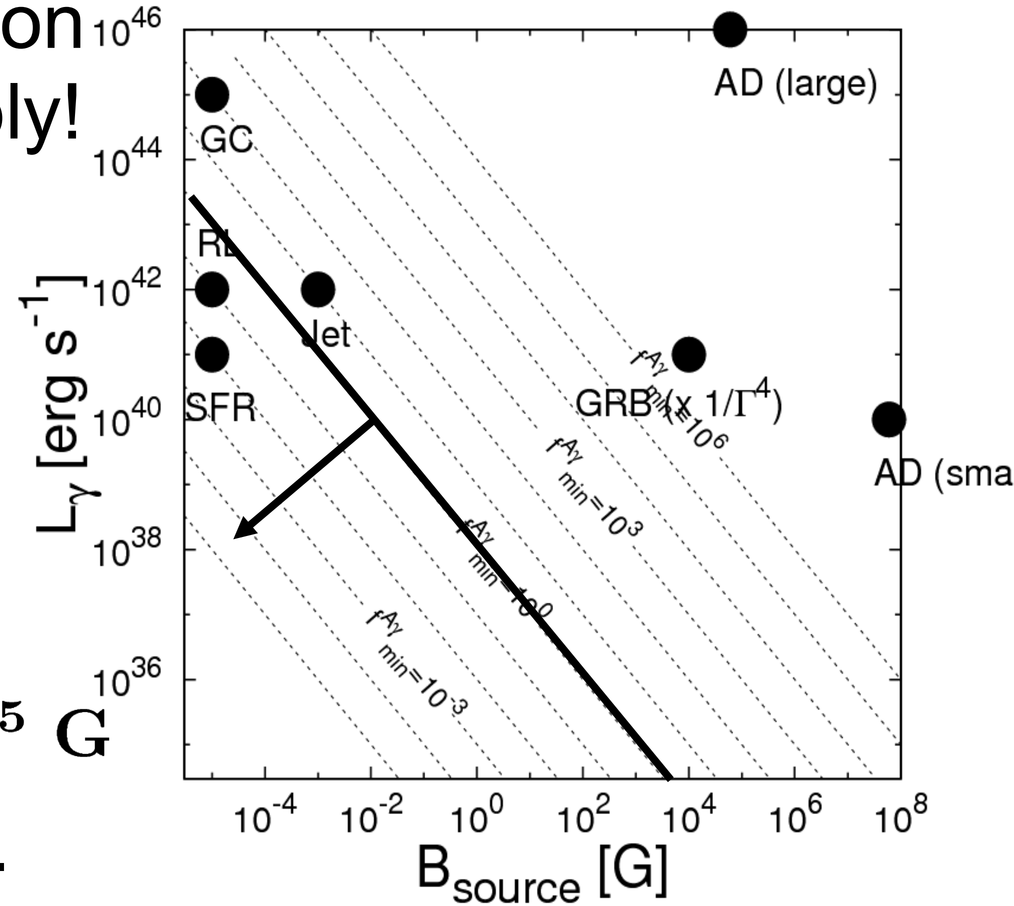
Since, $$\frac{L_\gamma^{\rm Edd.}\sigma_{{\rm CR}\gamma}B^{\rm Edd.}}{A} = 4\times 10^{23}\left(\frac{M}{M_\odot}\right)^{1/2}\ {\rm eV}^2$$

Only heavily sub-Eddington power objects need apply!

IF magnetic + photon luminosity are in equipartition:

$$L_\gamma \approx \beta R^2 B^2$$

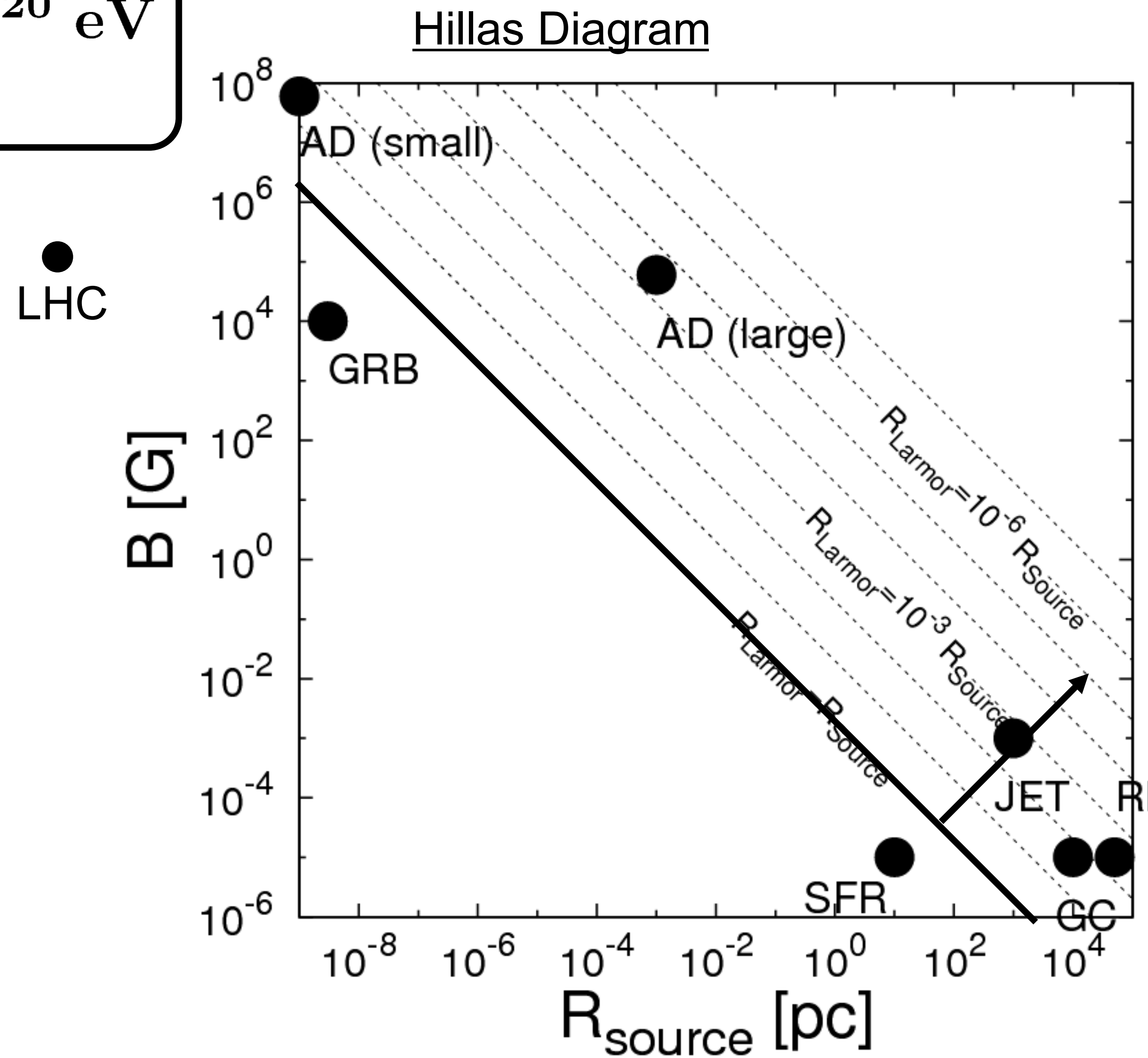
Requiring, $B < 4\times 10^{-5}\ {\rm G}$ to ensure safe passage.



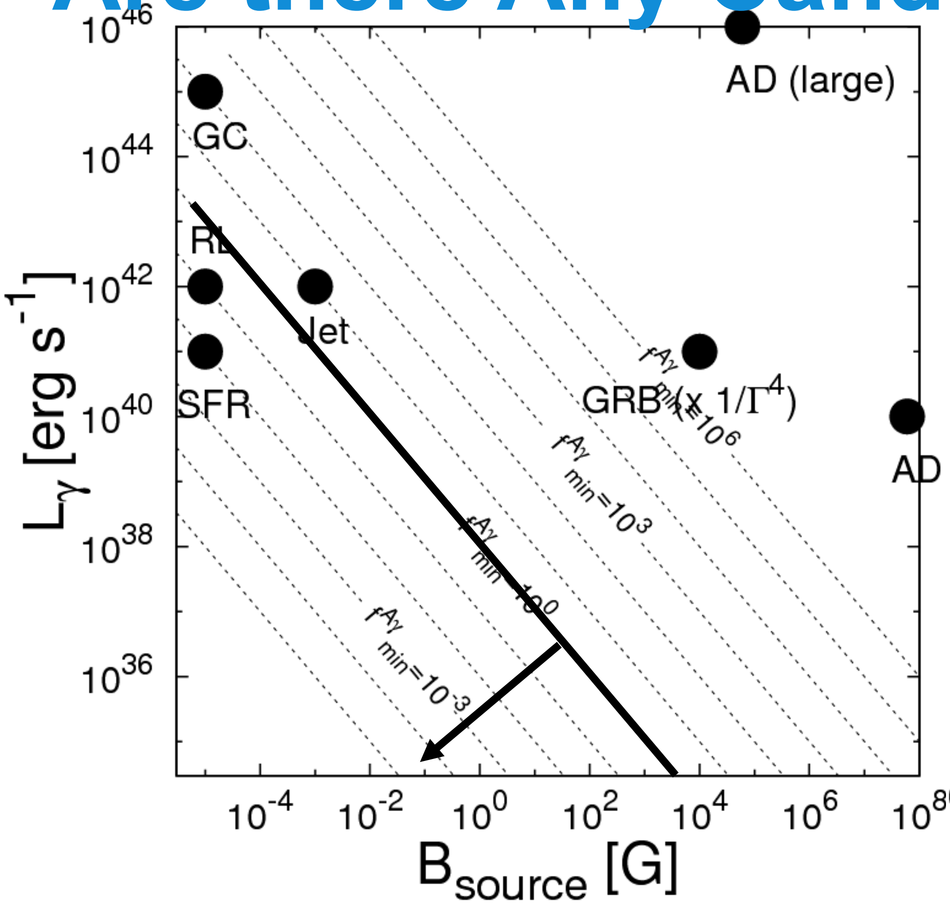
→ OVERALL MESSAGE: Compact Sources Disfavoured

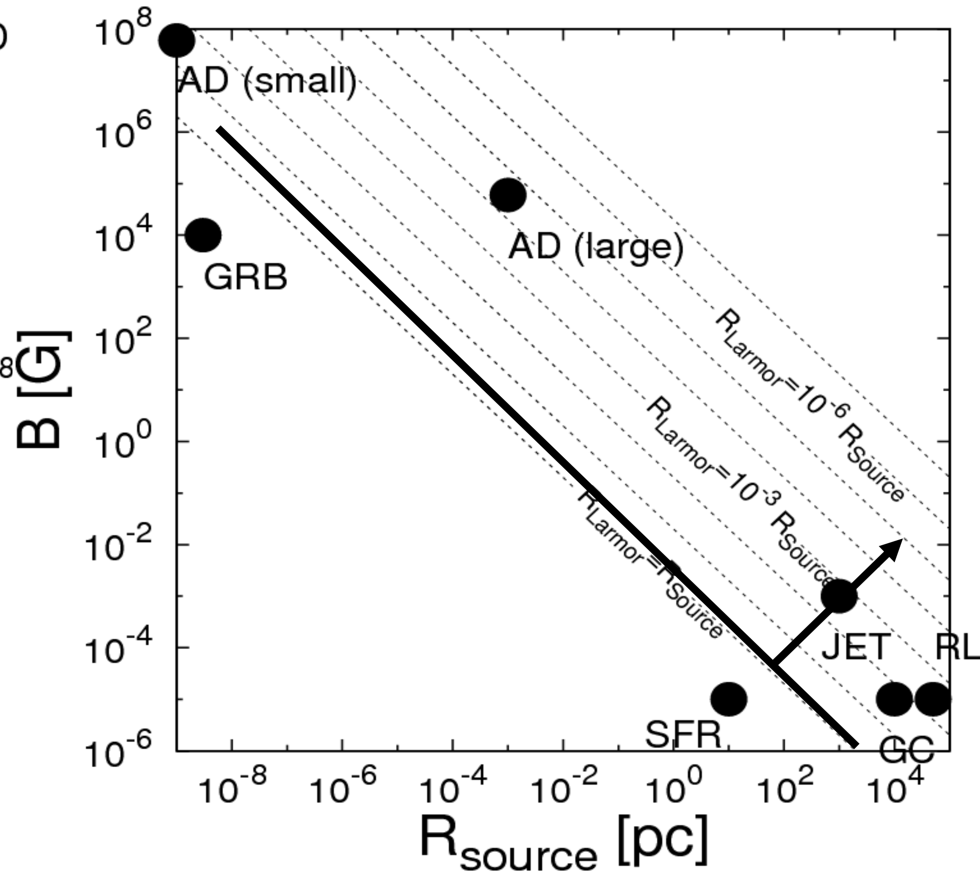
Are there Any Candidate Sources Left?

Accelerators of 10^{20} eV
Iron nuclei



Are there Any Candidate Sources Left?

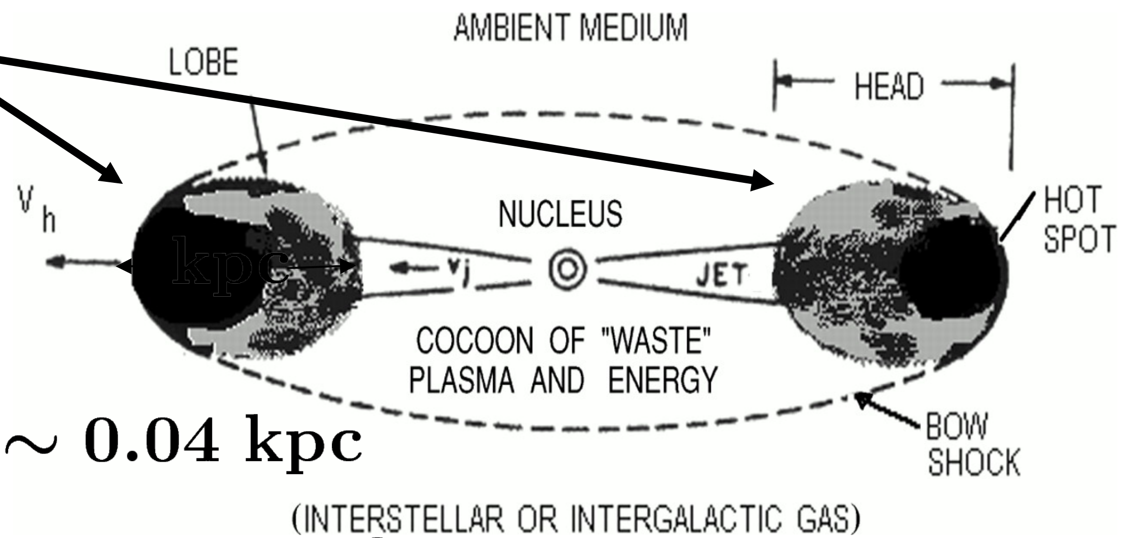




Example Candidate UHECR Source
(a Nuclei Friendly Environment)

Diagram taken from Ferrari -1998

Stochastic Acceleration in Radio Lobes:



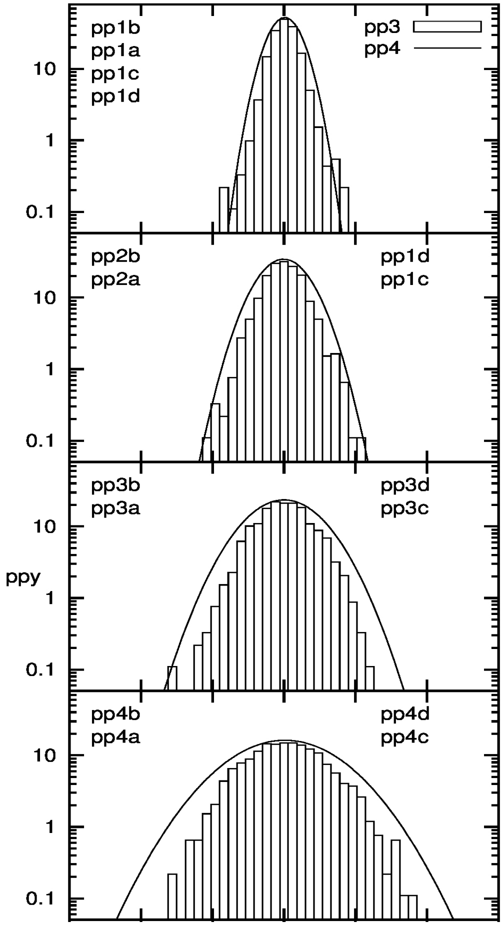
$$B_{\text{source}} \sim 10^{-4}\ \text{G}$$

$$R_{\text{Larmor}}(10^{20}\ \text{eV Fe}) \sim 0.04\ \text{kpc}$$

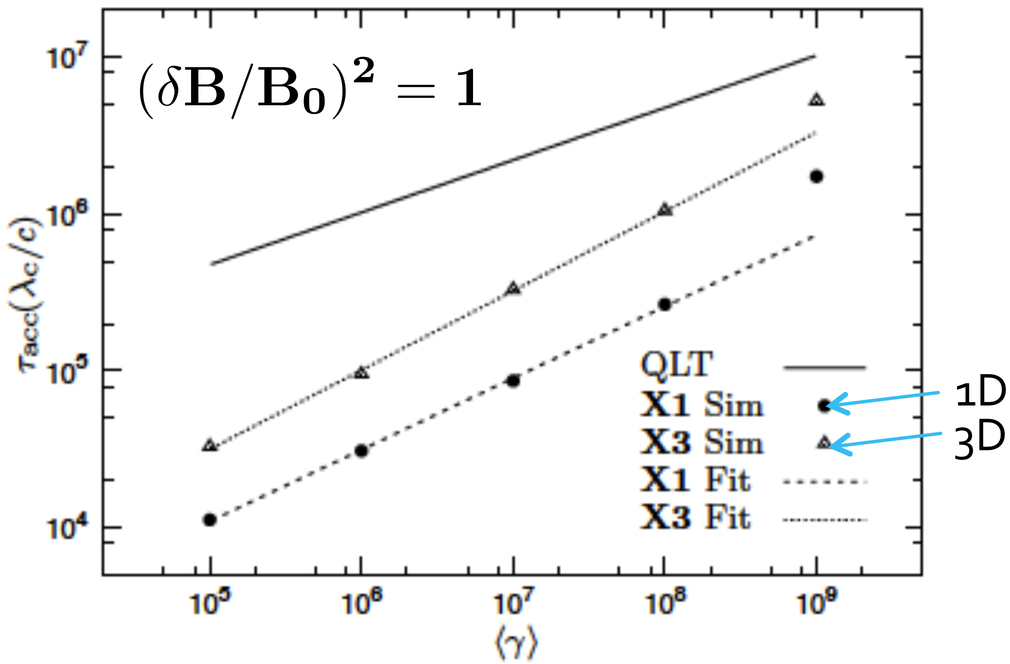
$t_{\text{acc}} < 10^{6}$ yrs for $\beta_{\text{scat.}} > 10^{-2}$

General PROBLEM for Large Accelerators- ACCELERATION TIME

Can Centaurus A's Radio Lobes Accelerate UHECR?



time



Yes, but requires:

$$\beta_A > 0.1$$

where

$$\beta_A = \frac{1}{c}\frac{B}{\sqrt{4\pi m_p n_p}}$$

astro-ph/0903.1259, O'Sullivan et al.

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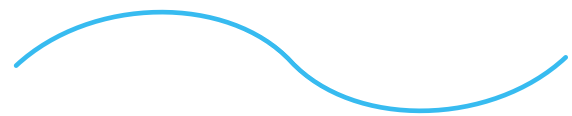
Diffusion Coefficient

- From resonant scattering between particles and magnetic field perturbations

With Larmor radius R_L



$B_o + \delta B(k)$ resonance for $k \sim R_L^{-1}$

$$\mathbf{P(k) \propto k^{-q}}$$

$$\frac{\mathbf{D_{xx}}}{\beta} = \left\langle \frac{\mathbf{B_0^2}}{(\delta \mathbf{B(k)})^2} \right\rangle \mathbf{R_L} = \frac{\mathbf{R_L}}{\mathbf{kP(k)}}$$

Probability to scatter off resonant mode within Larmor period

$$\propto \mathbf{p^{2-q}}$$

Since $$\frac{\mathbf{p^2}}{\mathbf{D_{pp}}} \sim \frac{\mathbf{D_{xx}}}{\beta^2_{\mathbf{scat}}}$$

$$\mathbf{D_{pp} \propto p^q}$$

- Bohm -> q=1
- Kolmogorov -> q=5/3
- Kraichnan -> q=3/2
- Hard-sphere -> q=2

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DESY.

Particle Transport Equation

- Cut-offs arise naturally in the general solution of the transport equation for particles

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \nabla_{\mathbf{p}} \cdot \left[(\mathbf{D_{pp}} \nabla_{\mathbf{p}} \mathbf{f}) - \frac{\mathbf{p}}{\tau_{\mathbf{loss}}(\mathbf{p})} \mathbf{f} \right] - \frac{\mathbf{f}}{\tau_{\mathbf{esc}}(\mathbf{p})} + \frac{\mathbf{Q}}{\mathbf{p^2}}$$

Acceleration

Radiative Losses

Escape

Source term

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Cut-off Shape

- Interplay of acceleration and cooling defines the value of the cut-off of the primary particles:

$$\frac{\mathbf{dN}}{\mathbf{dE_e}} \propto \mathbf{E_e^{-\Gamma} e^{-(E_e/E_{\max})^{\beta_e}}} \qquad \beta_\mathbf{e} = \mathbf{2 - q - r}$$

- In the following, demonstrations for this result will be shown for the case of stochastic acceleration scenarios. However, in reality, this result is more general, holding also for shock acceleration scenarios.

[see Schlickeisser et al. 1985, Zirakashvili et al. 2007, Stawarz et al. 2008]

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A Simple Case- q=1, only escape

- Bohm diffusion (**q=1**) + only escape results in simple exponential cutoff.
- Some simplifications to the transport equation:

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \nabla_{\mathbf{p}} \cdot \left[(\mathbf{D}_{\mathbf{pp}} \nabla_{\mathbf{p}} \mathbf{f}) - \frac{\mathbf{p}}{\tau_{\mathbf{loss}}(\mathbf{p})} \mathbf{f} \right] - \frac{\mathbf{f}}{\tau_{\mathbf{esc}}(\mathbf{p})} + \frac{\mathbf{Q}}{\mathbf{p}^2}$$

Steady state

No losses

Delta injection

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DESY.

A Simple Case (II)- q=1, only escape

- Rearranging the terms (and explicitly stating the dependences from p of the parameters):

$$\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_0 \frac{p}{p_0}\frac{\partial f}{\partial p}\right) - \frac{f}{\tau_{esc}(p)} = \delta(p), \qquad \tau_{esc}(p) \propto p^{-1}$$

$$\boxed{\frac{\partial^2 f}{\partial p^2} + \frac{3}{p}\frac{\partial f}{\partial p} - \left(\frac{1}{D_0 \tau_0}\right) f = \delta(p)}$$

Cutoff comes from balancing 1st and 3rd term → $f \propto A e^{-p/p_\tau}$

Recall generally, $\beta_e = 2 - q - r$

$$q = 1,\ r = 0,\ \rightarrow\ \beta_e = 1$$

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(Note- energy losses for the $r = 0$ case will not alter this result)

Intuitive Insights into Cut-off Shape Origin

Consider the steady-state case of diffusion (constant diffusion coefficient) of particles into an absorbing medium

$$\nabla \cdot (\mathbf{D_{xx}} \nabla \mathbf{f}) - \frac{\mathbf{f}}{\tau(\mathbf{x})} = \delta(\mathbf{r})$$

For $\tau(\mathbf{x}) = \tau_*(\mathbf{x}/\mathbf{x}_*)^{\mathbf{2}}$ $\quad \mathbf{f} \propto \mathrm{const.}$

For $\tau(\mathbf{x}) = \tau_*$ $\quad \mathbf{f} \propto \mathbf{e}^{-\mathbf{x}/\mathbf{x}_\tau}$

For $\tau(\mathbf{x}) = \tau_*(\mathbf{x}/\mathbf{x}_*)^{-\mathbf{2}}$ $\quad \mathbf{f} \propto \mathbf{e}^{-(\mathbf{x}/\mathbf{x}_\tau)^{\mathbf{2}}}$

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End of First Lecture

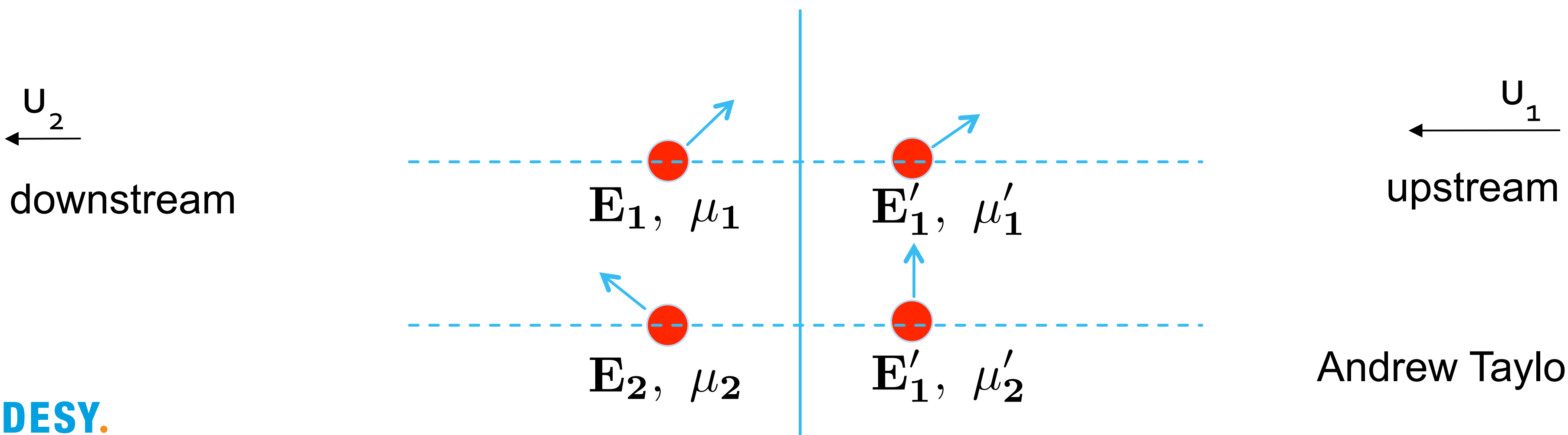
Shock Acceleration

$$\mathbf{E_2} = \mathbf{E_1}\left(\frac{\mathbf{1} + \beta\mu_\mathbf{1}}{\mathbf{1} + \beta\mu_\mathbf{2}}\right)$$

$$\mathbf{E_2} = \mathbf{\Gamma^2 E_1}(\mathbf{1} - \beta\mu_\mathbf{1})(\mathbf{1} + \beta\mu'_\mathbf{2})$$

$$\mu' = \frac{\mu - \beta}{\mathbf{1} - \beta\mu}$$

$$\mathbf{E_2} = \mathbf{\Gamma^2 E_1}(\mathbf{1} - \beta\mu_\mathbf{1})\left(\mathbf{1} + \beta\left(\frac{\mu_\mathbf{2} - \beta}{\mathbf{1} - \beta\mu_\mathbf{2}}\right)\right)$$



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Random Walks

$$\mathbf{f(x,t) = \gamma(t+1)/[\gamma([t-x]/2+1)\gamma([x+t]/2+1)]/(2^t)}$$

From Stirling's formula

$$\gamma(\mathbf{x}) \approx \frac{(\mathbf{x/e})^{\mathbf{x}}}{\pi^{\mathbf{1/2}}} \qquad \gamma(\mathbf{x+1}) \approx (\mathbf{2}\pi\mathbf{x})^{\mathbf{1/2}}(\mathbf{x/e})^{\mathbf{x}}$$

$$\mathbf{f(x,t)} \approx \frac{\mathbf{t^t e^{-t}}}{\mathbf{[(t-x)/2]^{(t-x)/2}[(t+x)/2]^{(t+x)/2}e^{-t}}}$$

$$\log[\mathbf{f(x,t)}] \approx \frac{\mathbf{t}}{[\frac{\mathbf{1}}{\mathbf{2}}(\mathbf{t-x})(\log\mathbf{t/2-x/2t})]+[\frac{\mathbf{1}}{\mathbf{2}}(\mathbf{t+x})(\log\mathbf{t/2+x/2t})]}$$

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Particle Acceleration with Cooling

$$\mathbf{\frac{dE_e}{cdt} = \frac{4}{3}\Gamma_e^2 \sigma_T U_B}$$

$$\mathbf{t_{cool} = \frac{9}{8\pi\alpha}\left(\frac{m_e}{E_\gamma^{sync}}\right) t_{lar}}$$

$$\mathbf{t_{cool} = E_e \frac{dt}{dE_e}}$$

$$\mathbf{\sigma_T U_{Bcrit} \frac{hc}{(m_e c^2)^2} = (2\pi/3)\alpha}$$

$$\mathbf{t_{cool} = \frac{9}{8\pi\alpha}\frac{h}{E_e}\frac{U_{B_{crit.}}}{U_B}}$$

Particle Acceleration with Cooling

$$\mathbf{t_{cool}} = \frac{9}{8\pi\alpha}\frac{\mathbf{h}}{\mathbf{E_e}}\frac{\mathbf{U_{B_{crit.}}}}{\mathbf{U_B}}$$

$$\mathbf{t_{cool}} = \frac{9}{8\pi\alpha}\left(\frac{\mathbf{m_e}}{\mathbf{E_\gamma^{sync}}}\right)\mathbf{t_{lar}}$$

$$\mathbf{t_{lar}} = \frac{2\pi\mathbf{E_e}}{\mathbf{eBc}} = \mathbf{\Gamma_e}\left(\frac{\mathbf{B_{crit}}}{\mathbf{B}}\right)\frac{\mathbf{h}}{\mathbf{m_e}}$$

$$\mathbf{E_\gamma^{sync}} = \mathbf{\Gamma_e^2}\left(\frac{\mathbf{B}}{\mathbf{B_{crit}}}\right)\mathbf{m_e}$$

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Intuitive Insights into Cut-off Shape Origin

Consider the steady-state case of diffusion (constant diffusion coefficient) of particles into an absorbing medium

$$\nabla \cdot (\mathbf{D_{xx}} \nabla \mathbf{f}) - \frac{\mathbf{f}}{\tau(\mathbf{x})} = \delta(\mathbf{r})$$

For $\tau(\mathbf{x}) = \tau_*(\mathbf{x}/\mathbf{x}_*)^{\mathbf{2}}$ $\quad \mathbf{f} \propto \mathrm{const.}$

For $\tau(\mathbf{x}) = \tau_*$ $\quad \mathbf{f} \propto \mathbf{e}^{-\mathbf{x}/\mathbf{x}_\tau}$

For $\tau(\mathbf{x}) = \tau_*(\mathbf{x}/\mathbf{x}_*)^{-\mathbf{2}}$ $\quad \mathbf{f} \propto \mathbf{e}^{-(\mathbf{x}/\mathbf{x}_\tau)^{\mathbf{2}}}$

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Intuitive Insights into Cut-off Shape Origin

$$\mathbf{D_{xx}\frac{\partial^2 f}{\partial x^2} + D_{xx}\frac{2}{x}\frac{\partial f}{\partial x} - \frac{f}{\tau(x)} = 0}$$

For $\tau(\mathbf{x}) = \tau_*$ $\mathbf{f} \propto \mathbf{e}^{-\mathbf{x}/\mathbf{x}_\tau}$

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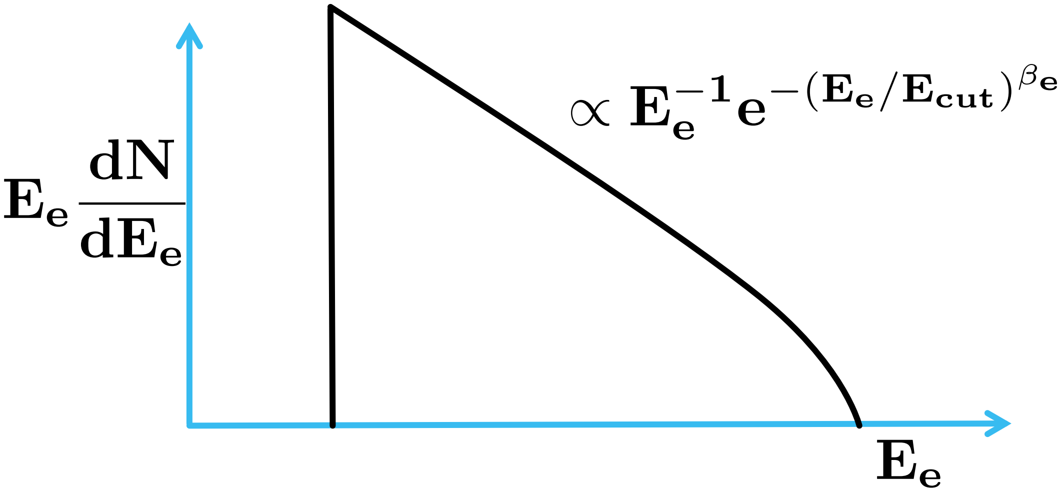
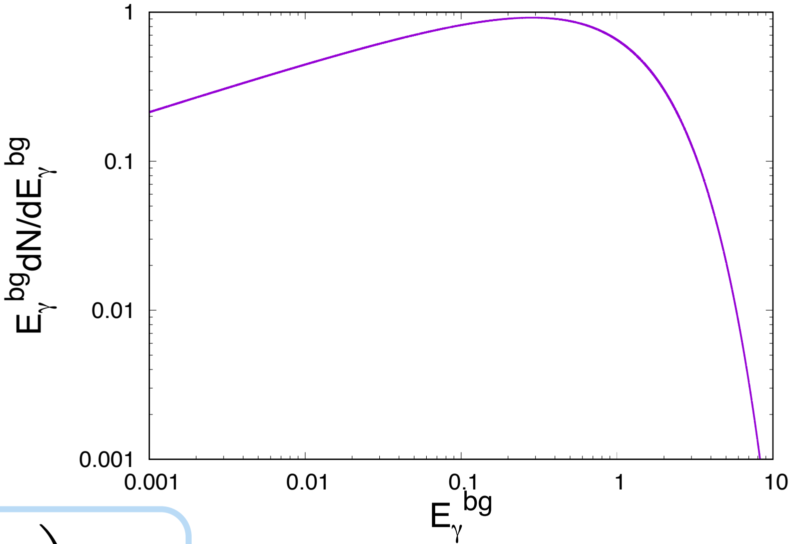
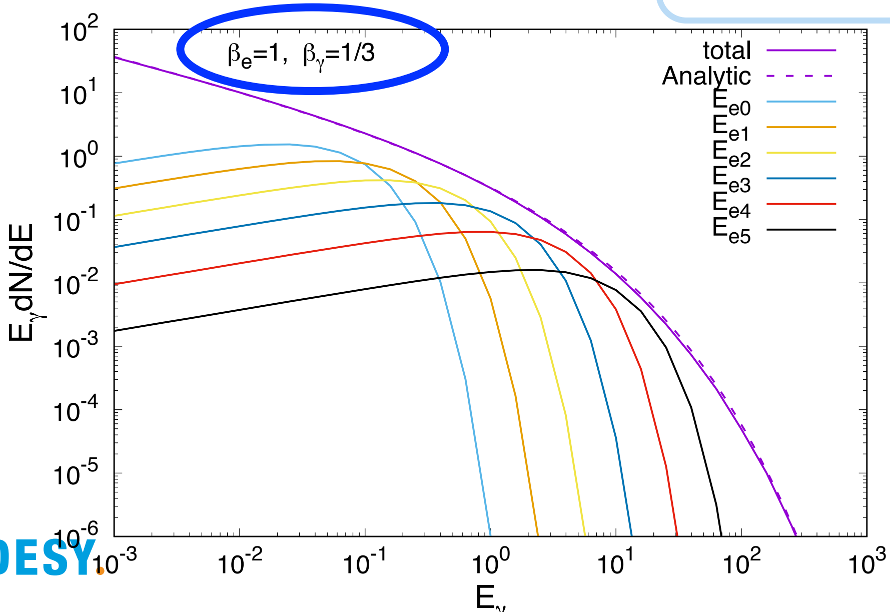
Cut-off Shape- Electrons & Photons





$$E_\gamma = \gamma_e^2 \left(\frac{B}{B_{crit}} \right) m_e$$



$$f(x) = \int_0^\infty e^{-(x/y^2)} e^{-y^{\beta_e}} dy$$

An

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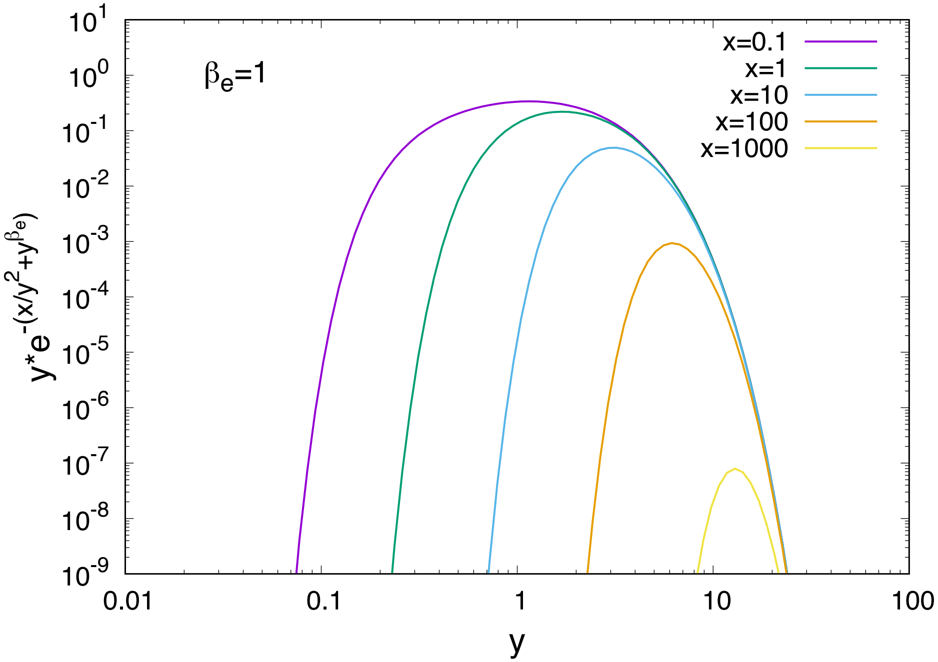
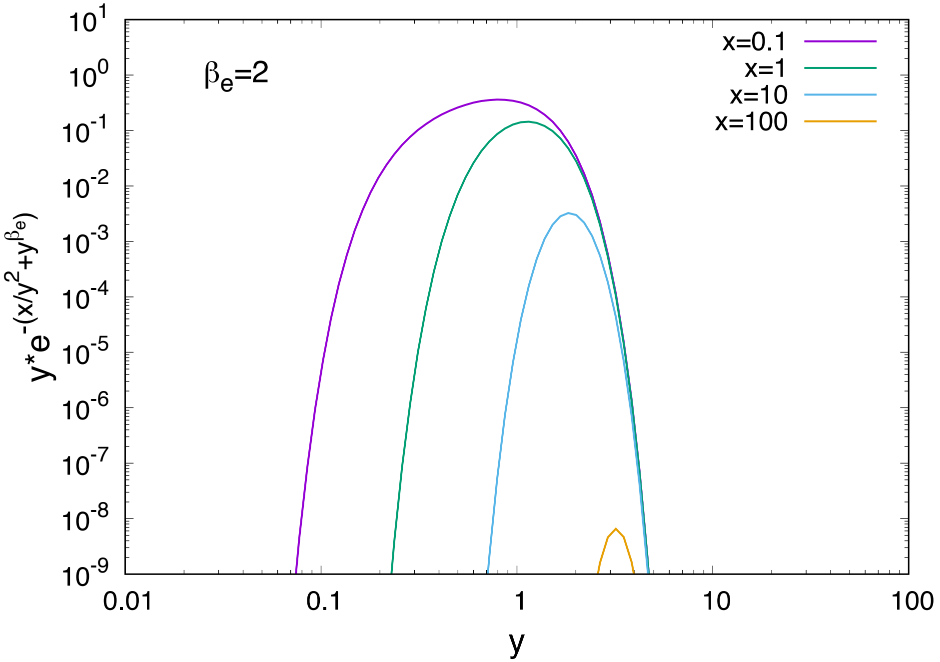
Integrand-





$$\mathbf{y^2}\left(\mathbf{y}^{\beta_\mathbf{e}} - \frac{\mathbf{1}}{\beta_\mathbf{e}}\right) = \frac{\mathbf{2x}}{\beta_\mathbf{e}}$$

$$\mathbf{y^2} \approx \left(\frac{\mathbf{2x}}{\beta_\mathbf{e}}\right)^{\frac{\mathbf{2}}{\beta_\mathbf{e}+\mathbf{2}}}$$

$$\frac{\mathbf{x}}{\mathbf{y^2}} \approx \mathbf{x}^{\frac{\beta_\mathbf{e}}{\beta_\mathbf{e}+\mathbf{2}}} \longrightarrow \qquad \beta_\gamma = \frac{\beta_\mathbf{e}}{\beta_\mathbf{e}+\mathbf{2}}$$

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Cut-off Shape- Emission Dependence

$$\frac{dN}{dE_e} \propto E_e^{-\Gamma} e^{-(E_e/E_{max})^{\beta_e}}$$

$$\frac{dN}{dE_\gamma} \propto E_\gamma^{-\Gamma} e^{-(E_\gamma/E_{max})^{\beta_\gamma}}$$

- Different emission processes dictate different relation between electrons and gamma rays

e.g.

- Synchrotron/IC Thomson: $$\beta_\gamma = \frac{\beta_e}{\beta_e + 2}$$
- SSC: $$\beta_\gamma = \frac{\beta_e}{\beta_e + 4}$$
- IC (Klein Nishina) $$\beta_\gamma = \beta_e$$

Good measurement of gamma ray cut-off can give insight on the cut-off region of primary electrons

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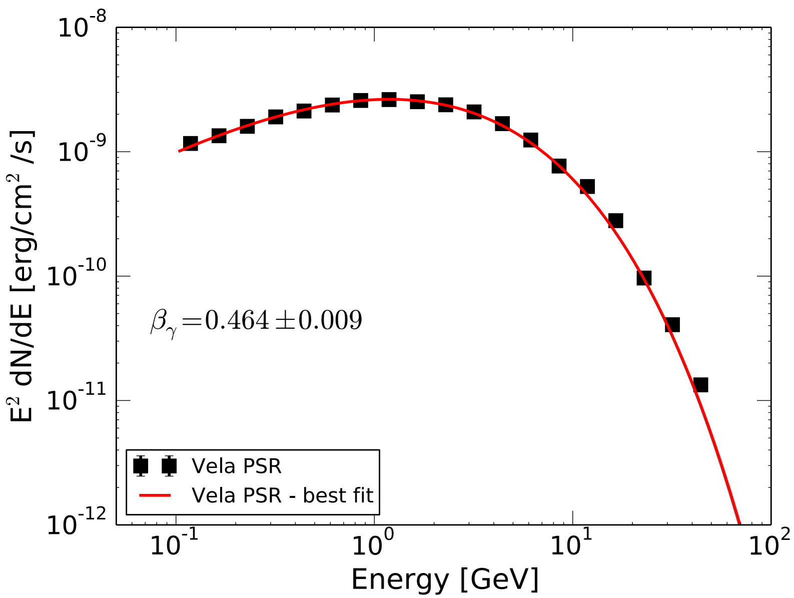
Observation of Cut-offs in Gamma-ray Spectra

- Test case- Vela Pulsar (brightest source)



$$\frac{\mathbf{dN}}{\mathbf{dE}_\gamma} \propto \mathbf{E}_\gamma^{-\boldsymbol{\Gamma}} \mathbf{e}^{-(\mathbf{E}_\gamma/\mathbf{E}_{\max})^{\beta_\gamma}}$$

Parameter	Value
N [ph/cm^2/s/GeV]	$\left(1.39^{+0.12}_{-0.10}\right) 10^{-5}$
Γ	1.019 ± 0.011
E_c [GeV]	0.238 ± 0.016
β_γ	0.464 ± 0.009
E_s (fixed) [GeV]	0.83255

Romoli et al., **Astropart.Phys. 88 38-45 (2017)**

- Note- MCMC method used to explore 'good-fit' region. This has the benefit of being stable on the landscape being explored

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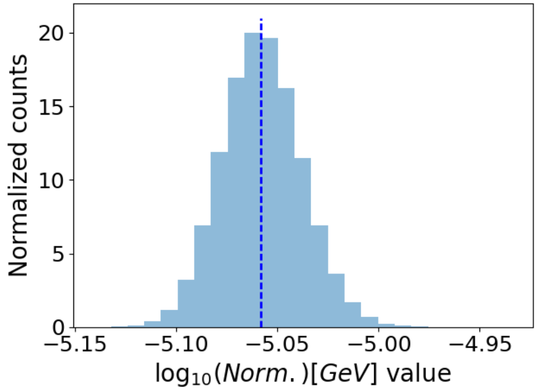
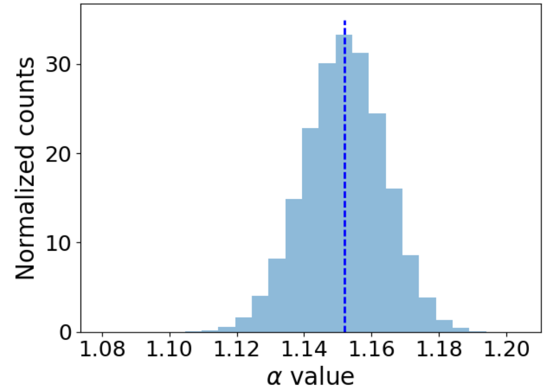
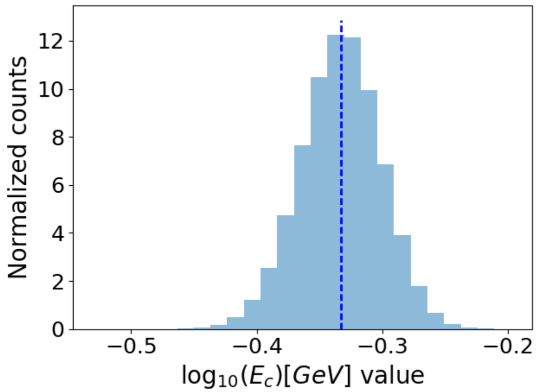
MCMC Parameter Constraints

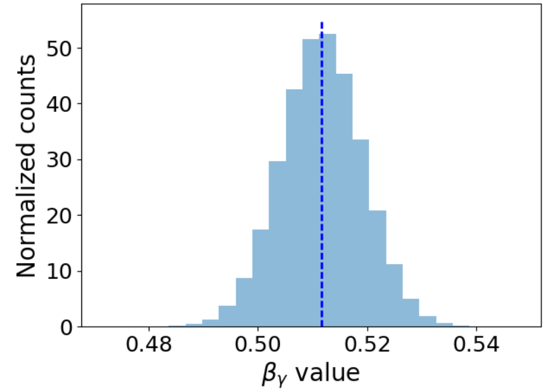
$$\frac{\mathbf{dN}}{\mathbf{dE_\gamma}} \propto \mathbf{E_\gamma^{-\Gamma} e^{-(E_\gamma/E_{max})^{\beta\gamma}}}$$

False minima









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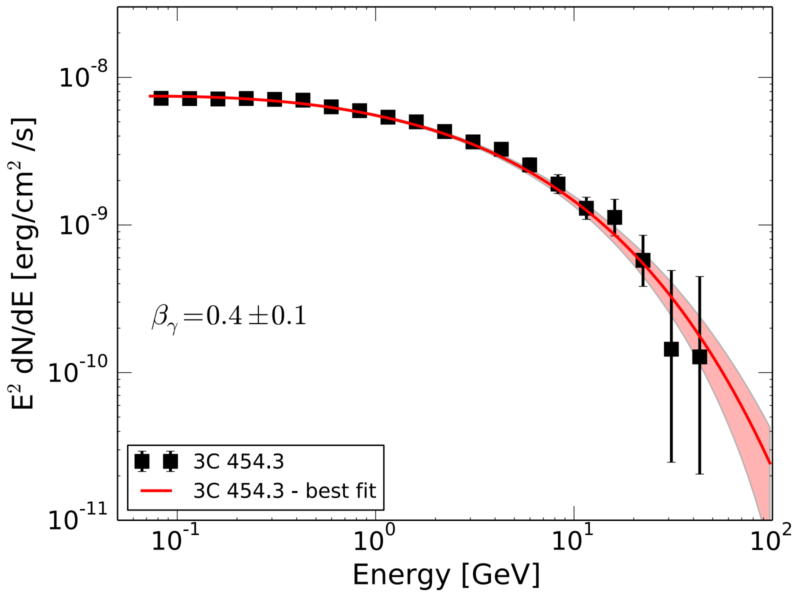
Observation of Cut-offs in Gamma-ray Spectra

- Brightest AGN Flare-

3C 454 Nov 2010



Romoli et al., **Astropart.Phys. 88 38-45 (2017)**

Parameter	Value
N [ph/cm^2/s/GeV]	$(4.7^{+3.9}_{-1.2})\ 10^{-5}$
Γ	$1.87^{+0.08}_{-0.12}$
E_c [GeV]	$1.1^{+1.6}_{-0.9}$
β_γ	0.4 ± 0.1
E_s (fixed) [GeV]	0.41275

- Indicating a cut-off value of the primary particles around 1 GeV
- **Caveats:**
  - Values obtained on a 7 days integration (for statistics)
  - Spectrum variable during the flare -> superposition effects?

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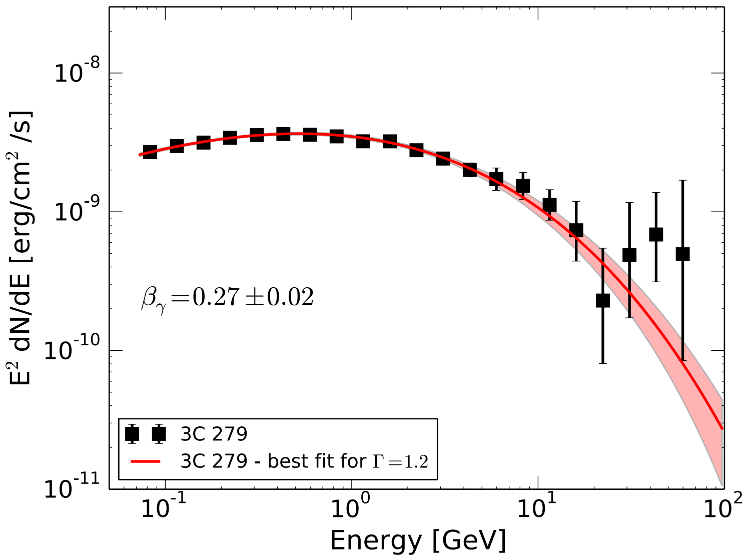
Observation of Cut-offs in Gamma-ray Spectra

- 2nd Brightest AGN Flare-

3C 279 June 2015



Parameter	$\Gamma = 1.2$
N [ph/cm^2/s/GeV]	$(2.8^{+0.8}_{-0.6})\ 10^{-4}$
Γ (fixed)	1.2
E_c [GeV]	$(8.4^{+6.6}_{-4.1})\ 10^{-3}$
β_γ	0.27 ± 0.02
E_s (fixed) [GeV]	

Values obtained on a 3 days integration
Note- X-ray observations during flare indicated that $\mathbf{\Gamma = 1.17 \pm 0.06}$

Romoli et al., **Astropart.Phys. 88 38-45 (2017)**

$$\beta_\gamma = \frac{\beta_\mathbf{e}}{\beta_\mathbf{e} + \mathbf{2}}$$

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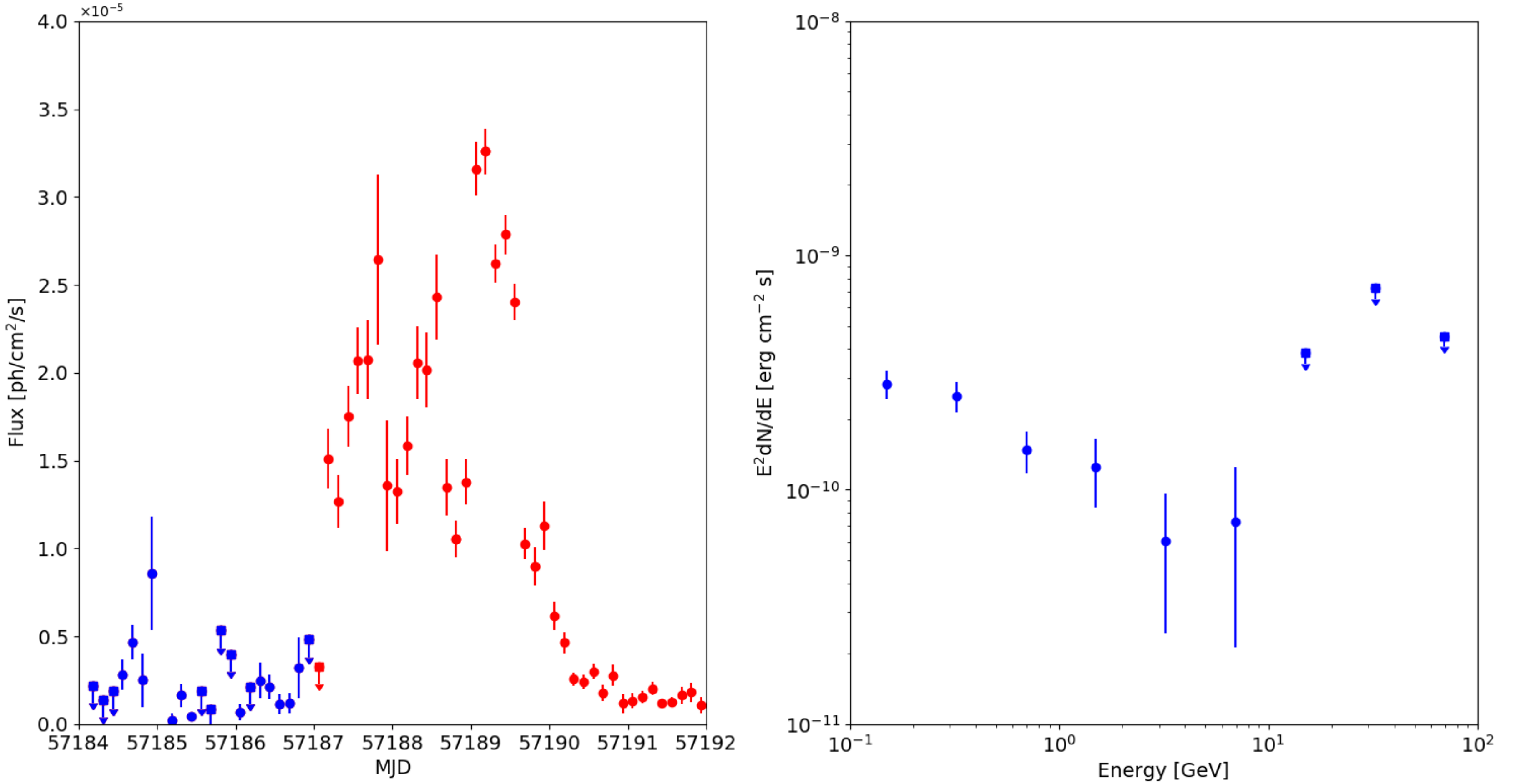
3C 279 June 2015 Flare-Temporal Evolution



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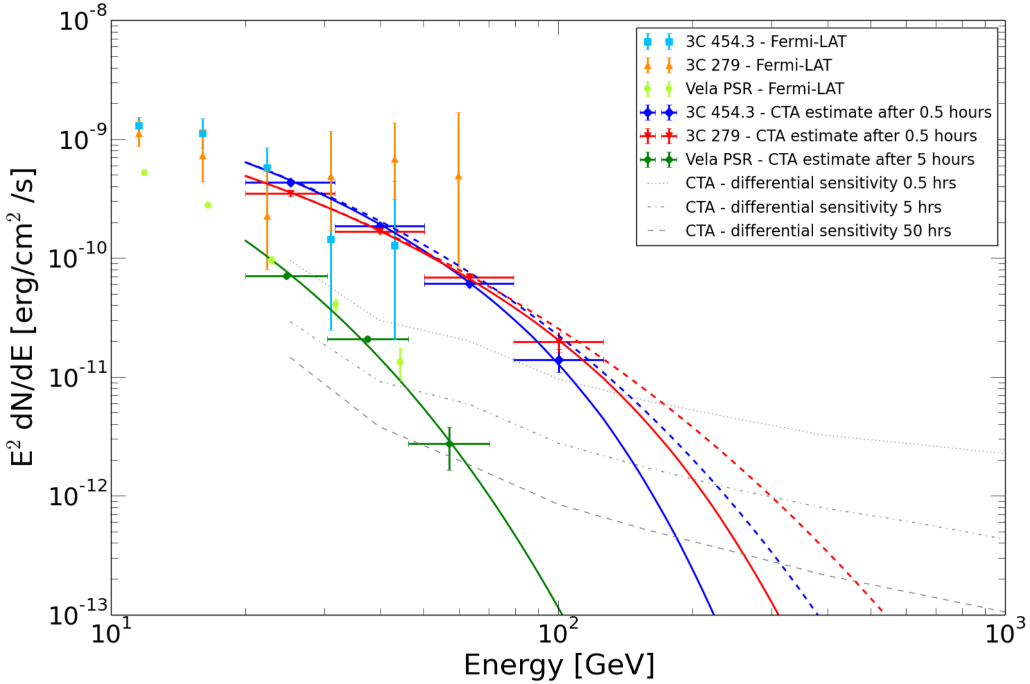
Prospects for CTA (South)



- Study using the expected CTA performance
- Fermi data integrated over 3 days
- Constraint on β_γ parameter at 10% level obtained during only 0.5 hr flare!

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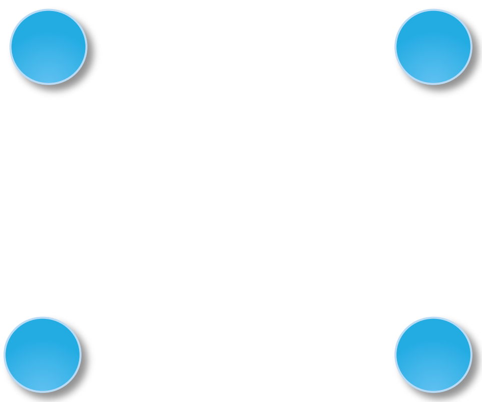
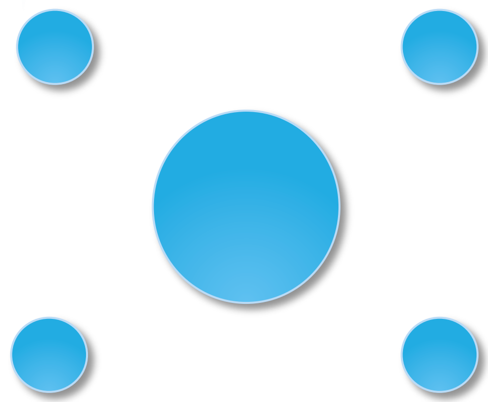
HESSI and HESSII Eras

H.E.S.S. Phase I: 2002-2012
- 4 telescopes of 12m
- 100 GeV - 100 TeV



H.E.S.S. Phase II: 2012-++
- Addition of CT5 to the array: 28m
- ~30 GeV - 100 TeV



CT5 allows E < 100 GeV measurements — best for:
- High redshift AGN + GRBs
- EBL studies at large z

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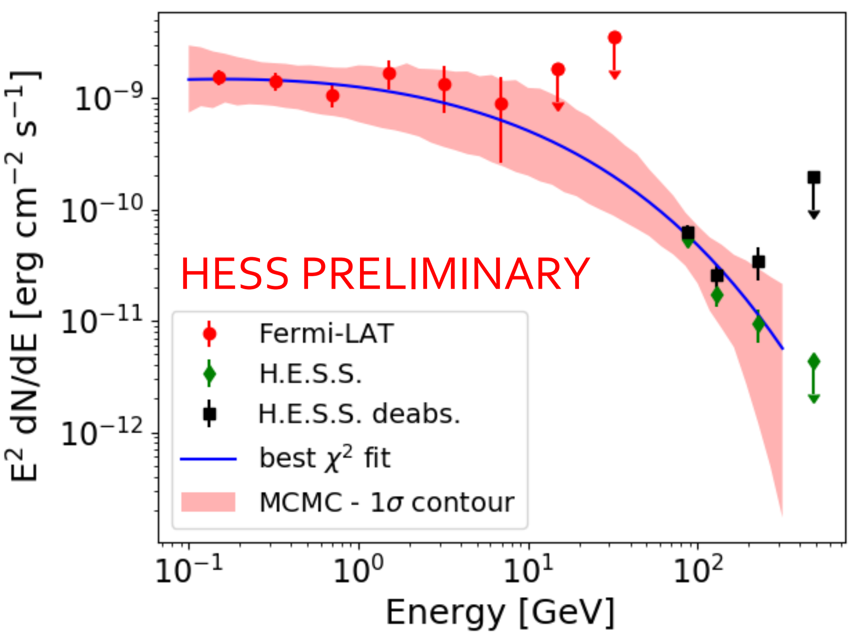
Can We Do Better Already? Fermi + H.E.S.S.II Fit



Parameter	MCMC fit
$\log_{10} N_0$ [ph/cm^2/s/GeV]	$(-4.75^{+0.91}_{-0.24}) \times 10^{-5}$
Γ	$(1.93^{+0.29}_{-0.41})$
$\log_{10} E_c$ [GeV]	$0.13^{+1.33}_{-2.82}$
β_γ	$0.34^{+0.32}_{-0.14}$

- Joint fit of Fermi-LAT data (9 hours centred on HESSII obs.) taken on night 2
- $$\beta_\gamma = \mathbf{0.34^{+0.32}_{-0.14}}$$

(HESSII data taken from ICRC2017 Presentation)

DESY. Fermi Gamma-ray Space Telescope H.E.S.S.

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The pp Cross-Section

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Cut-Offs for Primary and Secondaries

$$\Phi_\gamma(E_\gamma) = 4\pi n_{\mathrm{H}} \int \frac{d\sigma}{dE_\gamma}(p_p, E_\gamma) J(p_p) dp_p$$

For spectra of the form,

$$J_p(p_p) = \frac{A}{p_p^\alpha} \exp\left[-\left(\frac{p_p}{p_p^{\max}}\right)^\beta\right]$$

and the cutoff regions may be fit with a function of the form

$$\Phi_\gamma(E_\gamma) = \frac{A'}{E_\gamma^{\alpha'}} \exp\left[-\left(\frac{E_\gamma}{E_\gamma^{\max}}\right)^{\beta'}\right]$$

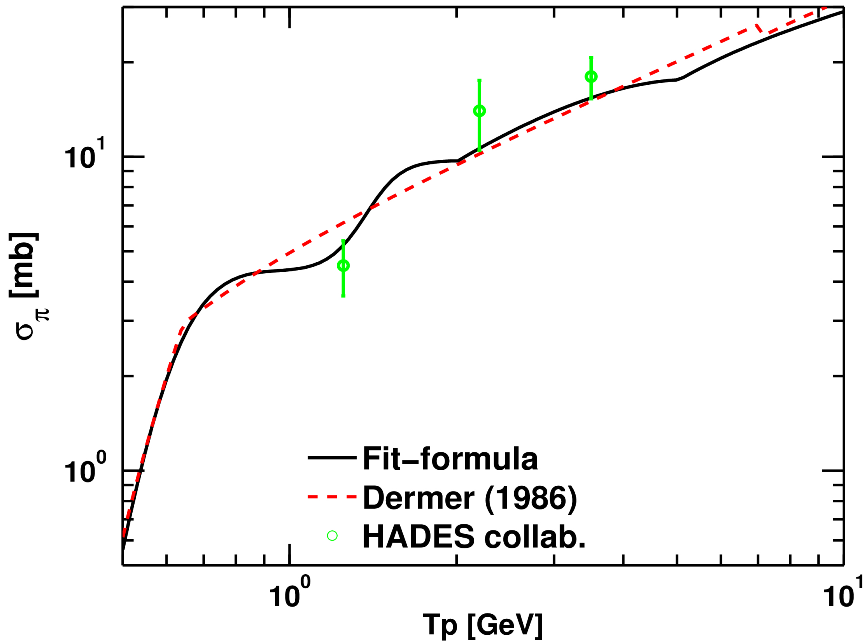
where $\beta' = \frac{a\beta}{\beta+b}$

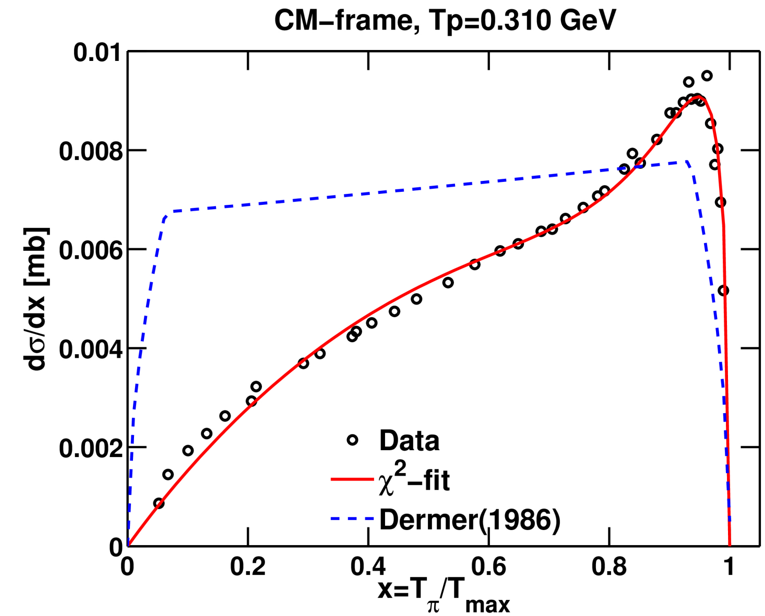
	Geant		Pythia		SIBYLL		QGSJET	
α	a	b	a	b	a	b	a	b
1.5	1.0	1.0	1.1	1.2	1.2	1.2	1.1	1.1
1.75	1.1	1.1	1.2	1.3	1.3	1.3	1.2	1.2
2.0	1.3	1.1	1.4	1.4	1.5	1.4	1.3	1.3
2.25	1.4	1.2	1.5	1.5	1.6	1.4	1.4	1.3
2.5	1.5	1.1	1.7	1.7	1.7	1.5	1.5	1.4

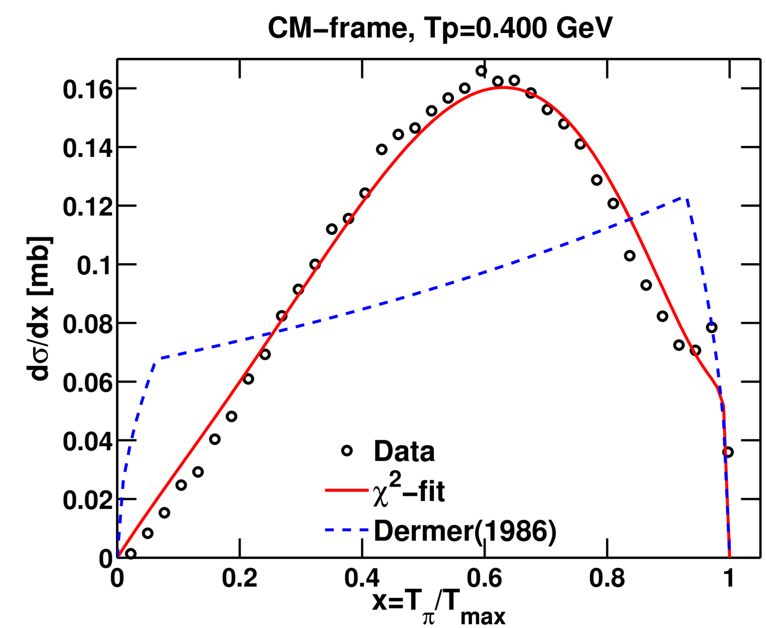
Kafexhiu et al., **Phys.Rev. D90 12, 123014 (2014)**

π Spectra for $T_p^{th}<T_p<1$ GeV

$$\sigma_\pi = \sigma_{\rm inel}\langle n_{\pi_0}\rangle$$







Kafexhiu et al.,
Phys.Rev. D90 12, 123014 (2014)

Note- Kamae description has artificially high threshold (~0.5 GeV)

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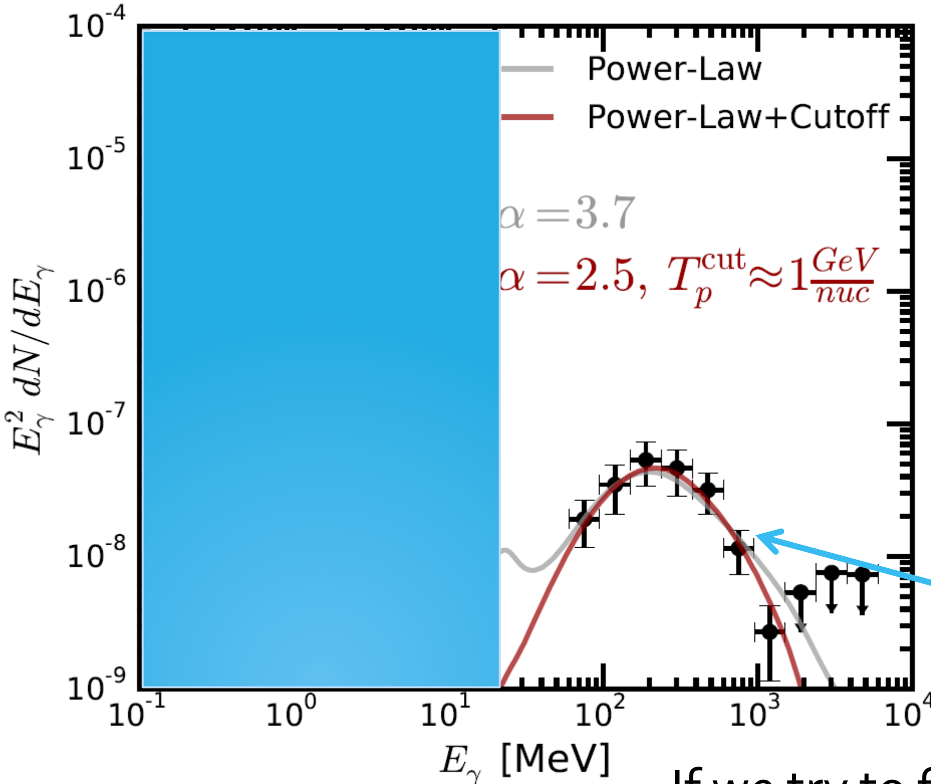
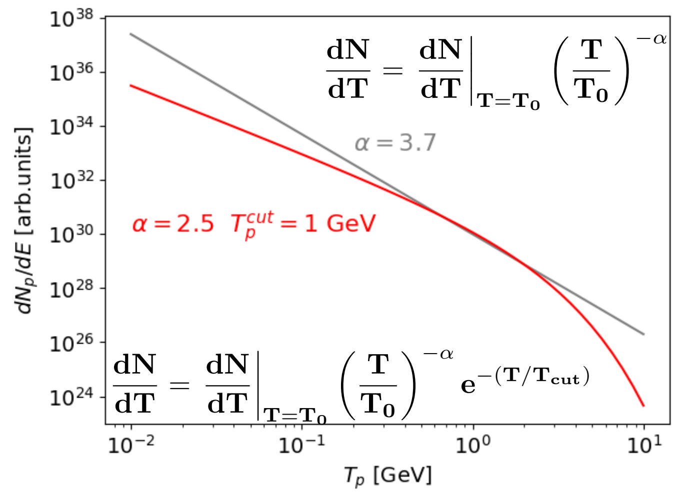
Constraining the Particle Spectra in Solar flares

- Optimal level of statistics (bright low energy transients, plenty of photons)
- Retrieve the primary particle spectrum (using the most up-to-date cross sections)





Fermi-LAT data

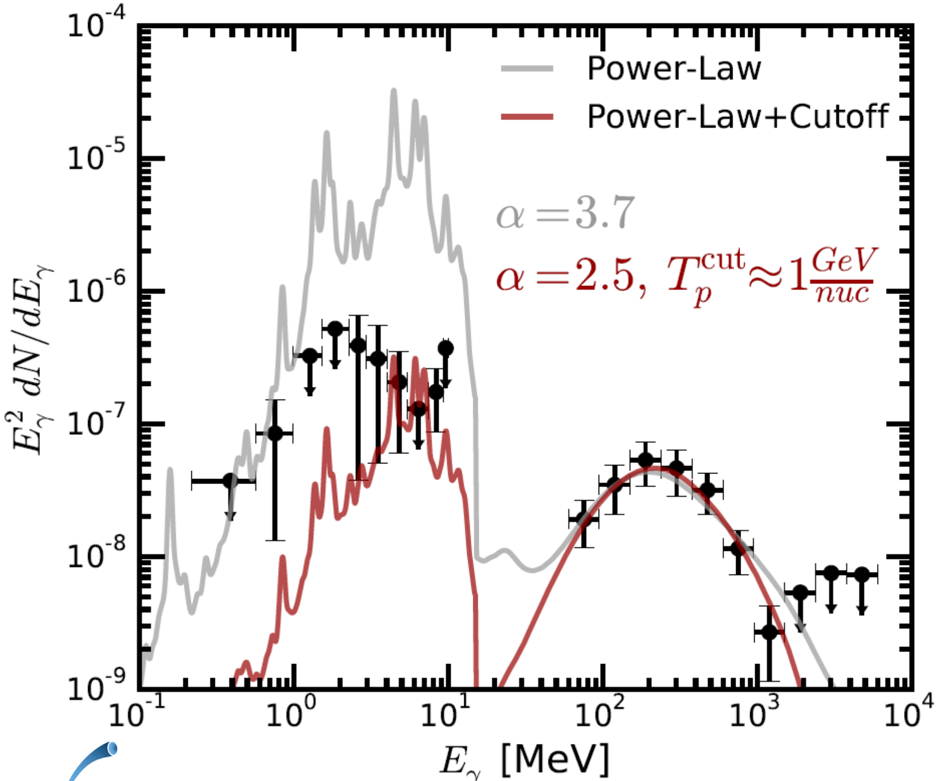
If we try to fit high energy cut-off, strong degeneracy exists with the spectral index

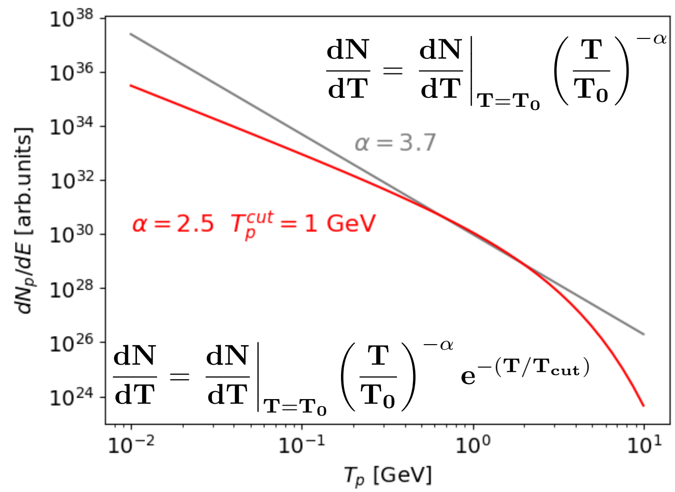
Taylor

DESY. Fermi Gamma-ray Space Telescope

Constraining the Particle Spectra in Solar flares

- Optimal level of statistics (bright low energy transients, plenty of photons)
- Retrieve the primary particle spectrum (using the most up-to-date cross sections)





This degeneracy can be broken by the lower energy emission detected by GBM, which nuclear de-excitation is expected to contribute/dominate

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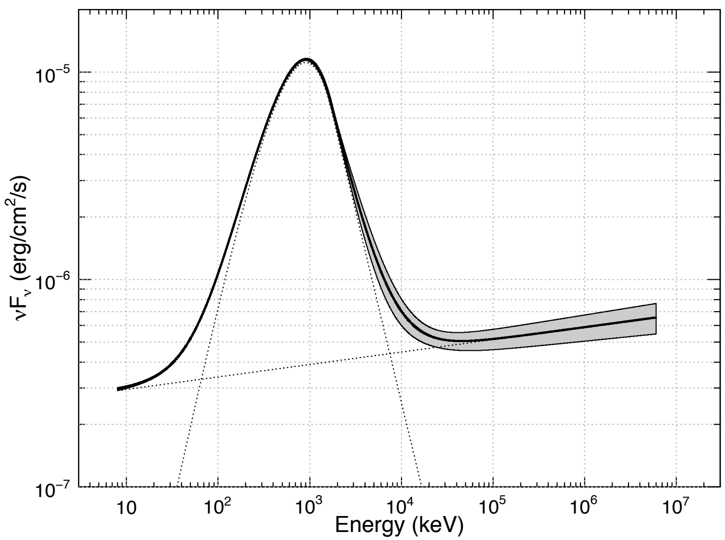
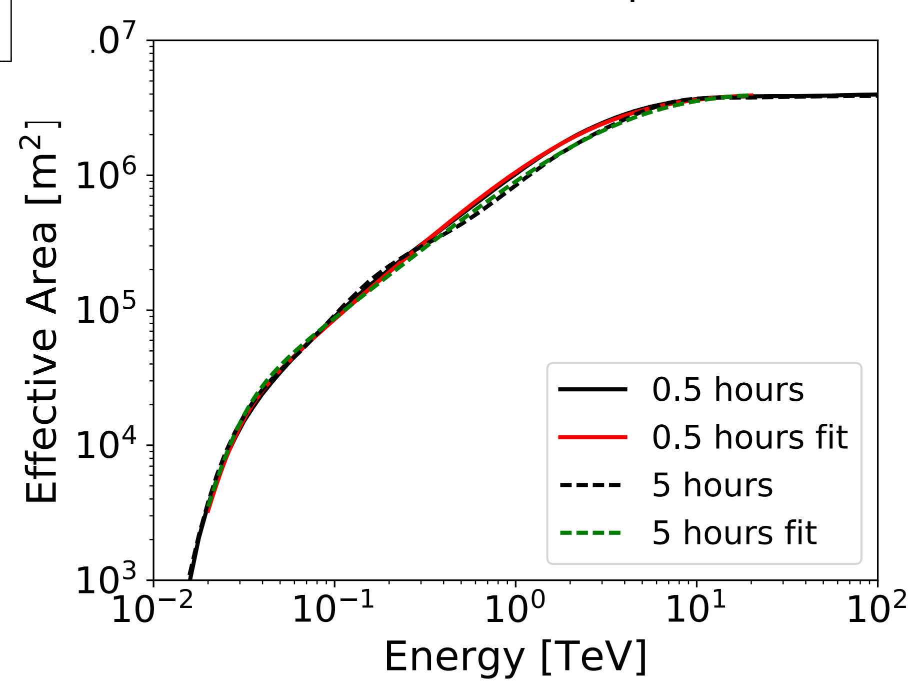
Future Sources to be Probed…..GRB
CTA (South)



- Evolution of spectra during flare
- Detection of as yet undetected VHE transients (eg. GRB)
- Detection of unexpected new VHE transient phenomena



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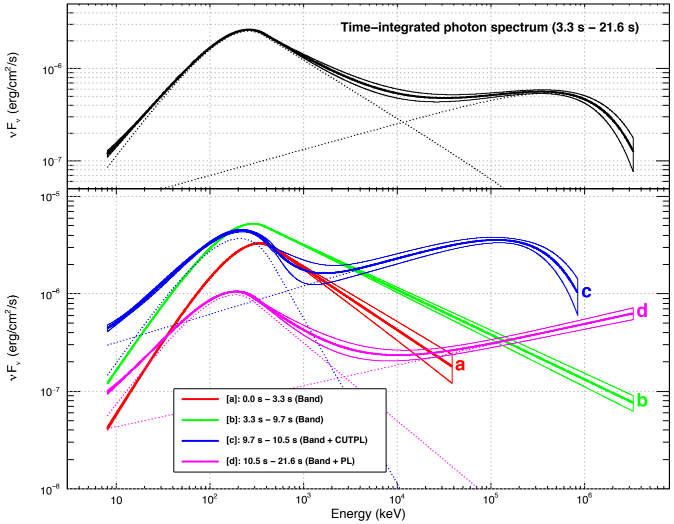
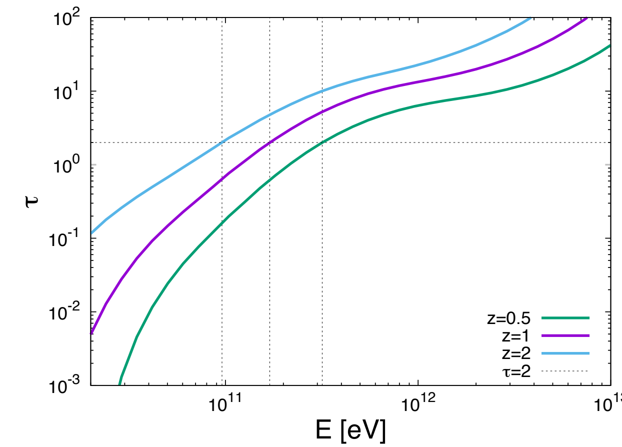
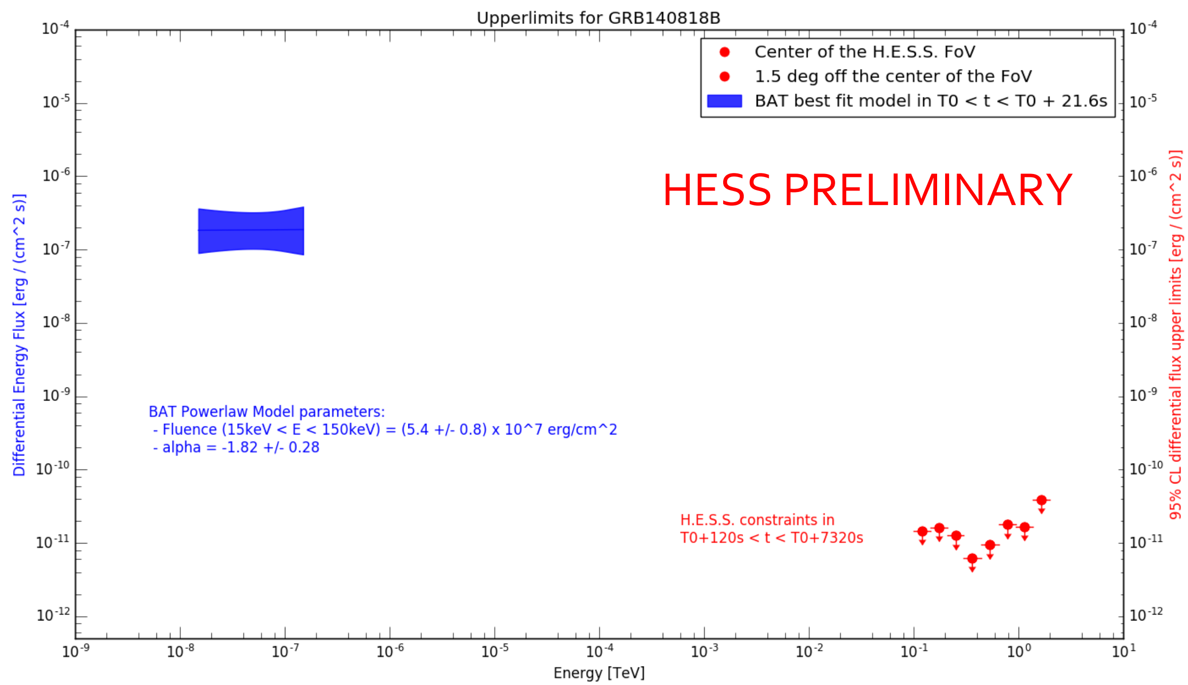
Fermi
Gamma-ray Space Telescope

Recent HESSII GRB Upper Limits

From Ackermann et al. 2011



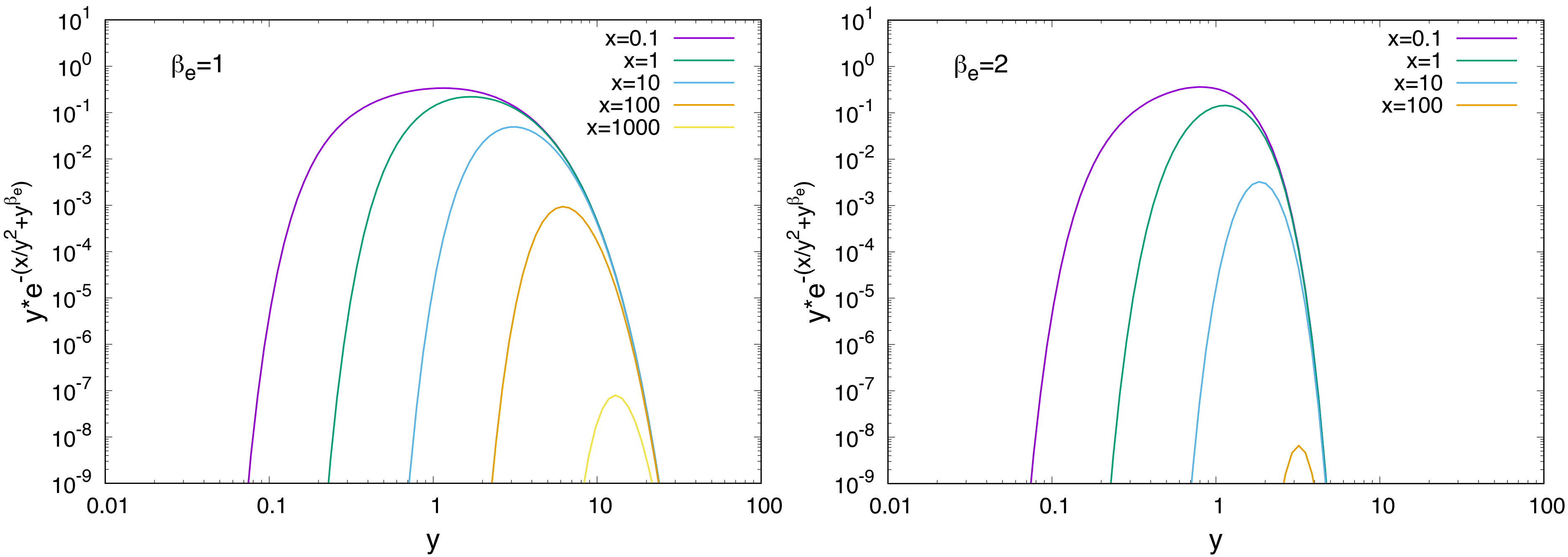




DE Fermi Gamma-ray Space Telescope

H.E.S.S.

Integrand



$$\mathbf{y^2}\left(\mathbf{y}^{\beta_\mathbf{e}} - \frac{\mathbf{1}}{\beta_\mathbf{e}}\right) = \frac{\mathbf{2x}}{\beta_\mathbf{e}}$$

$$\mathbf{y^2} \approx \left(\frac{\mathbf{2x}}{\beta_\mathbf{e}}\right)^{\frac{\mathbf{2}}{\beta_\mathbf{e}+\mathbf{2}}}$$

$$\frac{\mathbf{x}}{\mathbf{y^2}} \approx \mathbf{x}^{\frac{\beta_\mathbf{e}}{\beta_\mathbf{e}+\mathbf{2}}}$$

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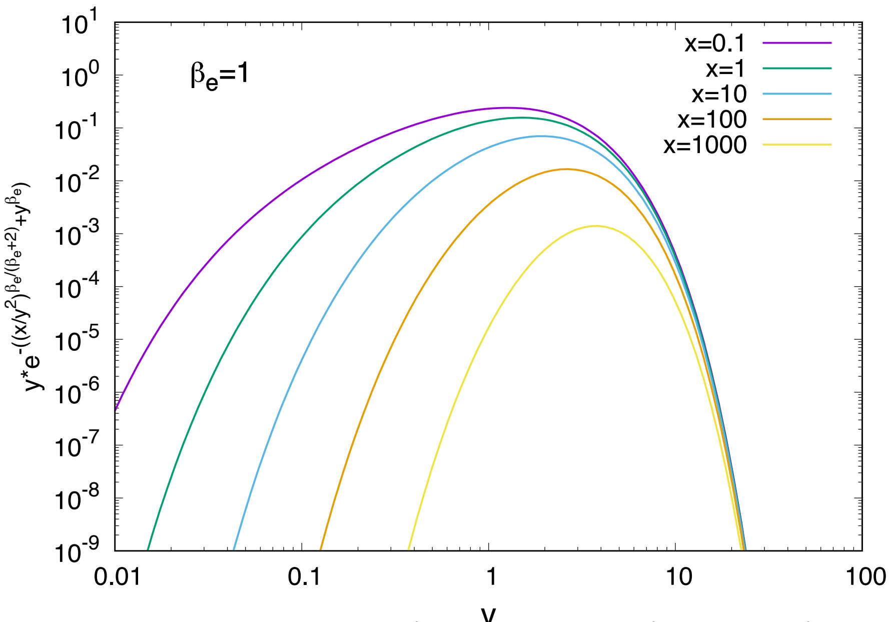
Integrand



$$\mathbf{y}^{\frac{2\beta_e}{\beta_e+2}}\left(\mathbf{y}^{\beta_e}-\frac{1}{\beta_e}\right)=\left(\frac{2}{2+\beta_e}\right)\mathbf{x}^{\frac{\beta_e}{\beta_e+2}}$$

$$\mathbf{y}^2\approx\mathbf{x}^{\frac{2}{\beta_e+4}}$$

$$\frac{\mathbf{x}}{\mathbf{y}^2}\approx\mathbf{x}^{\frac{\beta_e+2}{\beta_e+4}}$$

$$\left(\frac{\mathbf{x}}{\mathbf{y}^2}\right)^{\frac{\beta_e}{\beta_e+2}}\approx\mathbf{x}^{\frac{\beta_e}{\beta_e+4}}$$

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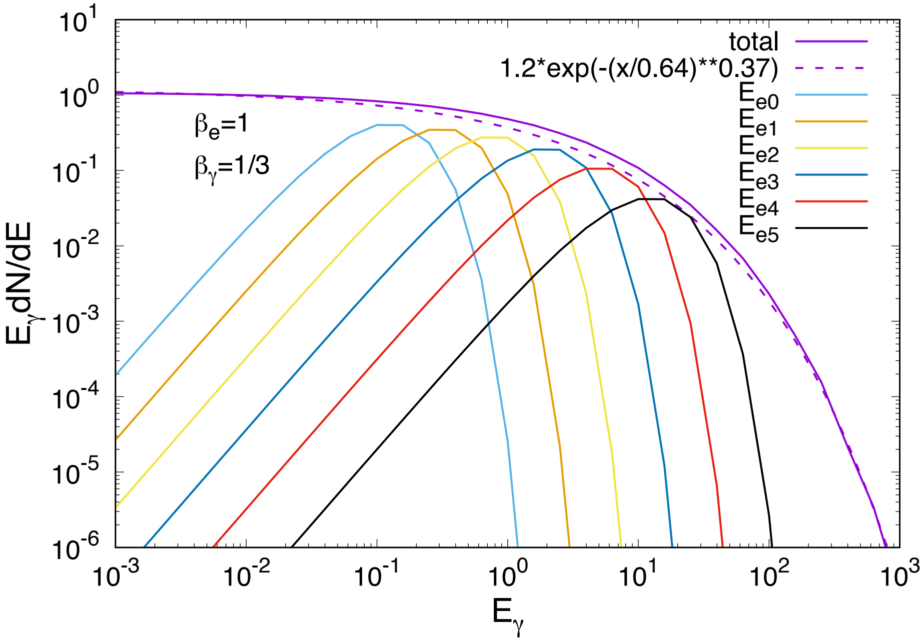

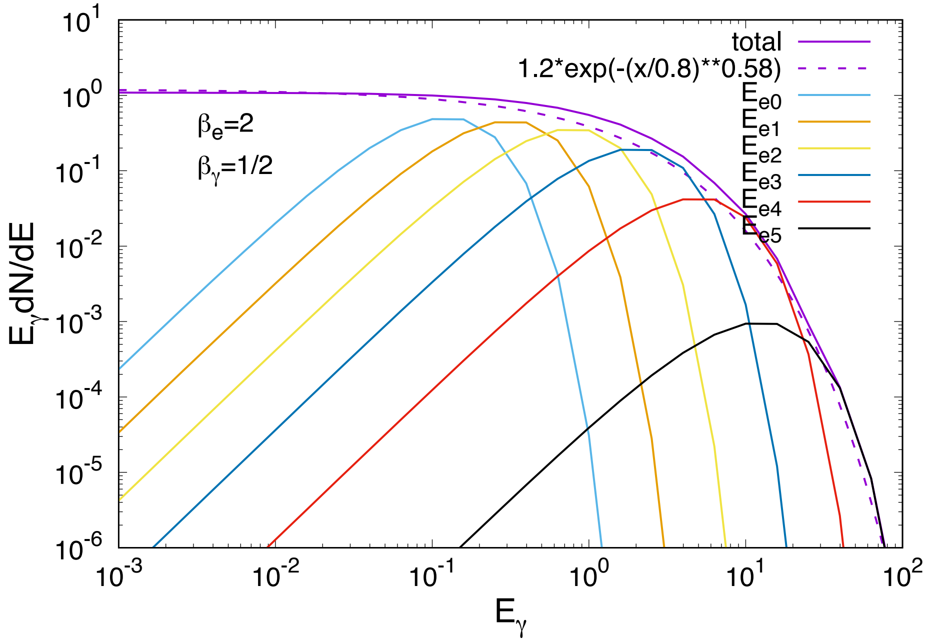

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pN Interactions

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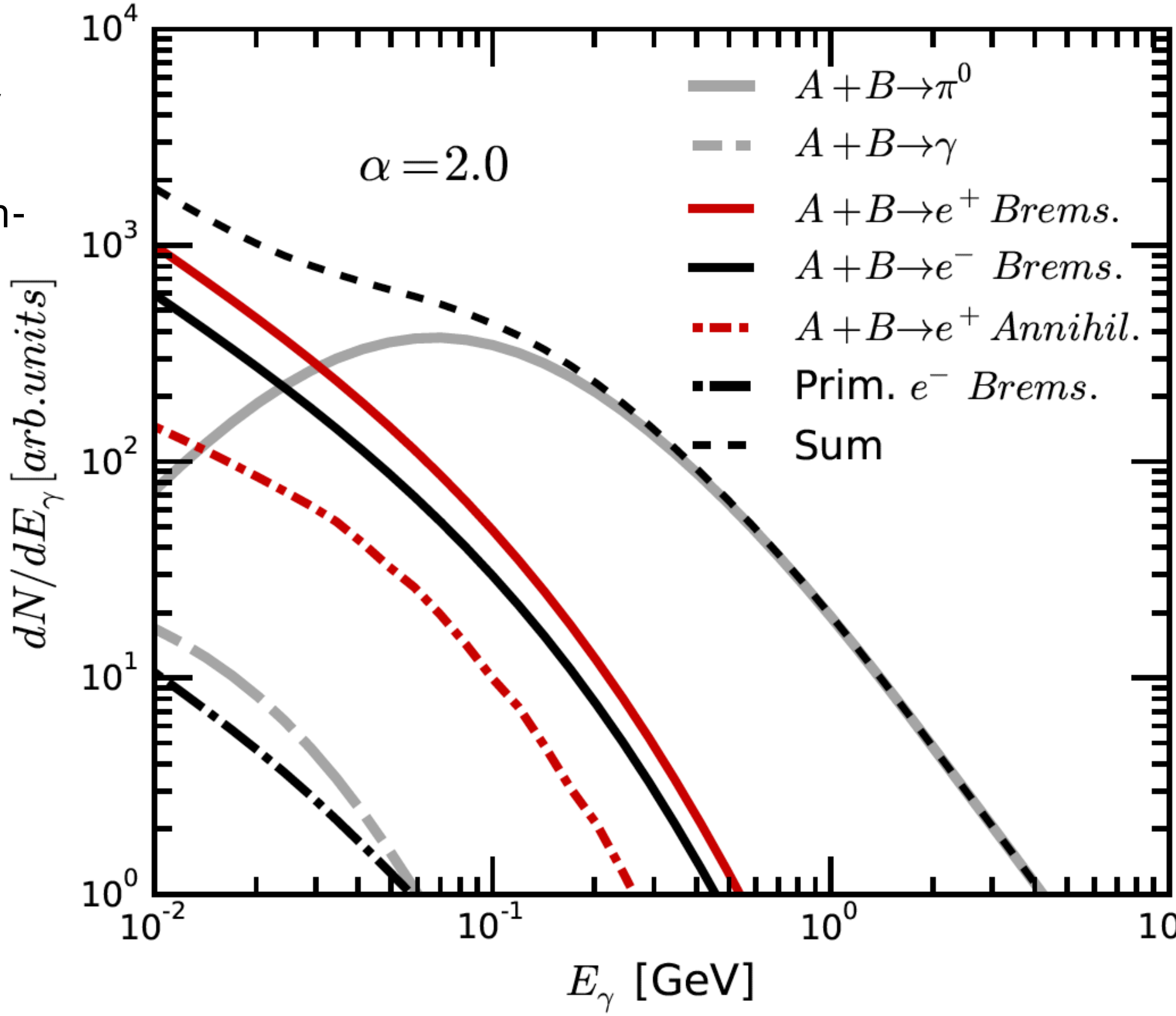
Multi-MeV Gamma-Ray Production Cross-Sections

There are also multiple channels by which multi-MeV gamma-ray emission can be produced from non-thermal **electrons**:

- Secondary Bremstrahlung
- Secondary Annihilation in flight
- Primary Bremstrahlung



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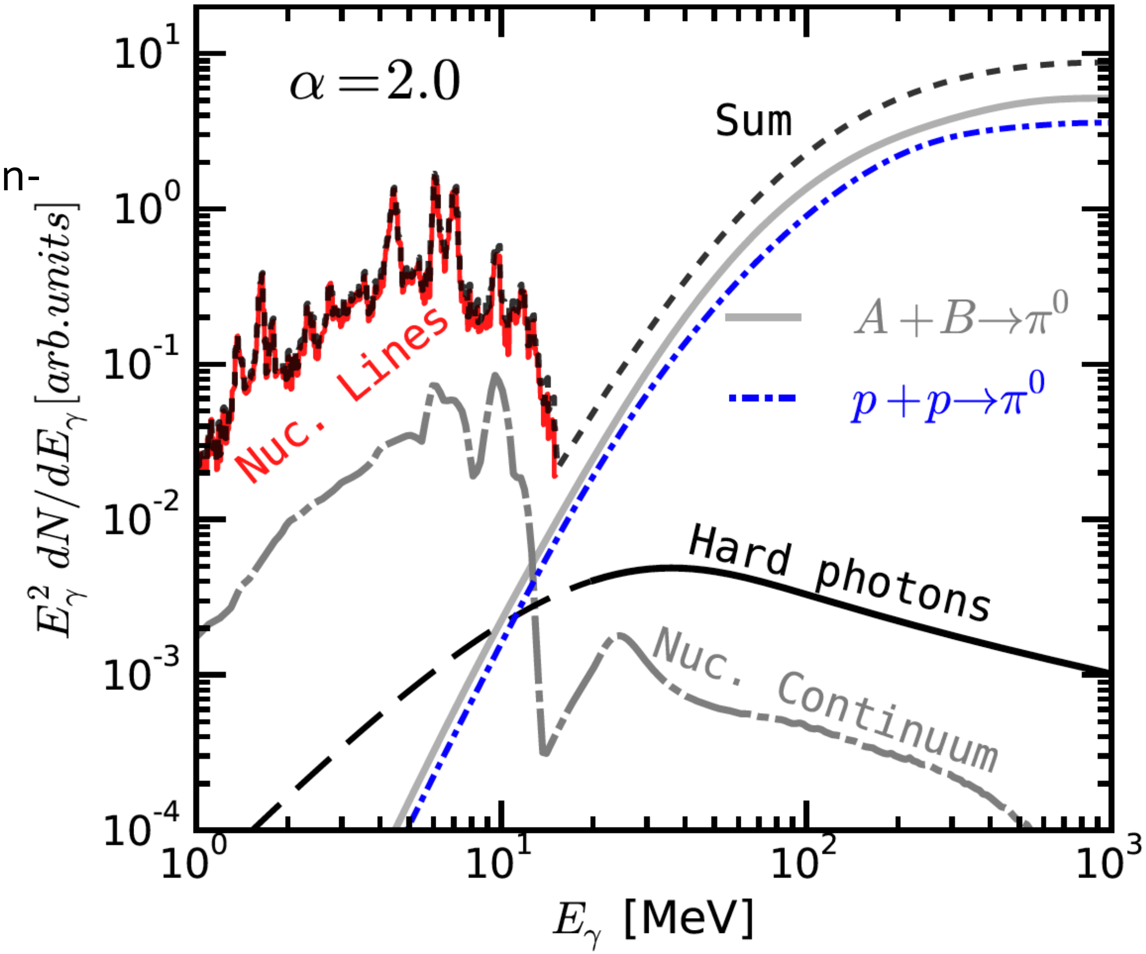
Multi-MeV Gamma-Ray Production Cross-Sections

There are multiple channels by which multi-MeV gamma-ray emission can be produced from non-thermal **protons**:

- Nuclear Line Emission
  - $a+B \rightarrow B^*$
- Nuclear Line Continuum:
  - -statistical photons
  - -direct photons
  - -pre-equilibrium processes
- Hard Photon Emission
  - (nuclear Bremstrahlung)



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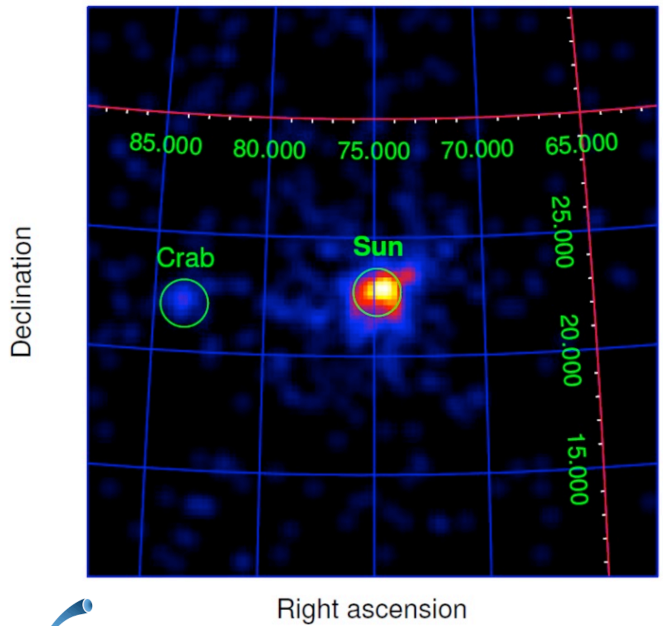
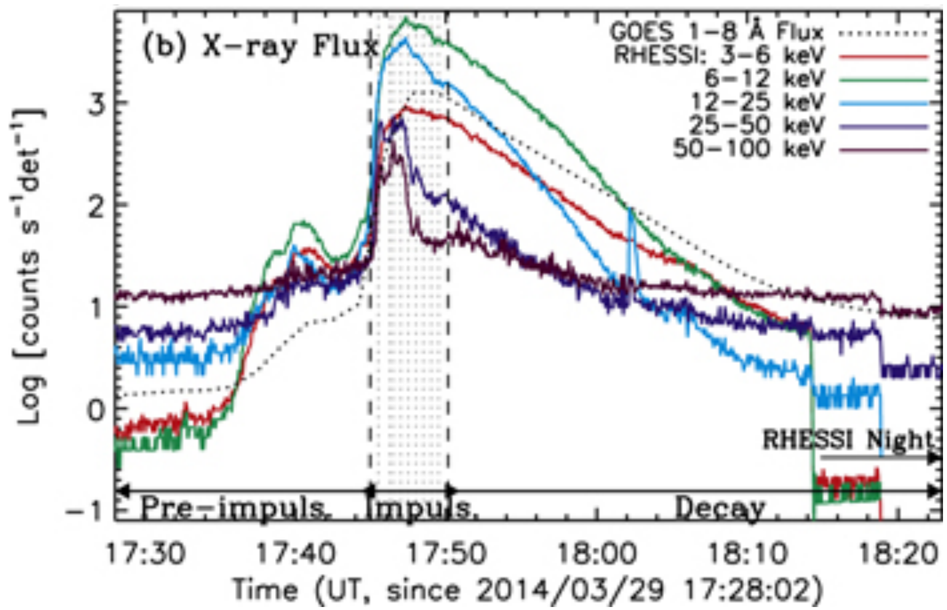
Other Bright Sources Seen By Fermi



- Gamma ray emission during flaring events
  - Most probable scenario, magnetic reconnection in Solar Corona



Emission of gamma rays!

Most important channels:

- De-excitation of atomic nuclei (low energy)
- Decay of neutral pions $\pi_0 \rightarrow \gamma\gamma$ (high energy)

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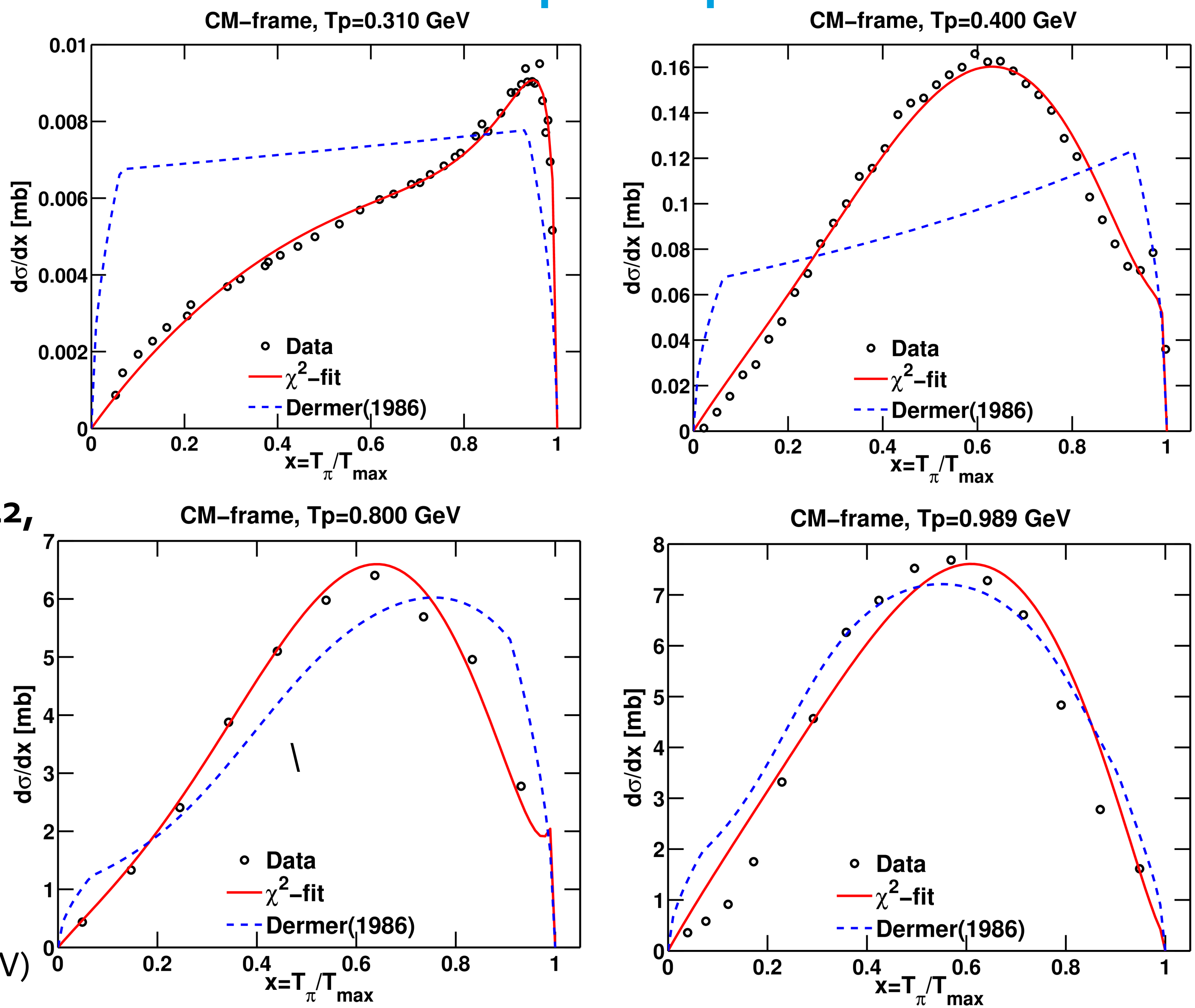
π Spectra for $T_p^{th}<T_p<1$ GeV



Kafexhiu et al., **Phys.Rev. D90 12, 123014 (2014)**

Note- Kamae description has artificially high threshold (~0.5 GeV)

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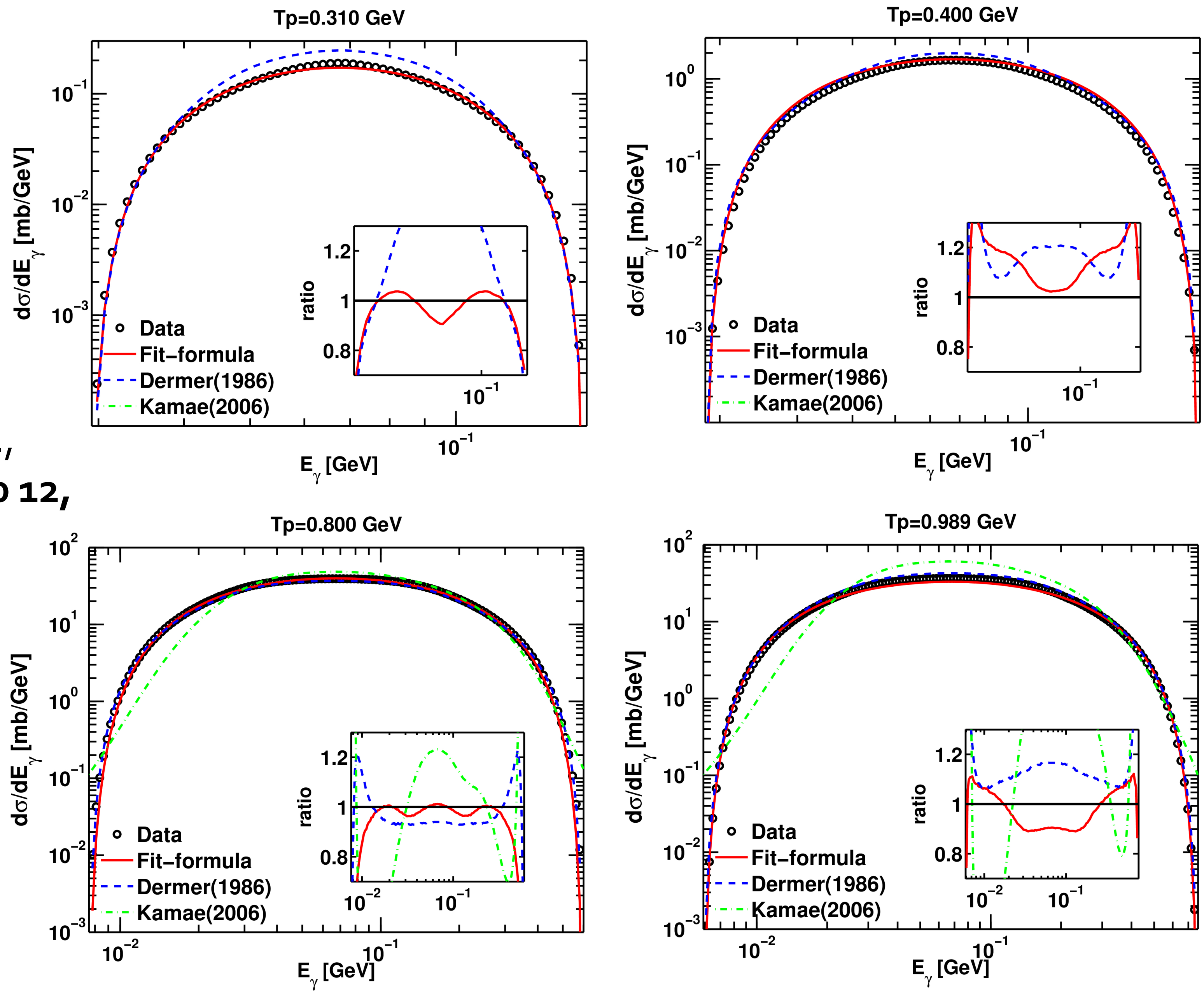
γ-ray Spectra for $T_p^{th} < T_p < 1$ GeV



Kafexhiu et al.,
Phys.Rev. D90 12, 123014 (2014)

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Stochastic Particle Acceleration- Random Walk Result (Spatial)

$$\nabla \cdot (\mathbf{D_{xx}} \nabla \mathbf{f}) = \delta(\mathbf{r})$$

Spherically symmetric case:

$$\frac{\mathbf{1}}{\mathbf{r^2}} \frac{\partial}{\partial \mathbf{r}} \left(\mathbf{r^2} \frac{\partial}{\partial \mathbf{r}} \mathbf{f} \right) = \delta(\mathbf{r})$$

$$\mathbf{u} = \mathbf{rf}$$

$$\frac{\mathbf{1}}{\mathbf{r}} \frac{\partial^{\mathbf{2}} \mathbf{u}}{\partial \mathbf{r^2}} = \delta(\mathbf{r})$$

Stochastic Particle Acceleration- Random Walk Result (Spatial)

$$\frac{\mathbf{1}}{\mathbf{r}}\frac{\partial^{\mathbf{2}}\mathbf{u}}{\partial\mathbf{r}^{\mathbf{2}}} = \delta(\mathbf{r})$$

$$\mathbf{u} = \mathbf{Ar} + \mathbf{B}$$

$$\mathbf{f} = \mathbf{A} + \frac{\mathbf{B}}{\mathbf{r}}$$

Radiative Loss Timescale

- Relativistic particle will loose its energy on a timescale that depends of the different processes

$$\tau_{\rm cool}(\mathbf{E}) \propto \mathbf{E}^{\mathbf{r}}$$

Synchrotron:
r = -1

Inverse Compton *(Thomson)*:
r = -1

Inverse Compton (K.N):
r = 1

Andrew Taylor

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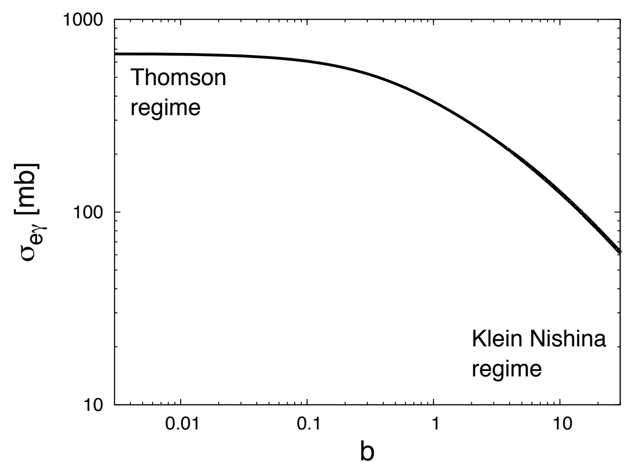

1.

$$\mathbf{E}_\gamma \approx \gamma_{\mathbf{e}}^{\mathbf{2}} \left(\frac{\mathbf{B}}{\mathbf{B}_{\rm crit}}\right) \mathbf{m}_{\mathbf{e}} = \mathbf{b}\mathbf{E}_{\mathbf{e}}$$

$$\mathbf{E}_\gamma = \left(\frac{\mathbf{b}}{\mathbf{1}+\mathbf{b}}\right) \mathbf{E}_{\mathbf{e}}$$

2. *N* Taylor

DESY.