Lecture (2) Plan:

- **UHECR- the observational status**
- **UHECR transport**
- **Hydro Turbulence and Magneto-Hydro Turbulence**
- **Non-thermal particle transport equation in magnetic turbulence**
- **The extragalactic magnetic field environment**

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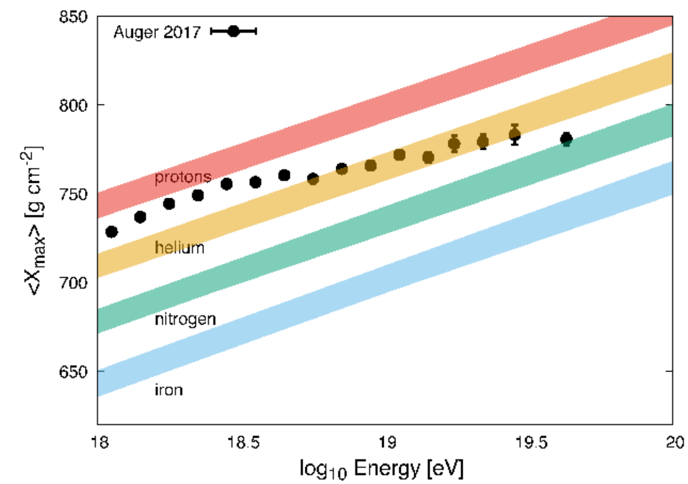
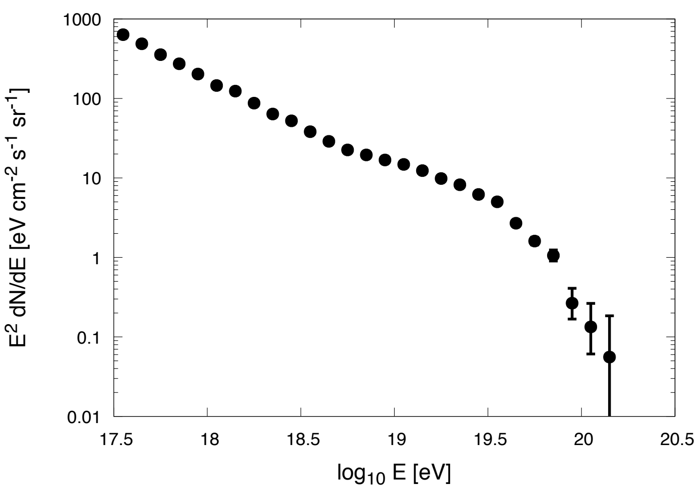
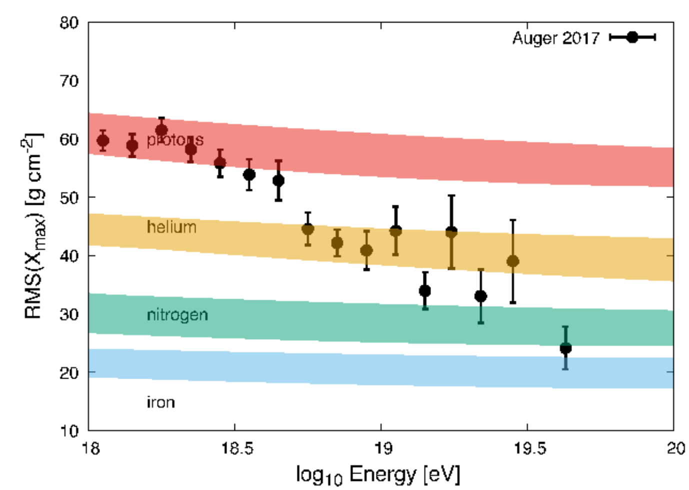
UHECR: The Observational Status

Composition

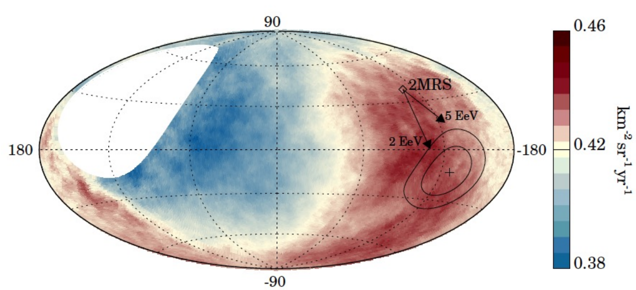


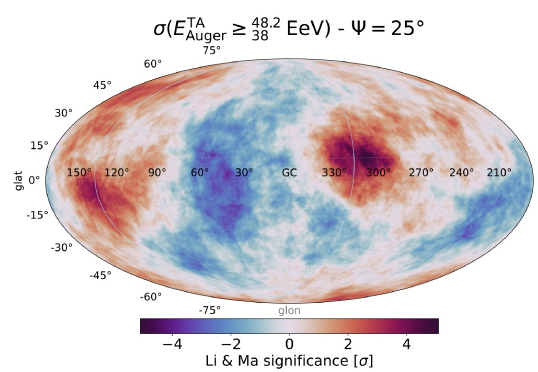
Spectrum





Anisotropy





Pierre Auger Collaboration. ApJ. 935 (2022)

Caccianiga et al. for the Auger and TA Collaborations. PoS (ICRC2023) 521

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Hydro Turbulence

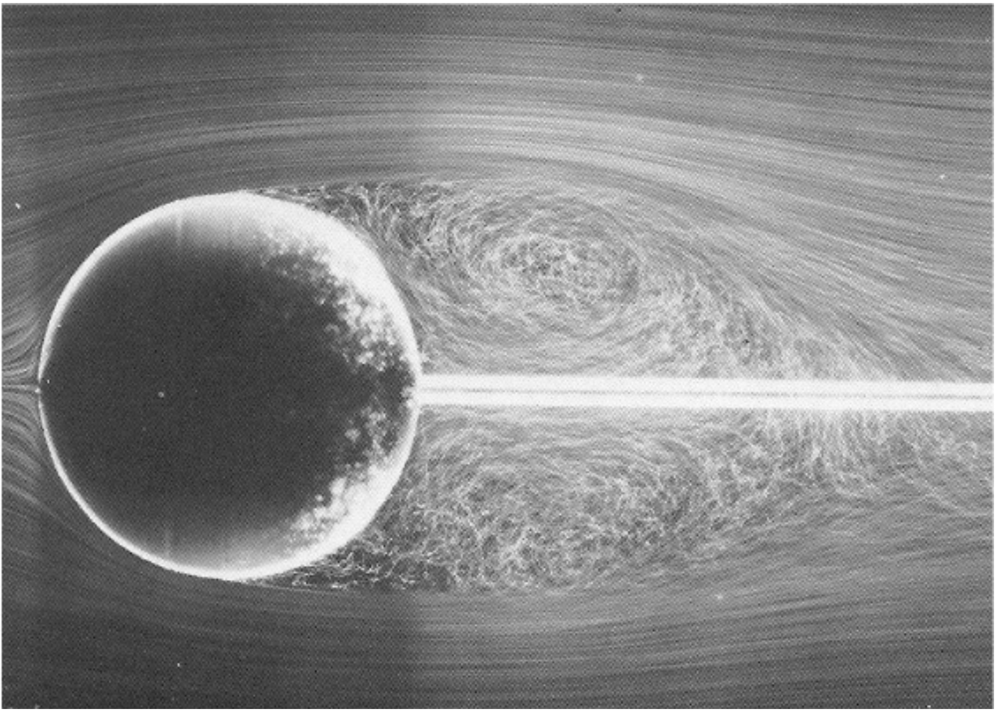
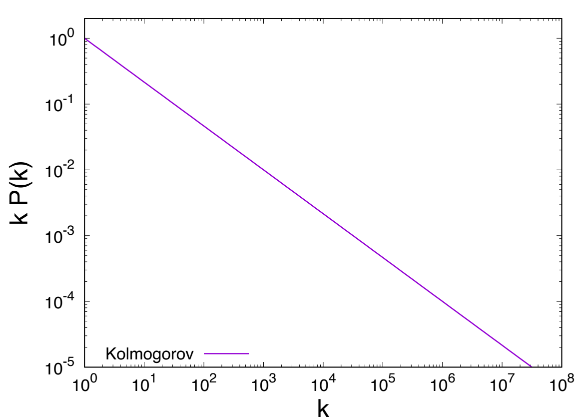
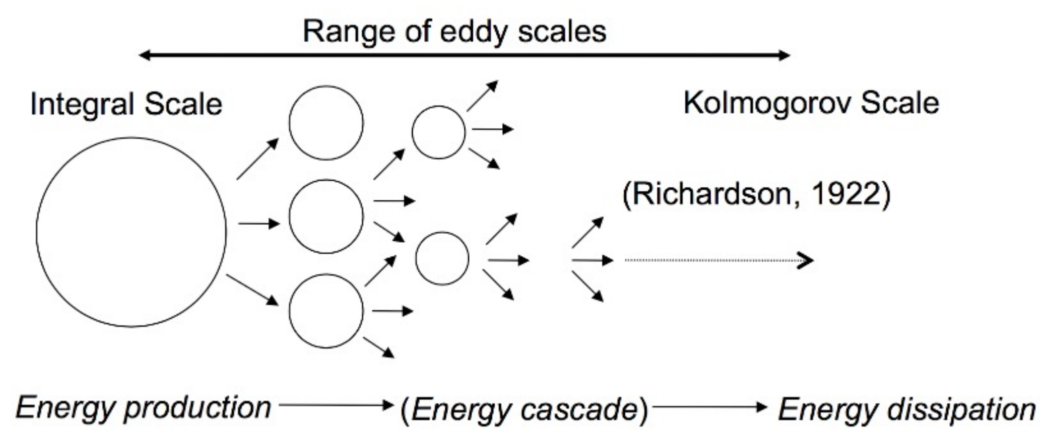


Image from University of Sydney

Richardson, 1922

> *Big whorls have little whorls*
> *That feed on their velocity;*
> *And little whorls have lesser whorls*
> *And so on to viscosity.*





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Hydrodynamics

A brief comment-

$$\frac{\partial \rho \mathbf{v}}{\partial \mathbf{t}} + \nabla \cdot \mathbf{P} = \rho \mathbf{g}$$

Momentum flux conservation

$$\mathbf{P} = \mathbf{pI} + \rho \mathbf{vv}$$

Spatial part of stress energy tensor

$$\rho \frac{\partial \mathbf{v}}{\partial \mathbf{t}} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla \mathbf{p} + \rho \mathbf{g}$$

Magneto-Hydrodynamics

A brief comment-

$$\frac{\partial \rho \mathbf{v}}{\partial \mathbf{t}} + \nabla \cdot (\mathbf{P} - \mathbf{P}_{\mathrm{M}}) = \rho \mathbf{g}$$

Momentum flux conservation

$$\mathbf{P} = \mathbf{pI} + \rho \mathbf{vv}$$

$$\mathbf{P}_{\mathrm{M}} = -\frac{\mathbf{B^2}}{\mathbf{8}\pi}\mathbf{I} + \frac{\mathbf{BB}}{\mathbf{4}\pi}$$

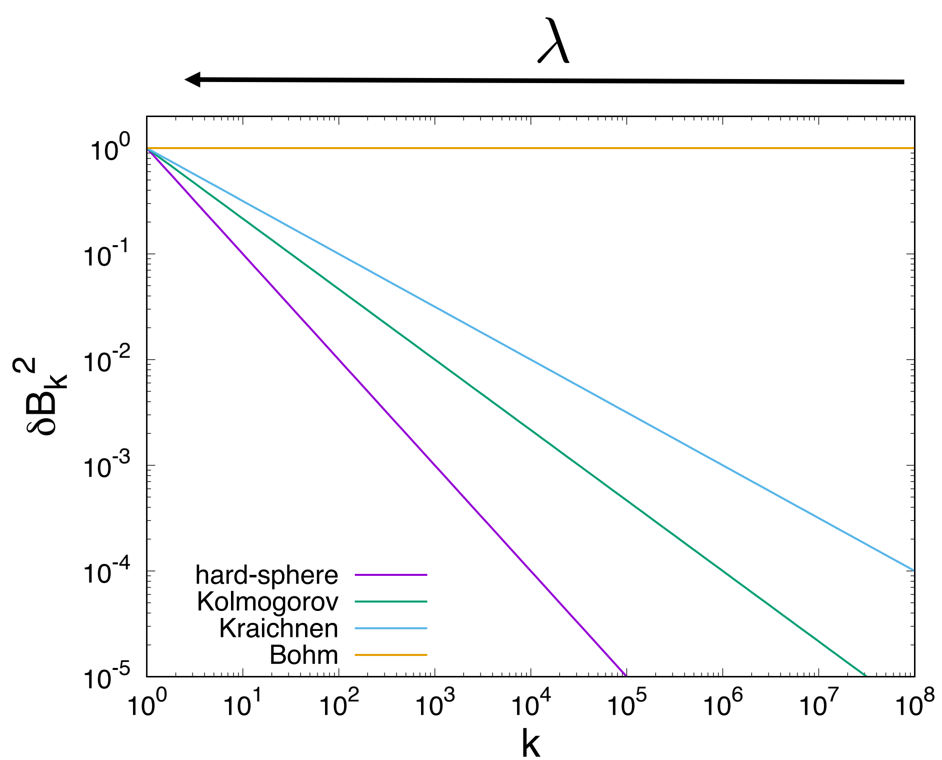
Maxwell stress tensor

Galactic Magneto-Hydro Turbulence

One of the key drivers is thought to be Supernova explosions

$$\delta \mathbf{B}^2 = \int \frac{\mathbf{d}(\delta \mathbf{B}^2)}{\mathbf{d} \ln \mathbf{k}} \mathbf{d} \ln \mathbf{k} = \int \delta \mathbf{B}_\mathbf{k}^2 \mathbf{d} \ln \mathbf{k}$$

$$\delta \mathbf{B}_\mathbf{k}^2 = \delta \mathbf{B}_\mathbf{0}^2 \left(\frac{\mathbf{k}}{\mathbf{k_0}} \right)^{\mathbf{1}-\alpha}$$



Note for MHD turbulence, the theoretically expected turbulence index is still debated

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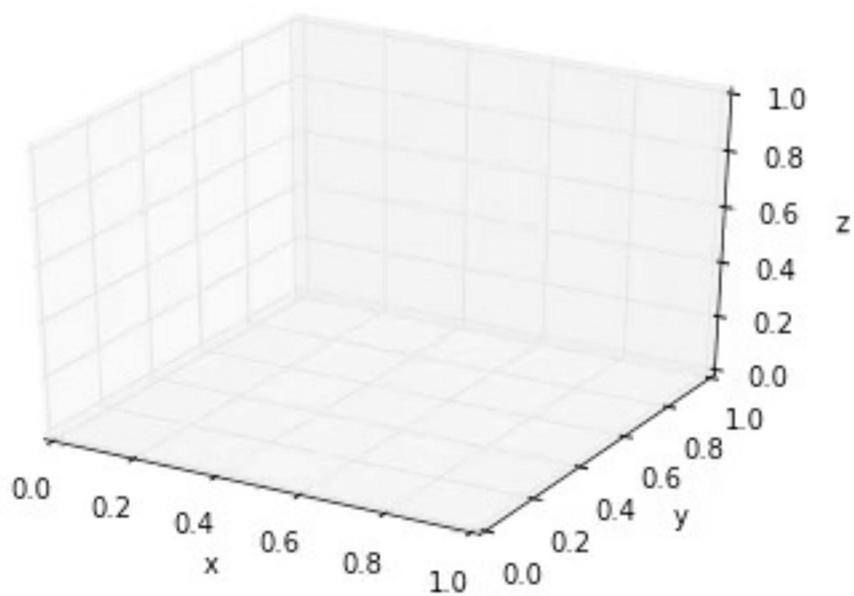
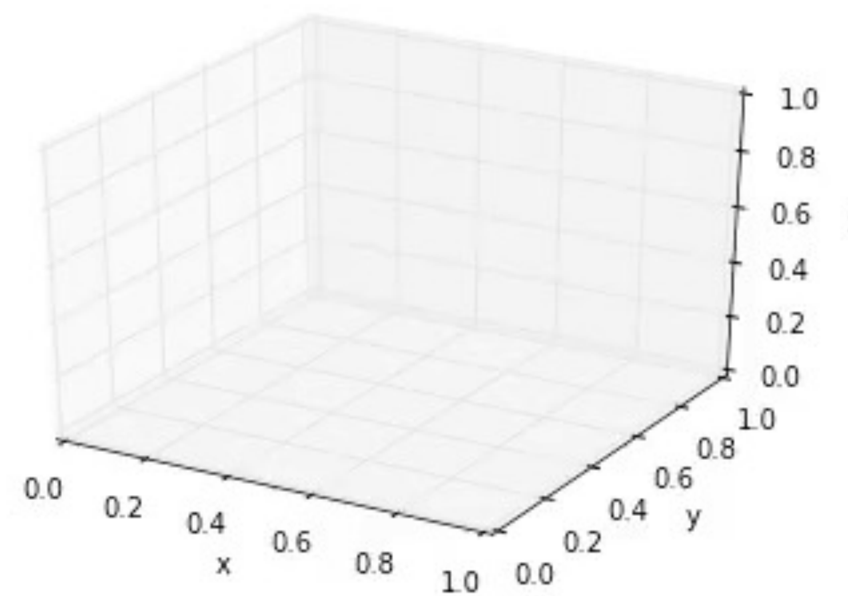
Charged Particles in Magnetic Fields

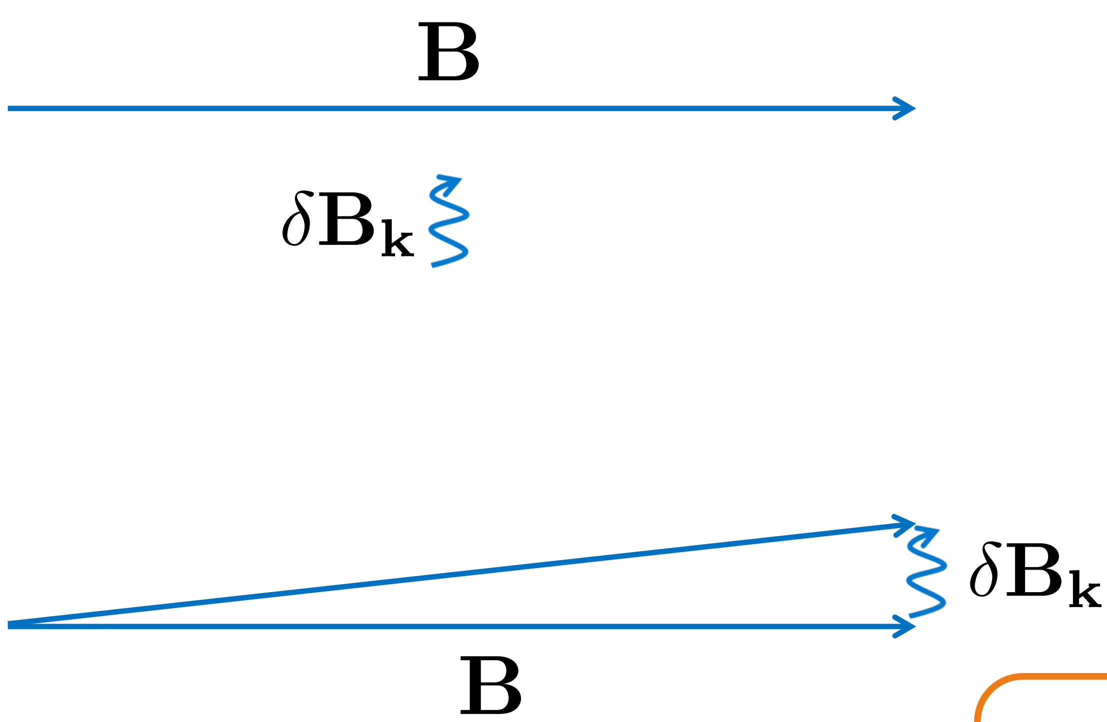
Note- a lot of what you **may have** studied about charged particle propagation in magnetic fields **likely** assumed magnetic field variation was on much longer length scales than particle Larmor radius.





Particle Diffusion in Magnetic Turbulence (Quasi-Linear Theory)?

The propagation of cosmic rays is dictated by the magnetic field landscape they live in.



$$\delta\theta = \frac{\delta \mathbf{B_k}}{\mathbf{B}}$$

$$\langle \boldsymbol{\Delta}\theta^2 \rangle = \mathbf{N} \langle \delta\theta^2 \rangle$$

$$= \left(\frac{\mathbf{t}}{\mathbf{t_{lar}}} \right) \langle \delta\theta^2 \rangle$$

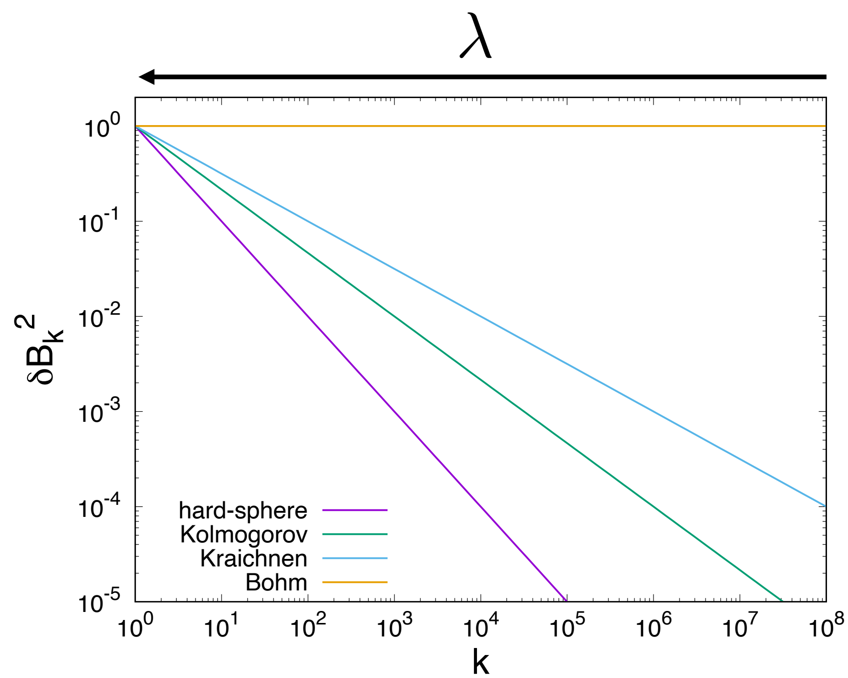
$$\mathbf{D}_{\theta\theta} = \frac{\boldsymbol{\Delta}\theta^2}{\mathbf{t}} = \frac{\mathbf{1}}{\mathbf{t_{lar}}} \left(\frac{\delta \mathbf{B_k^2}}{\mathbf{B^2}} \right)$$

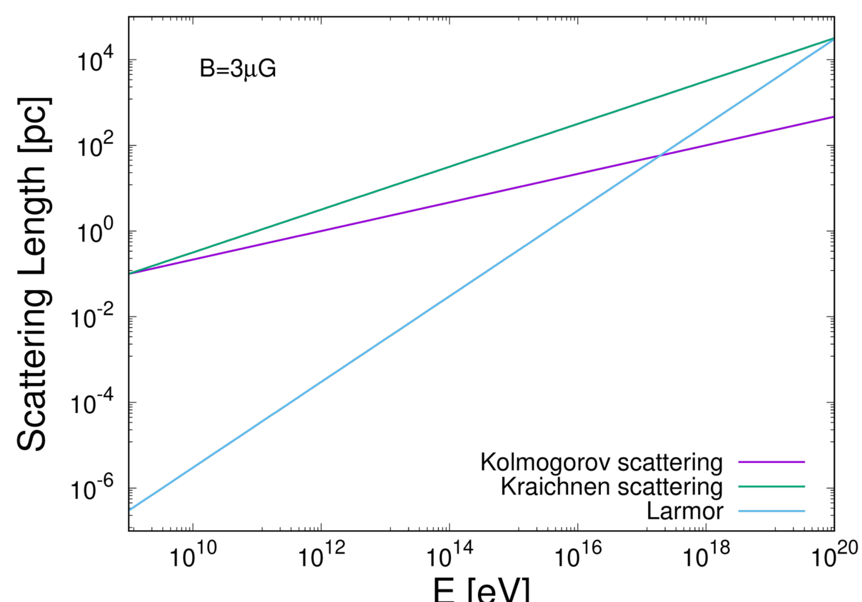
Propagation through Magnetic Fields

$$t_{scat} \approx \frac{1}{D_{\theta\theta}}$$

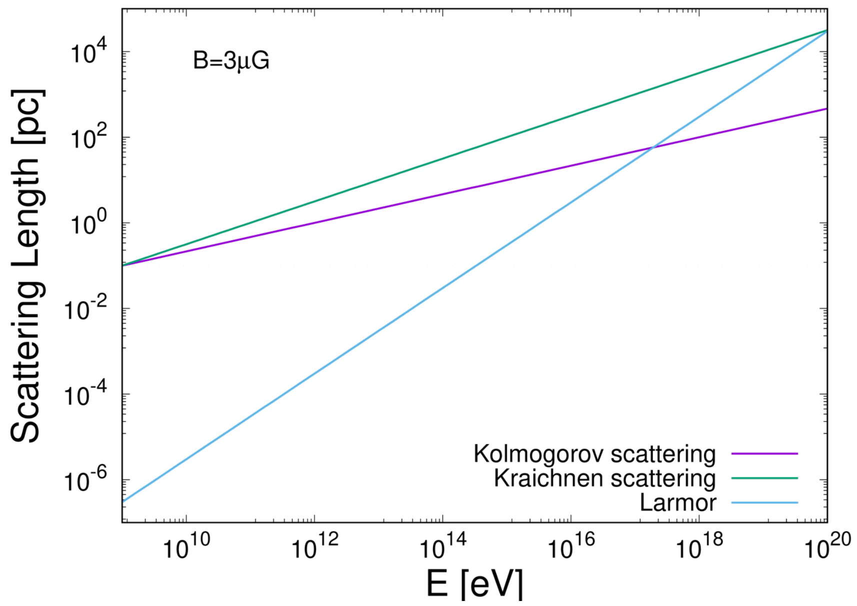
$$\frac{D_{xx}}{c} \approx l_{scat}$$

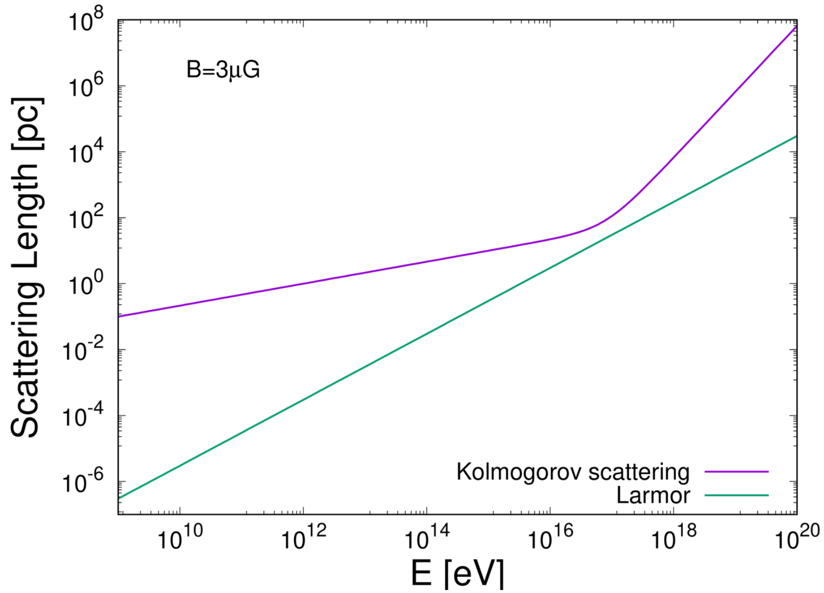
$$\frac{D_{xx}}{c} \approx R_{lar} \left(\frac{B^2}{\delta B_k^2} \right)$$





Propagation through Magnetic Fields





Transport (Continuity) Equation

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} + \nabla_{\mathbf{x}} \cdot \mathbf{j} = \mathbf{Q}$$

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \nabla_{\mathbf{x}} \cdot (\mathbf{D}_{\mathbf{xx}} \nabla_{\mathbf{x}} \mathbf{f}) + \mathbf{Q}$$

$$\mathbf{j} = -\mathbf{D}_{\mathbf{xx}} \nabla_{\mathbf{x}} \mathbf{f}$$

Charged Particle Motion in Turbulent Magnetic Fields

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \nabla_{\mathbf{x}} \cdot (\mathbf{D}_{\mathbf{xx}} \nabla_{\mathbf{x}} \mathbf{f}) + \mathbf{Q}$$

Diffusion

Source term

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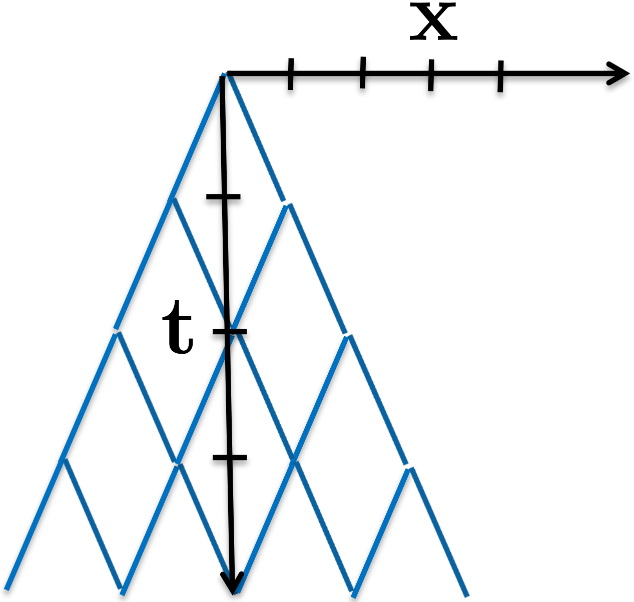
Random Walks



$$\mathbf{f(x,t)} = \frac{\mathbf{t!}}{([\mathbf{t}-\mathbf{x}]/2)!([\mathbf{x}+\mathbf{t}]/2)!(2^{\mathbf{t}})}$$

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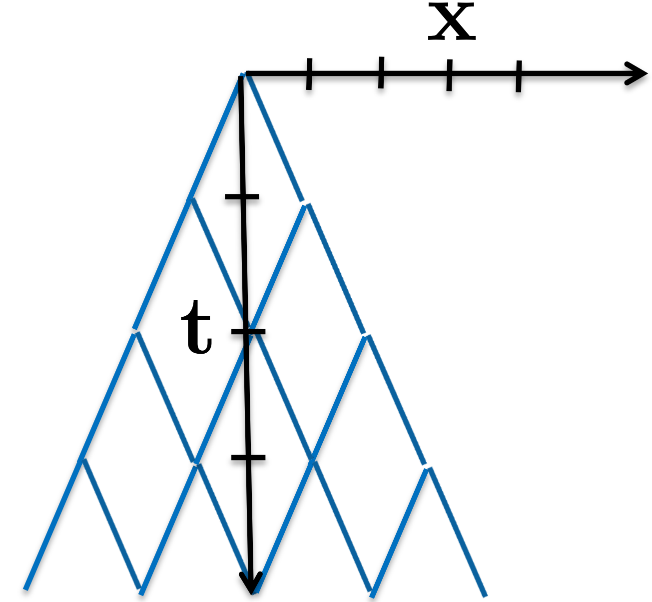
Random Walks



$$\gamma(\mathbf{t}+\mathbf{1}) = \mathbf{t}!$$

$$\gamma(\mathbf{t}+\mathbf{1}) = \int_{\mathbf{0}}^{\infty} \mathbf{x}^{\mathbf{t}} \mathbf{e}^{-\mathbf{x}} \mathbf{dx}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) = \frac{\gamma(\mathbf{t}+\mathbf{1})}{[\gamma([\mathbf{t}-\mathbf{x}]/\mathbf{2}+\mathbf{1})\gamma([\mathbf{x}+\mathbf{t}]/\mathbf{2}+\mathbf{1})](\mathbf{2}^{\mathbf{t}})}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) \approx \frac{\mathbf{e}^{-\mathbf{x}^{2}/(\mathbf{2t})}}{(\mathbf{2}\pi\mathbf{t})^{\mathbf{1}/\mathbf{2}}}$$

Suggest you all have a go at demonstrating this.

Steady State Distribution Around a Source of Diffusing Particles

cosmic rays diffuse in magnetic field turbulence

Note- expressions on previous slide in dimensionless units,

$$\mathbf{t \rightarrow 2Dt}$$

$$\mathbf{f(r,t) \approx \frac{e^{-r^2/(4Dt)}}{(4\pi Dt)^{3/2}}}$$

3D Green's function

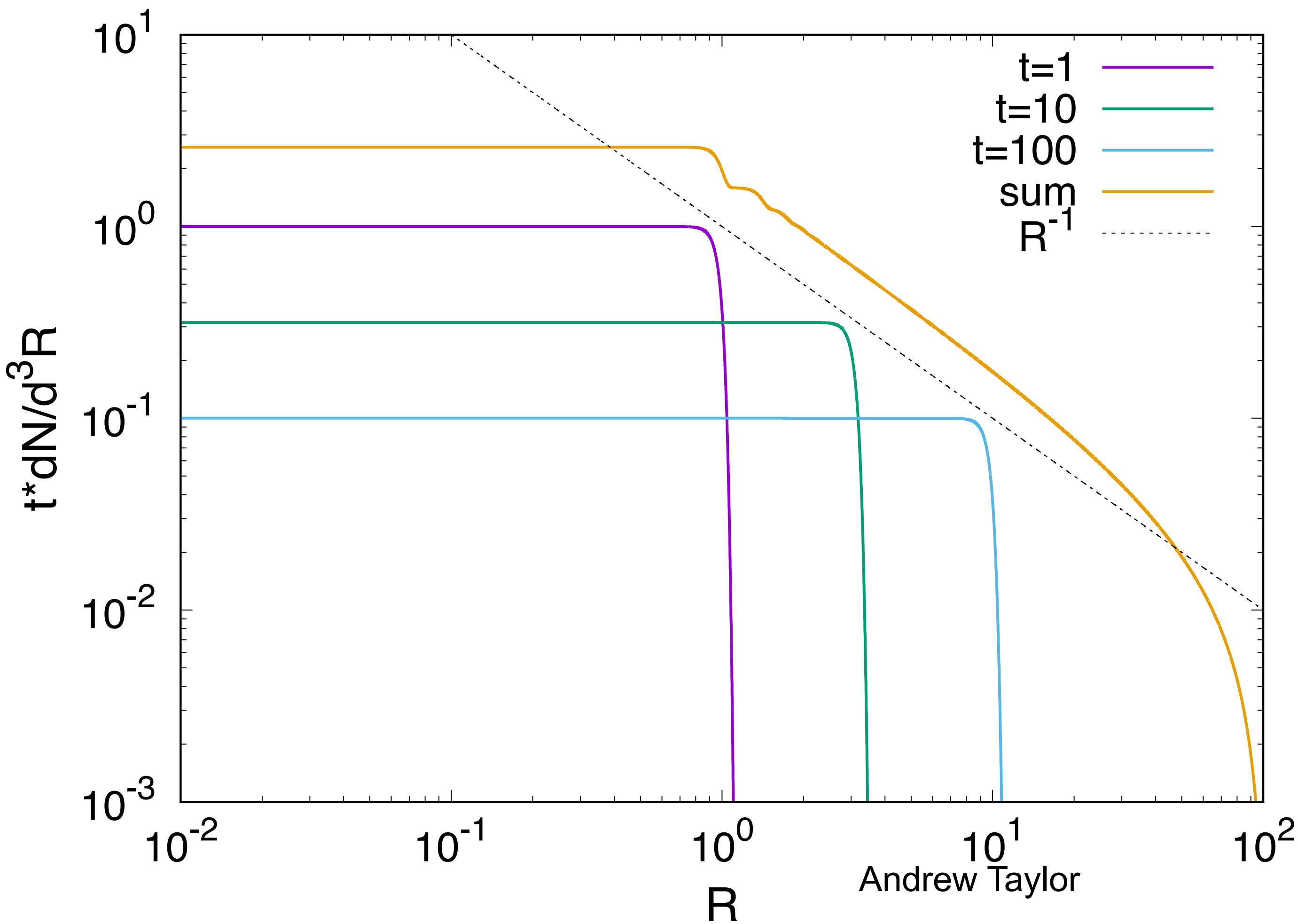
$$\mathbf{F(r) = \int_0^\infty f(r,t)dt}$$

$$\mathbf{= \frac{1}{Dr}}$$

Suggest you all have a go at demonstrating this!

Steady State Distribution Around a Source of Diffusing Particles

$$\int \mathbf{f(r,t)dt} = \int \mathbf{tf(r,t)dlnt}$$



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Smart to look at this

Spectral Effects of Magnetic Fields

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Energy Dependent Magnetic Horizon

$$\mathbf{l_{MH} = (D_{xx} t_H)^{1/2} = 60 \left(\frac{D_{xx}}{1\ Mpc}\right)^{1/2} \left(\frac{t_H}{4000\ Mpc}\right)^{1/2} Mpc}$$

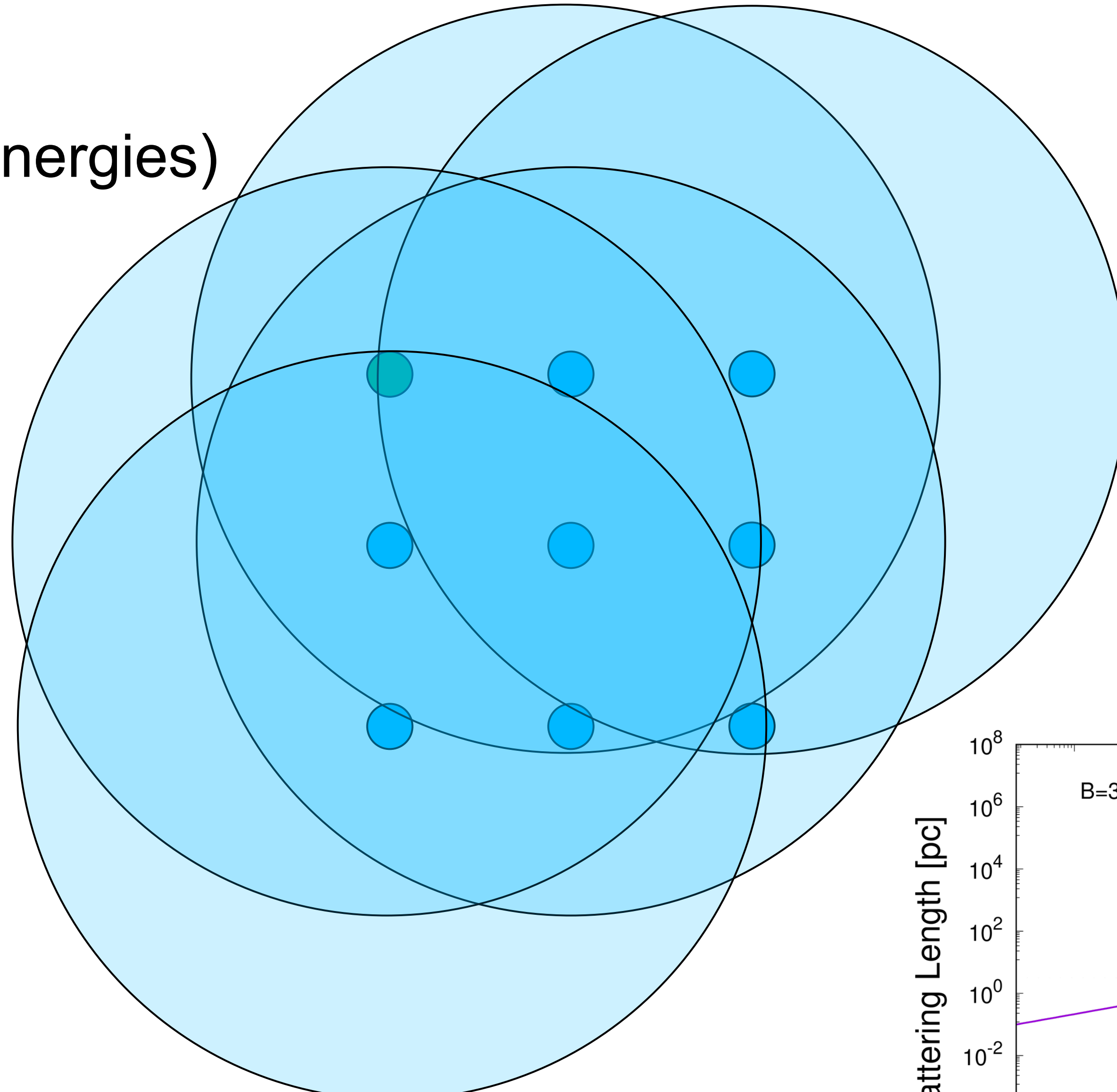
If the diffusion coefficient, D_{xx}, is energy dependent, the magnetic horizon is also energy dependent.

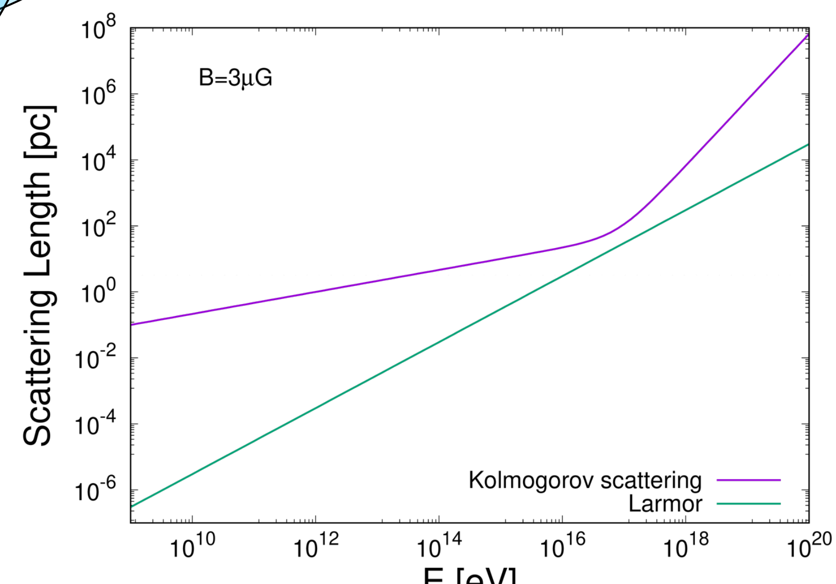
Extragalactic cosmic rays cannot arrive to the Milky Way at low energies within a Hubble time!

Aloisio, R. +, ApJ 612 (2004)

Magnetic Horizon Effect

(medium energies)





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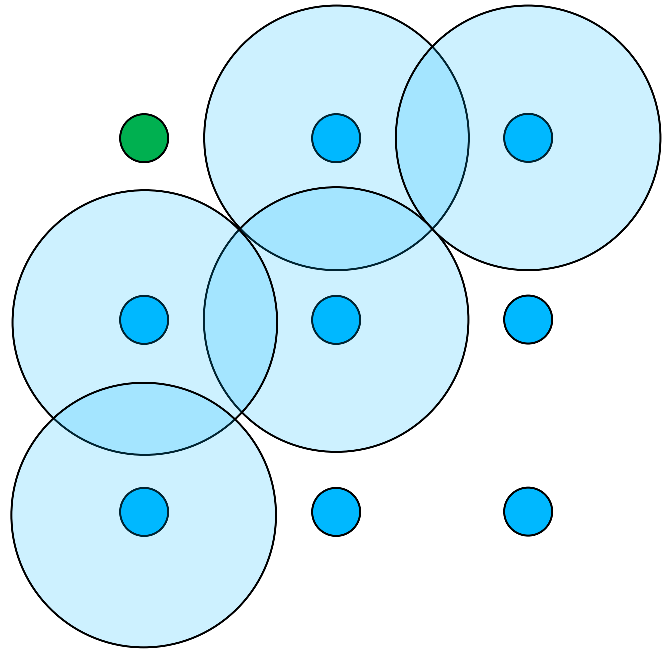
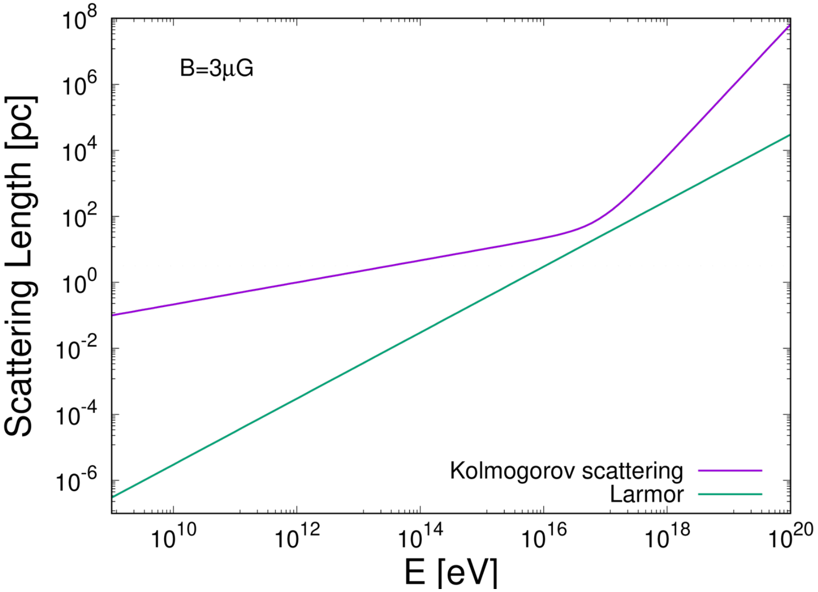
Magnetic Horizon Effect

$$l_{MH} = (D_{xx} c t_H)^{1/2}$$

(low energies)

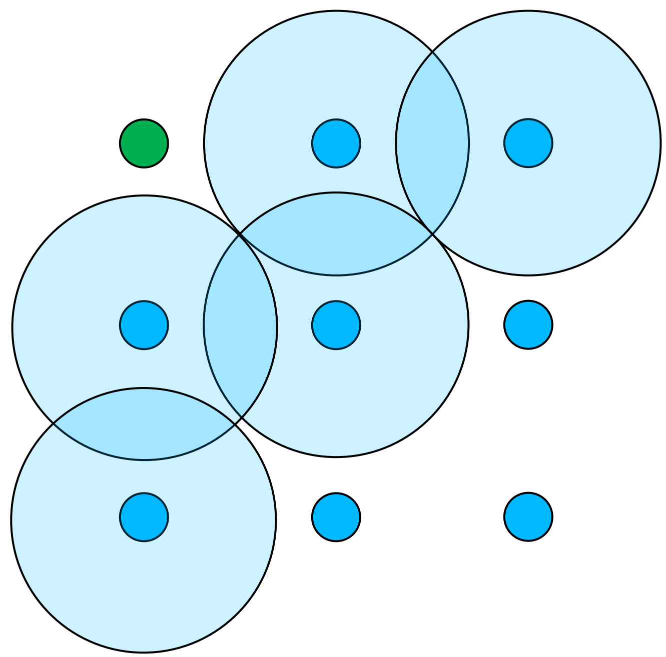




Once l_{MH} becomes smaller than r_s cosmic rays from the nearest sources become suppressed

Energy Loss Horizon

(high energies)



Propagation through Extragalactic Magnetic Fields

3 Phases of Propagation:

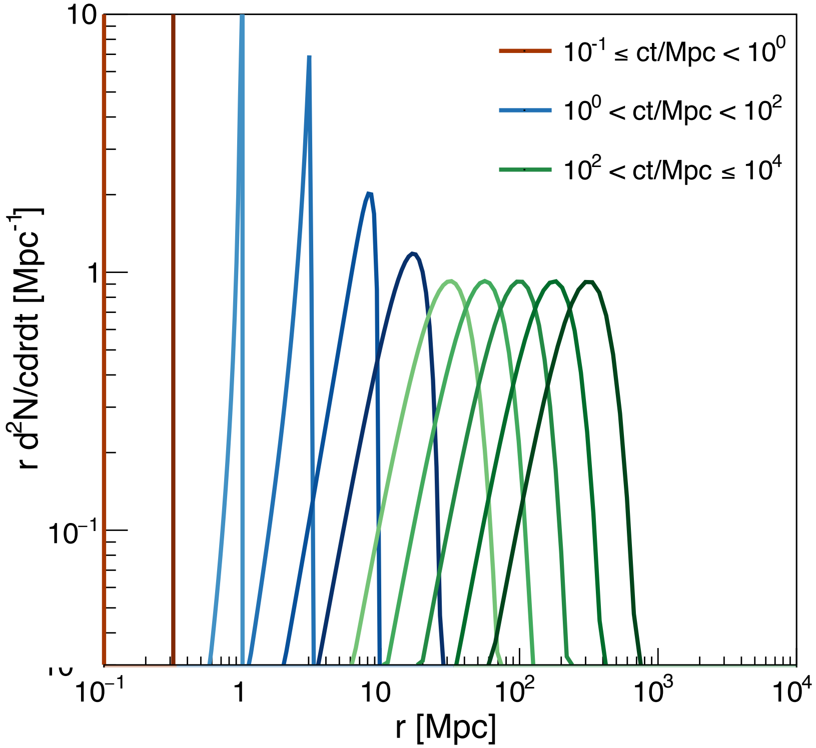
1. Ballistic
2. Ballistic/Gaussian
3. Gaussian

] Juttner

$$\frac{\mathbf{dN}}{\mathbf{dr}} = \frac{\mathbf{r^2}\alpha \mathbf{e}^{(-\alpha/\sqrt{[\mathbf{1}-(\mathbf{r}/\mathbf{ct})^{\mathbf{2}}]})}}{(\mathbf{ct})^{\mathbf{3}}\mathbf{K_1}(\alpha)[\mathbf{1}-(\mathbf{r}/\mathbf{ct})^{\mathbf{2}}]^{\mathbf{2}}}$$

$$\alpha = \mathbf{tc^2/2D}$$

Aloisio, R. +, ApJ 693 2009,

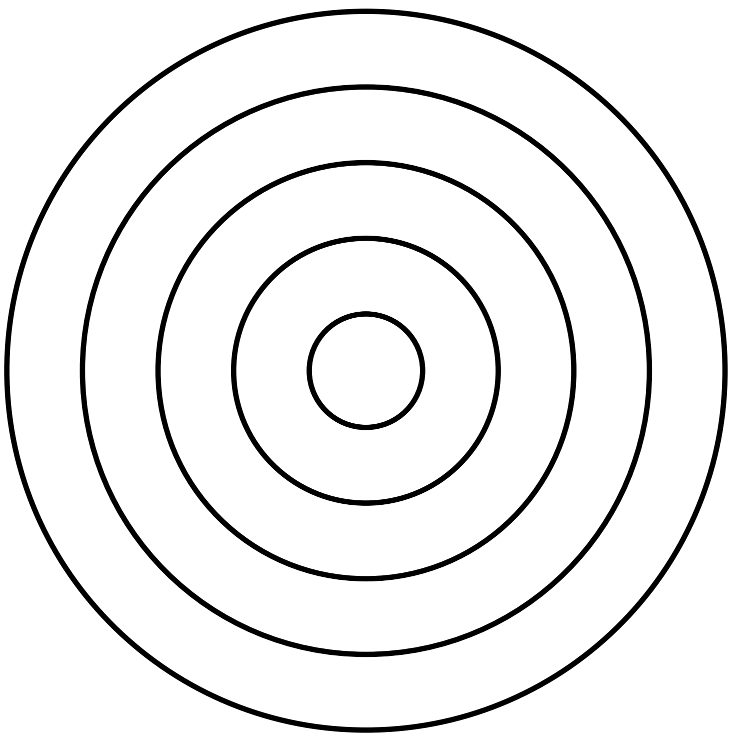


Lang, R. +, PRD 102 (2020)

Extragalactic Magnetic Field Effects

Olbers Paradox for extragalactic cosmic rays:

1) Without extragalactic magnetic fields (ie. ballistic propagation)
2) With extragalactic magnetic fields (ie. diffusive propagation)



$$\mathbf{dF = \frac{1}{r^2} n dV}$$

$$\mathbf{F_t = \int_0^{r_{max}} \frac{dF}{dr} dr}$$

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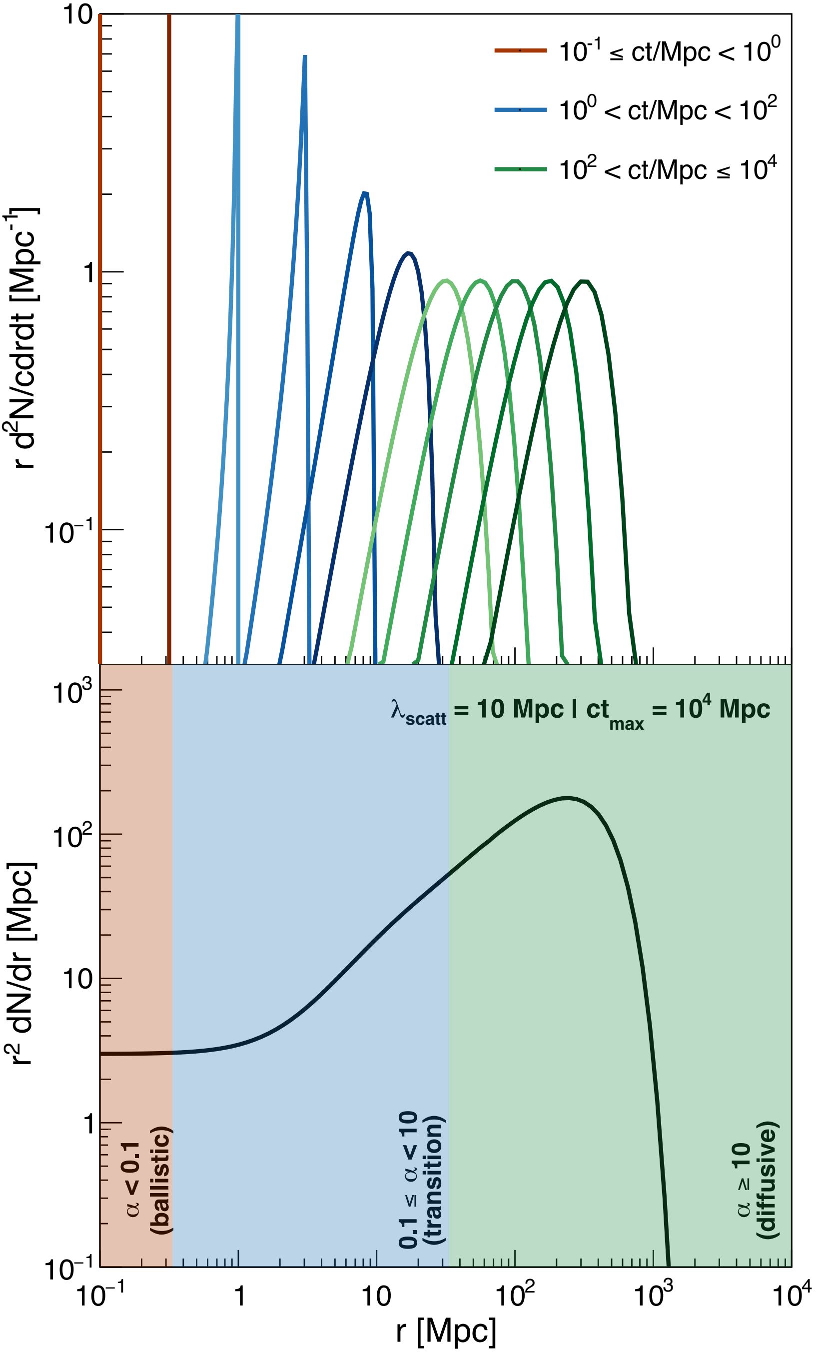
Magnetic Horizon Effect



$$\mathbf{F_t} = \int_0^{\mathbf{r_{max}}} \frac{\mathbf{dF}}{\mathbf{dr}} \mathbf{dr}$$

Constant for ballistic propagation

If cosmic ray sources were continuously distributed in space, magnetic fields wouldn't alter the total cosmic ray spectrum at Earth.

How does the discreet nature of cosmic ray sources alter this statement?

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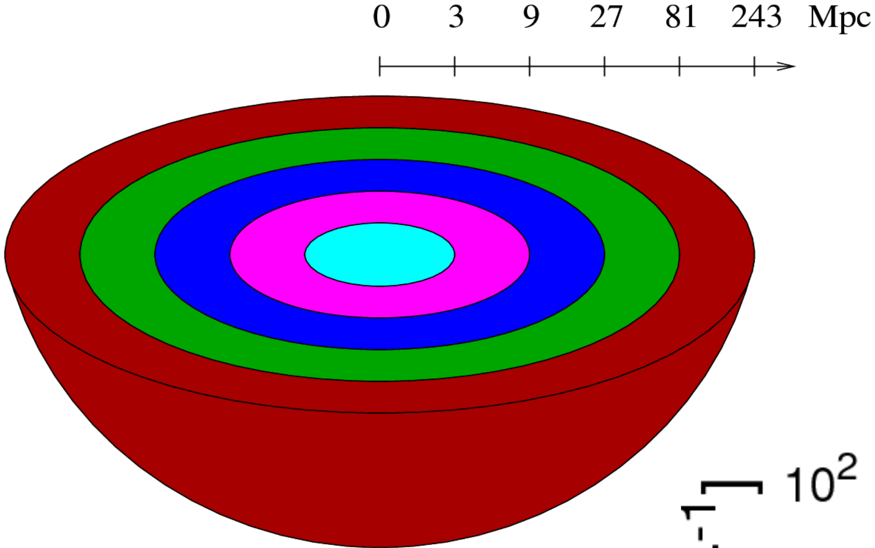
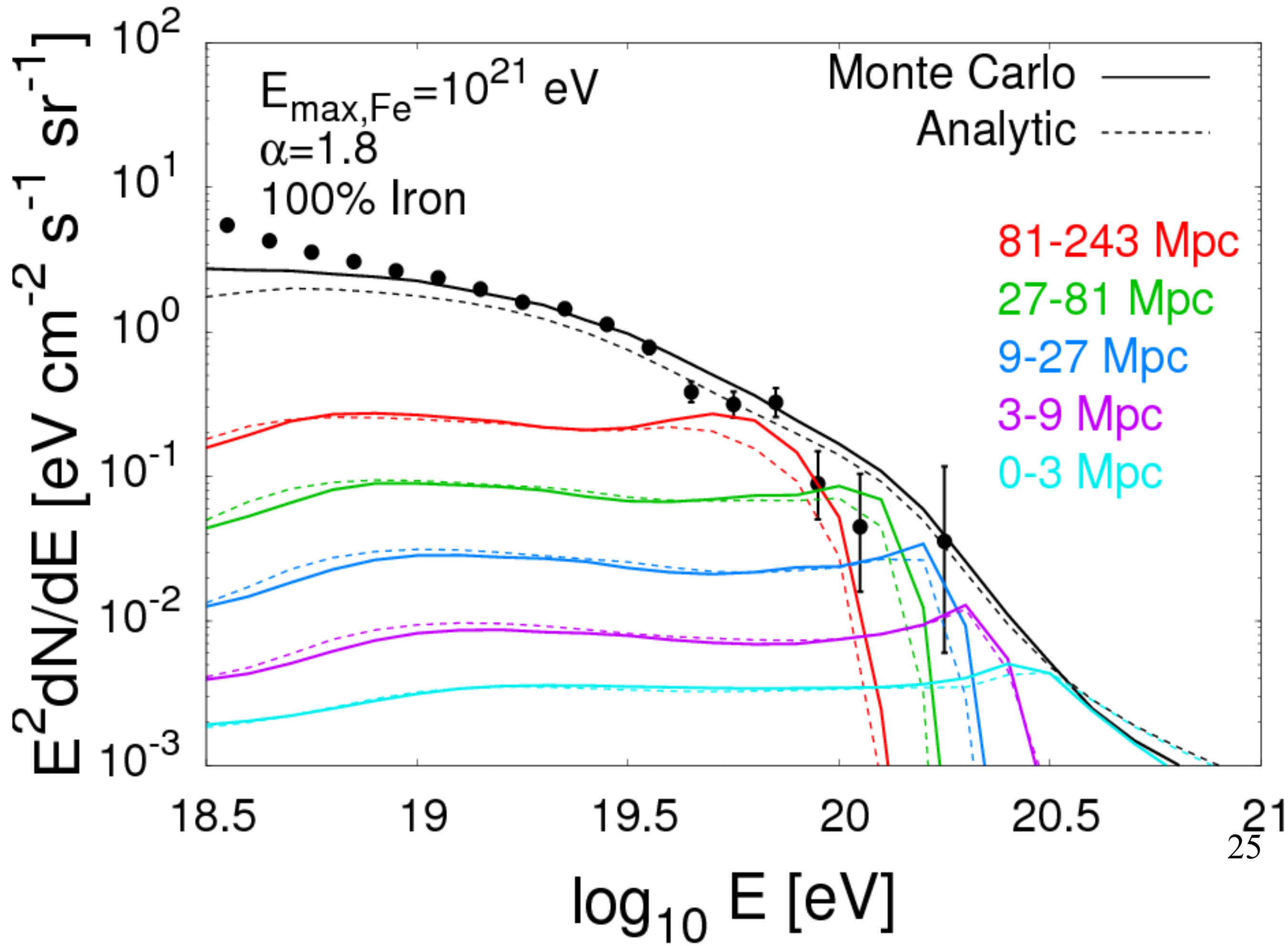
Local Scales Effect Highest Energies

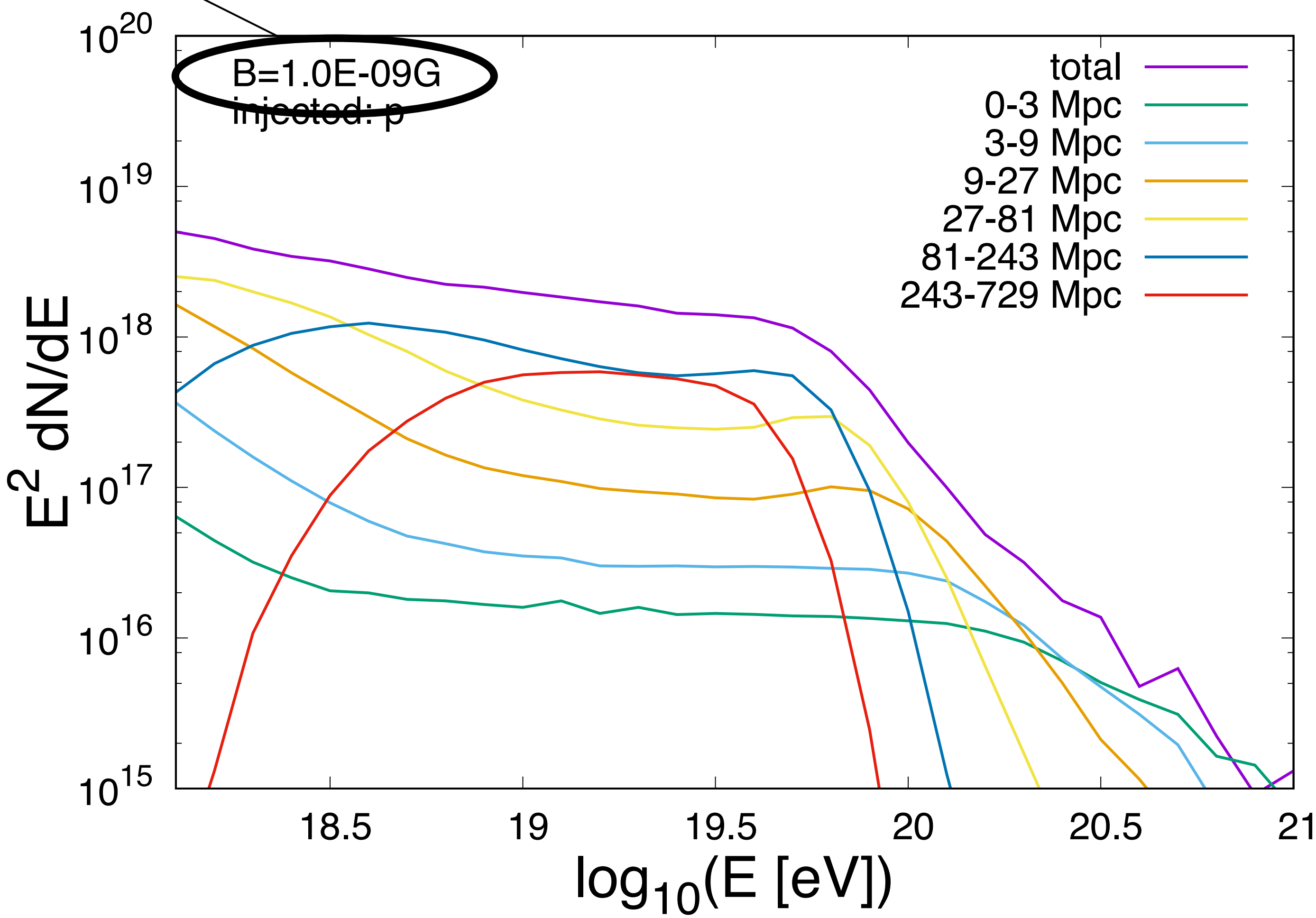


(logarithmic scale)



Magnetic Horizon Effect
-Local Scales Also Effect Low Energies

Note strong B-field strength considered



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Skymap Effects of Magnetic Fields

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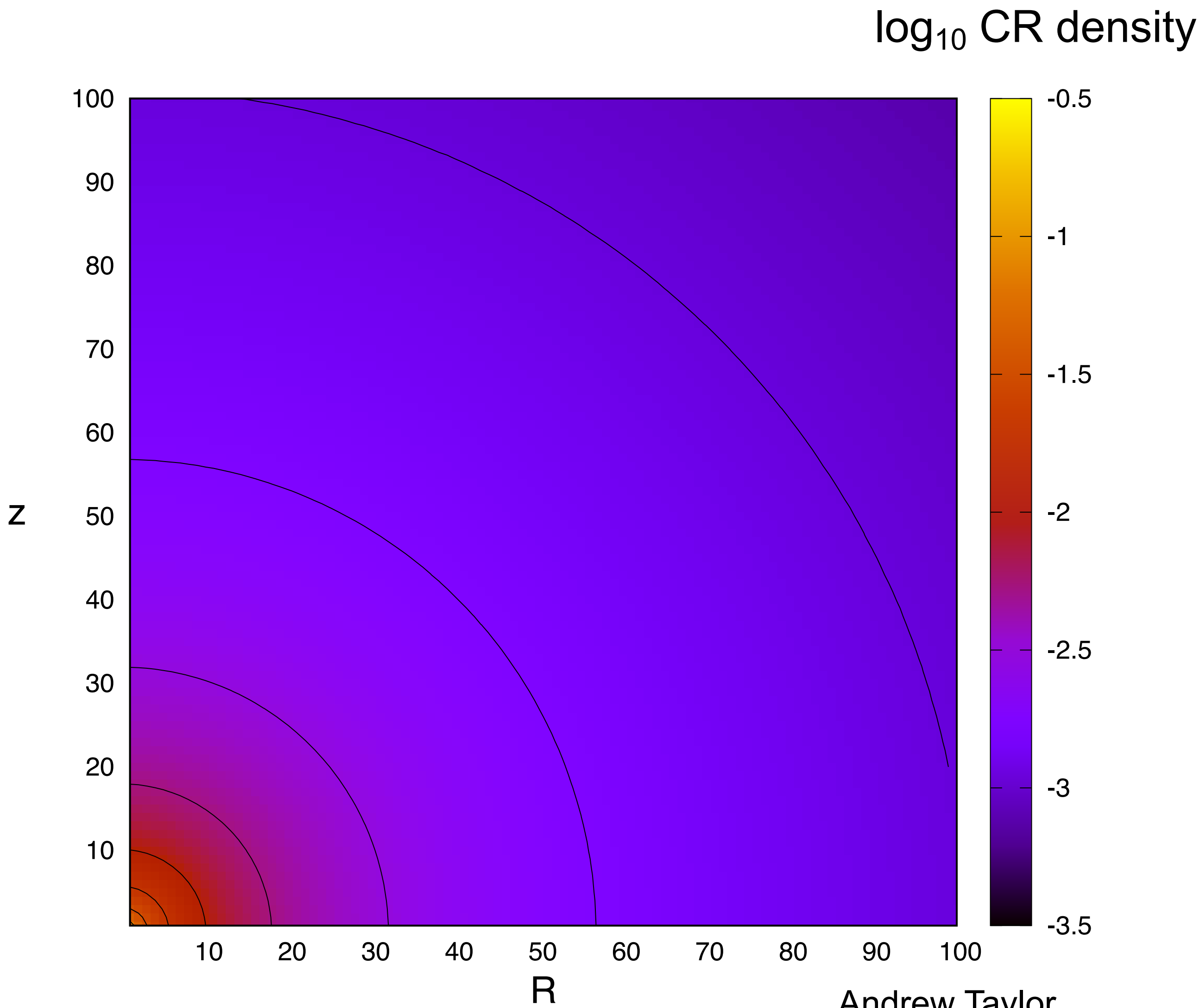
Steady State Distribution Around a Source of Diffusing Particles



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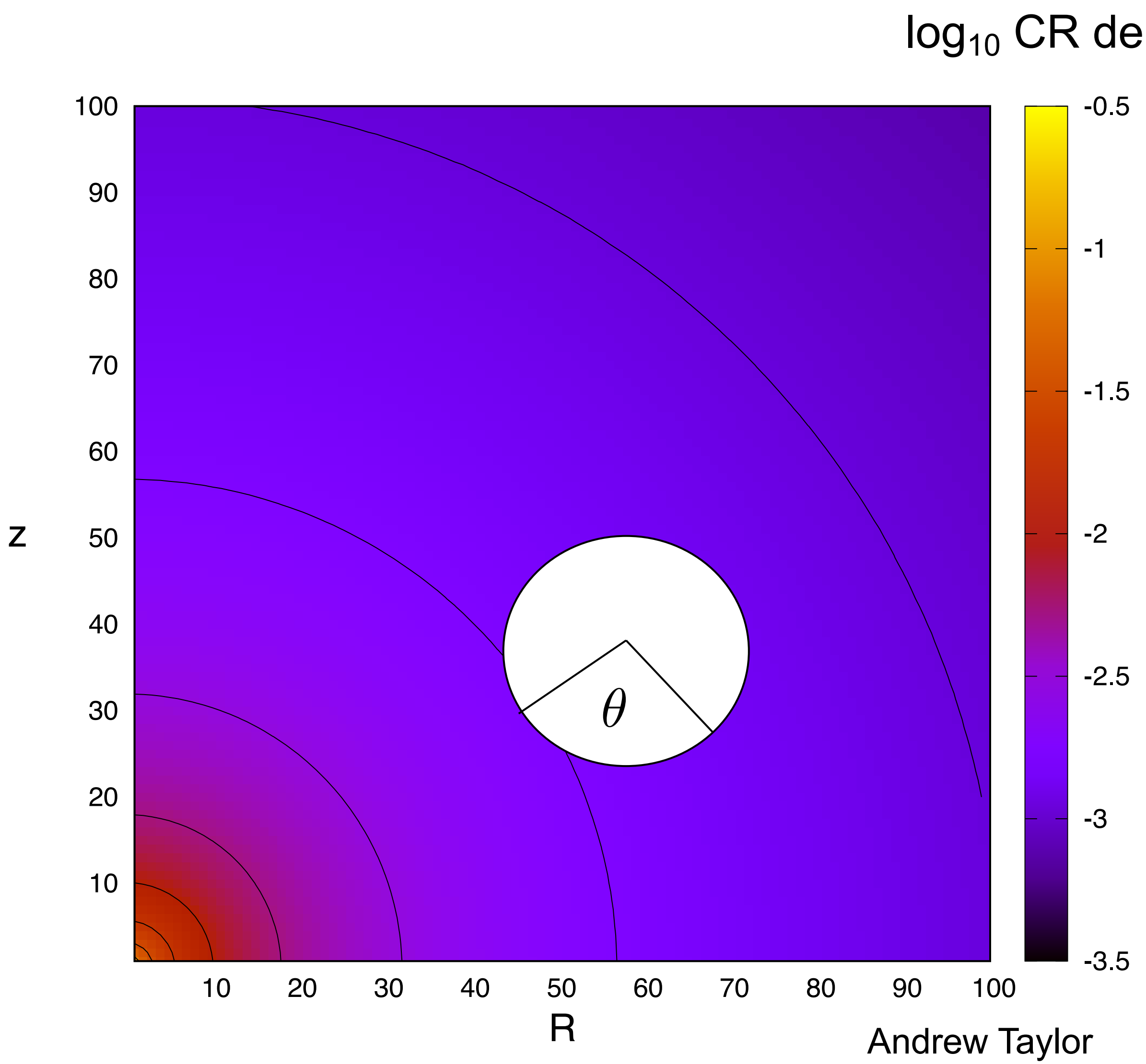
Steady State Distribution Around a Source of Diffusing Particles

Lang+, PRD 103, 063005



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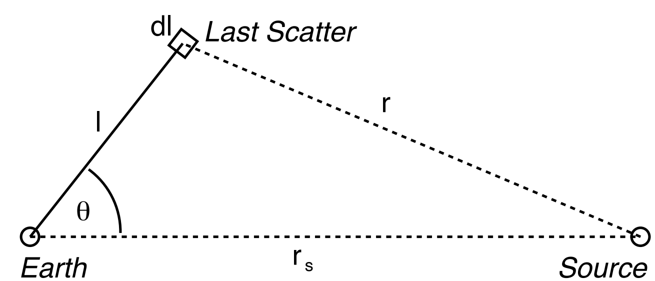
Dipole observed



$$\mathbf{r^2 = l^2 + r_s^2 - 2lr_s}\cos\theta$$

$$\frac{\mathbf{dN}}{\mathbf{d}\cos\theta} \propto \frac{\mathbf{1}}{\mathbf{r}}$$

$$\propto \frac{\mathbf{1}}{\mathbf{r_s}}\left(\mathbf{1} + \frac{\mathbf{l}}{\mathbf{r_s}}\cos\theta\right)$$

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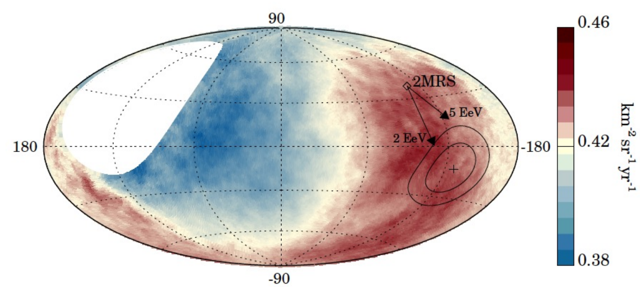
Diffusive and Ballistic Propagation of CR from Sources

$$\mathbf{U_{CR} = \frac{L_{CR}}{D r}}$$

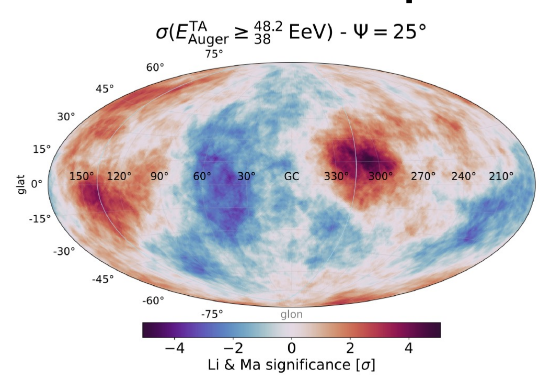
Dipole observed

$$\frac{\mathbf{dN}}{\mathbf{d}\cos\theta} \propto \left(\mathbf{1} + \frac{\lambda_{\mathbf{scat}}}{\mathbf{r_s}}\cos\theta\right)$$



$$\mathbf{U_{CR} = \frac{L_{CR}}{4\pi r^2 c}}$$

Objects look like a point sources



The (Considerable) Unknowns About Extragalactic Magnetic Fields

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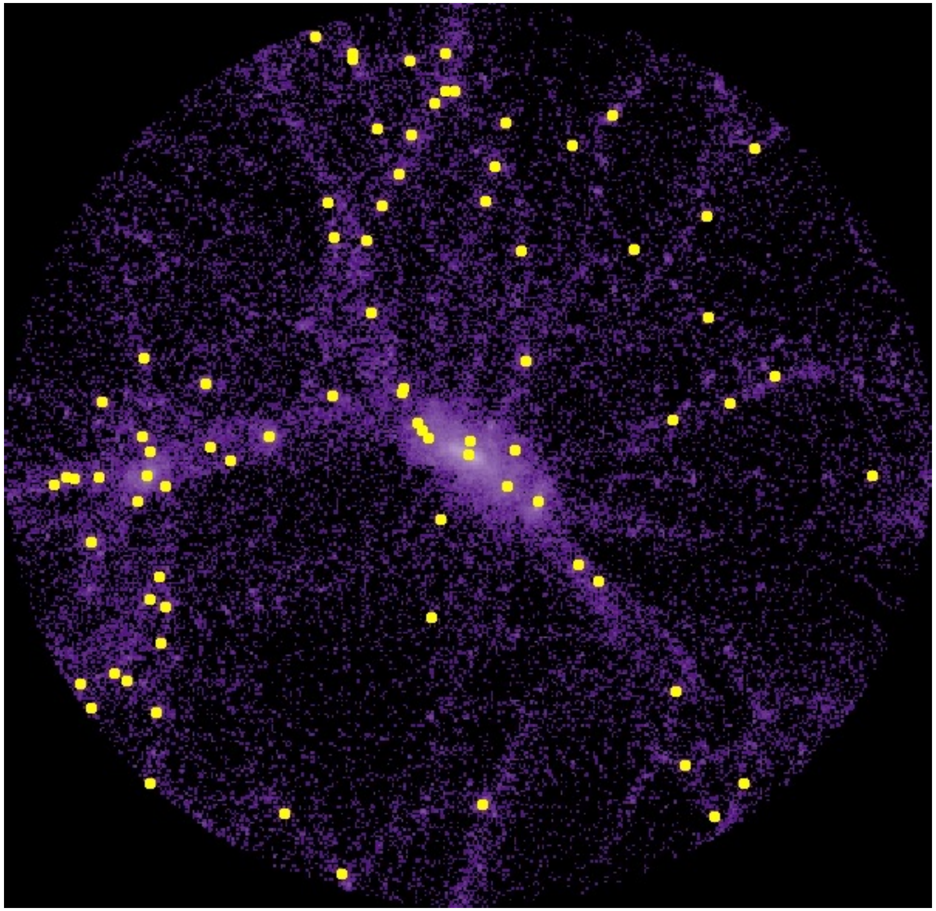
Extragalactic Magnetic Fields

The homogeneous scale for the Universe is thought to be 100 Mpc – is possible that the magnetic field in local extragalactic space is structured (the matter is structured on these scales).

What is the EGMF structure/strength in the inhomogeneous region around the Milky Way?



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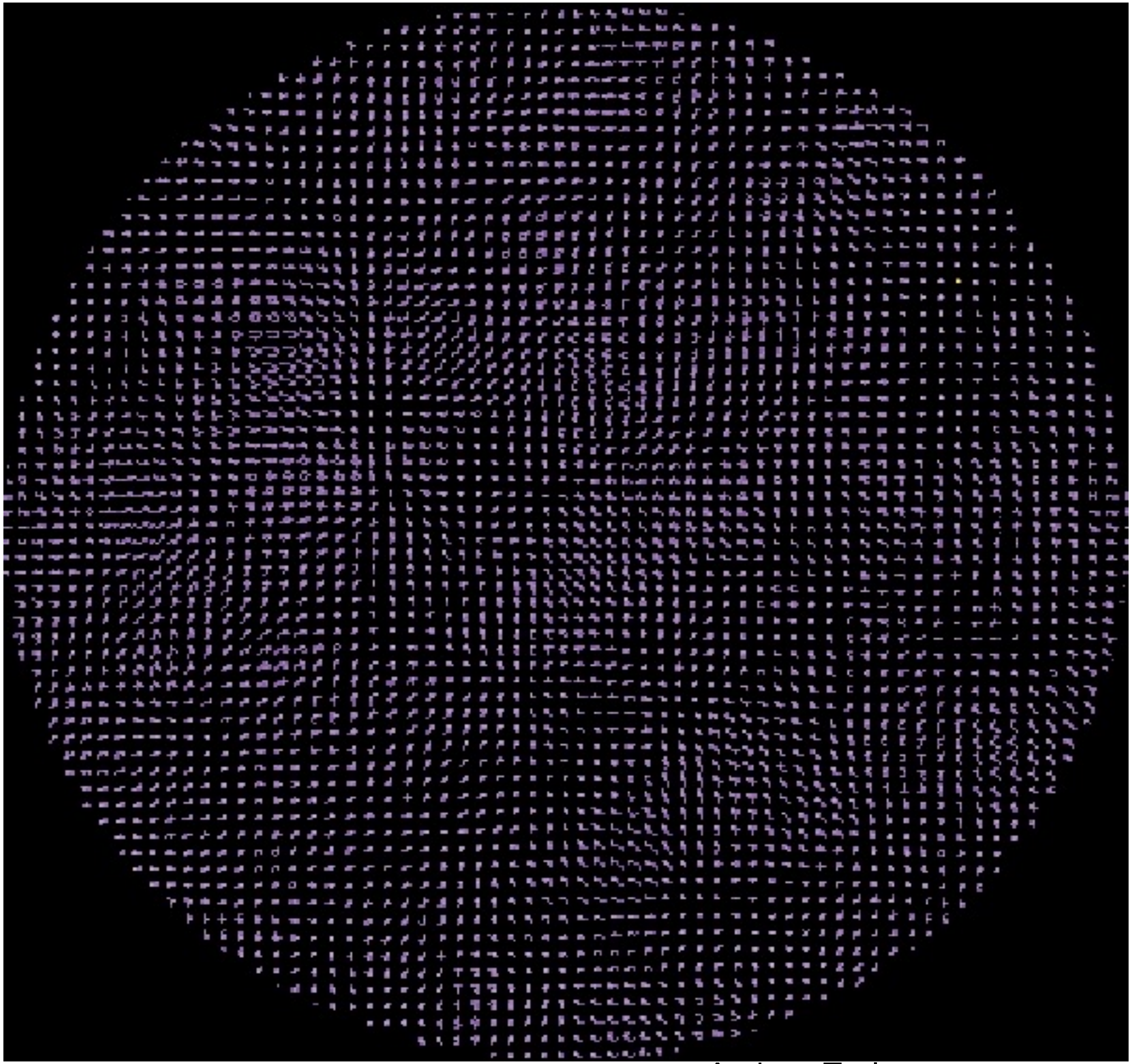
Extragalactic Magnetic Field Origin?



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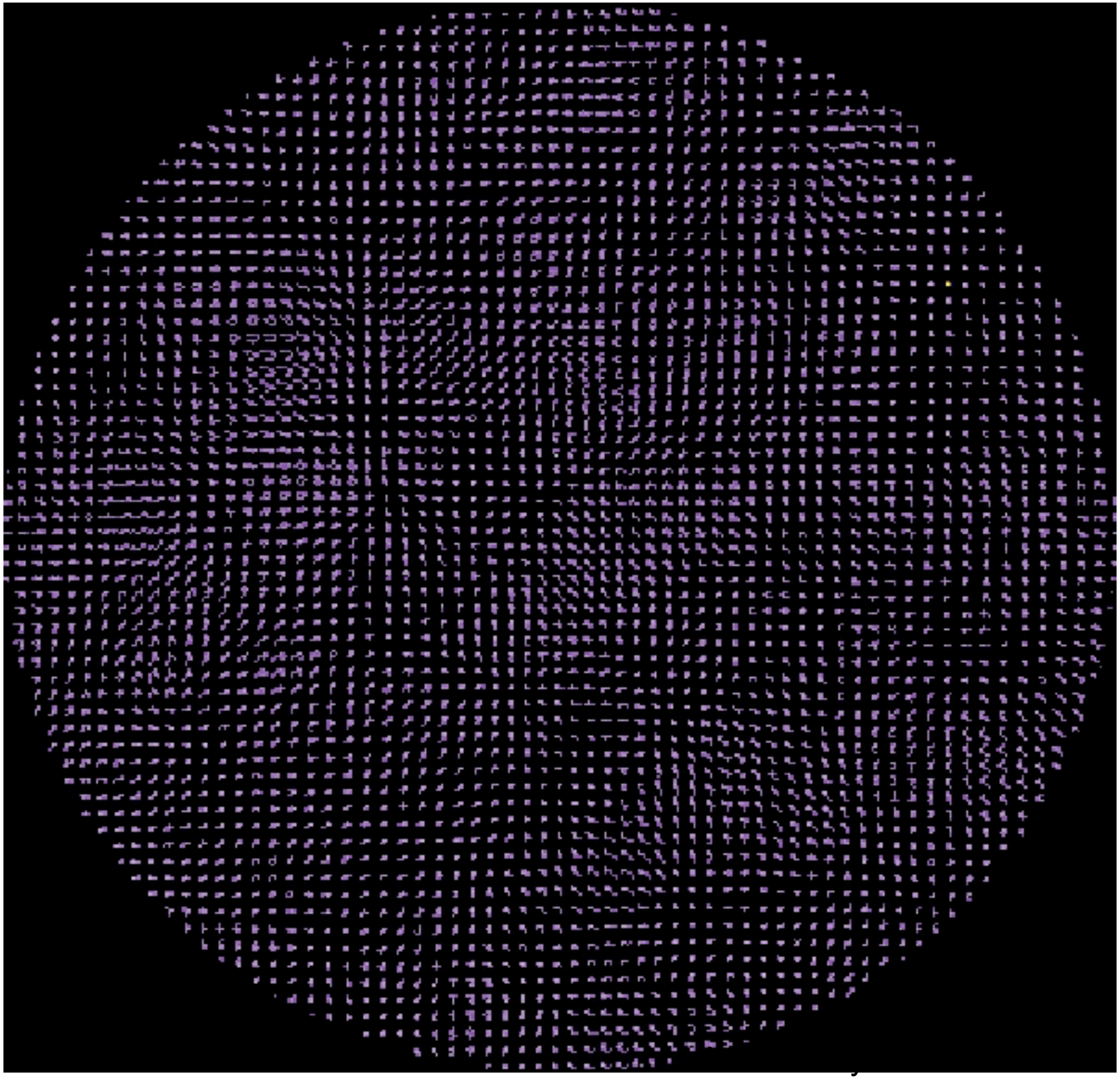
$z = 40$

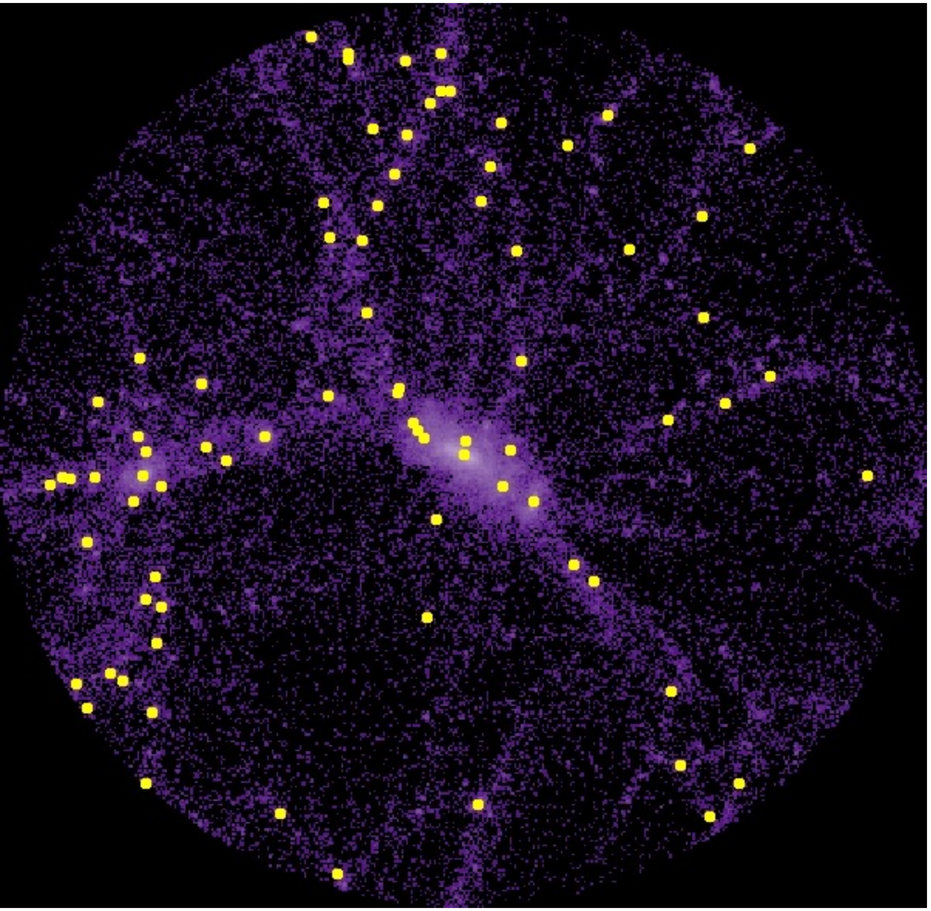
Seed B-field strength?

Extragalactic Magnetic Field Origin?



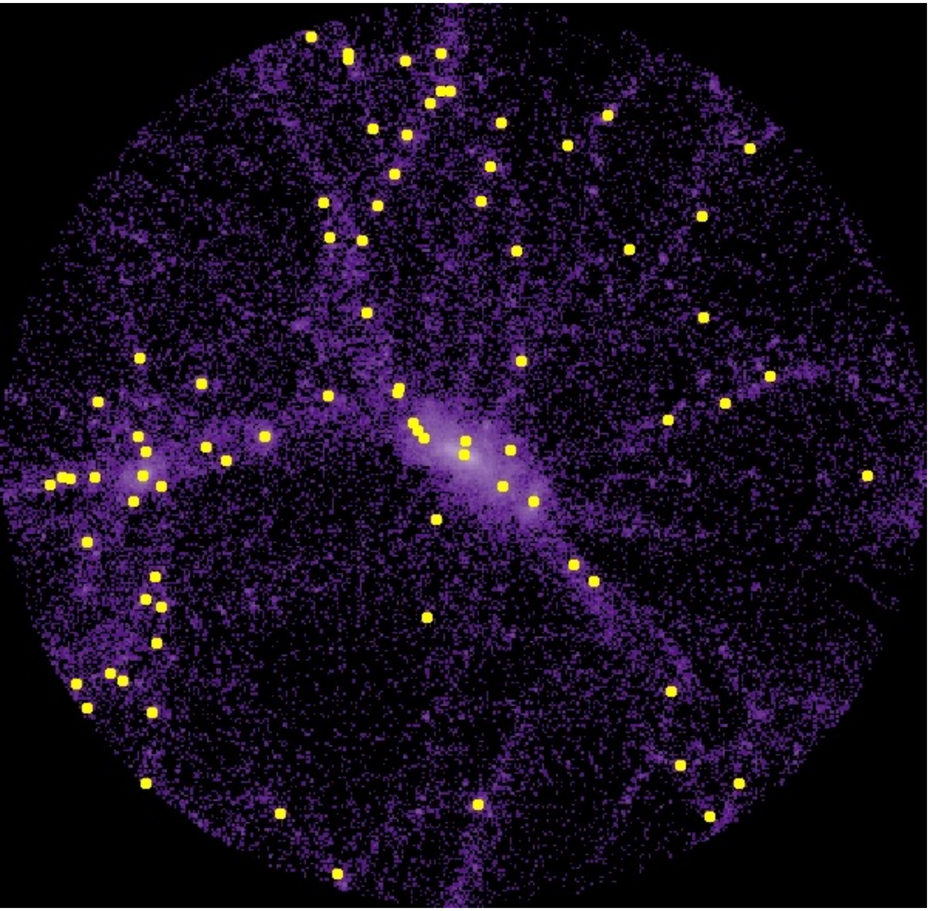
...compression and dynamo action lead to ~μG B-field strength growth on galactic scales

How Do Galactic and Extragalactic Magnetic Fields Merge?

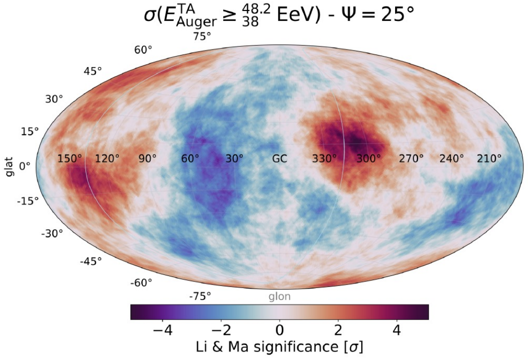


Question- do Galactic halo (out to the virial radius) or Extragalactic magnetic fields dominate the deflection of UHECR?

How Do Galactic and Extragalactic Magnetic Fields Merge?



Question- do Galactic halo (out to the virial radius) or Extragalactic magnetic fields dominate the deflection of UHECR?



Perhaps the UHECR skymap can be used to determine this

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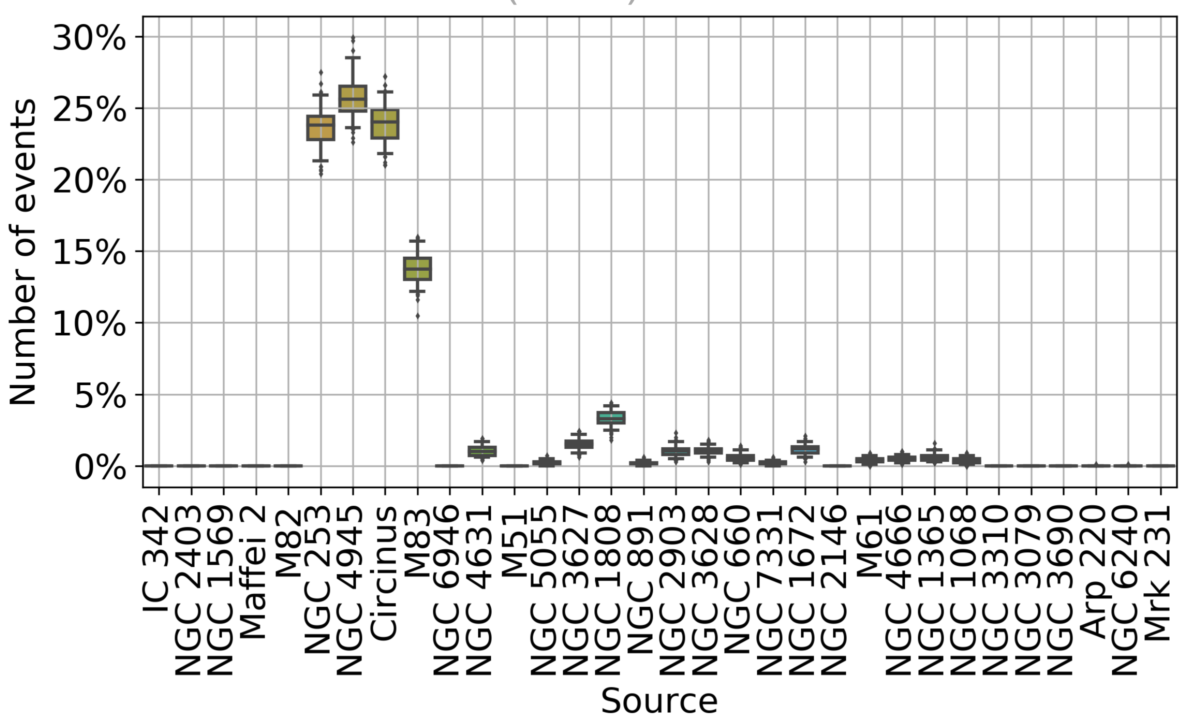
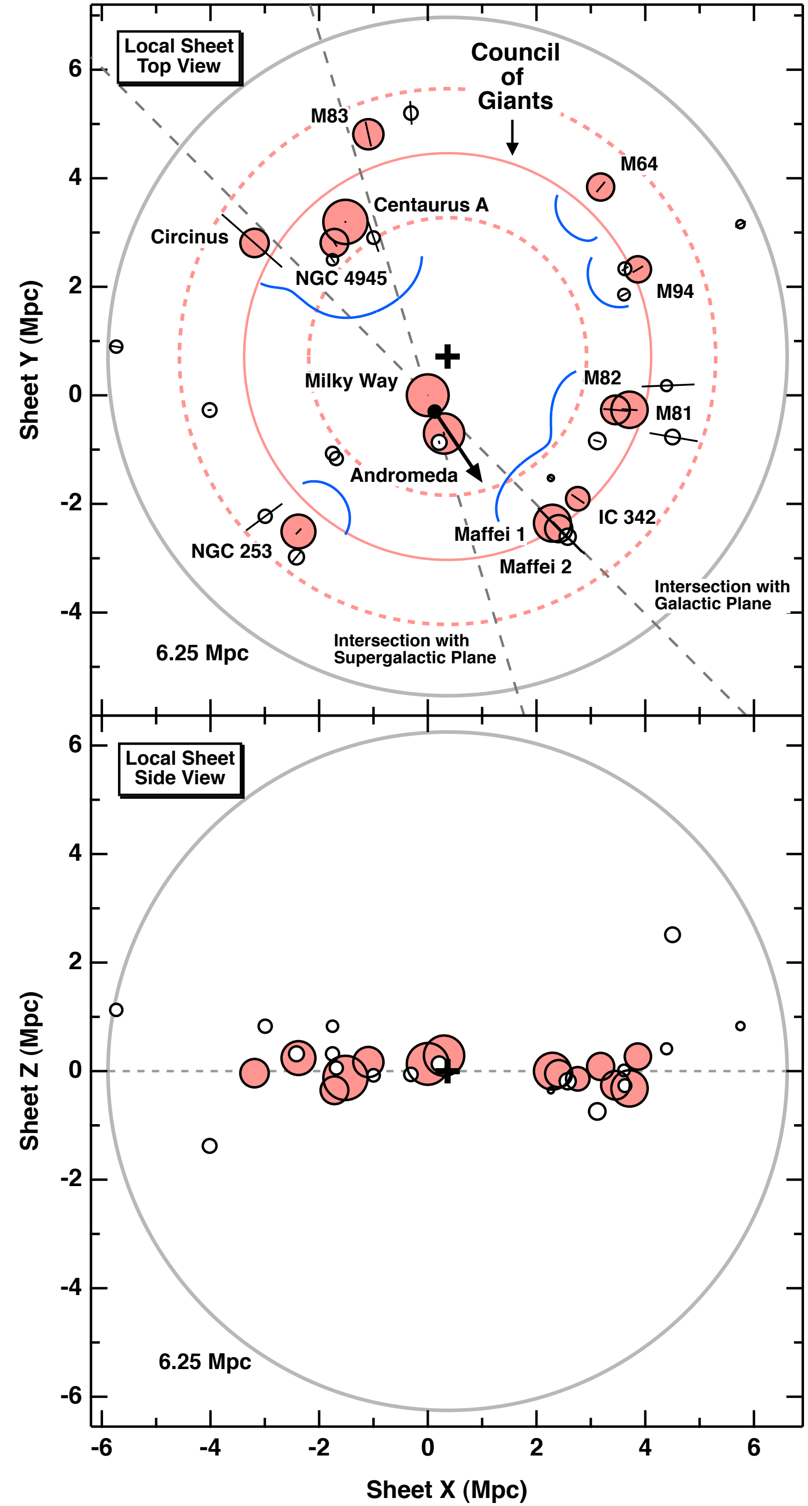
Our Local Extragalactic Neighbourhood

The local (<10 Mpc) extragalactic objects are structured, sitting in a roughly circular disk shape around the Milky Way

van Vliet MNRAS 510 (2021)





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Bregman et al. ApJ 928 (2022)

McCall MNRAS 440 (2014)

The Uniqueness of Cen A within the Council of Giants

$$\mathbf{t_{acc}} = \eta \frac{\mathbf{R_{lar}}}{\mathbf{c}\beta^2}$$

$$\mathbf{t_{esc.}} = \frac{\mathbf{R}}{\mathbf{c}\beta}$$

AM Hillas (1984)

$$\mathbf{E_{max}} = \beta \mathbf{eBR}$$

$$\mathbf{L_B} = \mathbf{U_B} 4\pi \mathbf{R}^2 \beta \mathbf{c}$$

Under the assumption of equipartition of energy between kinetic energy and magnetic field:

Lovelace et al. (1976)

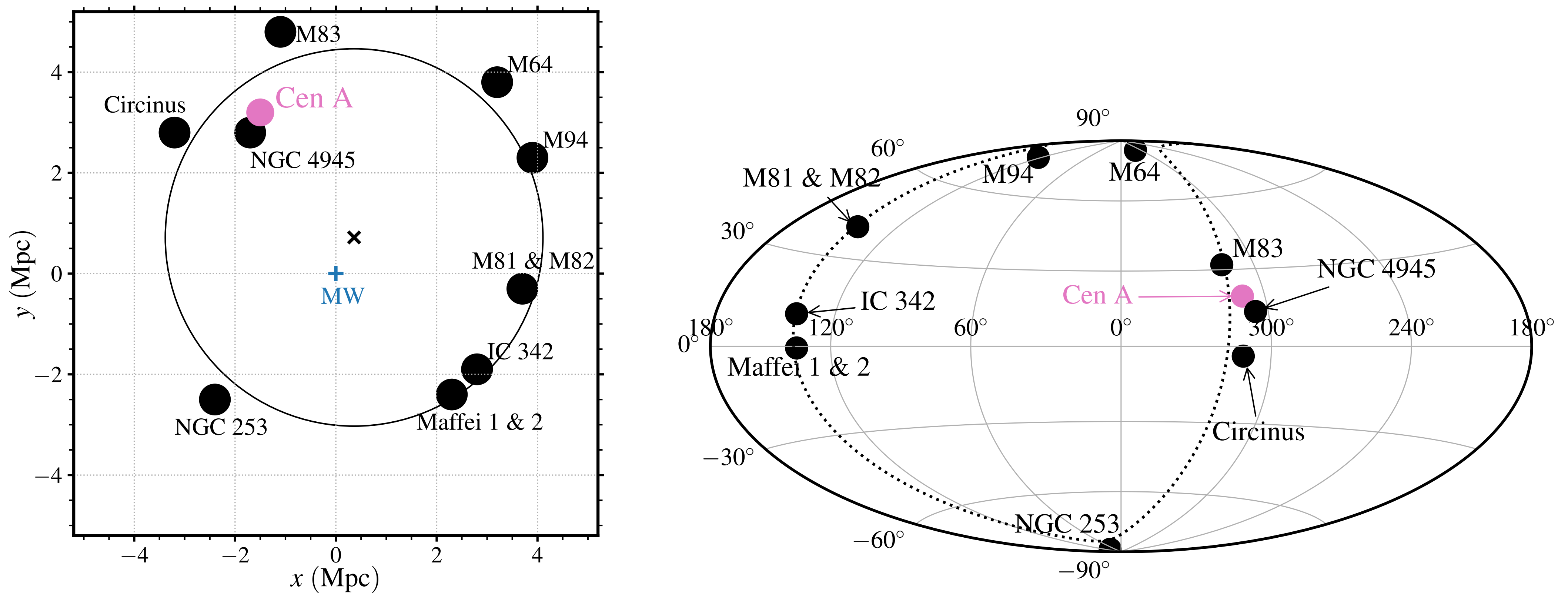
$$\mathbf{E_{max}} \lesssim \frac{\mathbf{Z}}{\eta} (\beta \mathbf{L_{KE}} \alpha \hbar)^{1/2} \approx \mathbf{10} \; \frac{\mathbf{Z}}{\eta} \left(\frac{\beta \mathbf{L_{KE}}}{\mathbf{3 \times 10^{43} \; erg \; s^{-1}}} \right)^{\mathbf{1/2}} \; \mathbf{EeV}$$

Local Extragalactic Structure The Council of Giants

Cen A is unique within the council of giant structure are being the only object proving a kinetic luminosity capable of giving rise to multi EeV acceleration

Lovelace et al. (1976)

$$\mathbf{E_{max}} \lesssim \frac{\mathbf{Z}}{\eta} \left(\beta \mathbf{L_{KE}} \alpha \hbar\right)^{\mathbf{1/2}} \approx \mathbf{10} \ \frac{\mathbf{Z}}{\eta} \left(\frac{\beta \mathbf{L_{KE}}}{\mathbf{3 \times 10^{43} \ erg \ s^{-1}}} \right)^{\mathbf{1/2}} \mathbf{EeV}$$



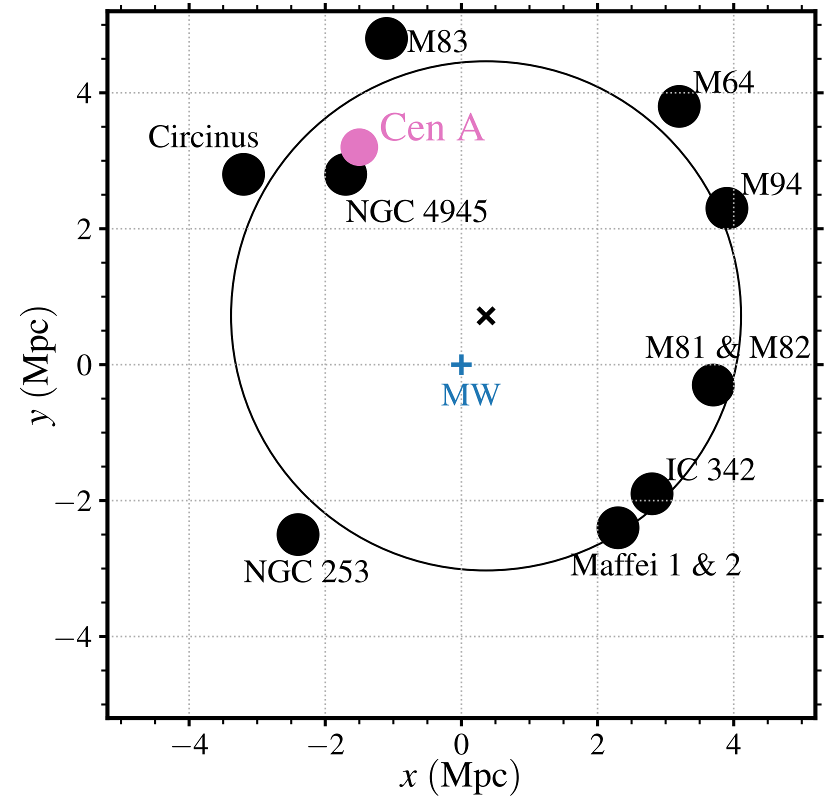
Simulation Setup

- Particles initially fill 300 kpc region surrounding Cen A (isotropic momentum distribution)
- Large angle particle scattering occurs within the virial region (< 300 kpc) of all members of the council of giant system
- Outside the virial radii of these galaxies the particle propagation is treated as ballistic
- Fundamental parameter of problem- optical depth of scattering regions

$$\tau = \frac{\mathbf{r_{vir}}}{\mathbf{l_{sc}}}$$

- Echo signals results are rather insensitive to optical depth of scattering regions, provided

$$\tau > \mathbf{1}$$



- Only He and Fe injected into the system (fragile and robust species compared to crossing time of system)
- Particles photo-disintegrate en-route in extragalactic radiation fields
- 30 EeV particles being focused on
- Deflections from MW magnetized halo intentionally left out

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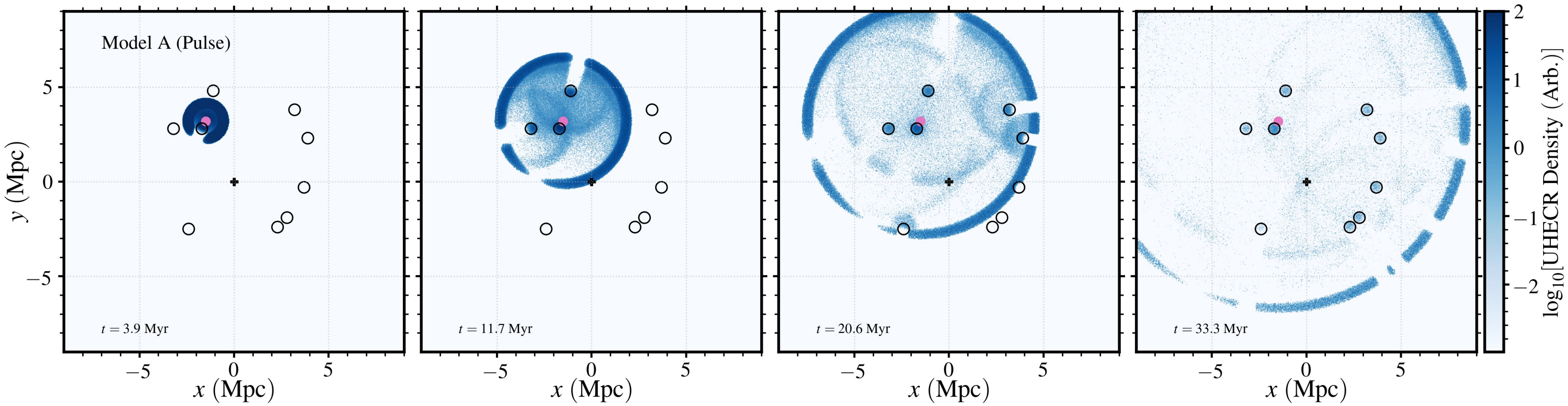
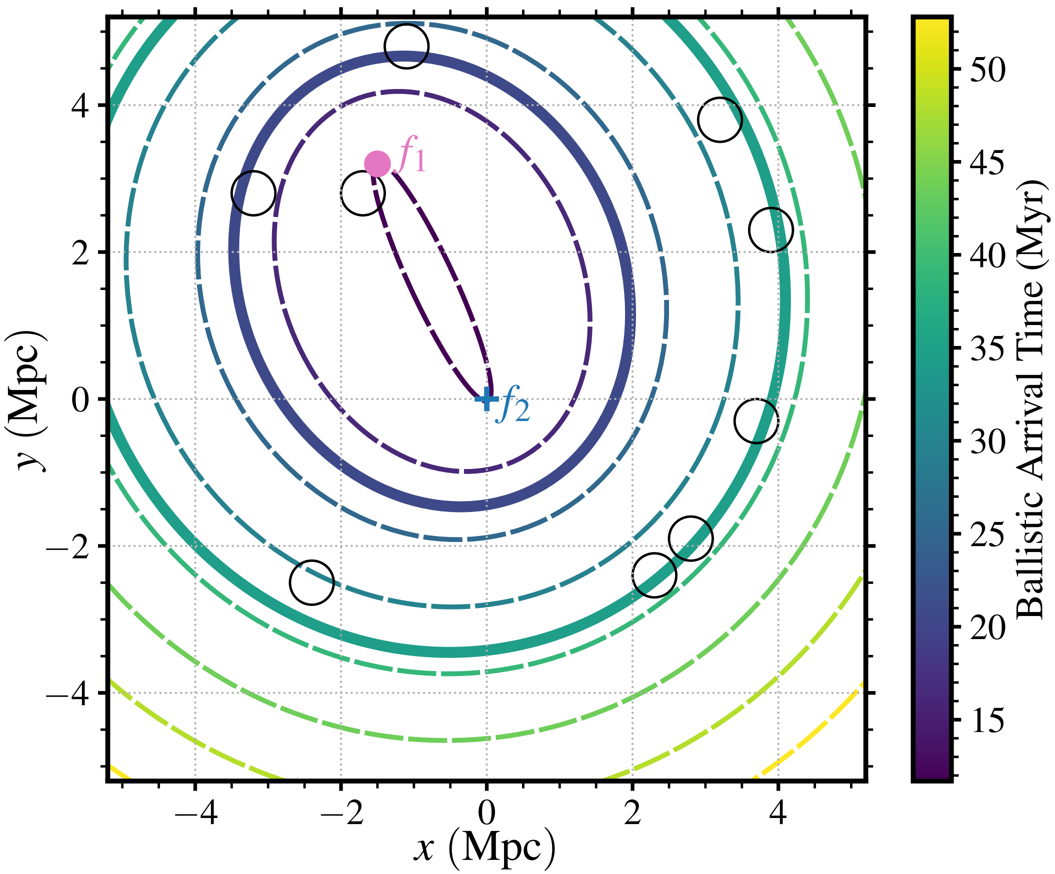
Simulations of UHECR Propagation Through the CoG Structure





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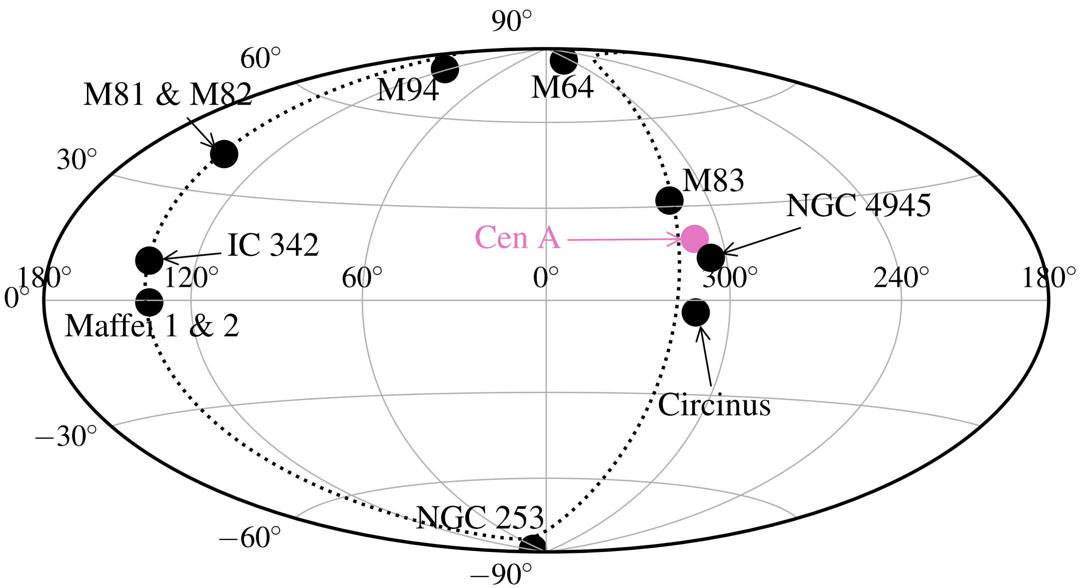
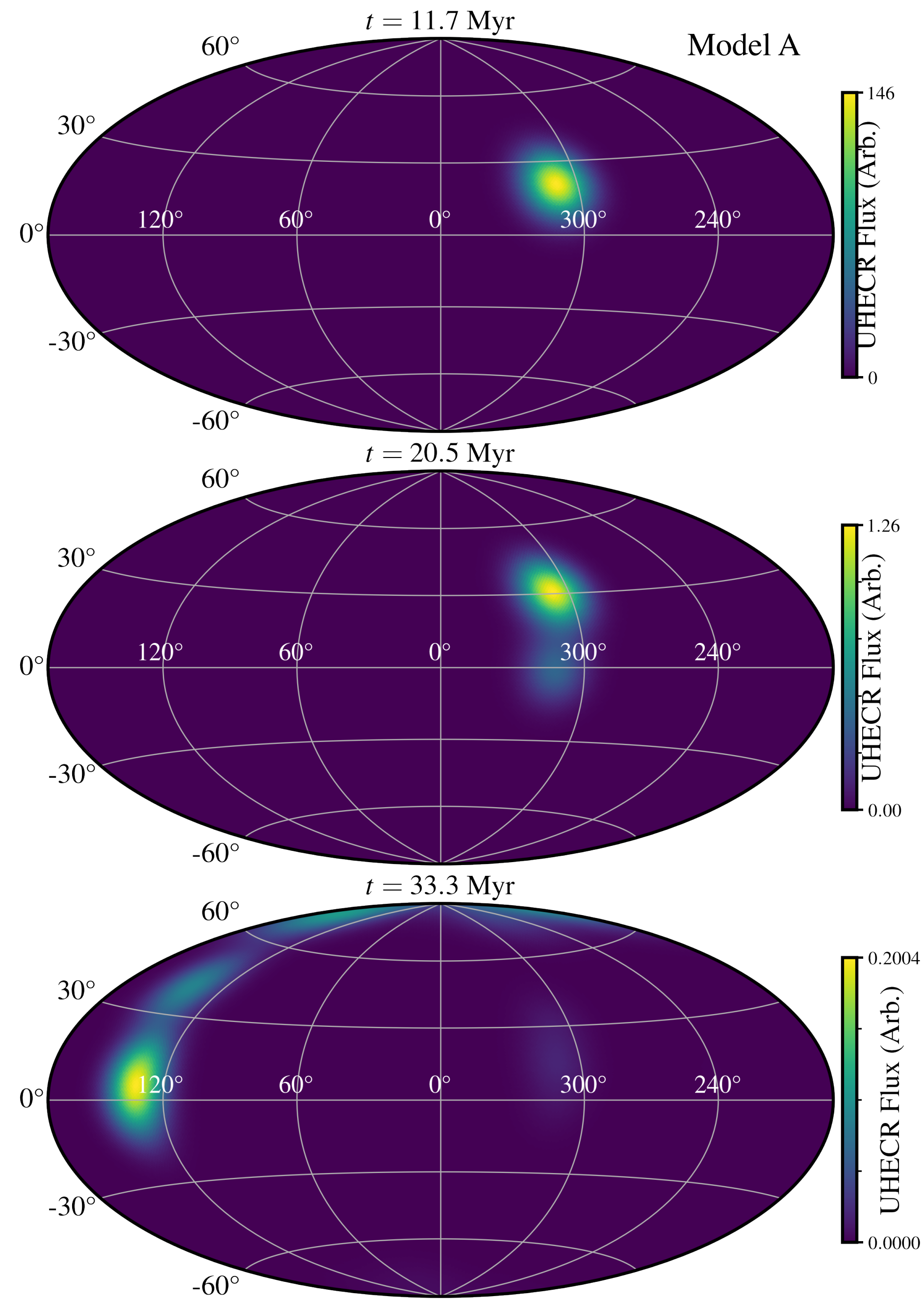
Milky Way Based Observers





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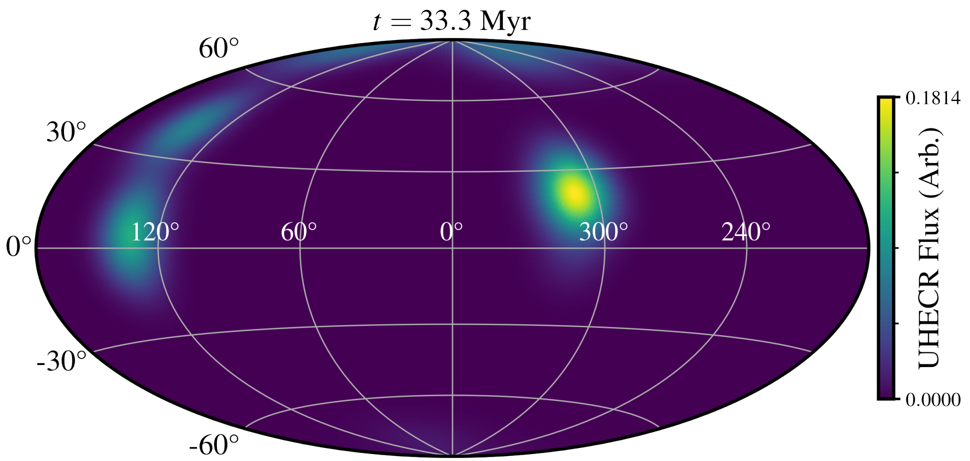
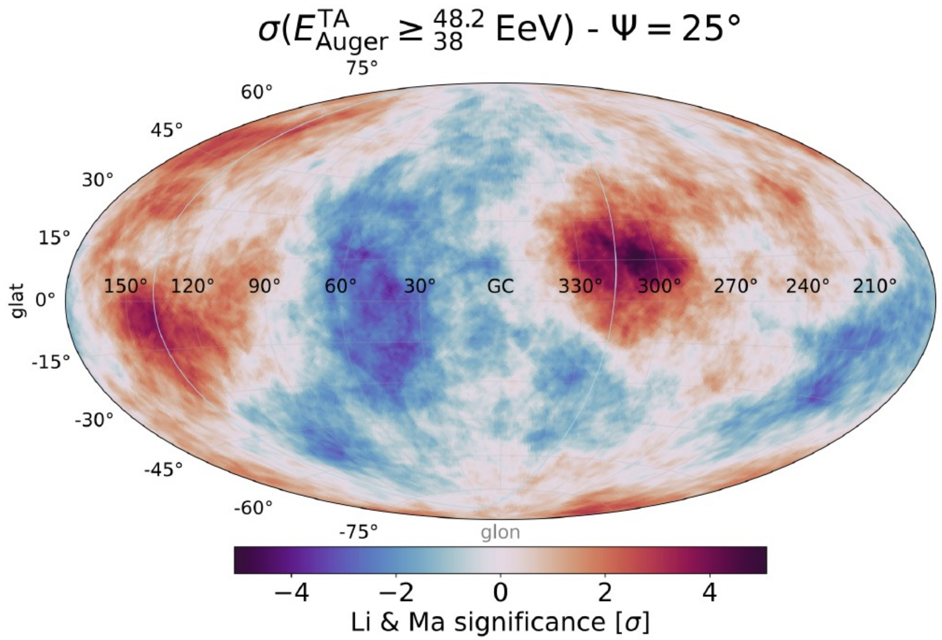
Is Local Magnetic Structure Imprinted on the UHECR Skymap?





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Conclusion

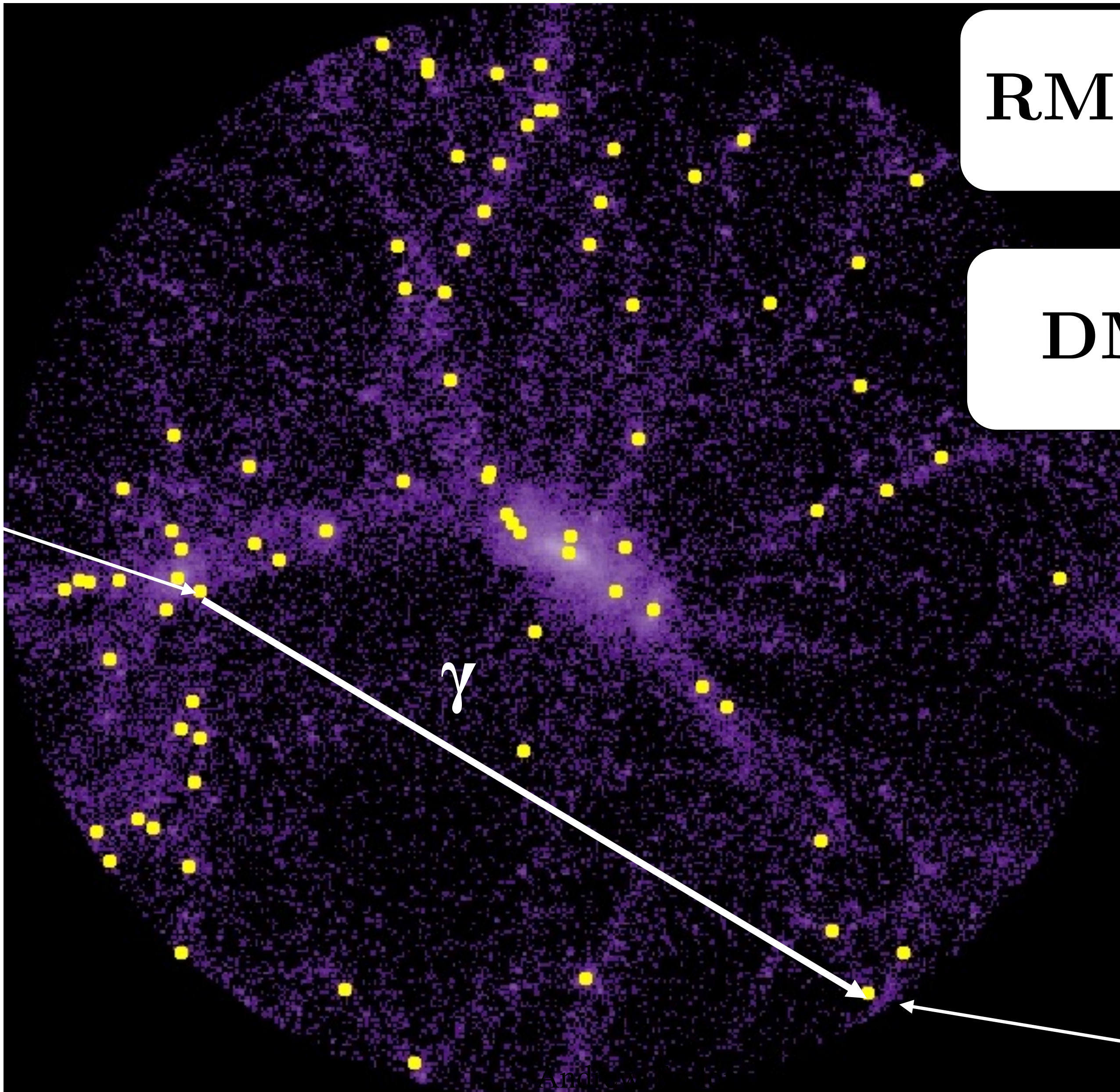
- Cascades in hydrodynamics and magneto-hydrodynamics lead to the formation of turbulence
- Charged particle propagation is dictated by magnetic structure, and in particular by magnetic turbulence structure
- Extragalactic magnetic fields can prevent the arrival of ”low” energy cosmic rays from even the most local sources (the magnetic horizon)
- Our knowledge of the magnetic structure of the Milky Way (+ other galaxies) is particularly poor in the Galactic halo region
- The magnetic structure in our local inhomogeneous patch of the Universe is even more poorly probed
- It seems possible that the arrival of extragalactic cosmic rays can pick up an imprint of the local extragalactic magnetic field structure

End of Lecture

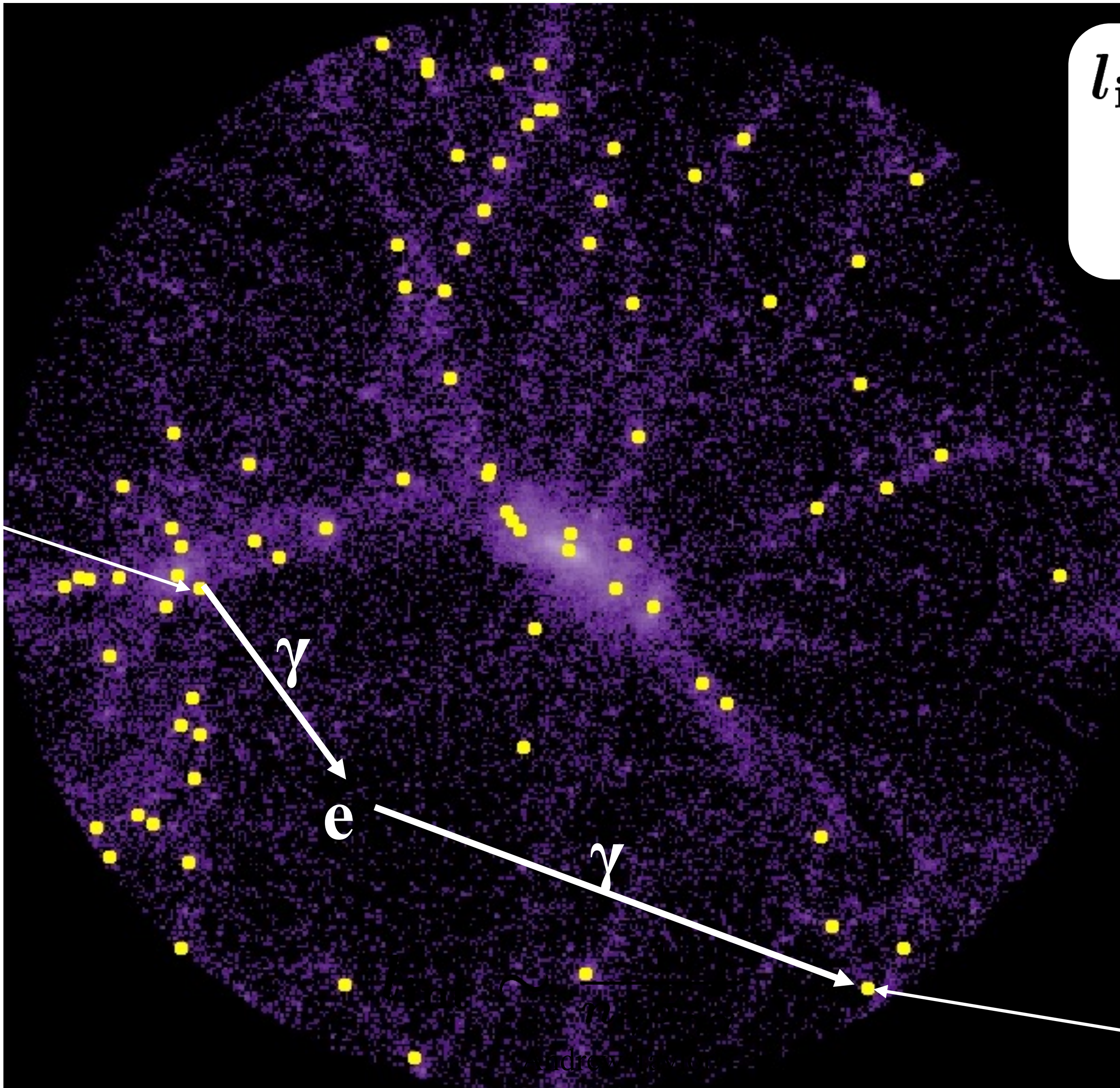
A Radio Probe

$$\mathbf{RM} = \int \mathbf{n_e B_{\parallel} dl}$$

$$\mathbf{DM} = \int \mathbf{n_e dl}$$



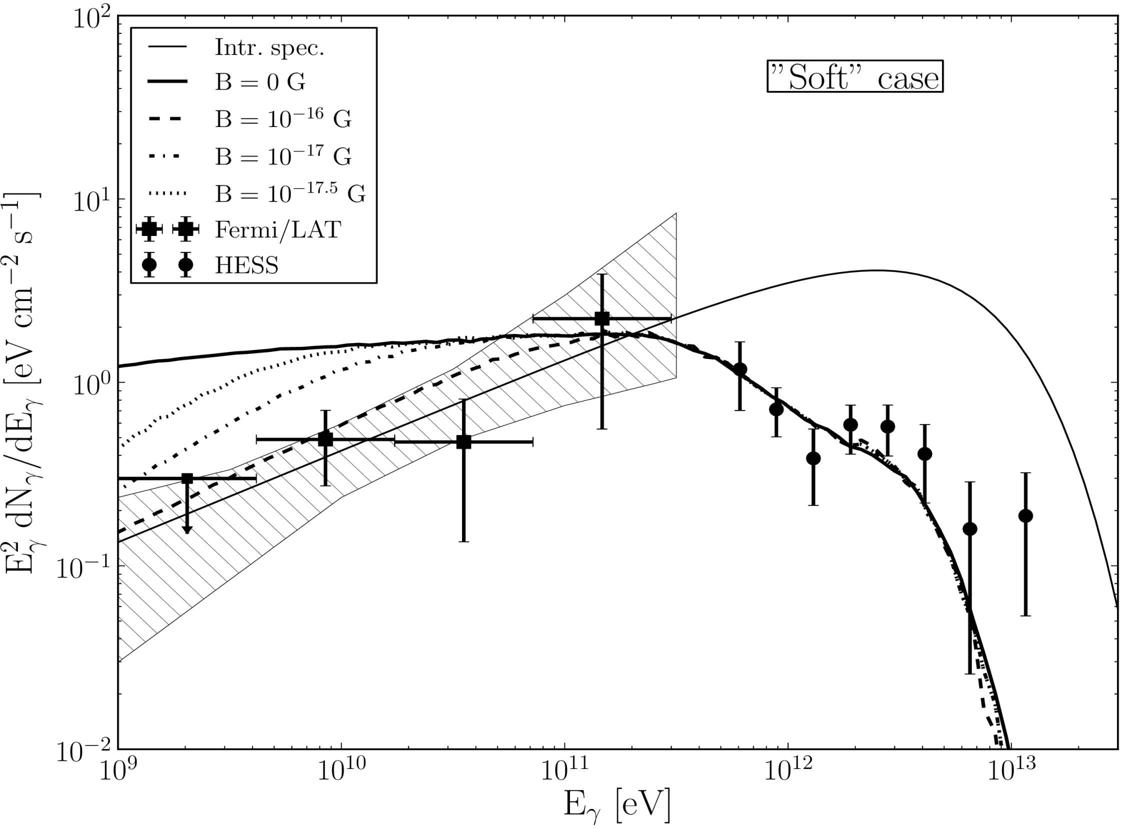
A Gamma-Ray Probe

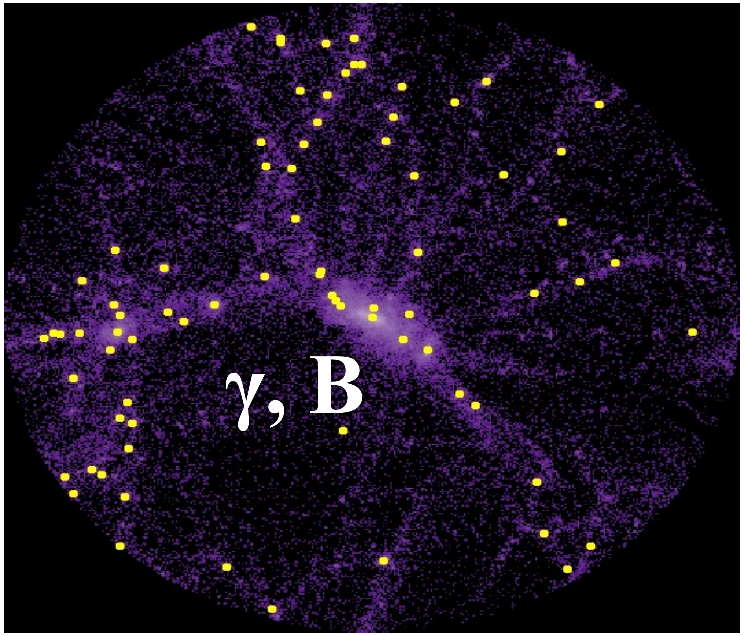


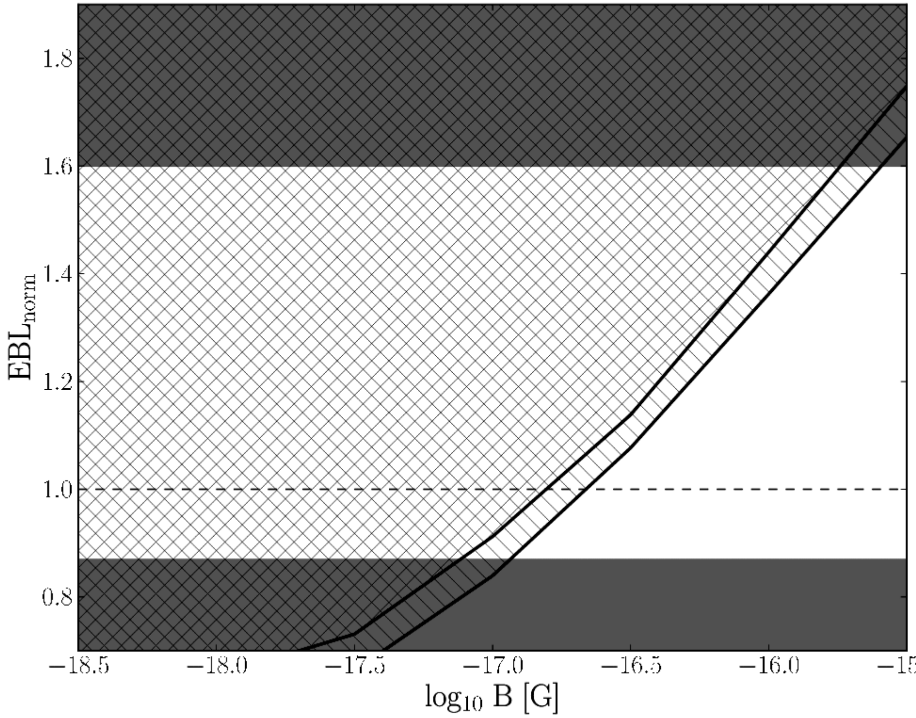
$$l_{\mathrm{int}} \sim \frac{1}{n_\gamma \sigma_{\gamma\gamma}}$$

$$\sim 30\ \mathrm{Mpc}$$

Probing Extragalactic Radiation + Magnetic Fields?







astro-ph/1101.0932 Taylor et al.

astro-ph/1112.2534 Vovk et al.

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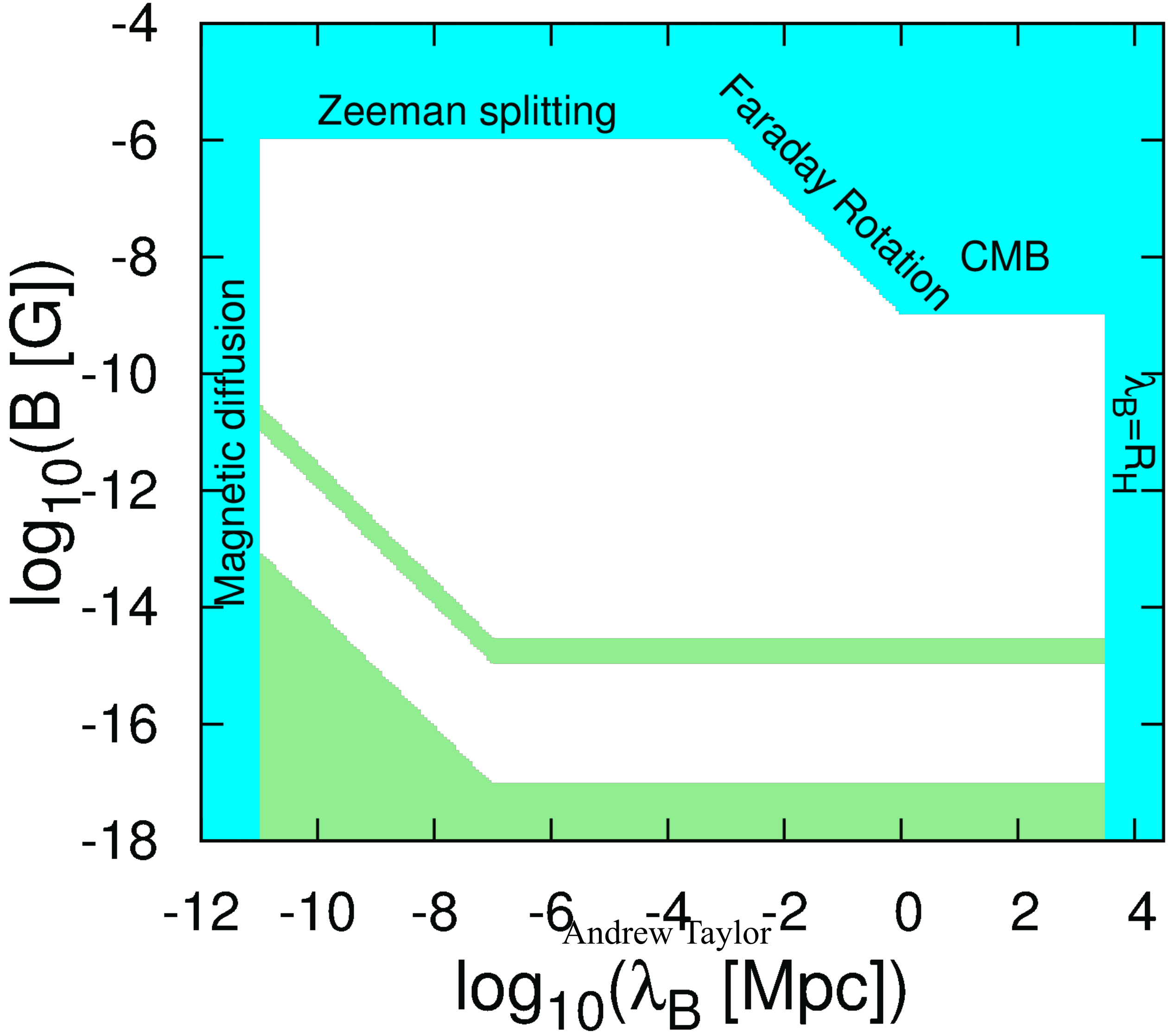
Extragalactic Magnetic Field is Hugely Uncertain



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Extra Slides

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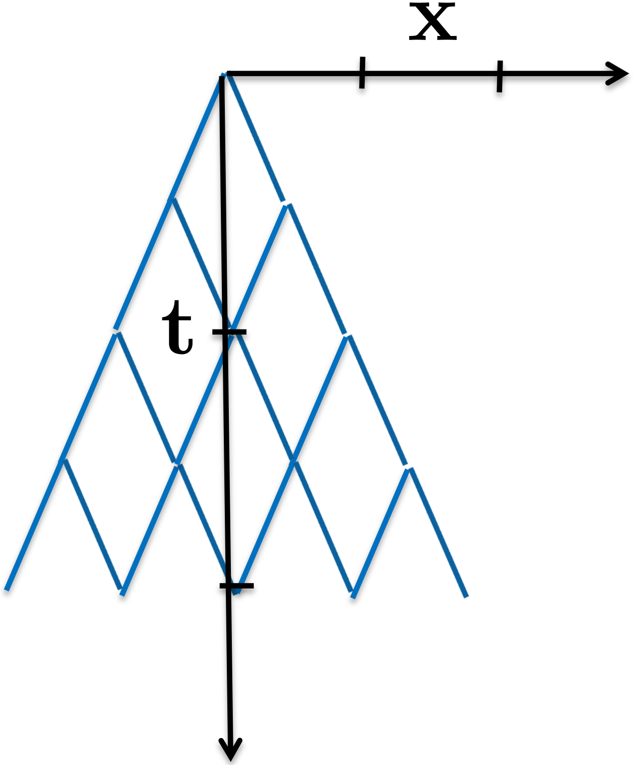
Random Walks



$$\gamma(t+1) = t!$$

$$\gamma(t+1) = \int_0^\infty x^t e^{-x} dx$$

$$f(x,t) = \frac{\gamma(t+1)}{[\gamma([t-x]/2+1)\gamma([x+t]/2+1)](2^t)}$$

$$f(x,t) \approx \frac{e^{-x^2/(2t)}}{(2\pi t)^{1/2}}$$

Random Walks

Stirling's approximation

$$\gamma(\mathbf{x}+\mathbf{1}) \approx (\mathbf{2}\pi\mathbf{x})^{\mathbf{1}/\mathbf{2}}(\mathbf{x}/\mathbf{e})^{\mathbf{x}}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) \approx \frac{\mathbf{2}^{-\mathbf{t}}}{(\mathbf{2}\pi)^{\mathbf{1}/\mathbf{2}}} \frac{\mathbf{t}^{\mathbf{1}/\mathbf{2}}\mathbf{t}^{\mathbf{t}}}{[(\mathbf{t}^{\mathbf{2}}-\mathbf{x}^{\mathbf{2}})/\mathbf{4}]^{\mathbf{t}/\mathbf{2}}[(\mathbf{t}^{\mathbf{2}}-\mathbf{x}^{\mathbf{2}})/\mathbf{4}]^{\mathbf{1}/\mathbf{2}}} \left(\frac{\mathbf{t}-\mathbf{x}}{\mathbf{t}+\mathbf{x}}\right)^{\mathbf{x}/\mathbf{2}}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) \approx \frac{\mathbf{2}}{(\mathbf{2}\pi\mathbf{t})^{\mathbf{1}/\mathbf{2}}} \left[\mathbf{1}-\frac{\mathbf{x}^{\mathbf{2}}}{\mathbf{t}^{\mathbf{2}}}\right]^{-\mathbf{t}/\mathbf{2}} \left[\mathbf{1}-\frac{\mathbf{x}^{\mathbf{2}}}{\mathbf{t}^{\mathbf{2}}}\right]^{-\mathbf{1}/\mathbf{2}} \left(\frac{\mathbf{1}+\mathbf{x}/\mathbf{t}}{\mathbf{1}-\mathbf{x}/\mathbf{t}}\right)^{-\mathbf{x}/\mathbf{2}}$$

Random Walks

Consider log of this expression

$$\log\left[\mathbf{1}-\frac{\mathbf{x^2}}{\mathbf{t^2}}\right]^{-\mathbf{t}/\mathbf{2}} \approx \frac{\mathbf{x^2}}{\mathbf{2t}}$$

$$\log\left[\mathbf{1}-\frac{\mathbf{x^2}}{\mathbf{t^2}}\right]^{-\mathbf{1}/\mathbf{2}} \approx \frac{\mathbf{x^2}}{\mathbf{2t^2}}$$

$$\log\left(\frac{\mathbf{1}+\mathbf{x}/\mathbf{t}}{\mathbf{1}-\mathbf{x}/\mathbf{t}}\right)^{-\mathbf{x}/\mathbf{2}} \approx \log\left(\mathbf{1}+\frac{\mathbf{2x}}{\mathbf{t}}\right)^{-\mathbf{x}/\mathbf{2}} \approx -\frac{\mathbf{x^2}}{\mathbf{t}}$$

Random Walks

Gathering, throwing away the second term, and re-exponentiating

$$\mathbf{f(x,t) \propto e^{-x^2/(2t)}}$$

$$\int_{-\infty}^{\infty} \mathbf{f(x,t)dx = 1} \quad \Longrightarrow \quad \mathbf{f(x,t) = \frac{e^{-x^2/2t}}{(2\pi t)^{1/2}}}$$

How would this calculation change for 2D and 3D random walks?

Random Walks

The distribution function shapes stay the same, only their normalization changes.

$$\mathbf{f(R,t) \propto e^{-R^2/2t}}$$

For 2D

$$\int_{-\infty}^{\infty} \mathbf{f(R,t)dA = 1} \quad \Longrightarrow \quad \mathbf{f(R,t) = \frac{e^{-R^2/2t}}{2\pi t}}$$

Random Walks

The distribution function shapes stay the same, only their normalization changes.

$$\mathbf{f(r,t) \propto e^{-r^2/2t}}$$

For 3D

$$\int_{-\infty}^{\infty} \mathbf{f(r,t)dV = 1} \quad \Rightarrow \quad \mathbf{f(r,t) = \frac{e^{-r^2/2t}}{(2\pi t)^{3/2}}}$$

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Saturation of Steady-State Integral

3D system-

$$\mathbf{f(r,t)} = \frac{\mathbf{e^{-r^2/(4Dt)}}}{\mathbf{(4\pi Dt)^{3/2}}}$$

$$\mathbf{F(r)} = \int_0^\infty \mathbf{f(r,t)dt}$$

Change of variable-

$$\mathbf{x} = \frac{\mathbf{r^2}}{\mathbf{4Dt}}$$

$$\int_0^{\mathbf{t_H}} \mathbf{f(r,t)dt} = \frac{\mathbf{1}}{\mathbf{Dr}} \int_{\mathbf{r^2/Dt_H}}^\infty \mathbf{x^{-1/2}e^{-x}dx}$$

Saturation of Steady-State Integral

$$\gamma(\mathbf{t+1}) = \int_0^\infty \mathbf{x^t e^{-x} dx}$$

$$\mathbf{F(r)} = \int_0^\infty \mathbf{f(r,t)dt}$$

$$= \frac{\mathbf{1}}{\mathbf{Dr}} \int_{\mathbf{r^2/Dt_H}}^\infty \mathbf{x^{-1/2} e^{-x} dx}$$

$$= \frac{\mathbf{1}}{\mathbf{Dr}} [\mathbf{1.0 - \Gamma(1/2, r^2/Dt_H)}]$$

Incomplete gamma function

Repeat this for 1D and 2D systems

Saturation of Steady-State Integral

2D system-

$$\gamma(\mathbf{t}+\mathbf{1}) = \int_{\mathbf{0}}^{\infty} \mathbf{x}^{\mathbf{t}} \mathbf{e}^{-\mathbf{x}} \mathbf{dx}$$

$$\mathbf{f}(\mathbf{R}, \mathbf{t}) = \frac{\mathbf{e}^{-\mathbf{R}^2/(4\mathbf{Dt})}}{(4\pi\mathbf{Dt})}$$

$$\mathbf{F}(\mathbf{R}) = \frac{\mathbf{1}}{\mathbf{DR}} \int_{\mathbf{R}^2/\mathbf{Dt}_{\mathrm{H}}}^{\infty} \mathbf{x}^{-\mathbf{1}} \mathbf{e}^{-\mathbf{x}} \mathbf{dx}$$

$$= \frac{\mathbf{1}}{\mathbf{DR}} [\mathbf{1.0} - \mathbf{\Gamma}(\mathbf{0}, \mathbf{R}^2/\mathbf{Dt}_{\mathrm{H}})]$$

Saturation of Steady-State Integral

1D system-

$$\gamma(\mathbf{t}+\mathbf{1}) = \int_{\mathbf{0}}^{\infty} \mathbf{x}^{\mathbf{t}} \mathbf{e}^{-\mathbf{x}} \mathbf{dx}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) = \frac{\mathbf{e}^{-\mathbf{x}^2/(4\mathbf{Dt})}}{(4\pi\mathbf{Dt})^{1/2}}$$

$$\mathbf{F}(\mathbf{x}) = \frac{\mathbf{1}}{\mathbf{Dx}} \int_{\mathbf{x}^2/\mathbf{Dt}_{\mathrm{H}}}^{\infty} \mathbf{y}^{-3/2} \mathbf{e}^{-\mathbf{y}} \mathbf{dy}$$

$$= \frac{\mathbf{1}}{\mathbf{Dx}} [\mathbf{1.0} - \Gamma(-\mathbf{1}/\mathbf{2}, \mathbf{x}^2/\mathbf{Dt}_{\mathrm{H}})]$$

Extragalactic Deflections

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Random Walks

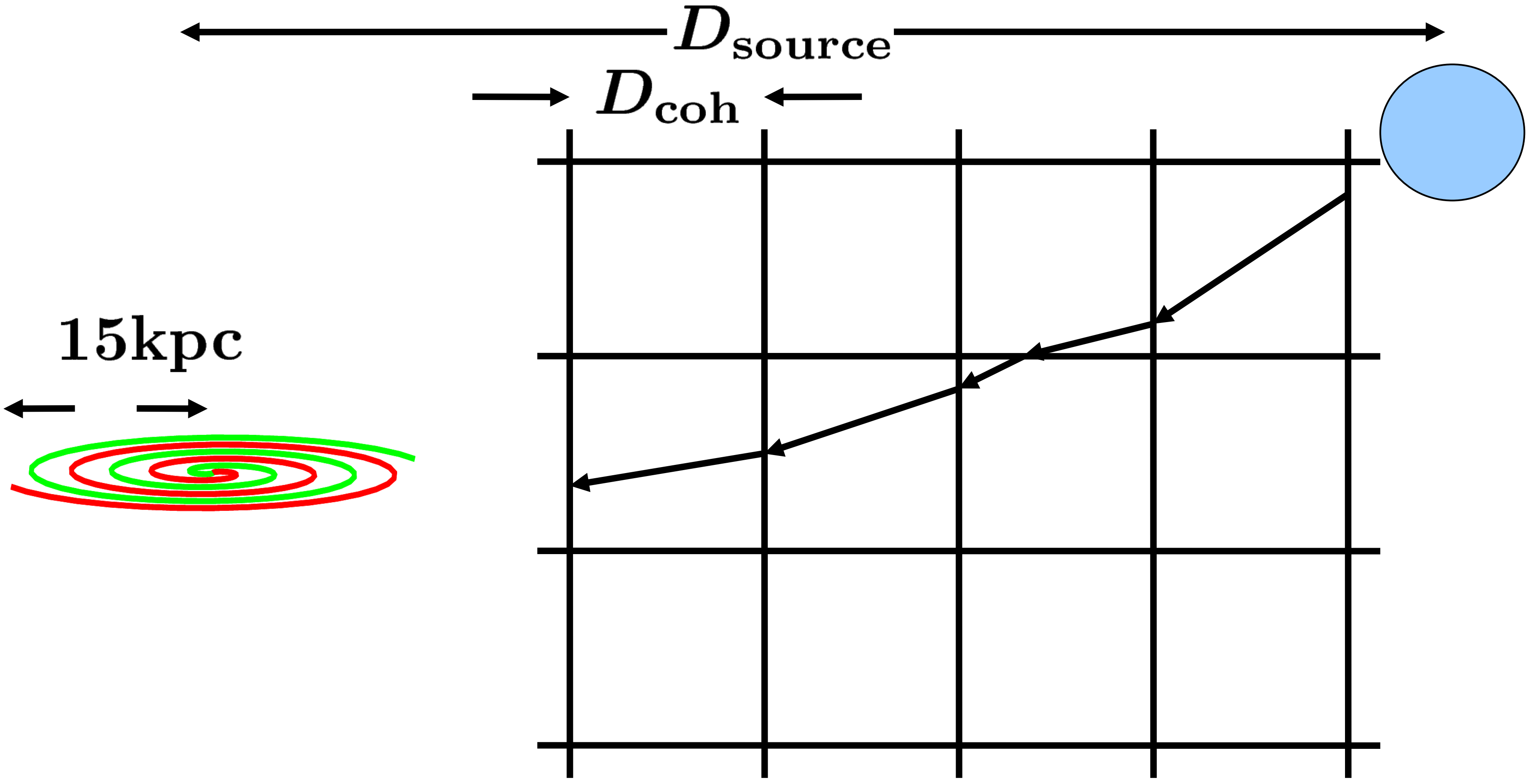
Consider log of this expression

$$\log\left[1-\frac{\mathbf{x^2}}{\mathbf{t^2}}\right]^{-\mathbf{t}} \approx \frac{\mathbf{x^2}}{\mathbf{t}}$$

$$\log\left[1-\frac{\mathbf{x^2}}{\mathbf{t^2}}\right]^{-\mathbf{1/2}} \approx \frac{\mathbf{x^2}}{\mathbf{2t^2}}$$

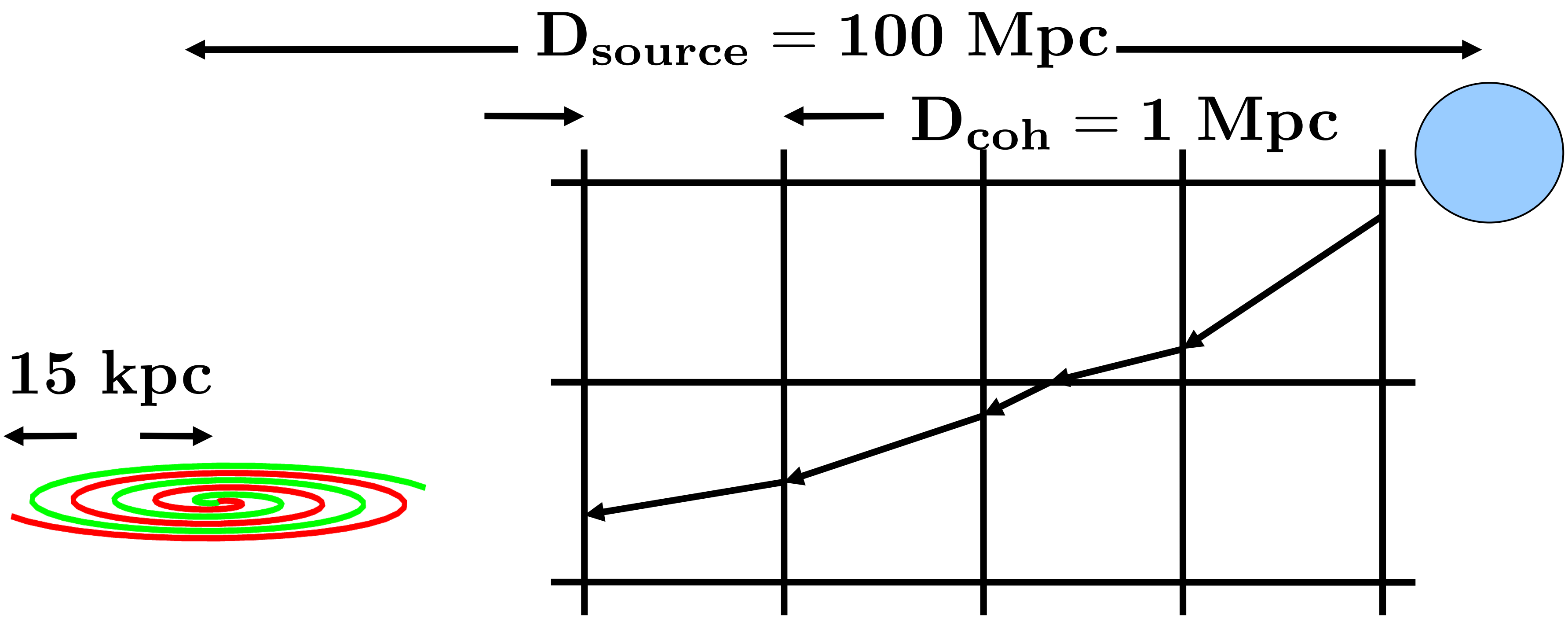
$$\log\left(\frac{\mathbf{1+x/t}}{\mathbf{1-x/t}}\right)^{-\mathbf{x}} \approx \log\left(\mathbf{1}+\frac{\mathbf{2x}}{\mathbf{t}}\right)^{-\mathbf{x/2}} \approx -\frac{\mathbf{2x^2}}{\mathbf{t}}$$

Those that Leave are Replaced by those that Arrive



$$N_{\rm cells} = \frac{D_{\rm source}}{D_{\rm coh}}, \; \Delta\theta_{\rm cell} = \frac{D_{\rm coh}}{R_{\rm Larmor}}, \quad \Delta\theta_{\rm tot} \approx N_{\rm cells}^{1/2} \Delta\theta_{\rm cell}$$

Those that Leave are Replaced by those that Arrive



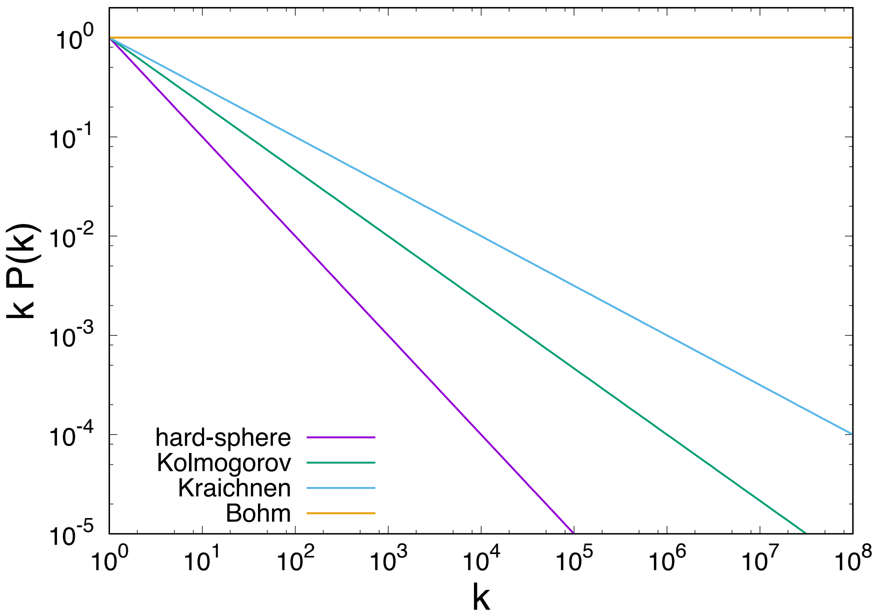
$$R_{Larmor} = 1\ \text{Gpc} \left(\frac{1}{Z}\right)\left(\frac{E}{10^{20}\ \text{eV}}\right)\left(\frac{0.01\ \text{nG}}{B}\right)$$

$$\Delta\theta_{tot} \approx N_{cells}^{1/2} \Delta\theta_{cell} \qquad [N_{cells} = 100]$$

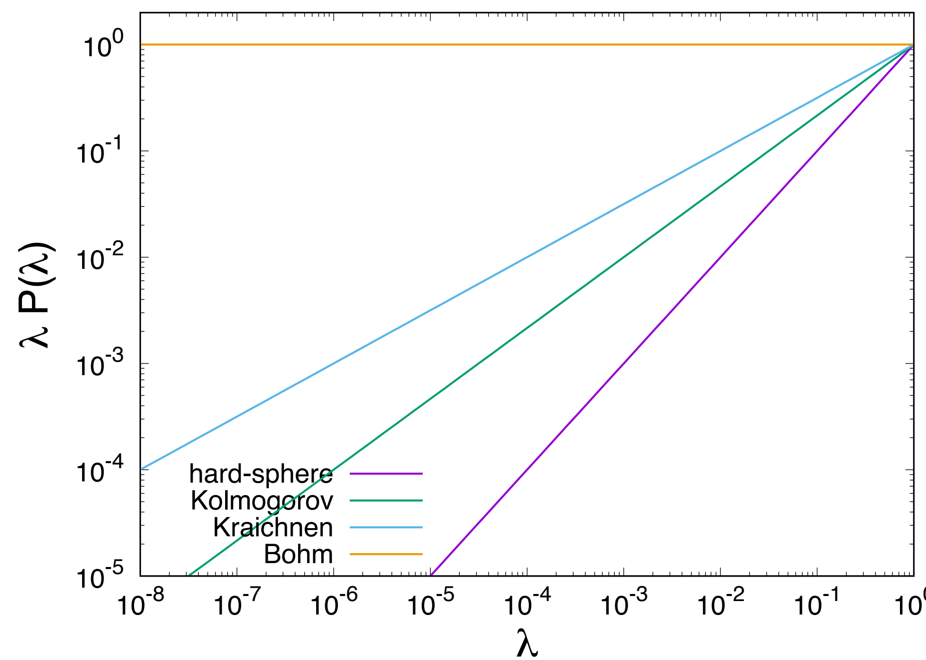
For 10^{20} eV protons: $\Delta\theta_{tot} = 0.01$ rad. (ie. 0.6°)

For 10^{20} eV iron: $\Delta\theta_{tot} = 0.26$ rad. (ie. 15°)

Supernovae as Drivers of Galactic Turbulence



$$\mathbf{P(k)} = \frac{\mathbf{dP}}{\mathbf{dk}} = \mathbf{P_0}\left(\frac{\mathbf{k}}{\mathbf{k_0}}\right)^{-\alpha}$$



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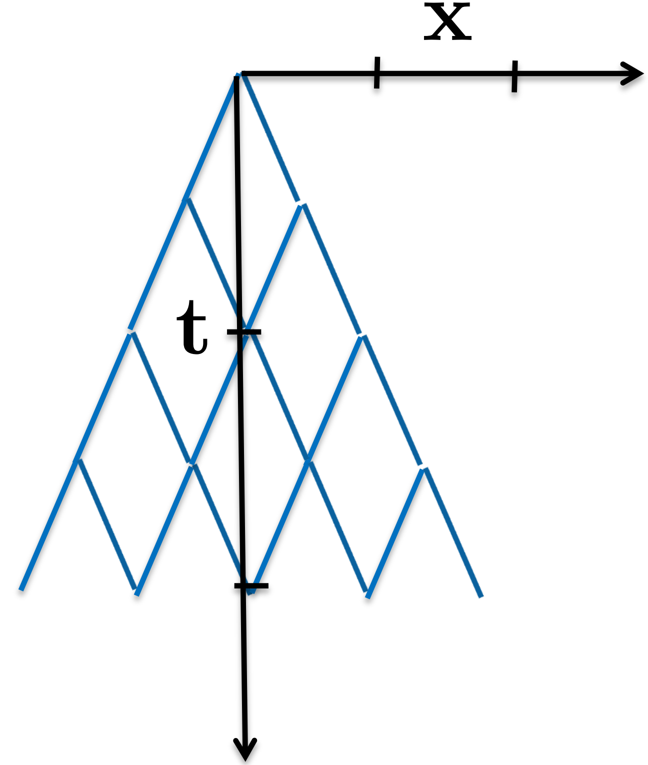
Random Walks



$$\gamma(\mathbf{t}+\mathbf{1}) = \mathbf{t}!$$

$$\gamma(\mathbf{t}+\mathbf{1}) = \int_{\mathbf{0}}^{\infty} \mathbf{x}^{\mathbf{t}} \mathbf{e}^{-\mathbf{x}} \mathbf{dx}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) = \frac{\gamma(\mathbf{2t}+\mathbf{1})}{[\gamma([\mathbf{t}-\mathbf{x}]+\mathbf{1})\gamma([\mathbf{x}+\mathbf{t}]+\mathbf{1})](\mathbf{2}^{\mathbf{2t}})}$$

Random Walks

Stirling's approximation

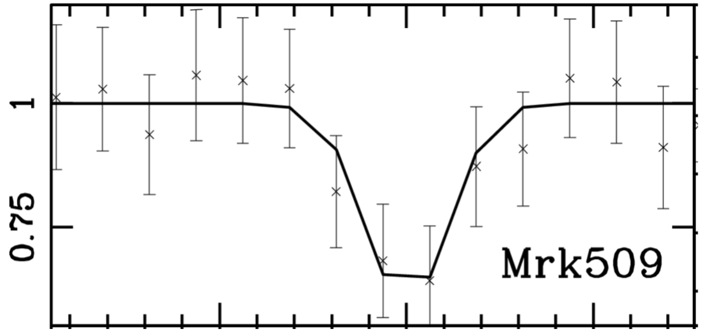
$$\gamma(\mathbf{x}+\mathbf{1}) \approx (\mathbf{2}\pi\mathbf{x})^{\mathbf{1}/\mathbf{2}}(\mathbf{x}/\mathbf{e})^{\mathbf{x}}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) = \frac{\gamma(\mathbf{2t}+\mathbf{1})}{[\gamma([\mathbf{t}-\mathbf{x}]+\mathbf{1})\gamma([\mathbf{x}+\mathbf{t}]+\mathbf{1})](\mathbf{2}^{\mathbf{2t}})}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) \approx \frac{\mathbf{2}^{-\mathbf{2t}}}{(\mathbf{2}\pi)^{\mathbf{1}/\mathbf{2}}} \frac{(\mathbf{2t})^{\mathbf{1}/\mathbf{2}}(\mathbf{2t})^{\mathbf{2t}}}{(\mathbf{t}^{\mathbf{2}}-\mathbf{x}^{\mathbf{2}})^{\mathbf{t}}(\mathbf{t}^{\mathbf{2}}-\mathbf{x}^{\mathbf{2}})^{\mathbf{1}/\mathbf{2}}} \left(\frac{\mathbf{t}-\mathbf{x}}{\mathbf{t}+\mathbf{x}}\right)^{\mathbf{x}}$$

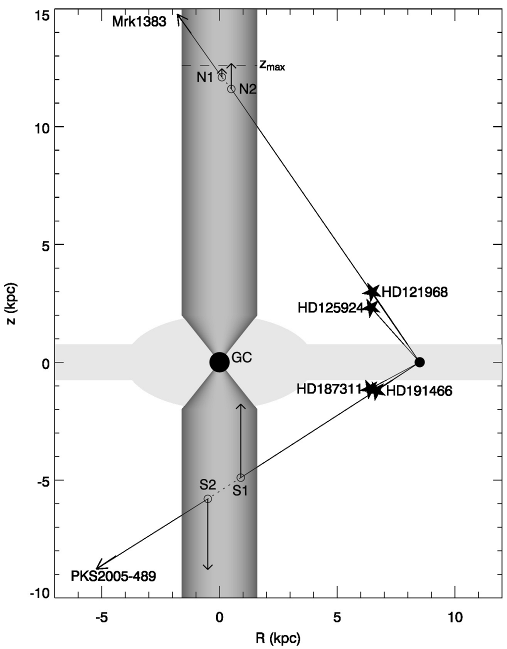
$$\mathbf{f}(\mathbf{x},\mathbf{t}) \approx \frac{\mathbf{2}}{(\mathbf{2}\pi\mathbf{t})^{\mathbf{1}/\mathbf{2}}} \left[\mathbf{1}-\frac{\mathbf{x}^{\mathbf{2}}}{\mathbf{t}^{\mathbf{2}}}\right]^{-\mathbf{t}} \left[\mathbf{1}-\frac{\mathbf{x}^{\mathbf{2}}}{\mathbf{t}^{\mathbf{2}}}\right]^{-\mathbf{1}/\mathbf{2}} \left(\frac{\mathbf{1}+\mathbf{x}/\mathbf{t}}{\mathbf{1}-\mathbf{x}/\mathbf{t}}\right)^{-\mathbf{x}}$$

Advection With the Bubbles?



Suzaku and Chandra X-ray observations of bright AGN (Mkr 501, PKS 2155, NGC 3783) indicated the presence of a hot local absorber surrounding the Milky Way

The Doppler shift of absorption lines for lines-of-sight which pass through these bubbles reveals evidence of a coherent velocity flow



Keeney et al. 2006