Lecture (1) Plan- RADIATION FIELDS:

- UHECR Observational Status
- Extragalactic radiation fields
- Cosmic ray proton interaction rates with extragalactic radiation fields
- Cosmic ray nuclei interaction rates with extragalactic radiation fields
- Application- what one can infer from spectral and composition information alone

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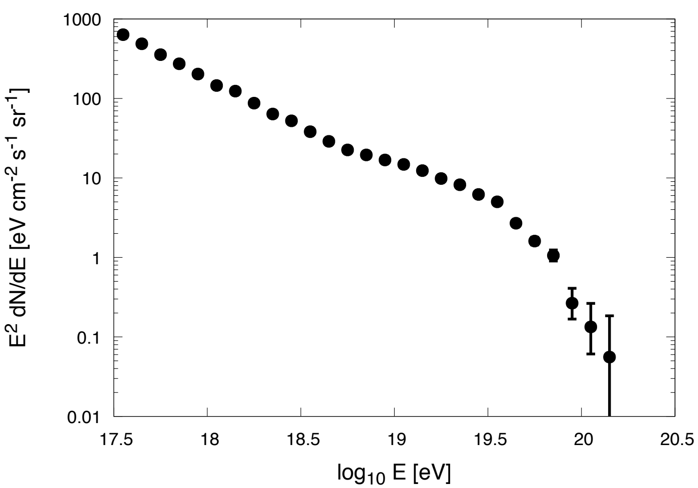
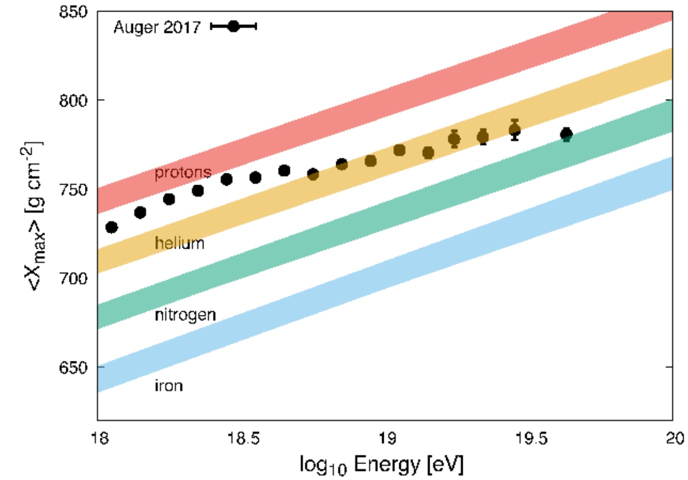
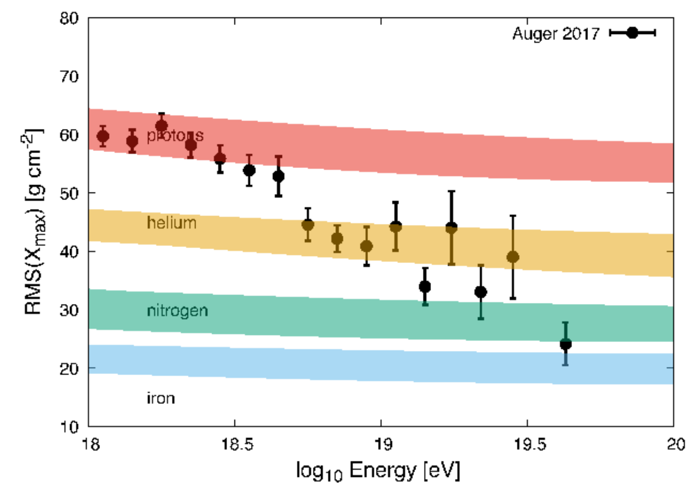
UHECR: The Observational Status

Spectrum

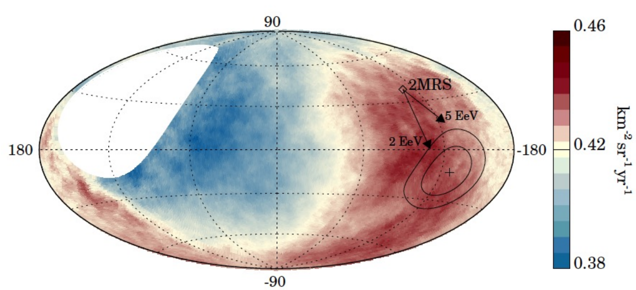


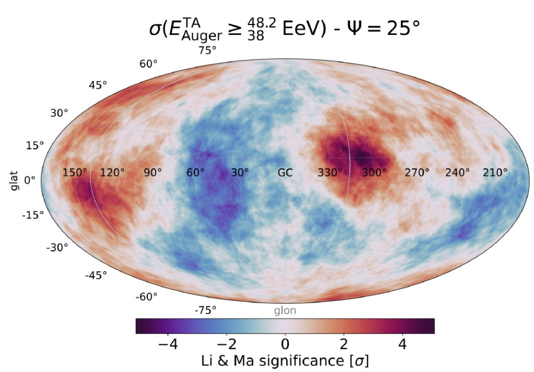
Composition





Anisotropy





Pierre Auger Collaboration. ApJ. 935 (2022)

Caccianiga et al. for the Auger and TA Collaborations. PoS (ICRC2023) 521

Cosmic Radiation Fields- Energy Density

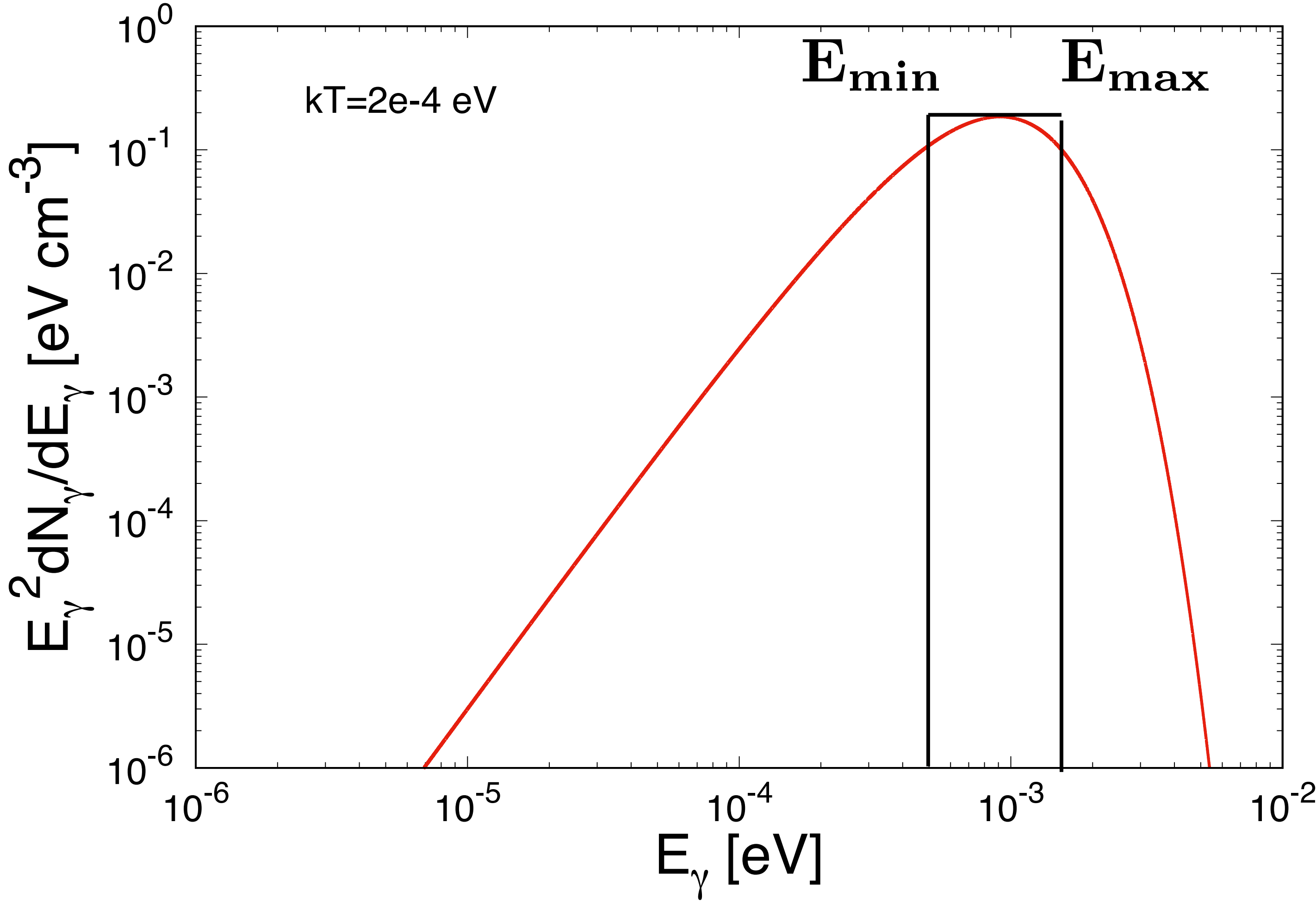
$$\mathbf{U}_\gamma = \int_{\mathbf{0}}^{\infty} \mathbf{E}_\gamma \frac{\mathbf{dn}}{\mathbf{dE}_\gamma} \mathbf{dE}_\gamma$$

$$= \int_{\mathbf{0}}^{\infty} \mathbf{E}_\gamma^{\mathbf{2}} \frac{\mathbf{dn}}{\mathbf{dE}_\gamma} \mathbf{dlnE}_\gamma$$

Note- this amounts to a visual inspection version of Laplace’s integral method

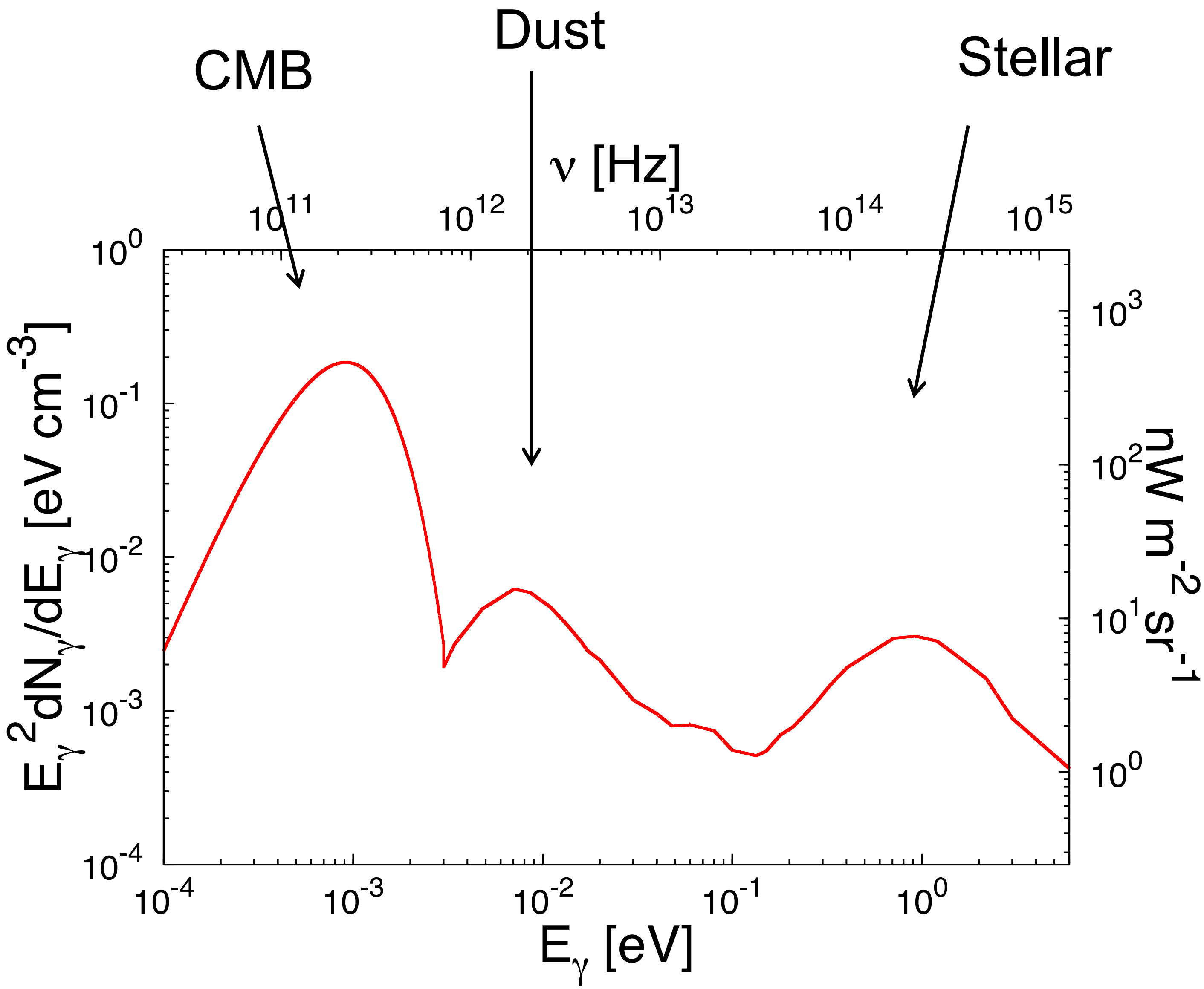
Cosmic Radiation Fields- Energy Density

Estimating $\mathbf{U_\gamma}$



$$\mathbf{U_\gamma \approx E^2 \frac{dN}{dE}\bigg|_{peak} \ln\left(\frac{E_{max}}{E_{min}}\right) \approx 0.2\ eV\ cm^{-3}}$$

Cosmic Radiation Fields- Energy Density

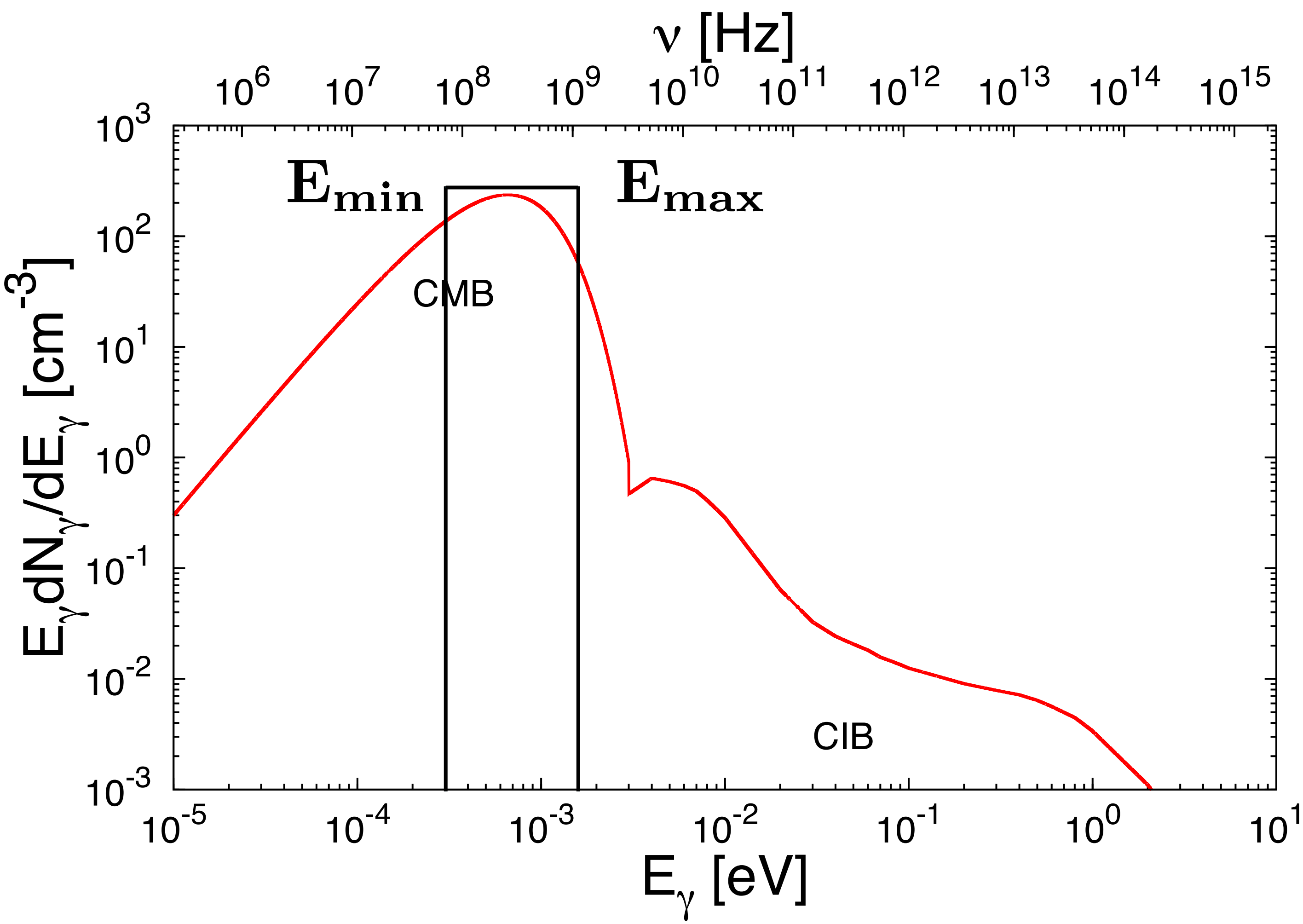


Cosmic Radiation Fields- Number Density

$$\mathbf{n}_\gamma = \int_{\mathbf{0}}^{\infty} \frac{\mathbf{dn}}{\mathbf{dE}_\gamma} \mathbf{dE}_\gamma$$

$$= \int_{\mathbf{0}}^{\infty} \mathbf{E}_\gamma \frac{\mathbf{dn}}{\mathbf{dE}_\gamma} \mathbf{d\ln E}_\gamma$$

Cosmic Radiation Fields- Number Density

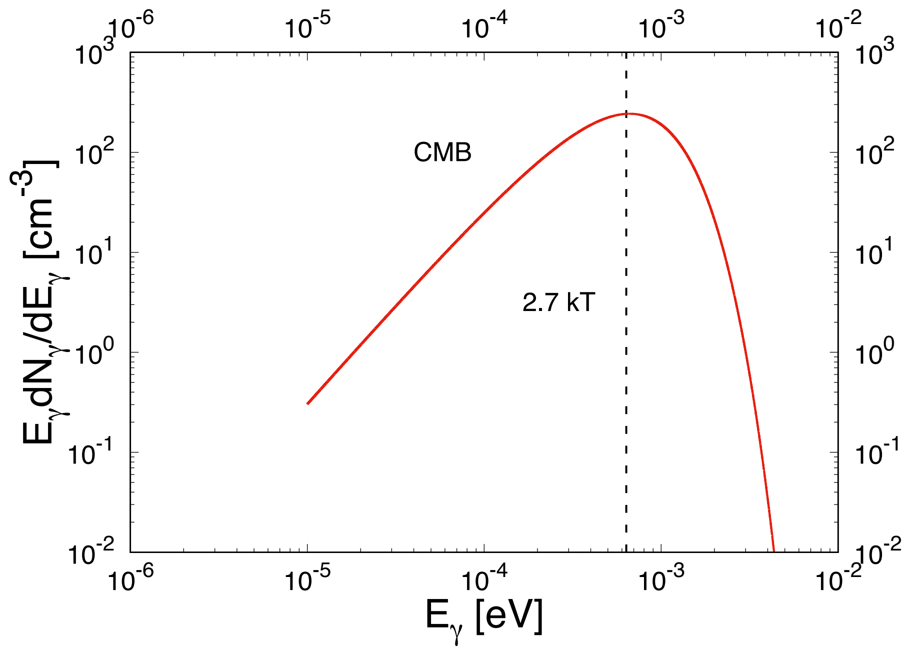


Estimating $\mathbf{n_\gamma}$

$$\mathbf{n_\gamma \approx E\frac{dN}{dE}\bigg|_{peak} \ln\left(\frac{E_{max}}{E_{min}}\right) \approx 400\ cm^{-3}}$$

CMB- Total Number Density

$$\frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} = \frac{\mathbf{8}\pi}{(\mathbf{hc})^\mathbf{3}} \frac{\epsilon_\gamma^\mathbf{2}}{\mathbf{e}^{\epsilon_\gamma/\mathbf{kT}} - \mathbf{1}}$$



$$\mathbf{n}_\gamma^\mathbf{BB} = \frac{\mathbf{8}\pi(\mathbf{kT})^\mathbf{3}}{(\mathbf{hc})^\mathbf{3}} \int_\mathbf{0}^\infty \frac{\mathbf{x}^\mathbf{2}}{\mathbf{e}^\mathbf{x} - \mathbf{1}} \mathbf{dx}$$

$$\frac{\mathbf{8}\pi(\mathbf{kT}_\mathbf{CMB})^\mathbf{3}}{(\mathbf{hc})^\mathbf{3}} \approx \mathbf{170\ cm}^{-\mathbf{3}}$$

$$\zeta(\mathbf{x}) = \sum_{\mathbf{n}=\mathbf{1}}^\infty \frac{\mathbf{1}}{\mathbf{n}^\mathbf{x}}$$

$$\mathbf{n}_\gamma^\mathbf{CMB} = \mathbf{8}\pi \frac{(\mathbf{kT}_\mathbf{CMB})^\mathbf{3}}{(\mathbf{hc})^\mathbf{3}} \gamma(\mathbf{3})\zeta(\mathbf{3}) \approx \mathbf{400\ cm}^{-\mathbf{3}}$$

CMB- Total Number Density

For a blackbody radiation field distribution, with temperature T,

$$\mathbf{n_\gamma = 8\pi \frac{(kT)^3}{(hc)^3} \gamma(3)\zeta(3) \approx 400\ cm^{-3}}$$

$$\mathbf{U_\gamma = 8\pi \frac{(kT)^4}{(hc)^3} \gamma(4)\zeta(4) = 0.25\ eV\ cm^{-3}}$$

$$\mathbf{\langle E_\gamma \rangle = \frac{\int_0^\infty E_\gamma \frac{dn}{dE_\gamma} dE_\gamma}{\int_0^\infty \frac{dn}{dE_\gamma} dE_\gamma} = \frac{\Gamma(4)\zeta(4)}{\Gamma(3)\zeta(3)} kT \approx 2.7\ kT}$$

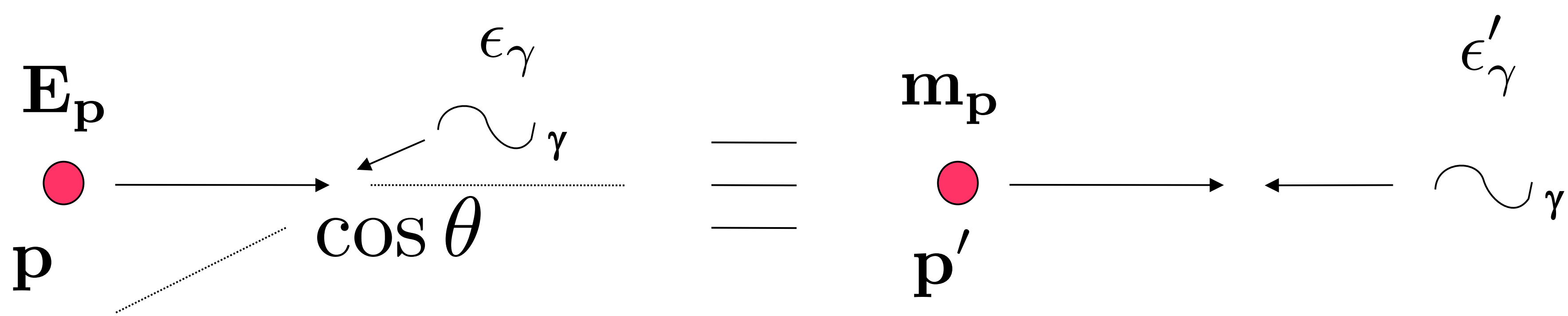
Cosmic Ray Proton Energy Losses

The Interaction Rate

radiation field → cross-section

$$\mathbf{R} = \int_{\mathbf{0}}^{\infty} \mathbf{d}\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_{-\mathbf{1}}^{\mathbf{1}} \frac{\mathbf{1}}{\mathbf{2}} \mathbf{d}(\cos\theta)\sigma(\cos\theta)\mathbf{1} + \beta\cos\theta)$$

All values above in lab frame



The Interaction Rate

$$\mathbf{R} = \int_{\mathbf{0}}^{\infty} \mathbf{d}\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_{-\mathbf{1}}^{\mathbf{1}} \frac{\mathbf{1}}{\mathbf{2}} \mathbf{d}(\cos\theta)\sigma(\cos\theta)(\mathbf{1} + \beta\cos\theta)$$

Since, $\epsilon'_\gamma \mathbf{m_p} = \epsilon_\gamma \mathbf{E_p}(\mathbf{1} + \beta\cos\theta)$

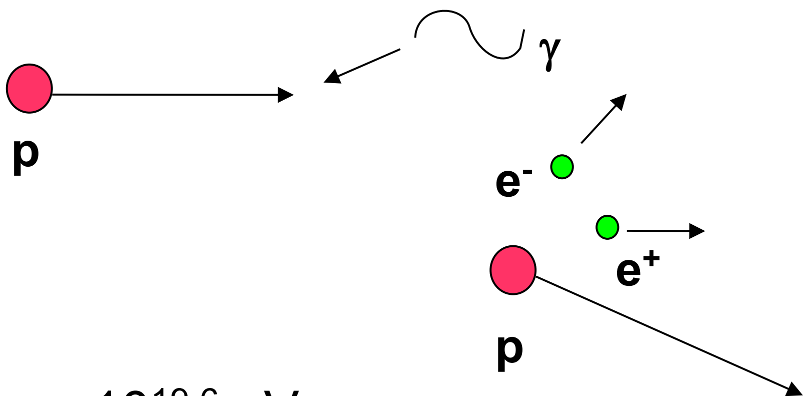
$$(\mathbf{1} + \beta\cos\theta)\mathbf{d}\cos\theta = \frac{\epsilon'_\gamma \mathbf{m_p}}{\epsilon_\gamma \mathbf{E_p}} \frac{\mathbf{d}(\epsilon'_\gamma \mathbf{m_p})}{\epsilon_\gamma \mathbf{E_p}}$$

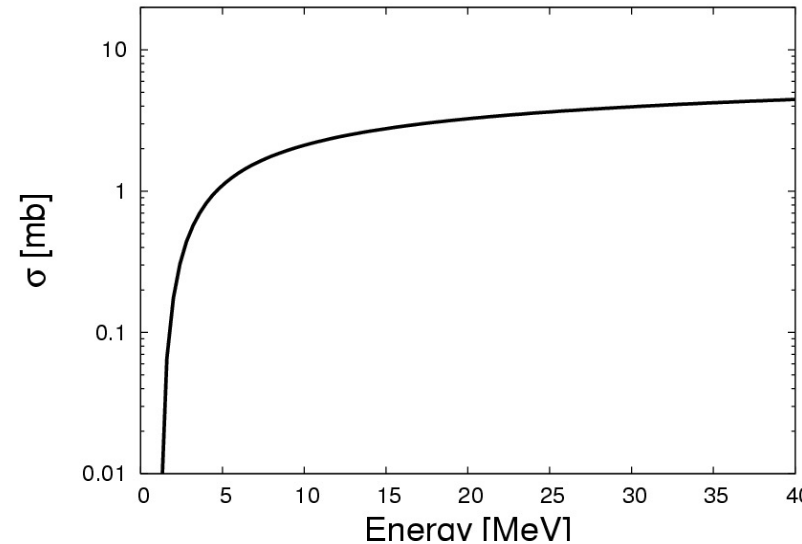
$$\mathbf{R} = \frac{\mathbf{1}}{\mathbf{2}} \int_{\mathbf{0}}^{\infty} \mathbf{d}\epsilon_\gamma \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_{\mathbf{0}}^{\mathbf{2}\epsilon_\gamma \mathbf{E_p}} \mathbf{d}(\epsilon'_\gamma \mathbf{m_p}) \frac{\epsilon'_\gamma \mathbf{m_p}}{\epsilon_\gamma^{\mathbf{2}} \mathbf{E_p^2}} \sigma(\epsilon')$$

$$= \frac{\mathbf{m_p^2}}{\mathbf{2E_p^2}} \int_{\mathbf{0}}^{\infty} \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_{\mathbf{0}}^{\mathbf{2}\epsilon_\gamma \frac{\mathbf{E_p}}{\mathbf{m_p}}} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma(\epsilon'_\gamma)$$

Cosmic Ray Proton Interactions

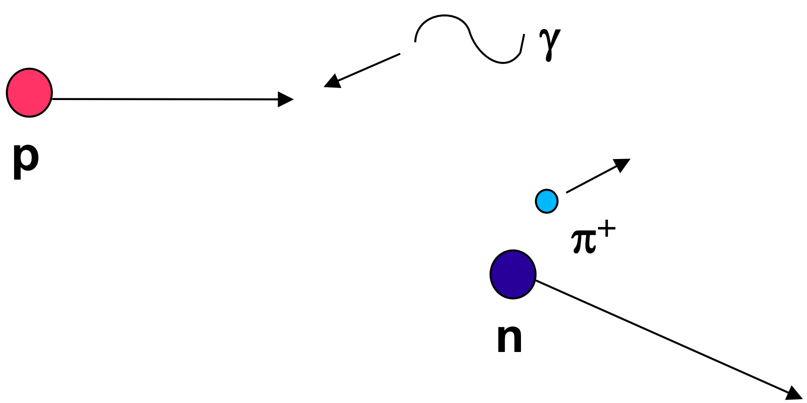
For $E_{proton}<10^{19.6}$ eV

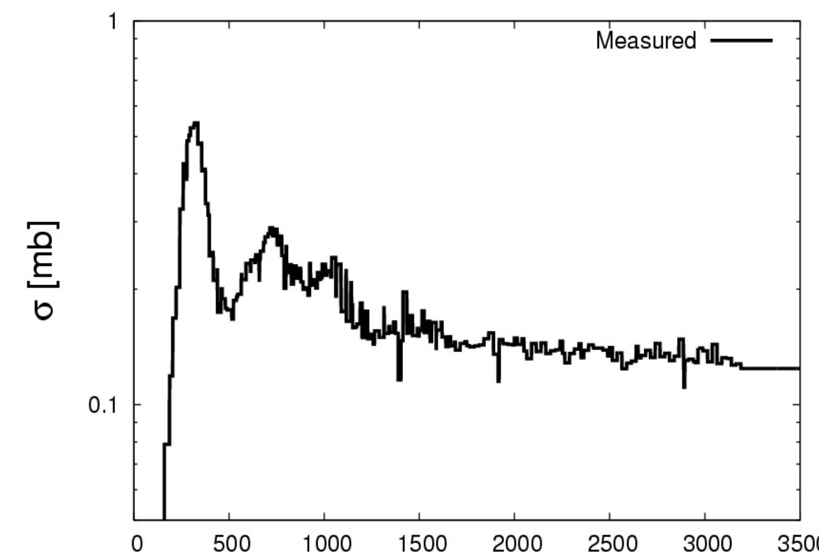




$E_{\gamma}^{th} \sim 1\text{MeV}$

For $E_{proton}>10^{19.6}$ eV





$E_{\gamma}^{th} \sim 140\text{ MeV}$

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Threshold Energy- Proton Pair Production

$$\mathbf{(E_p + E_\gamma)^2 - (p_p - E_\gamma)^2 = (m_p + 2m_e)^2}$$

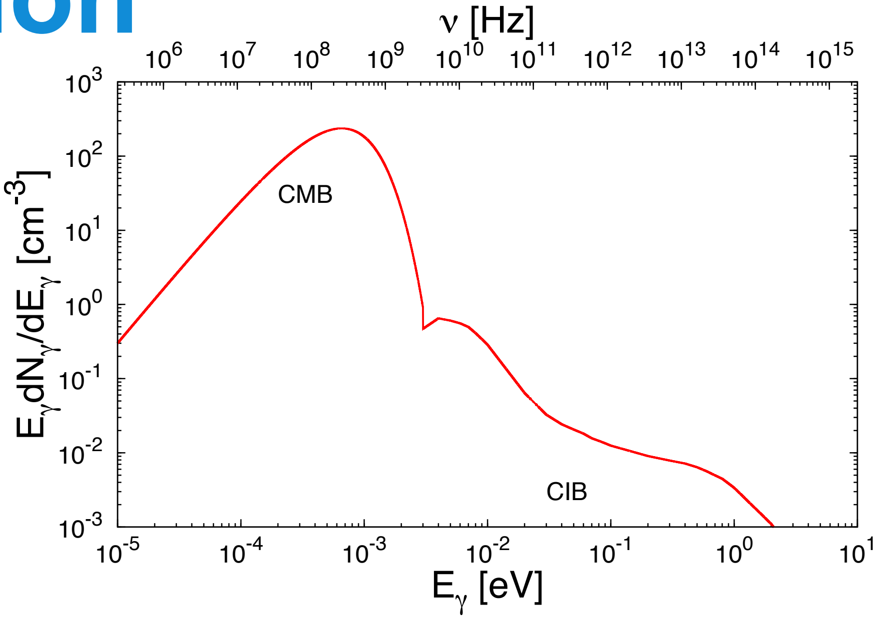
$$\mathbf{m_p^2 + 2E_pE_\gamma + 2p_pE_\gamma \approx m_p^2 + 4m_pm_e}$$

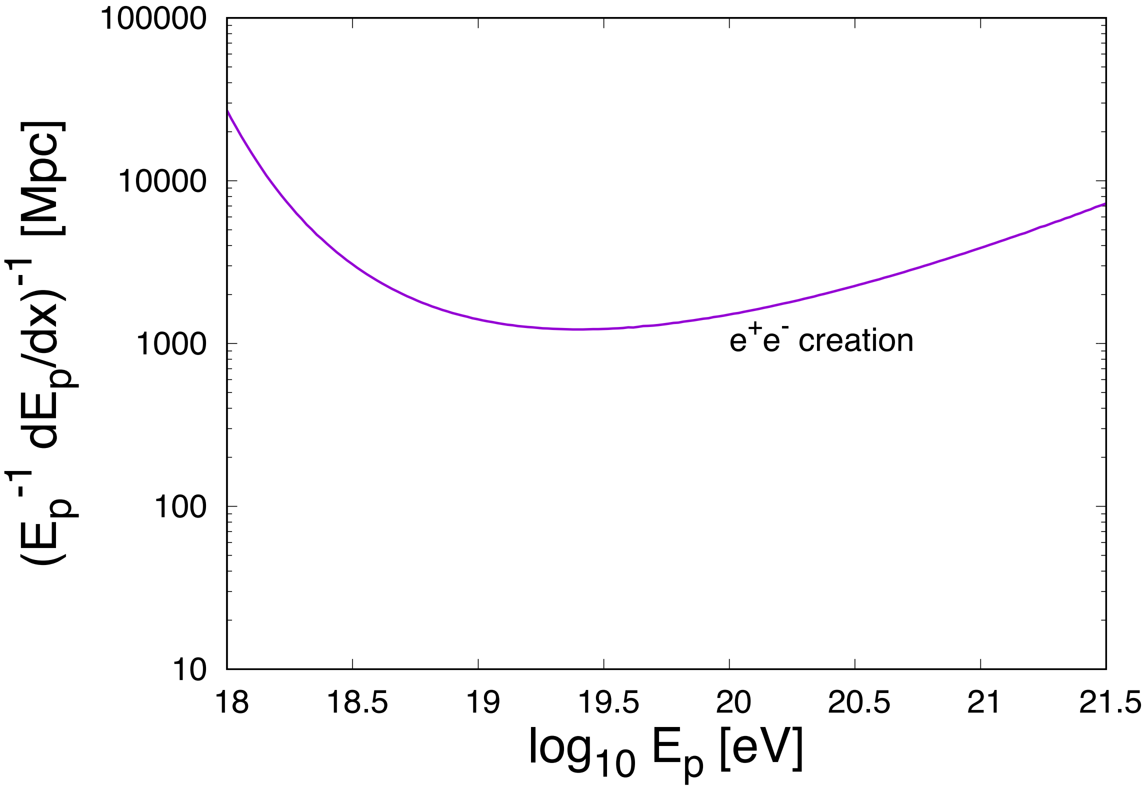
$$\boxed{\mathbf{E_p \approx \frac{m_e}{E_\gamma} m_p}} \approx \left(\frac{\mathbf{0.5 \times 10^6}}{\mathbf{6 \times 10^{-4}}}\right) \mathbf{0.9 \times 10^9 = 8 \times 10^{17}\ eV}$$

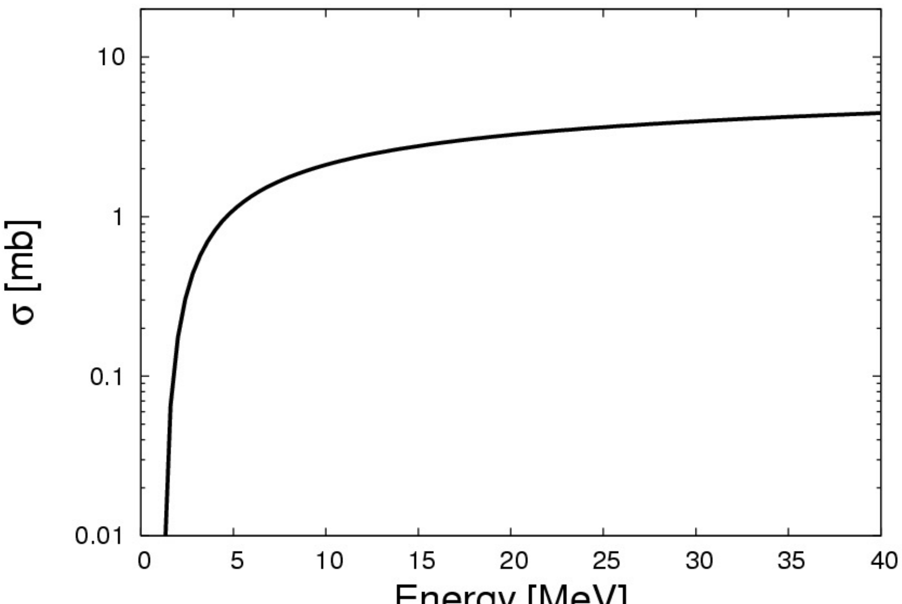
Repeat this calculation for pion production

Energy Loss Rate- Pair Production

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^2} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$







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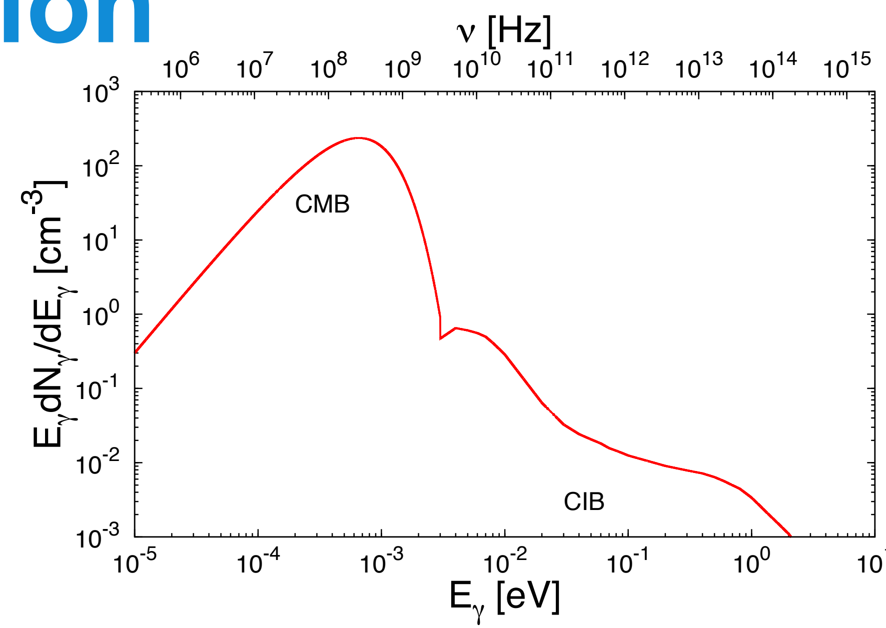
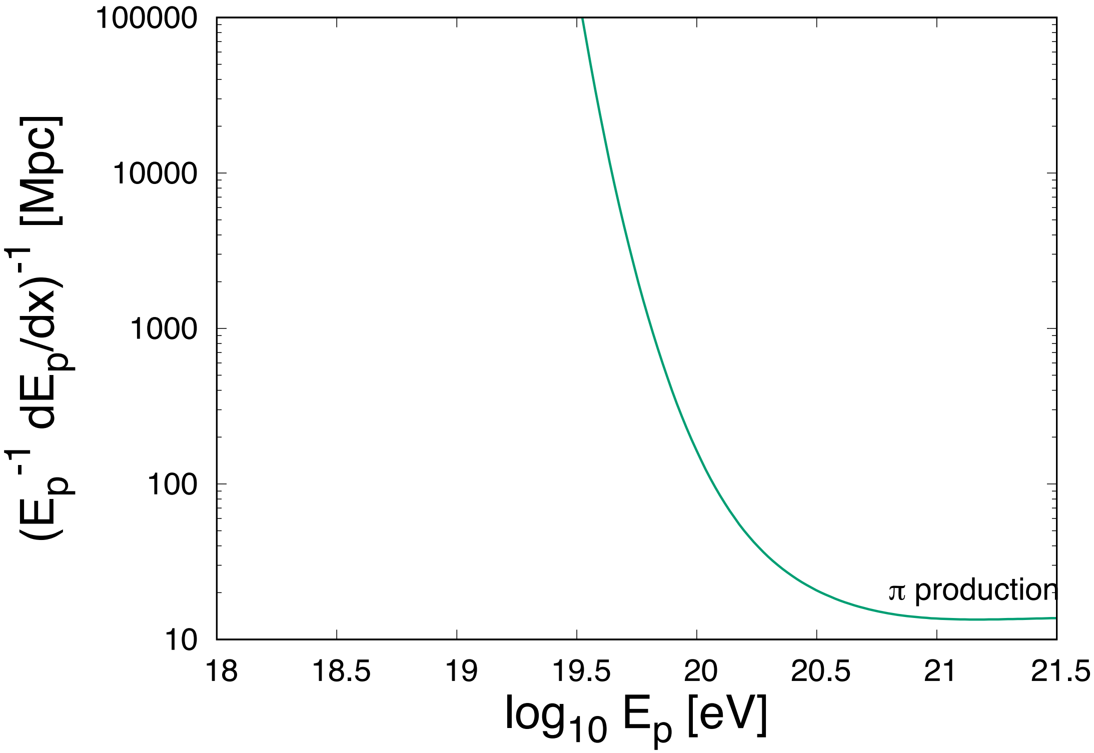
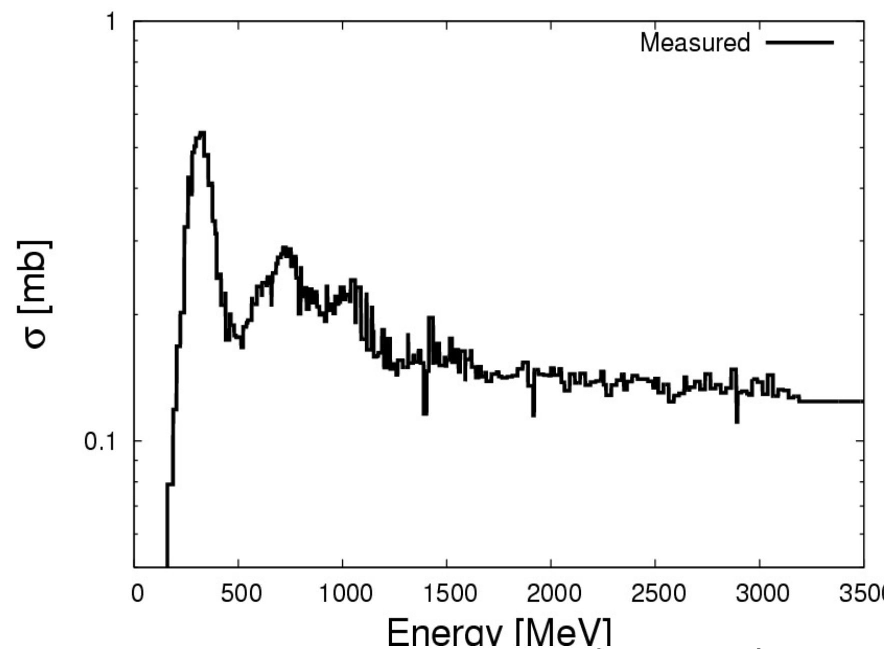
Energy Loss Rate- Pion Production

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^2} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$







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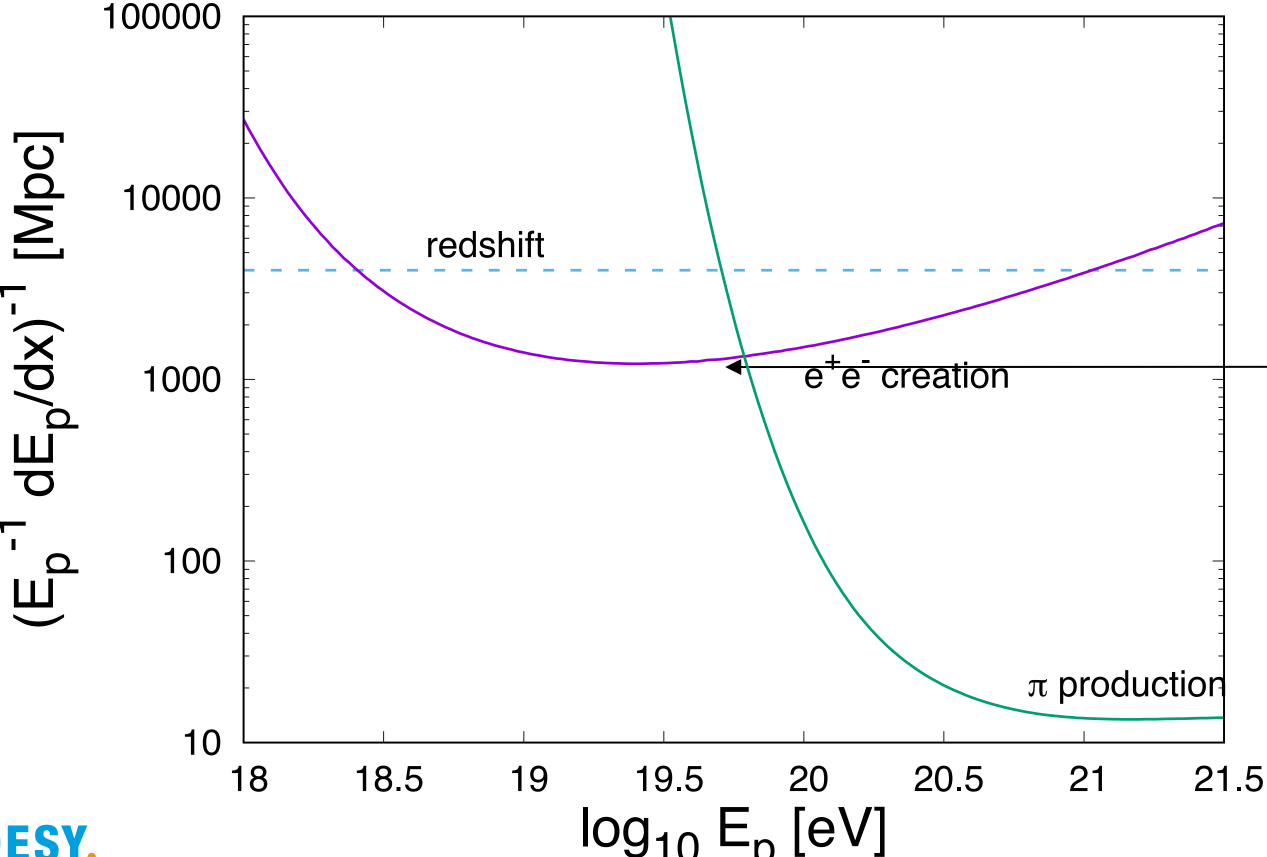
Energy Loss Rates due to Proton Interactions

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^2} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma)\mathbf{K_p}$$

where R is the energy loss rate

where K_p is the inelasticity



$$\approx \frac{\mathbf{m_p}}{\mathbf{m_e}} \frac{\mathbf{1}}{\mathbf{n_{CMB}}\sigma_{\mathbf{p}\gamma}}$$

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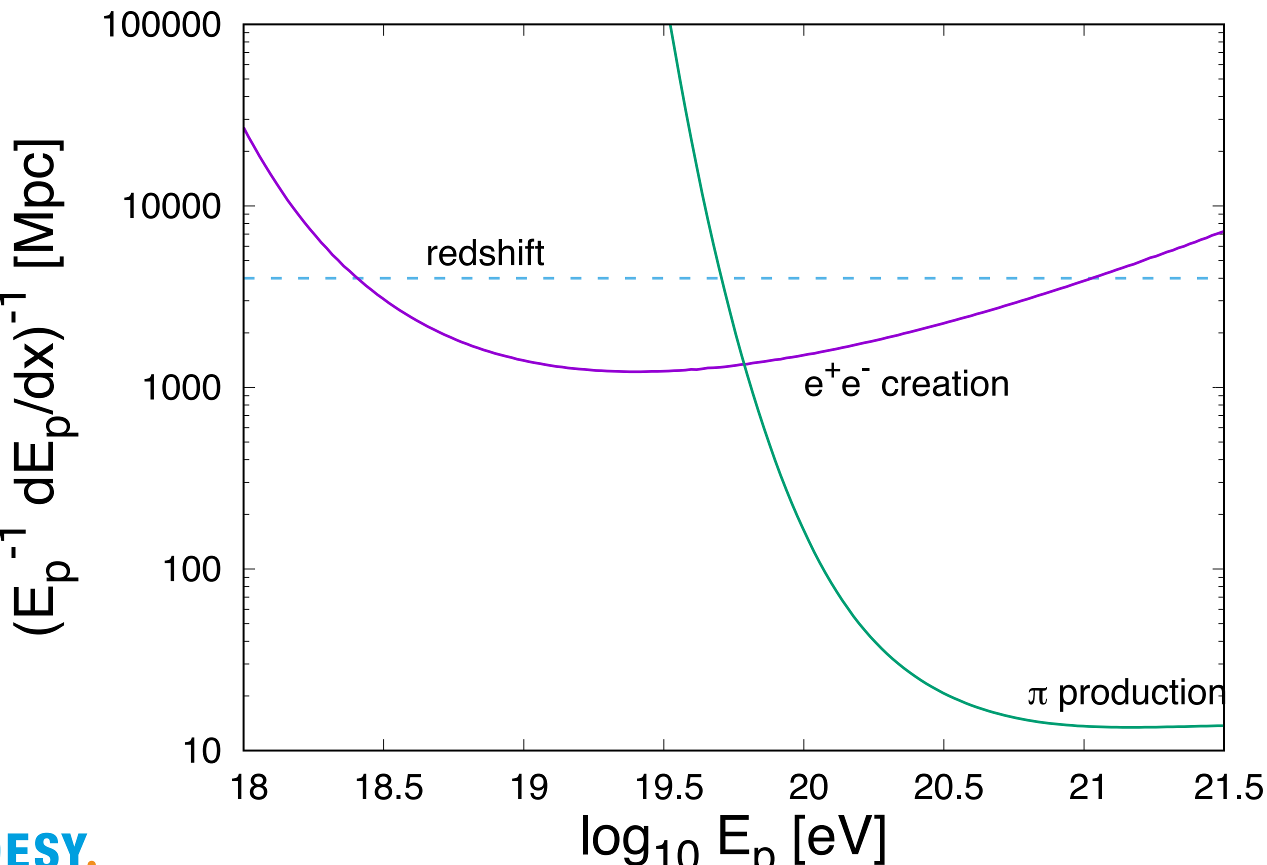
Energy Loss Rates due to Proton Interactions

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

where R is the energy loss rate

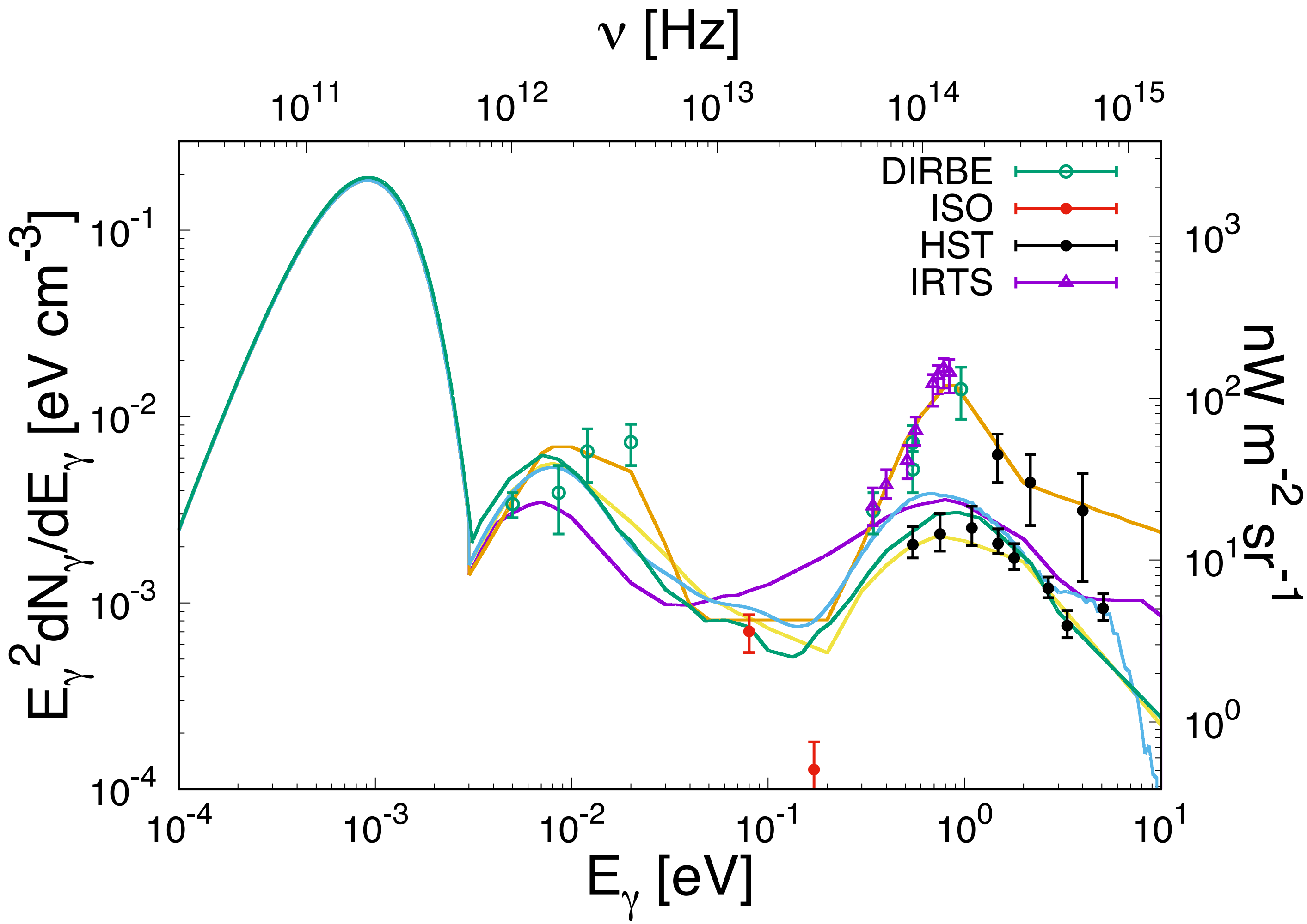
where K_p is the inelasticity



$$\approx \frac{\mathbf{m_p}}{\mathbf{m_\pi}} \frac{\mathbf{1}}{\mathbf{n_{CMB}}\sigma_{\mathbf{p}\gamma}}$$

EBL Radiation Field Models

The EBL isn't actually known with very great accuracy (since it is difficult to measure directly)



This uncertainty should be propagated into theoretical calculations of interaction rates

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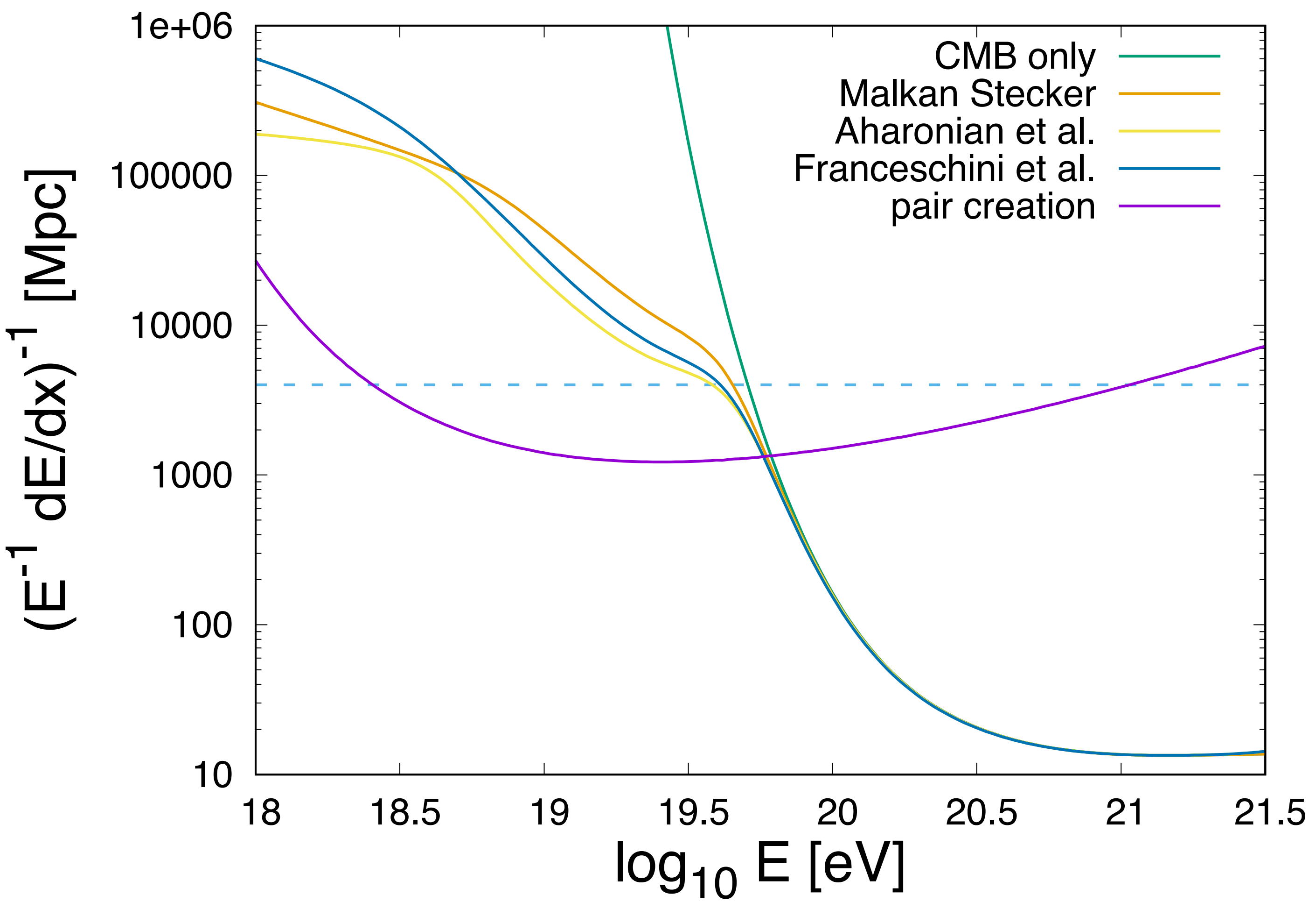
....with Different IR Backgrounds



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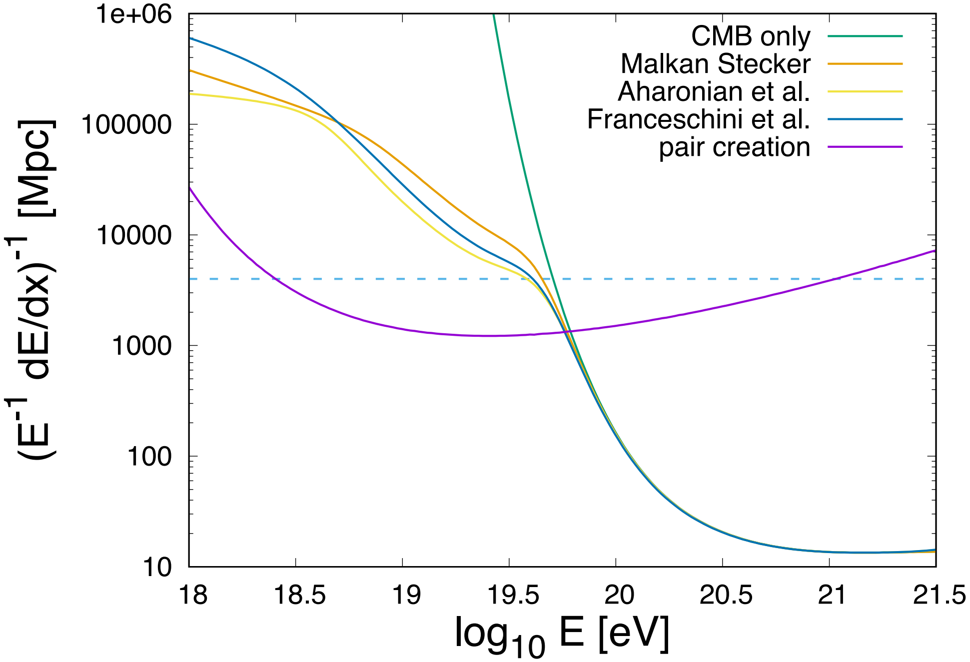
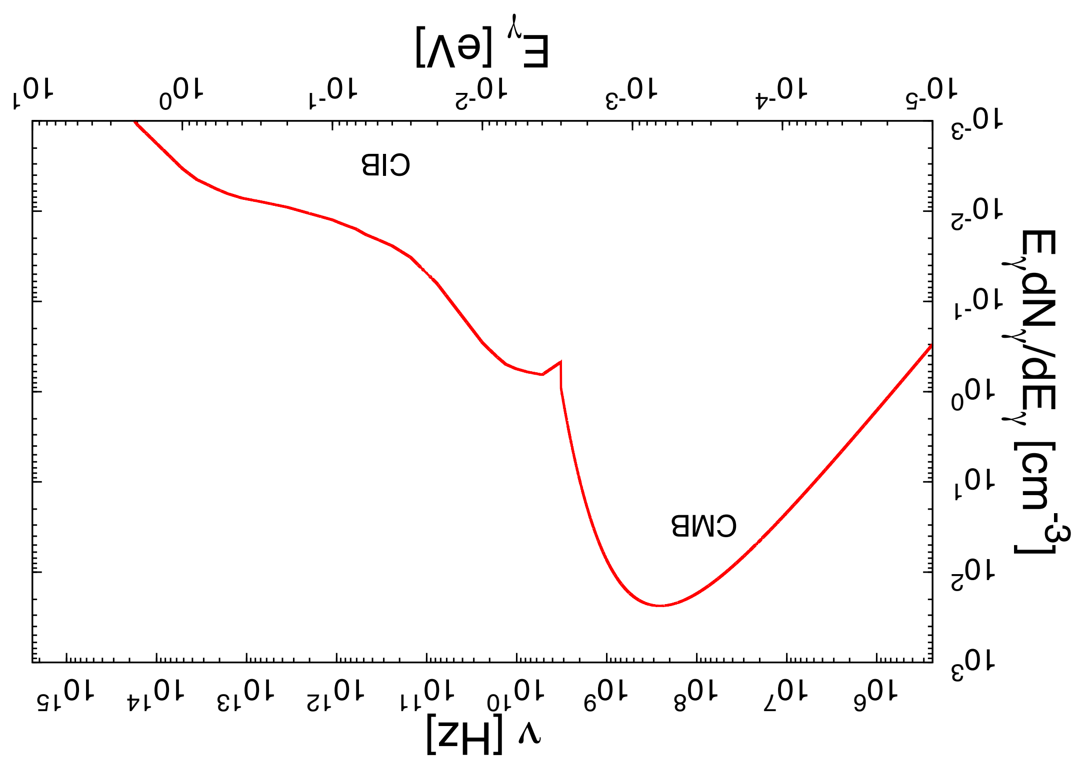
....with Different IR Backgrounds





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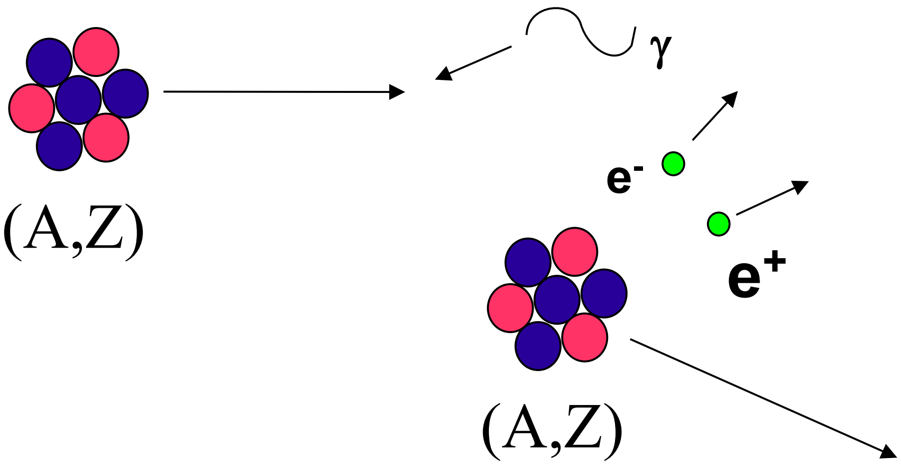
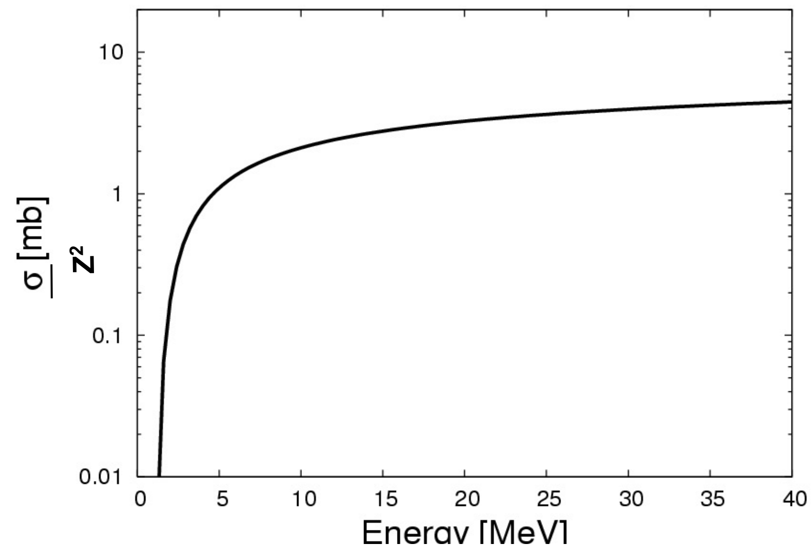
Cosmic Ray Nuclei Energy Losses

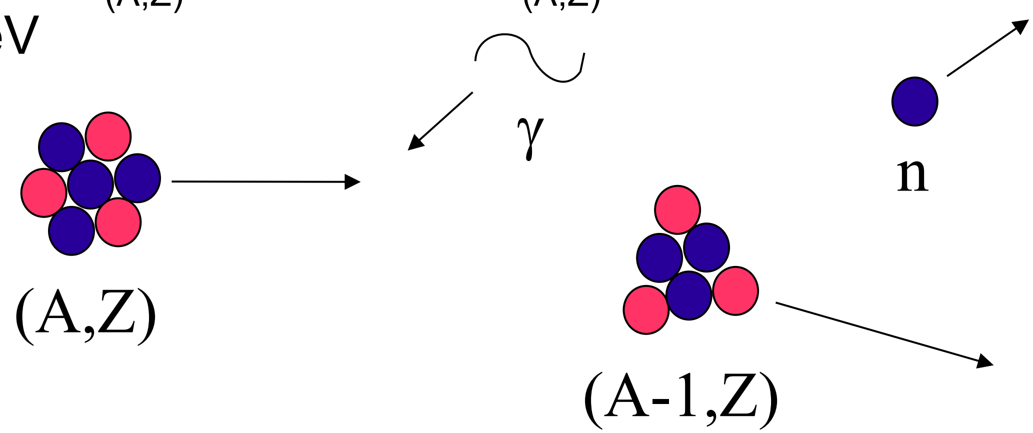
Cosmic Ray Nuclei Interactions

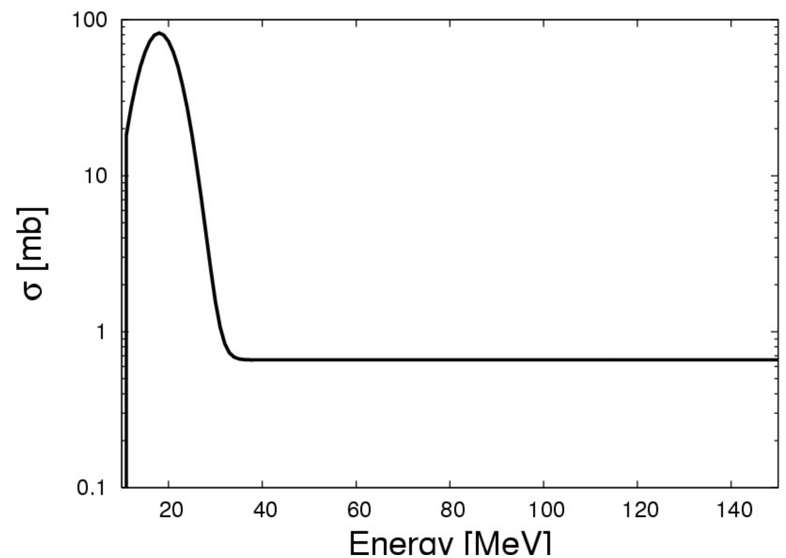
For $10^{19.7} < E_{(A,Z)} < 10^{20.2}$ eV





For $E_{(A,Z)} < 10^{19.7}$ and $E_{(A,Z)} < 10^{20.2}$ eV





Cosmic Ray Nuclei Interactions

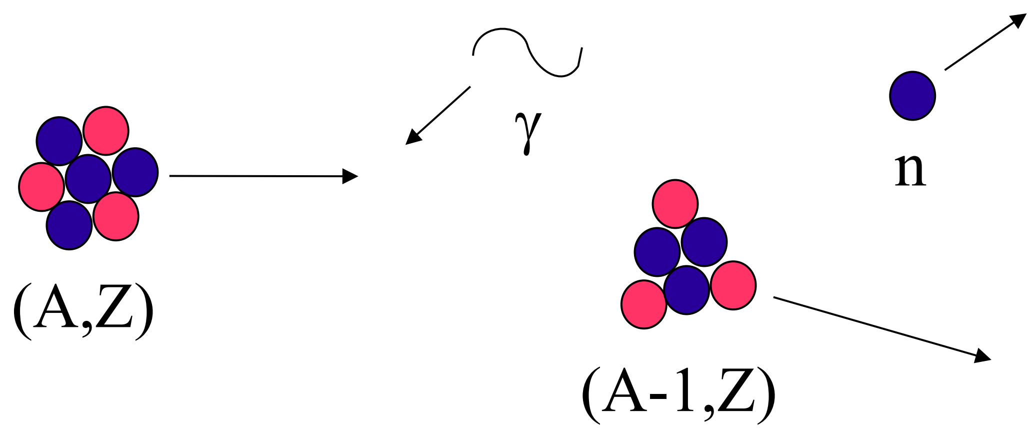


Photo-disintegration-

$$N_{(A,Z)} + \gamma \longrightarrow N'_{(A',Z')} + (Z-Z')p + (A-A'+Z'-Z)n, \quad E_\gamma \sim 30\,\mathrm{MeV}$$

$$n \longrightarrow p + e^- + \bar{\nu}_e$$

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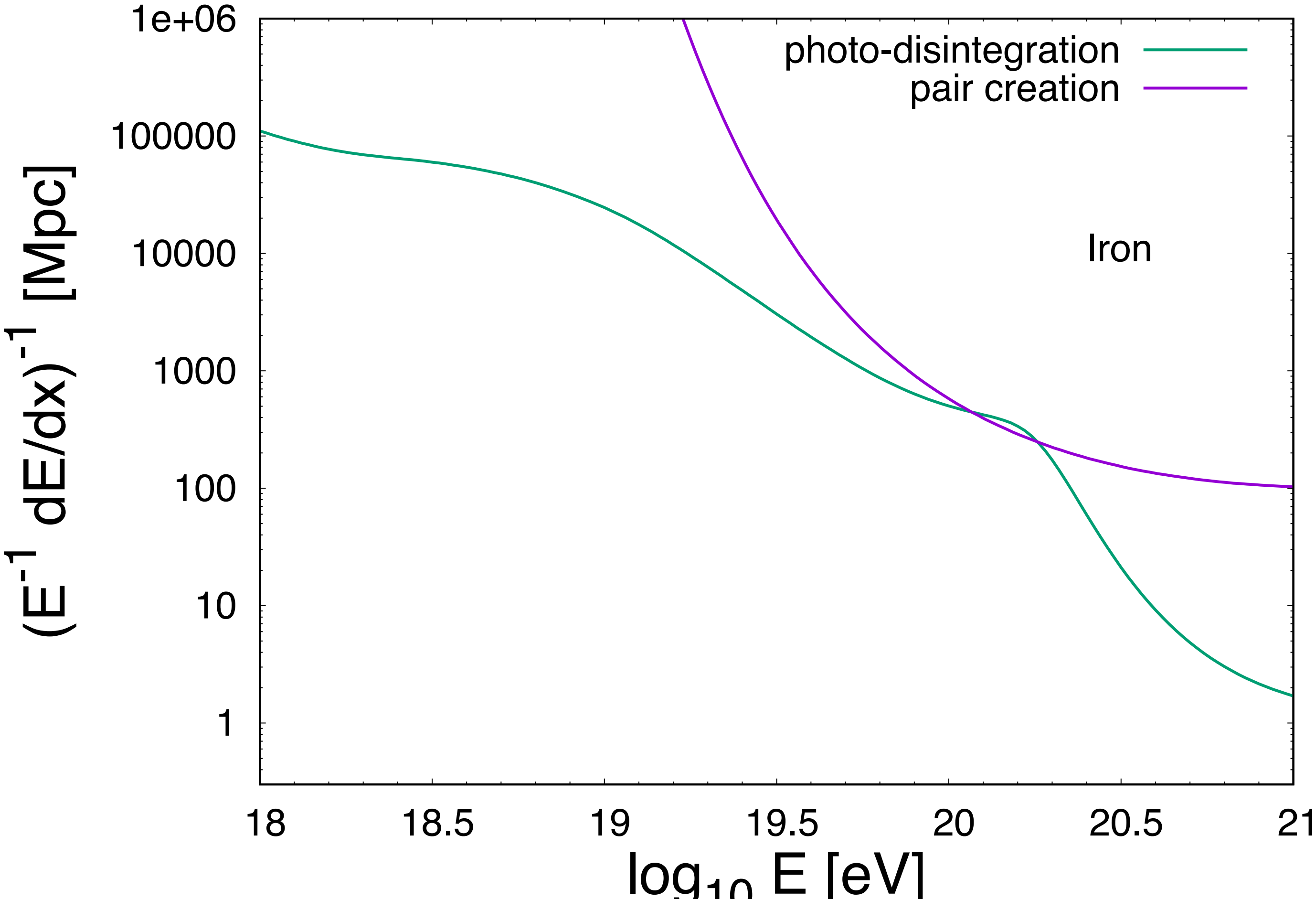
Energy Loss Rates due to Nuclei Interactions

$$\mathbf{R} = \frac{\mathbf{A^2 m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{Am_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{N}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

where R is the energy loss rate



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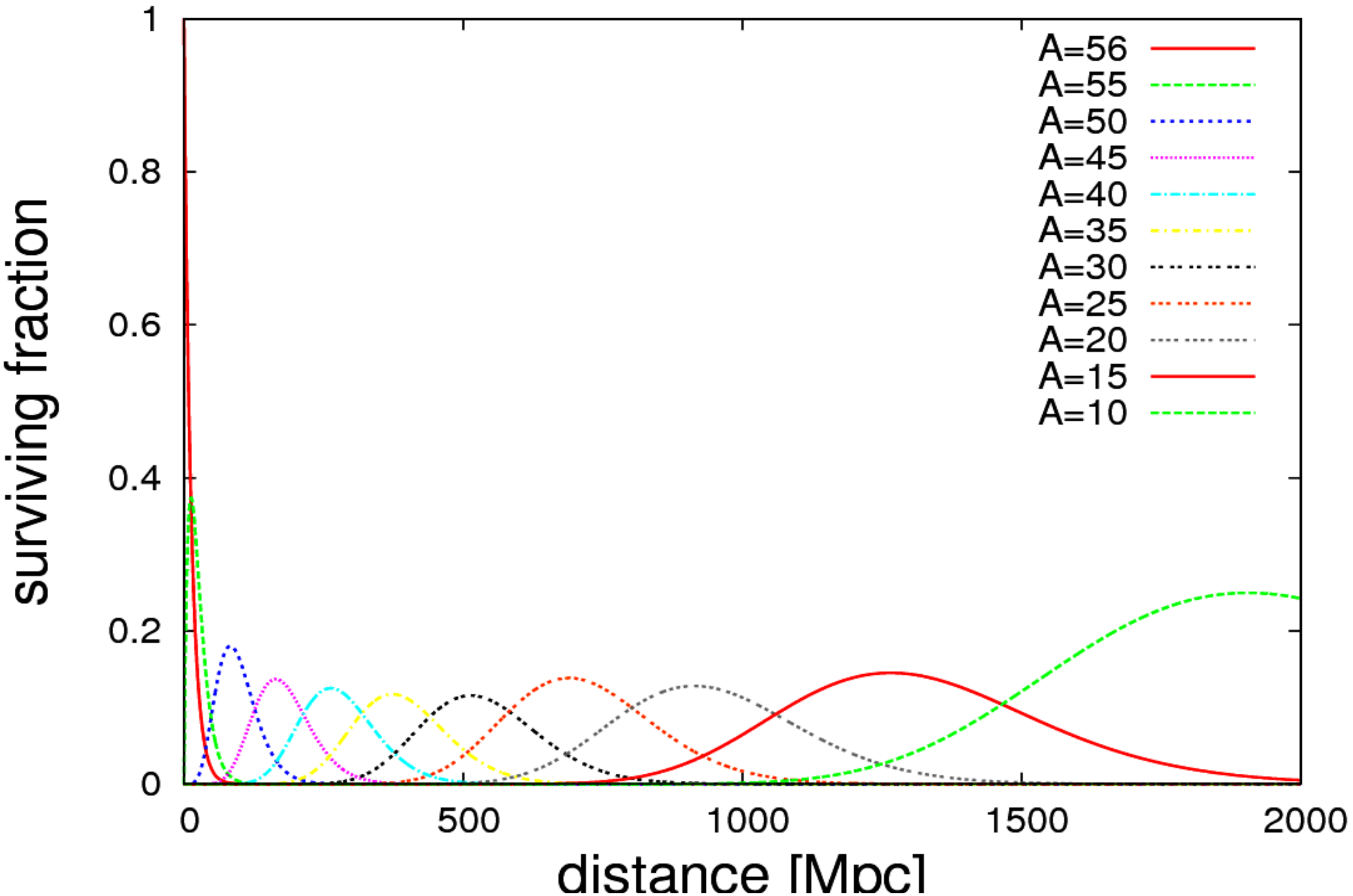
Cosmic Ray Disintegration During Propagation



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Cosmic Ray Spectra

Assumptions on Source Population

Spatial Distribution

$$\frac{dN}{dV_C} \propto (1+z)^n \qquad z < z_{max}$$

$$n = -6,\ -3,\ 0,\ 3$$

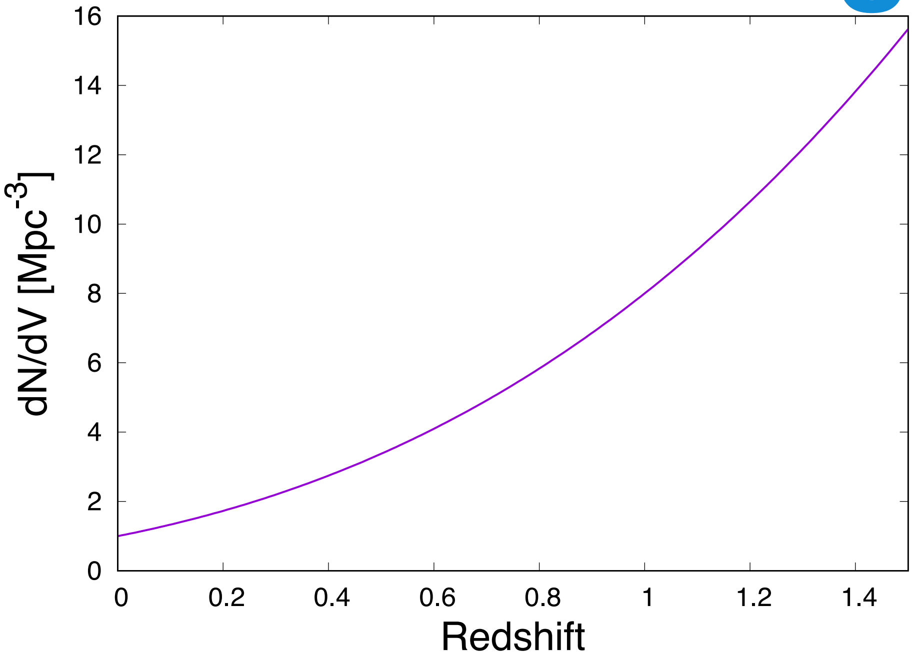
Energy Distribution

$$\frac{dN}{dE} \propto E^{-\alpha} \exp[-E/E_{Z,max}]$$

$$E_{Z,max} = (Z/26) \times E_{Fe,max}$$

Note- magnetic field horizon effects are neglected in the following.
This amounts to assuming: $d_s < (c t_H \lambda_{scat})^{1/2}$
ie. the source distribution may be approximated to be spatially continuous (also note, presence of t_H term comes from temporally continuous assumption)

A Cosmological Distribution of Sources

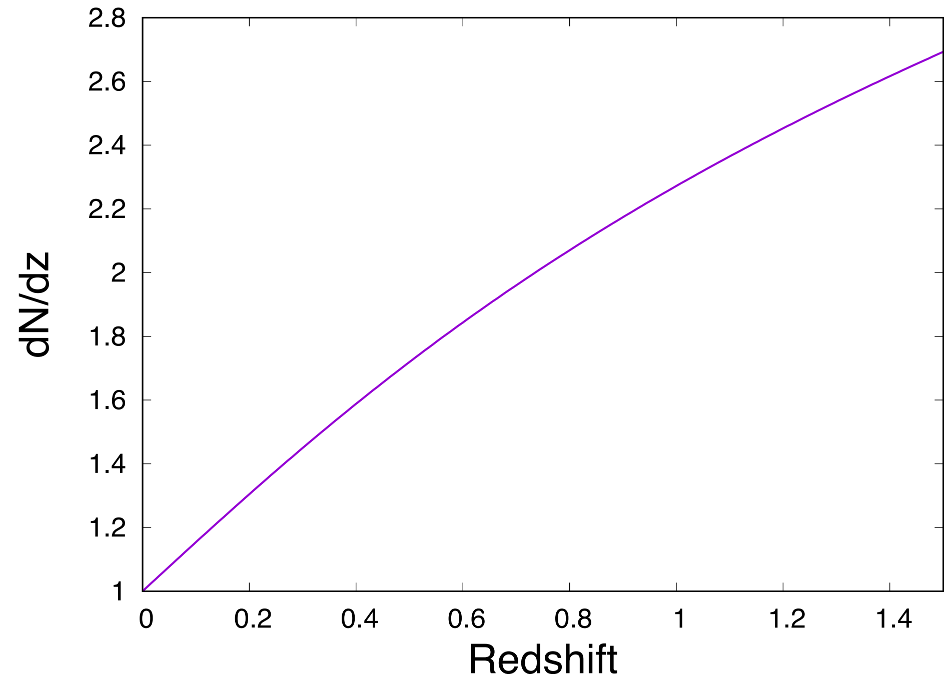


Distribution of sources in a comoving volume

$$\mathbf{dV_c = 4\pi\chi^2 d\chi}$$

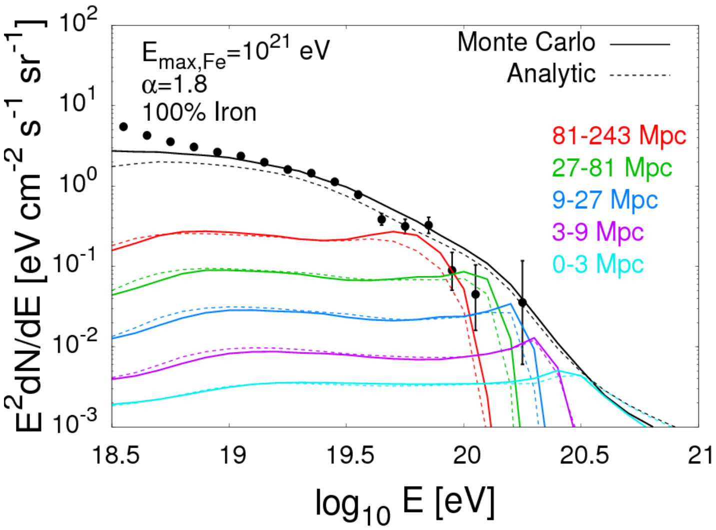
$$\mathbf{d\chi = \frac{dz}{H}}$$

$$\approx \frac{\mathbf{dz}}{\mathbf{H_0(\Omega_M(1+z)^3 + \Omega_\Lambda)^{1/2}}}$$

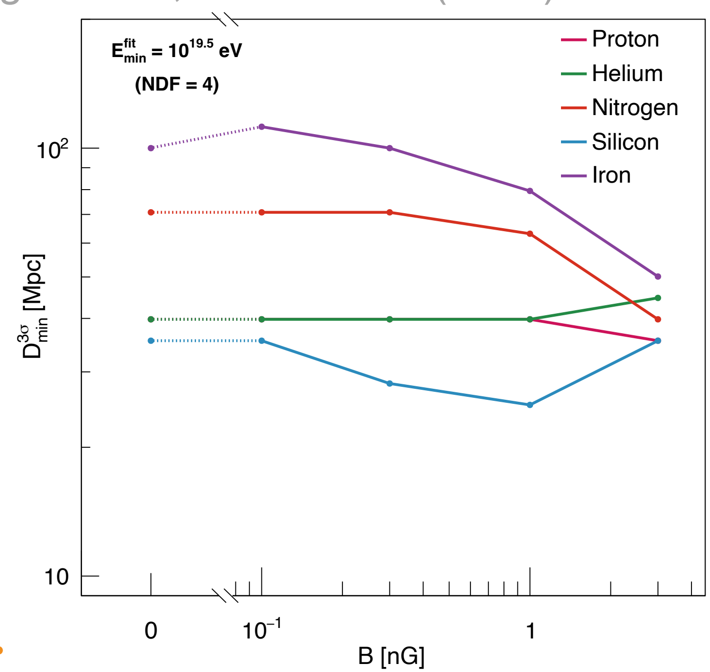


Proximity of Local Sources?

Taylor PRD, 84 105007 (2011)



Lang et PRD, 102 063012 (2020)

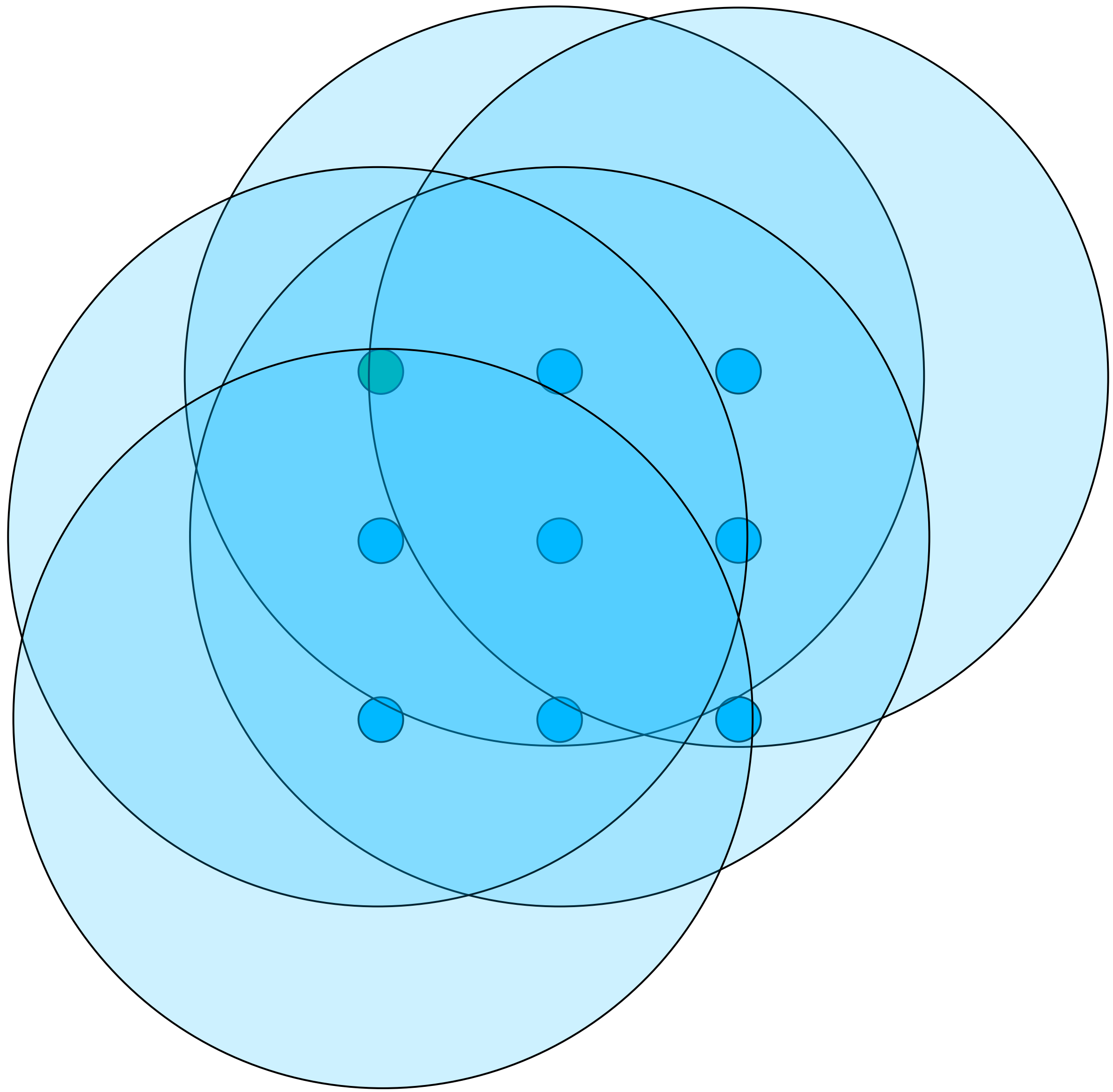


Note- magnetic field horizon effects are neglected here. This amounts to assuming:

$$\mathbf{d_s < (ct_H \lambda_{scat})^{1/2}}$$

ie. the source distribution may be approximated to be spatially continuous (also note, presence of t_H term comes from temporally continuous assumption)

Magnetic Horizon Effect



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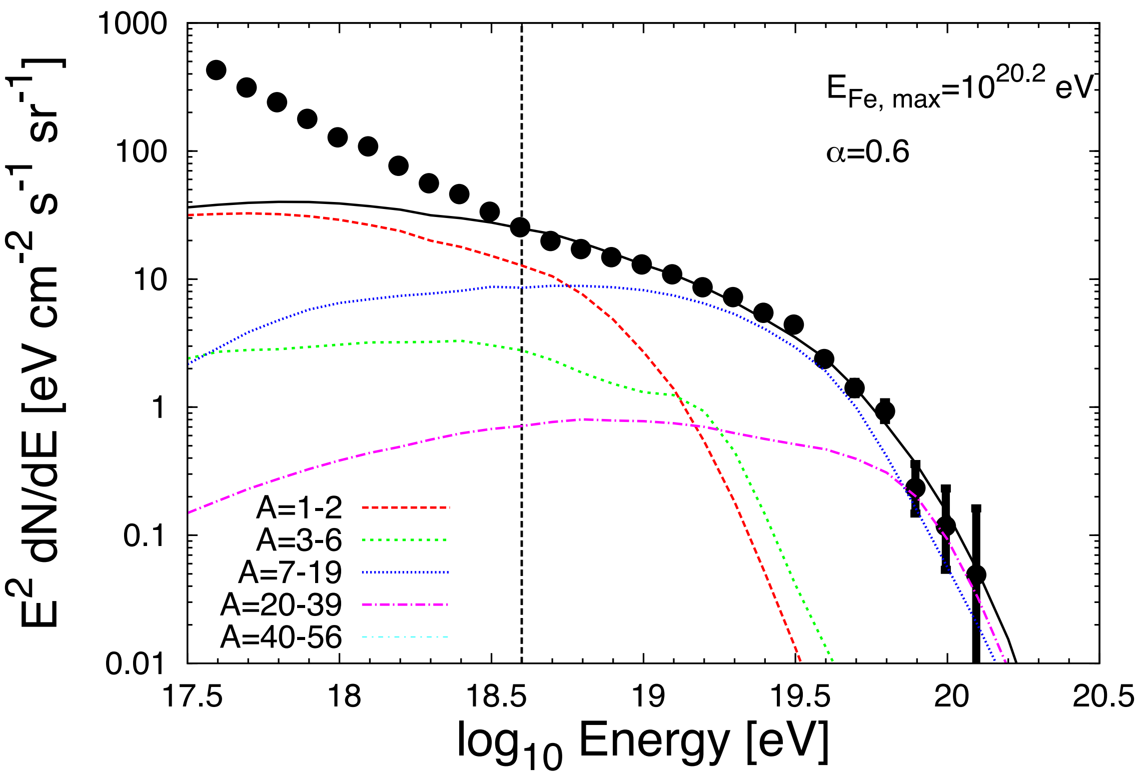
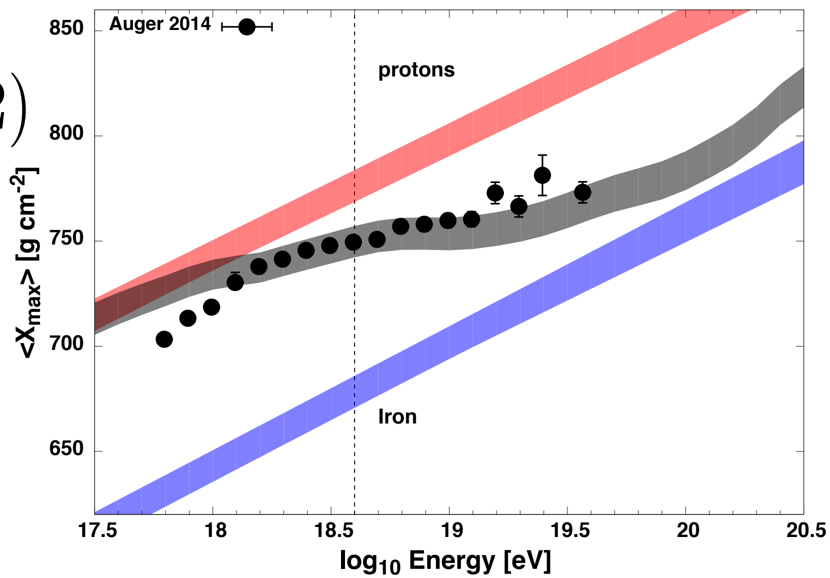
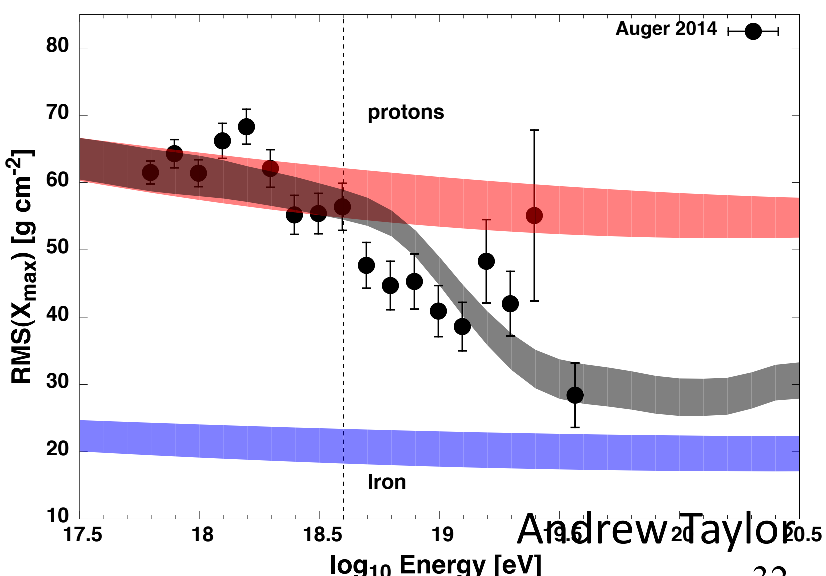
MCMC Likelihood Scan: Spectral + Composition Fits

$$\mathbf{L}(\mathbf{f_p}, \mathbf{f_{He}}, \mathbf{f_N}, \mathbf{f_{Si}}, \mathbf{E_{max}}, \alpha) \propto \exp(-\chi^2/2)$$

n=3 evolution result







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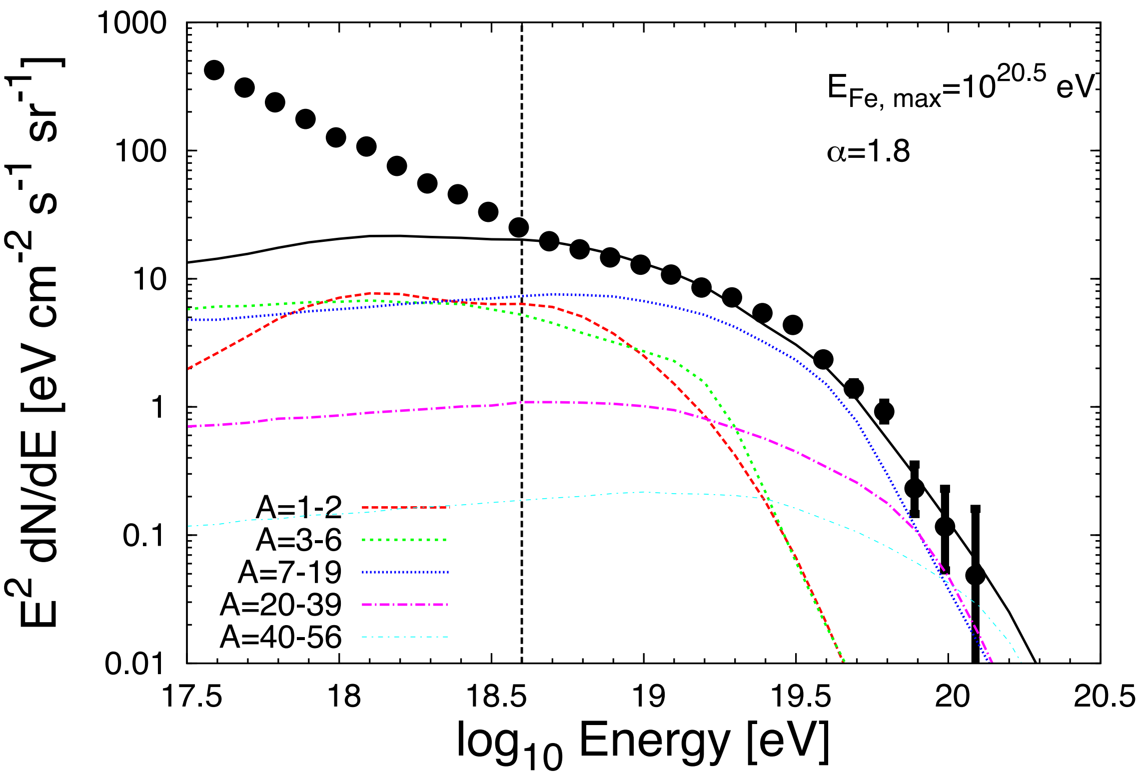
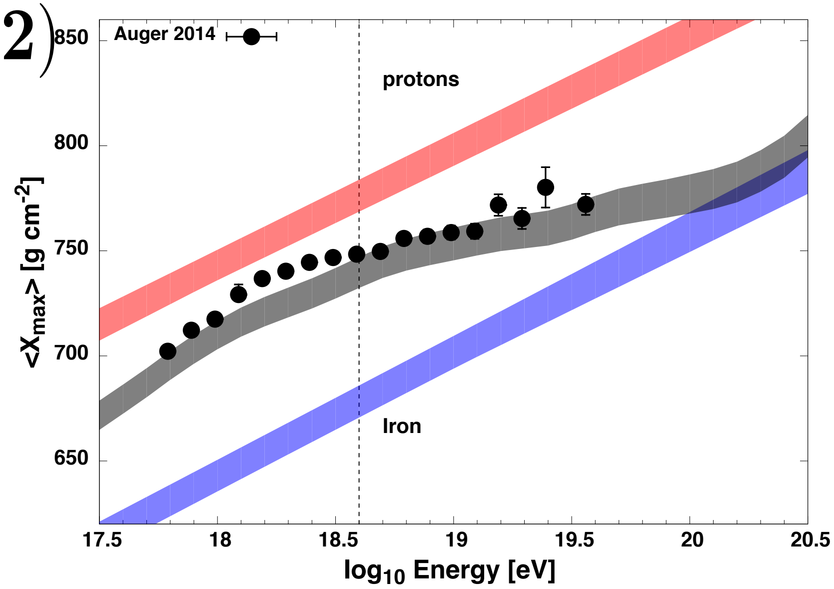
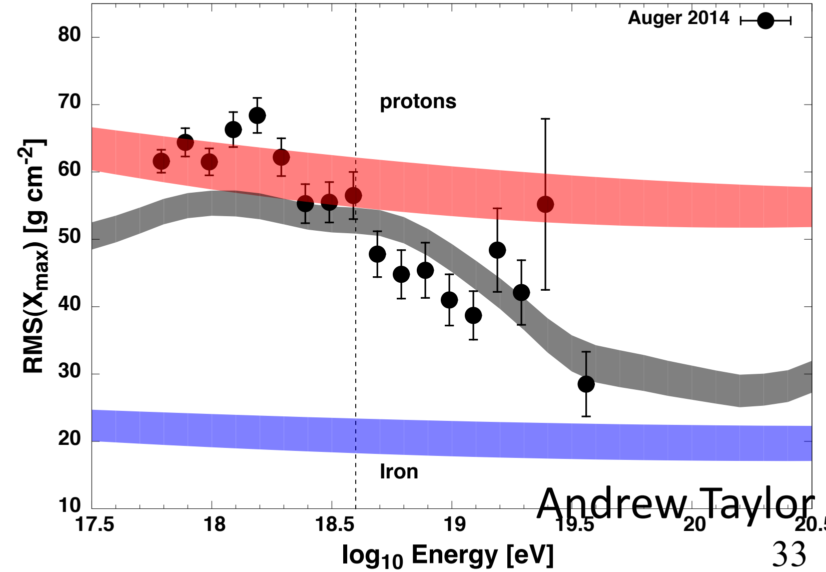
MCMC Likelihood Scan: "Soft" Spectra Solutions

$$\mathbf{L}(\mathbf{f_p}, \mathbf{f_{He}}, \mathbf{f_N}, \mathbf{f_{Si}}, \mathbf{E_{max}}, \alpha) \propto \exp(-\chi^2/2)$$

n=-6 evolution result







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Proximity-Spectral Index Relation

Taylor, PRD 92 (2015) 6

Parameter	$n=-6$		$n=-3$		$n=0$		$n=3$	
	Best-fit Value	Posterior Mean & Standard Deviation	Best-fit Value	Posterior Mean & Standard Deviation	Best-fit Value	Posterior Mean & Standard Deviation	Best-fit Value	Posterior Mean & Standard Deviation
α	1.8	1.83 ± 0.31	1.6	1.67 ± 0.36	1.1	1.33 ± 0.41	0.6	0.64 ± 0.44
$\log_{10}\left(\frac{E_{\mathrm{Fe,max}}}{\mathrm{eV}}\right)$	20.5	20.55 ± 0.26	20.5	20.52 ± 0.27	20.2	20.38 ± 0.25	20.2	20.16 ± 0.18

note trend in index

PAO, JCAP 04 (2017) 038

note trend in index

source evolution		γ	$\log_{10}(R_{\mathrm{cut}}/\mathrm{V})$	D	$D(J)$	$D(X_{\max})$
$(1+z)^m$	$m=+3$	$-1.40^{+0.35}_{-0.09}$	$18.22^{+0.05}_{-0.02}$	179.1	7.5	171.7
	$m=0$	$+0.96^{+0.08}_{-0.13}$	$18.68^{+0.02}_{-0.04}$	174.3	13.2	161.1
	$m=-3$	$+1.42^{+0.06}_{-0.07}$	$18.85^{+0.04}_{-0.07}$	173.9	19.3	154.6
	$m=-6$	$+1.56^{+0.06}_{-0.07}$	18.74 ± 0.03	182.4	19.1	163.3
	$m=-12$	$+1.79 \pm 0.06$	18.73 ± 0.03	182.1	18.1	164.0
	$z \leq 0.02$	$+2.69 \pm 0.01$	$19.50^{+0.08}_{-0.07}$	178.6	15.3	163.3

Local source solution calls upon a more acceptable spectral index

Proximity-Spectral Index Relation

Taylor, PRD 92 (2015) 6

Parameter	$n=-6$ Best-fit Value	$n=-6$ Posterior Mean & Standard Deviation	$n=-3$ Best-fit Value	$n=-3$ Posterior Mean & Standard Deviation	$n=0$ Best-fit Value	$n=0$ Posterior Mean & Standard Deviation	$n=3$ Best-fit Value	$n=3$ Posterior Mean & Standard Deviation
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note trend in index →

PAO, JCAP 04 (2017) 038

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Evidence that either there aren't many such sources, or that these sources (spectrally) are copies of each other (ie. stability of solution issues) Ehlert PRD, 107 103045 (2020)

Proximity-Spectral Index Relation

Taylor, PRD 92 (2015) 6

Parameter	$n=-6$ Best-fit Value	$n=-6$ Posterior Mean & Standard Deviation	$n=-3$ Best-fit Value	$n=-3$ Posterior Mean & Standard Deviation	$n=0$ Best-fit Value	$n=0$ Posterior Mean & Standard Deviation	$n=3$ Best-fit Value	$n=3$ Posterior Mean & Standard Deviation
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PAO, JCAP 04 (2017) 038

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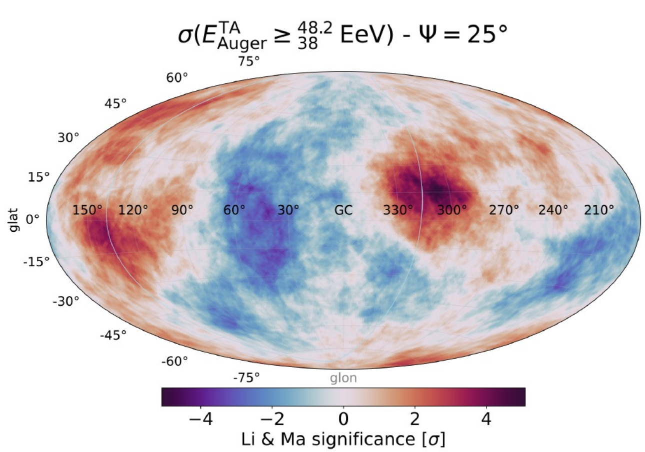
note trend in index



A single/few local sources solution calls upon a more acceptable spectral index/resolves the - how to square this with the anisotropy data?

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Conclusions

- The attenuation of cosmic ray protons/nuclei due to the presence of background radiation fields is reasonably well understood
- The largest limitation presently is the EBL (dust and stellar emission components)
- Despite these limitations, calculations for the propagation ultra high energy cosmic rays in these background radiation fields are predictive
- A negative evolution of sources allows for softer source injection spectra (more consistent with the Fermi acceleration model)
- The current cosmic ray data at the highest energies is suggestive that the nearest sources should be no further than a few 10s of Mpc

End of Lecture

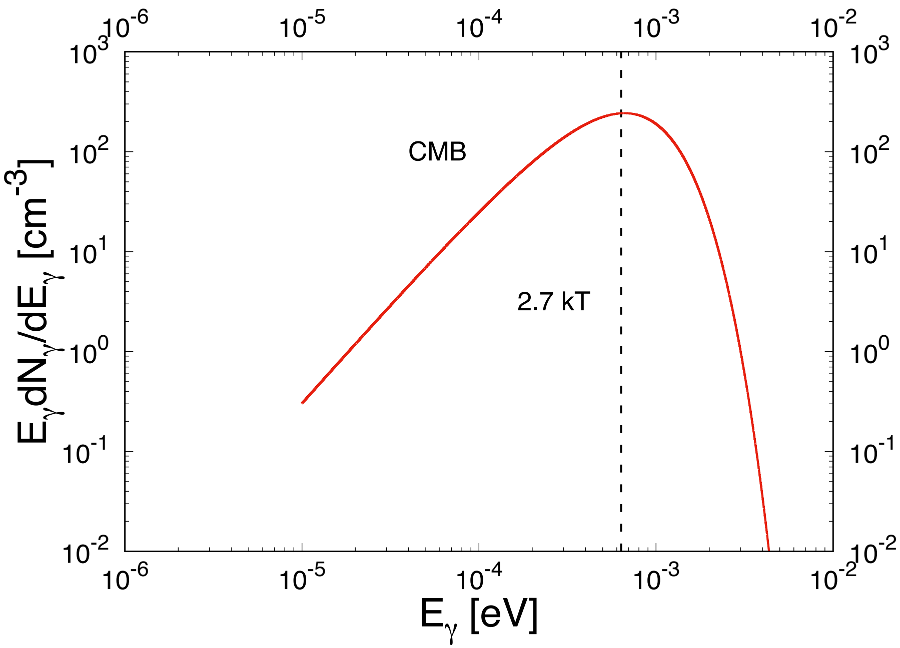
Blackbody- Total Number Density

$$\mathbf{n_\gamma^{BB}} = \mathbf{8}\pi\frac{\mathbf{(kT)^3}}{\mathbf{(hc)^3}}\gamma(\mathbf{3})\zeta(\mathbf{3})$$

$$\mathbf{n_\gamma^{BB}} = \frac{\mathbf{8}\pi\mathbf{(kT)^3}}{\mathbf{(hc)^3}}\int_{\mathbf{0}}^{\infty}\frac{\mathbf{x^2}}{\mathbf{e^x - 1}}\mathbf{dx}$$

$$\int_{\mathbf{0}}^{\infty}\mathbf{x^2 e^{-x} dx} = \gamma(\mathbf{3})$$

$$\frac{\mathbf{x^n}}{\mathbf{e^x - 1}} = \frac{\mathbf{e^{-x} x^n}}{\mathbf{1 - e^{-x}}}$$



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CMB- Total Number Density

$$\mathbf{n_\gamma^{BB}} = 8\pi \frac{(\mathbf{kT})^3}{(\mathbf{hc})^3}\gamma(\mathbf{3})\zeta(\mathbf{3})$$

$$\frac{\mathbf{x^n}}{\mathbf{e^x - 1}} = \frac{\mathbf{e^{-x}x^n}}{\mathbf{1 - e^{-x}}}$$

$$= \sum_{\mathbf{m=0}}^{\infty} \mathbf{e^{-mx}e^{-x}x^n}$$

$$= \sum_{\mathbf{m=1}}^{\infty} \mathbf{e^{-mx}x^n}$$

CMB- Total Number Density

$$n_\gamma^{BB} = 8\pi \frac{(kT)^3}{(hc)^3} \gamma(3)\zeta(3)$$

$$\int \frac{x^n}{e^x - 1} dx = \sum_{m=1}^{\infty} \int e^{-mx} x^n dx$$

Let $y = mx$

$$\int \frac{x^n}{e^x - 1} dx = \sum_{m=1}^{\infty} \int e^{-y} \left(\frac{y}{m}\right)^n d\left(\frac{y}{m}\right)$$

$$\int \frac{x^n}{e^x - 1} dx = \sum_{m=1}^{\infty} \frac{1}{m^{n+1}} \int y^n e^{-y} dy = \gamma(n+1)\zeta(n+1)$$

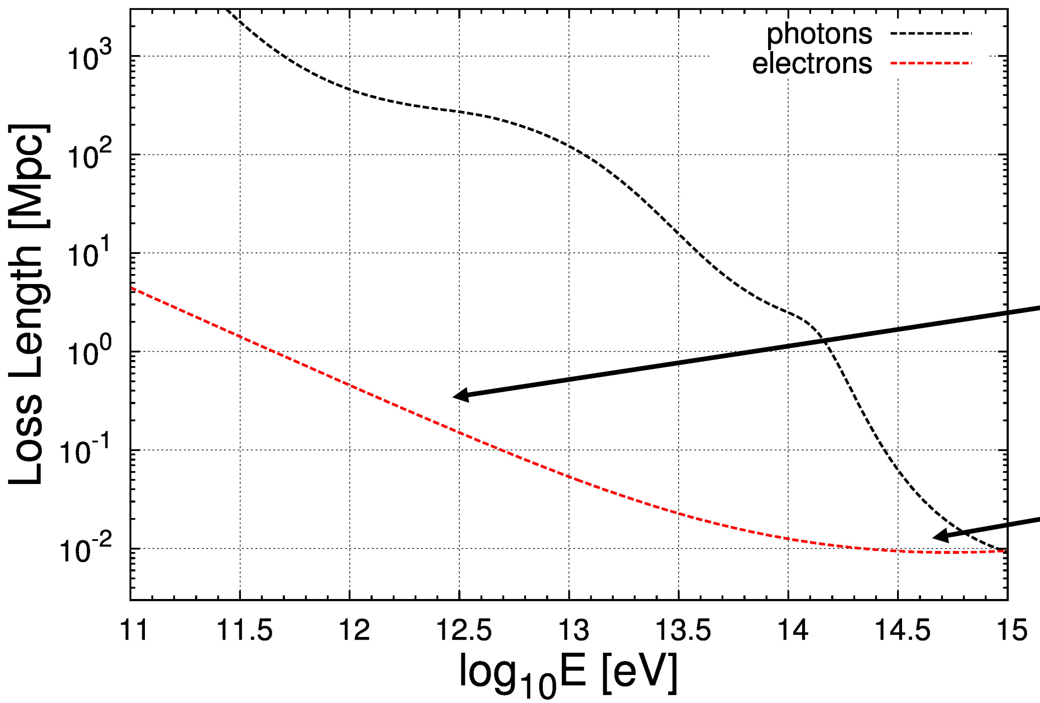
Threshold Energy- Proton Pion Production

$$\mathbf{(E_p + E_\gamma)^2 - (p_p - E_\gamma)^2 = (m_p + m_\pi)^2}$$

$$\mathbf{m_p^2 + 2E_pE_\gamma + 2p_pE_\gamma \approx m_p^2 + 2m_pm_\pi}$$

$$\mathbf{E_p \approx \frac{m_\pi}{2E_\gamma} m_p \approx \left(\frac{135 \times 10^6}{2 \times 6 \times 10^{-4}}\right) 0.9 \times 10^9 = 10^{20}\ eV}$$

Energy Loss Rates of Electrons and Photons



$\tau_e \propto E_e^{-1}$

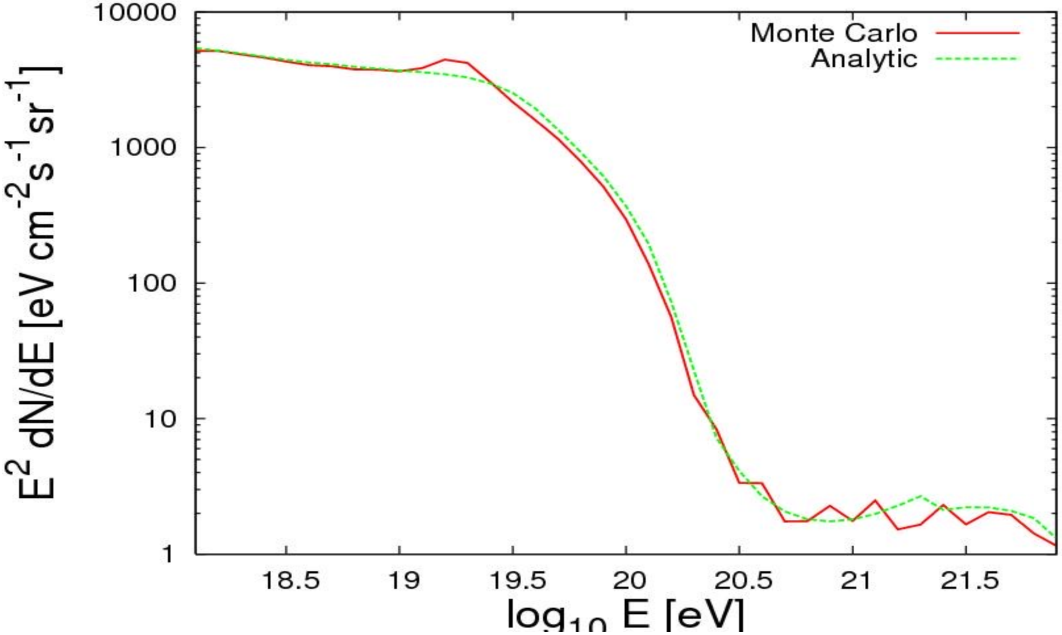
onset of Klein Nishina effect

Thomson regime electron cooling:

$$\tau_e^{-1} = \frac{m_e^2 c^4}{2E_e^2} \int_0^\infty d\epsilon_\gamma \frac{1}{\epsilon_\gamma^2} \frac{dn}{d\epsilon_\gamma} K_e \int_0^{2E_e\epsilon_\gamma/(m_e c^2)} d\epsilon'_\gamma \epsilon'_\gamma \sigma(\epsilon'_\gamma)$$

$$\approx \sigma_T \int_0^\infty b \frac{dn}{d\epsilon_\gamma} d\epsilon_\gamma = \sigma_T \int_0^\infty \frac{E_e \epsilon_\gamma}{(m_e c^2)^2} \frac{dn}{d\epsilon_\gamma} d\epsilon_\gamma = \sigma_T \frac{E_e}{(m_e c^2)^2} U_\gamma$$

Comparison of Analytic and Monte Carlo Results



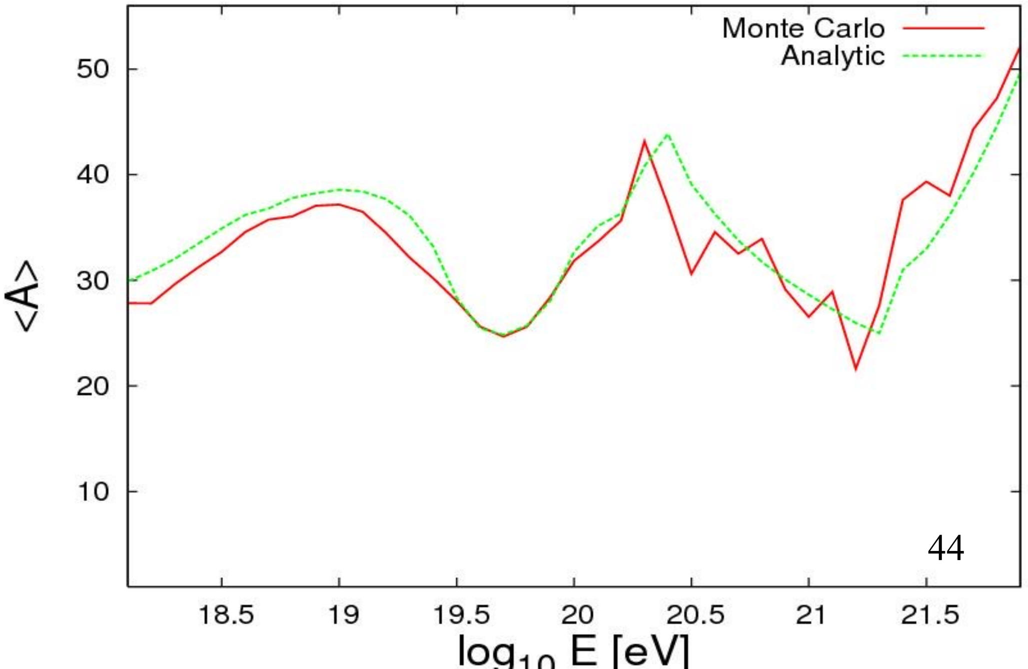


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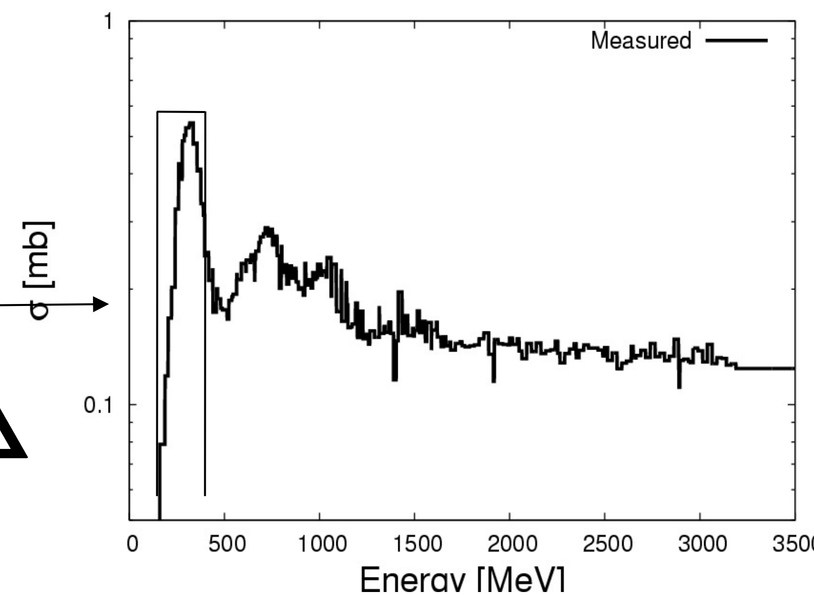
Photo-Pion Production Rate

$$\mathbf{R} = \frac{\mathbf{m_p^2 c^4}}{\mathbf{2E^2}} \int_0^\infty \mathbf{d}\epsilon_\gamma \frac{\mathbf{1}}{\epsilon_\gamma^{\mathbf{2}}} \frac{\mathbf{dn}}{\mathbf{d}\epsilon_\gamma} \int_0^{\mathbf{2E}\epsilon_\gamma/(\mathbf{m_p c^2})} \mathbf{d}\epsilon'_\gamma \epsilon'_\gamma \sigma_{\mathbf{p}\gamma}(\epsilon'_\gamma) \mathbf{K_p}$$

Assuming the cross-section is approximately:

$$\sigma_{\mathbf{p}\gamma}(\epsilon_\gamma) = \mathbf{0} \qquad \begin{array}{l} \epsilon_\gamma < \mathbf{E} - \mathbf{\Delta} \\ \epsilon_\gamma > \mathbf{E} + \mathbf{\Delta} \end{array}$$

$$\sigma_{\mathbf{p}\gamma}(\epsilon_\gamma) = \sigma_{\mathbf{p}\gamma} \quad \mathbf{E} - \mathbf{\Delta} < \epsilon_\gamma < \mathbf{E} + \mathbf{\Delta}$$



Where $\sigma_{\mathbf{p}\gamma} = \mathbf{0.5\ mb}$, $\mathbf{E} = \mathbf{300\ MeV}$, $\mathbf{\Delta} = \mathbf{100\ MeV}$

DESY.

Photo-Pion Production Rate

$$\mathbf{R}(\mathbf{\Gamma}) \approx \sigma_0 \int_{(\mathbf{E_0}-\mathbf{\Delta_0})/2\mathbf{\Gamma}}^{(\mathbf{E_0}+\mathbf{\Delta_0})/2\mathbf{\Gamma}} \left(\frac{\epsilon^2 - [(\mathbf{E_0}-\mathbf{\Delta_0})/2\mathbf{\Gamma}]^2}{\epsilon^2} \right) \frac{\mathbf{dn}}{\mathbf{d}\epsilon} \mathbf{d}\epsilon +$$

$$\sigma_0 \int_{(\mathbf{E_0}+\mathbf{\Delta_0})/2\mathbf{\Gamma}}^{\infty} \left(\frac{[(\mathbf{E_0}+\mathbf{\Delta_0})/2\mathbf{\Gamma}]^2 - [(\mathbf{E_0}-\mathbf{\Delta_0})/2\mathbf{\Gamma}]^2}{\epsilon^2} \right) \frac{\mathbf{dn}}{\mathbf{d}\epsilon} \mathbf{d}\epsilon$$

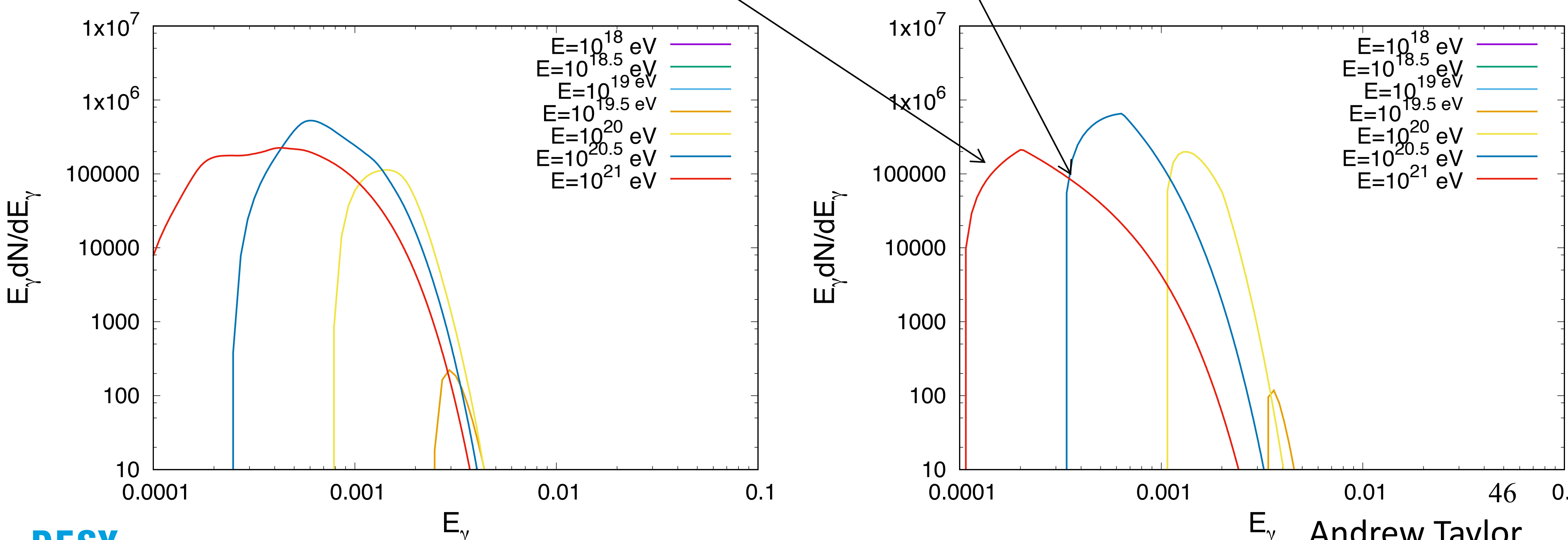


Photo-Pion Production Rate

$$\mathbf{R(\Gamma) \approx n_0 \sigma_0 \int_{x_1(\Gamma)}^{x_2(\Gamma)} \frac{\left(x^2 - x_1(\Gamma)^2\right)}{e^x - 1} dx +}$$

$$\mathbf{n_0 \sigma_0 \int_{x_2(\Gamma)}^{\infty} \frac{\left(x_2^2(\Gamma) - x_1^2(\Gamma)\right)}{e^x - 1}}$$

$$\mathbf{R(\Gamma) \approx \frac{1}{l_0} \left[\, (\gamma_i(3, x_2(\Gamma)) - \gamma_i(3, x_1(\Gamma))) - x_1(\Gamma)^2 (\gamma_i(1, x_2(\Gamma)) - \gamma_i(1, x_1(\Gamma))) + \right.}$$
$$\mathbf{\left. x_2(\Gamma)^2 (1 - \gamma_i(1, x_2(\Gamma))) - x_1(\Gamma)^2 (1 - \gamma_i(1, x_2(\Gamma))) \, \right]}$$

$$\mathbf{\gamma_i(3, x) = 2 - (2 + 2x + x^2) \exp(-x) \qquad \gamma_i(1, x) = 1 - \exp(-x)}$$

$$\mathbf{R(\Gamma) \approx \frac{2}{l_0} \left[e^{-x_1} (1 - e^{-x_1} + x_1 (1 - 2e^{-x_1})) \right]}$$

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Photo-Pion Production Rate: Blackbody Interactions

$$\mathbf{R(\Gamma) \approx n_0\sigma_0 \int_{x_1(\Gamma)}^{x_2(\Gamma)} \frac{\left(x^2 - x_1(\Gamma)^2\right)}{e^x - 1} dx +}$$

$$\mathbf{n_0\sigma_0 \int_{x_2(\Gamma)}^{\infty} \frac{\left(x_2^2(\Gamma) - x_1^2(\Gamma)\right)}{e^x - 1}}$$

$$\mathbf{R(\Gamma) \approx \frac{2}{l_0} \left[e^{-x_1}(1 - e^{-x_1} + x_1(1 - 2e^{-x_1}))\right]}$$

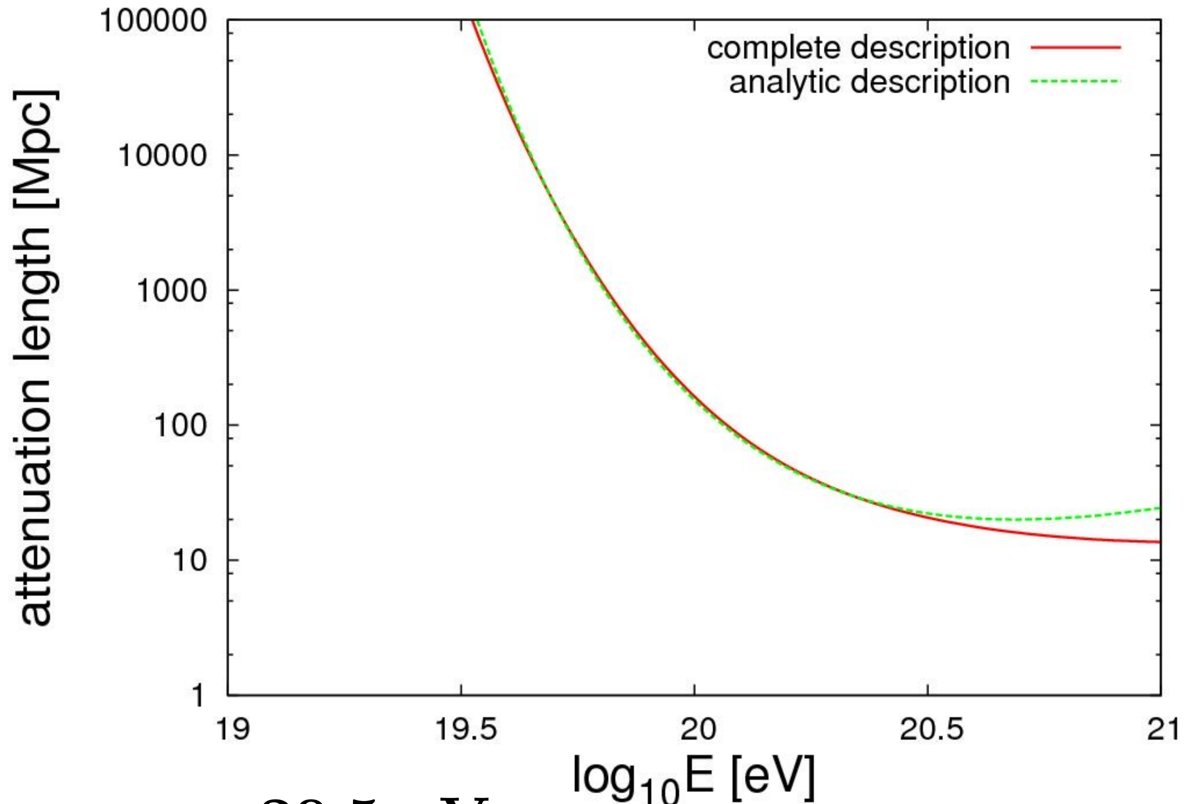
Where, $\mathbf{l_0 = 10\ Mpc}$ $\mathbf{x_1 = \frac{(E - \Delta)m_p}{2kT_{CMB}E_p} = \frac{10^{20.5\ eV}}{E_p}}$

Photo-Pion Production Rate: Blackbody Interactions

With, $\mathbf{kT_{CMB} \approx 2 \times 10^{-4}\ eV}$

$$\mathbf{R \approx 0.2\sigma_{p\gamma} \int_{\frac{E-\Delta}{2\Gamma}}^{\frac{E+\Delta}{2\Gamma}} d\epsilon_\gamma \frac{dn}{d\epsilon_\gamma}}$$

$$\mathbf{\approx \left(\frac{l_0}{e^{-x_1}(1-e^{-x_1})}\right)^{-1}}$$



Where $\mathbf{l_0}$ is 5 Mpc and $\mathbf{x_1 = \frac{10^{20.5}\ eV}{E_p}}$

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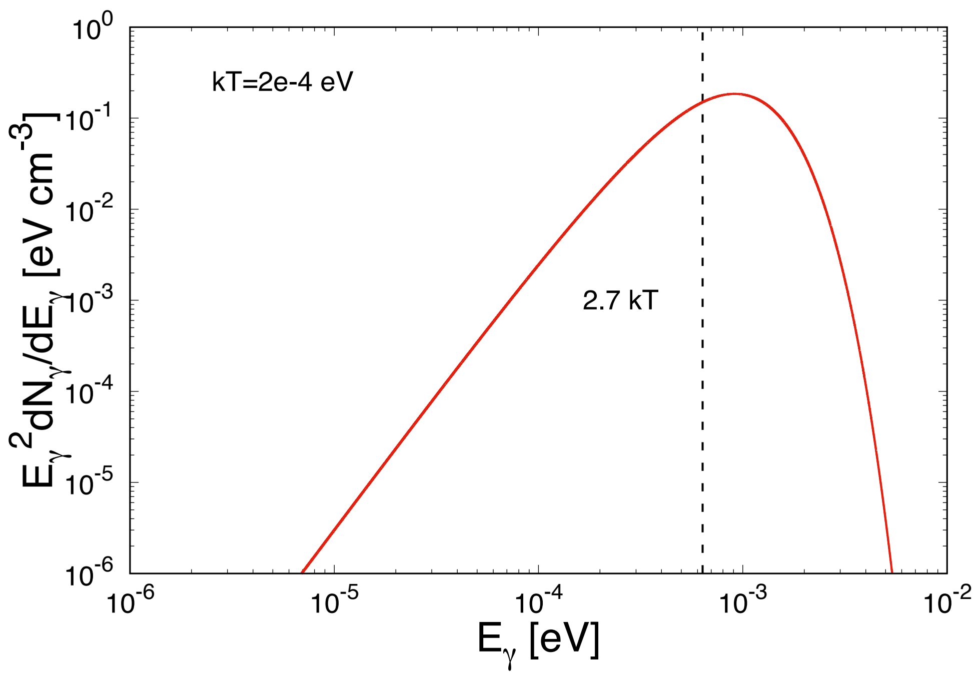
CMB- Total Energy Density

$$\rho_\gamma^{\mathbf{BB}} = \frac{8\pi(\mathbf{kT})^4}{(\mathbf{hc})^3}\int_0^\infty \frac{\mathbf{x^3}}{\mathbf{e^x - 1}}\mathbf{dx}$$

$$\rho_\gamma^{\mathbf{CMB}} = 8\pi\frac{(\mathbf{kT_{CMB}})^4}{(\mathbf{hc})^3}\gamma(4)\zeta(4) \approx \mathbf{0.25\ eV\ cm^{-3}}$$



An Analytic Description of these Results

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Differential Equation Describing System State

$$\frac{\mathbf{d}}{\mathbf{dt}}\begin{pmatrix} f_{56} \\ f_{55} \\ f_{54} \end{pmatrix} = \mathbf{\Lambda}\begin{pmatrix} f_{56} \\ f_{55} \\ f_{54} \end{pmatrix}$$

$$\mathbf{\Lambda} = \begin{pmatrix} -\left(\frac{1}{\tau_{56\to55}} + \frac{1}{\tau_{56\to54}} + \ldots\right) & 0 & 0 \\ \frac{1}{\tau_{56\to55}} & -\left(\frac{1}{\tau_{55\to54}} + \frac{1}{\tau_{55\to53}} + \ldots\right) & 0 \\ \frac{1}{\tau_{56\to54}} & \frac{1}{\tau_{55\to54}} & -\left(\frac{1}{\tau_{54\to53}} + \frac{1}{\tau_{54\to52}} + \ldots\right) \end{pmatrix}$$

by $$\mathbf{f_q(t)} = \sum_{\mathbf{n=q}}^{\mathbf{56}} \mathbf{A_n f_n(t)}$$

then $$\mathbf{f_q(t)} = \sum_{\mathbf{n=q}}^{\mathbf{56}} \mathbf{A_n e^{-\lambda_n t} f_n(0)}$$

(where A_n values are set by the initial conditions)

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Only Considering Single Nucleon Losses

$$\boldsymbol{\Lambda} = \begin{pmatrix} -\frac{1}{\tau_{56\rightarrow 55}} & 0 & 0 \\ \frac{1}{\tau_{56\rightarrow 55}} & -\frac{1}{\tau_{55\rightarrow 54}} & 0 \\ 0 & \frac{1}{\tau_{55\rightarrow 54}} & -\frac{1}{\tau_{54\rightarrow 53}} \end{pmatrix}$$

and

$$\mathbf{f_q(t)} = \sum_{\mathbf{n=q}}^{\mathbf{56}} \mathbf{f_{56}(0)} \frac{\tau_{\mathbf{q}} \tau_{\mathbf{n}}^{\mathbf{56-q-1}}}{\prod_{\mathbf{p=q}}^{\mathbf{56}} (\tau_{\mathbf{n}} - \tau_{\mathbf{p}})} \mathbf{e}^{-\frac{\mathbf{t}}{\tau_{\mathbf{n}}}}$$

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Nuclear Cascade Description

Consider

$$\frac{df_q}{dt} + \frac{f_q}{\tau_q} = \frac{f_{q+1}}{\tau_{q+1}}$$

$$e^{\left(\frac{-t}{\tau_q}\right)} \frac{d}{dt}\left[e^{\left(\frac{t}{\tau_q}\right)} f_q\right] = \frac{f_{q+1}}{\tau_{q+1}}$$

$$f_q = e^{\left(\frac{-t}{\tau_q}\right)} \int e^{\left(\frac{t}{\tau_q}\right)} \frac{f_{q+1}}{\tau_{q+1}} dt$$

Assume solution is true for q, apply to q+1

$$\frac{f_{q+1}(t)}{f_{56}(0)} = \sum_{n=q+1}^{56} \frac{\tau_{q+1}\tau_n^{56-q-2}}{\prod_{p=q+1}^{56}(\tau_n - \tau_p)} e^{-\frac{t}{\tau_n}}$$

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Nuclear Cascade Description

Assume solution is true

$$\frac{f_{q+1}(t)}{f_{56}(0)} = \sum_{n=q+1}^{56} \frac{\tau_{q+1}\tau_n^{56-q-2}}{\prod_{p=q+1}^{56}(\tau_n - \tau_p)} e^{-\frac{t}{\tau_n}}$$

$$f_q = e^{\left(\frac{-t}{\tau_q}\right)} \int e^{\left(\frac{t}{\tau_q}\right)} \frac{f_{q+1}}{\tau_{q+1}} dt$$

$$\frac{f_q(t)}{f_{56}(0)} = \sum_{n=q+1}^{56} \frac{\tau_n^{56-q-2}}{\prod_{p=q+1}^{56}(\tau_n - \tau_p)} \left[\left(\frac{1}{\tau_q} - \frac{1}{\tau_n}\right)^{-1} e^{\frac{-t}{\tau_n}}\right] - ce^{\frac{-t}{\tau_q}}$$

Since $f_q(0) = 0$

$$c = \sum_{n=q+1}^{56} \frac{\tau_q \tau_n^{56-q-1}}{\prod_{p=q}^{56}(\tau_n - \tau_p)}$$

Nuclear Cascade Description

$$\frac{f_q(t)}{f_{56}(0)} = \sum_{n=q+1}^{56} \frac{\tau_n^{56-q-2}}{\prod_{p=q}^{56}(\tau_n - \tau_p)} e^{\frac{-t}{\tau_n}} - \sum_{n=q+1}^{56} \frac{\tau_q \tau_n^{56-q-1}}{\prod_{p=q}^{56}(\tau_n - \tau_p)} e^{\frac{-t}{\tau_q}}$$

$$\frac{f_q(t)}{f_{56}(0)} = \sum_{n=q}^{56} \frac{\tau_q \tau_n^{56-q-1}}{\prod_{p=q}^{56}(\tau_n - \tau_p)} e^{-\frac{t}{\tau_n}}$$

These are equivalent if:

$$\sum_{n=q+1}^{56} \frac{\tau_q \tau_n^{56-q-1}}{\prod_{p=q}^{56}(\tau_n - \tau_p)} = \frac{\tau_q \tau_q^{56-q-1}}{\prod_{p=q}^{56}(\tau_q - \tau_p)}$$

Consider:

$$\frac{w^2}{(w-x)(w-y)(w-z)} + \frac{x^2}{(x-w)(x-y)(x-z)} + \frac{y^2}{(y-w)(y-x)(y-z)} = -\frac{z^2}{(z-w)(z-x)(z-y)}$$

Nuclear Cascade Description

$$\sum_{\mathbf{n=q+1}}^{\mathbf{56}} \frac{\tau_{\mathbf{q}}\tau_{\mathbf{n}}^{\mathbf{56-q-1}}}{\prod_{\mathbf{p=q}}^{\mathbf{56}}(\tau_{\mathbf{n}}-\tau_{\mathbf{p}})} = \frac{\tau_{\mathbf{q}}\tau_{\mathbf{q}}^{\mathbf{56-q-1}}}{\prod_{\mathbf{p=q}}^{\mathbf{56}}(\tau_{\mathbf{q}}-\tau_{\mathbf{p}})}$$

Consider the case

$$\frac{\mathbf{w^2}}{(\mathbf{w}-\mathbf{x})(\mathbf{w}-\mathbf{y})(\mathbf{w}-\mathbf{z})} + \frac{\mathbf{x^2}}{(\mathbf{x}-\mathbf{w})(\mathbf{x}-\mathbf{y})(\mathbf{x}-\mathbf{z})} + \frac{\mathbf{y^2}}{(\mathbf{y}-\mathbf{w})(\mathbf{y}-\mathbf{x})(\mathbf{y}-\mathbf{z})} = -\frac{\mathbf{z^2}}{(\mathbf{z}-\mathbf{w})(\mathbf{z}-\mathbf{x})(\mathbf{z}-\mathbf{y})}$$

$$\begin{vmatrix} 1 & w & w^2 & w^2 \\ 1 & x & x^2 & x^2 \\ 1 & y & y^2 & y^2 \\ 1 & z & z^2 & z^2 \end{vmatrix} = \mathbf{0}$$

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Cascade of Nuclei Through Species- single nucleon loss

Since nuclei Lorentz factor remains ~conserved, and cross-section varies mildly with A (nuclear mass)

$$\tau_{56\rightarrow 55} \approx \tau_{55\rightarrow 54} \ldots$$

For the case $\tau_{56\rightarrow 55} = \tau_{55\rightarrow 54} \ldots$

$$f_q = \frac{t^{(q_{max}-q)}}{\tau_q (q_{max}-q)!} e^{-t/\tau_q}$$

ie. Gaisser-Hillas type function! (used to describe air showers)

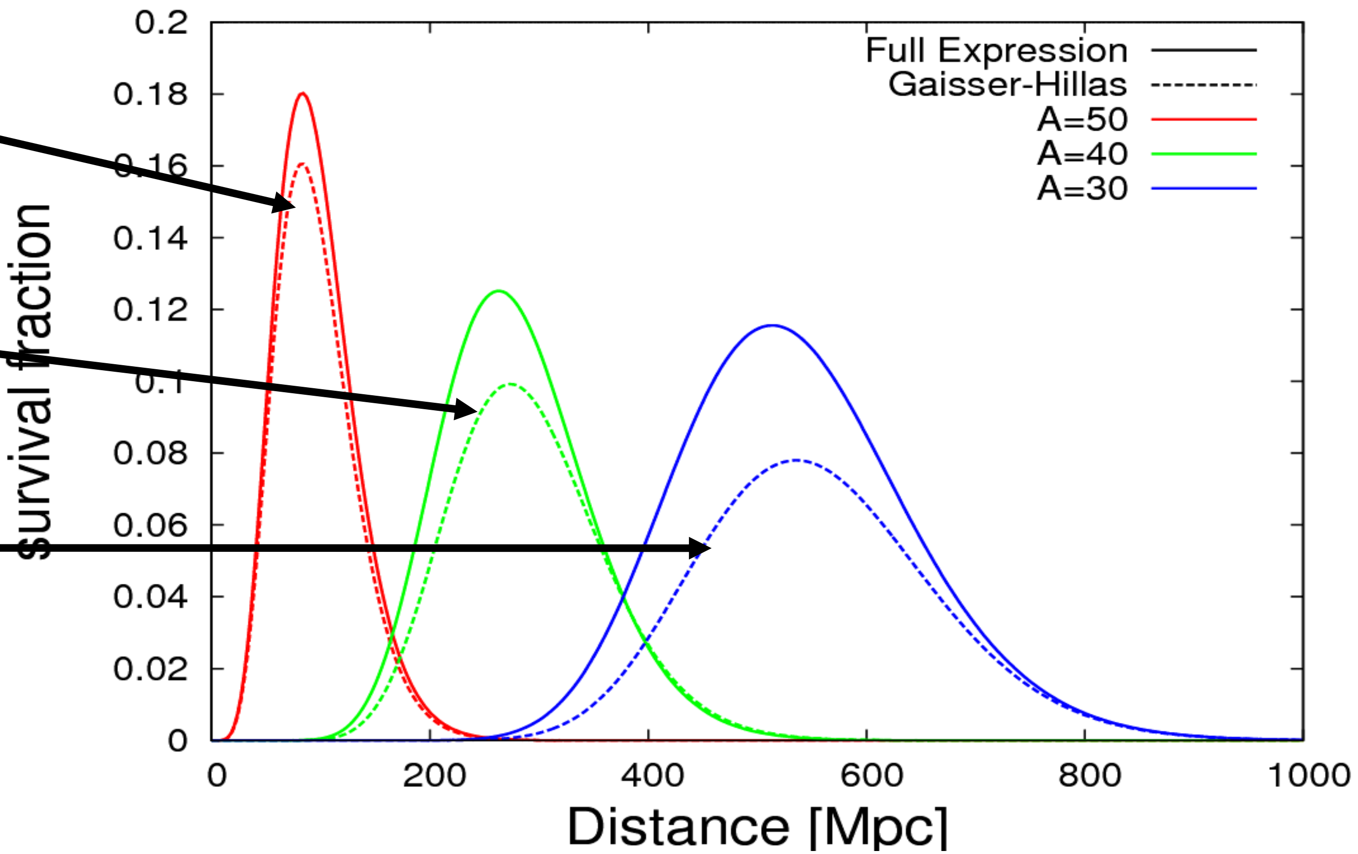
Cascade of Nuclei Through Species- Comparison of Approximation

Starting with Fe, $q_{\mathbf{max}} = 56$

$$f_{50} = \frac{t^6}{6!} e^{-\frac{t}{\tau_{50}}}$$

$$f_{40} = \frac{t^{16}}{16!} e^{-\frac{t}{\tau_{40}}}$$

$$f_{30} = \frac{t^{26}}{26!} e^{-\frac{t}{\tau_{30}}}$$



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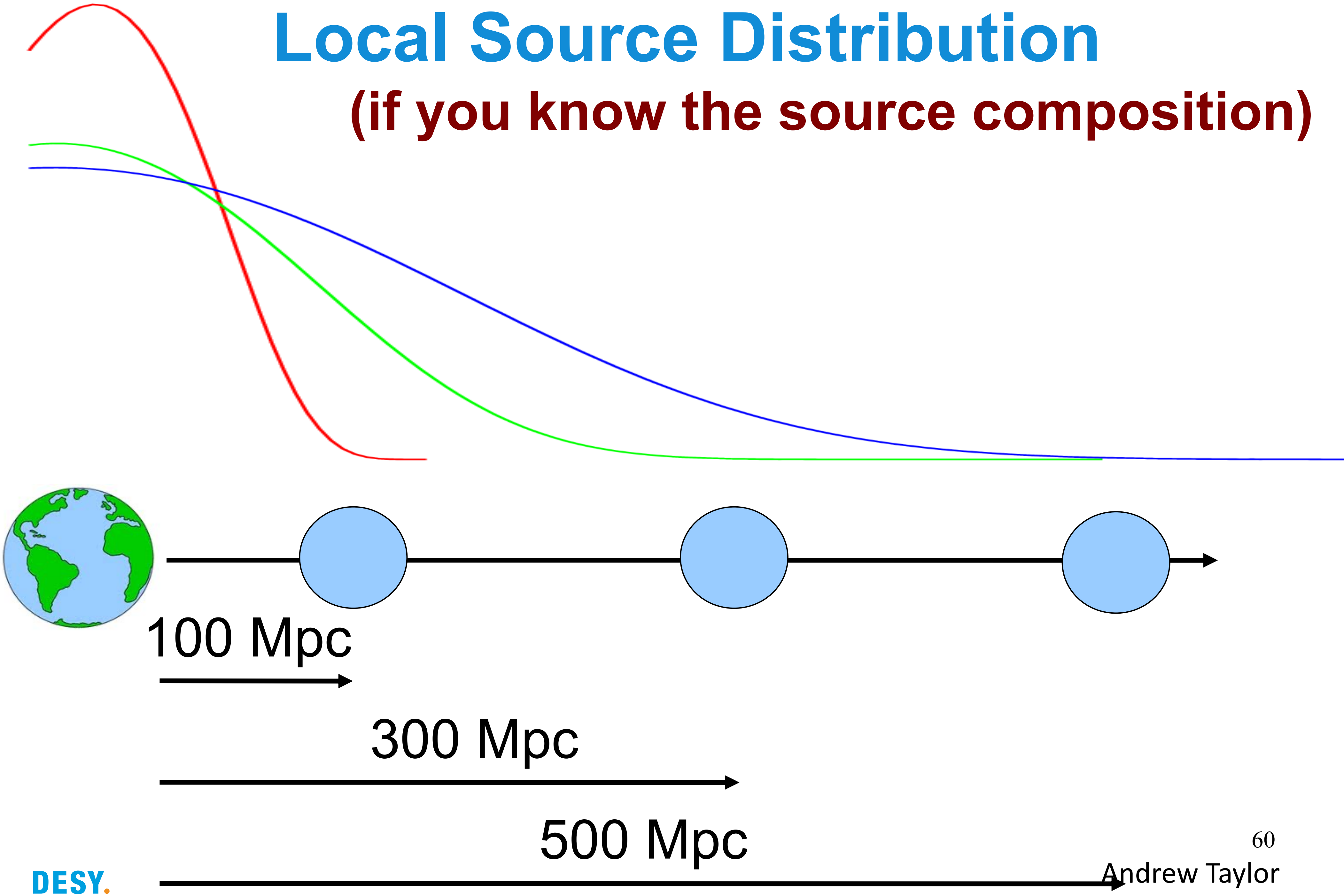
Composition – an Excellent Probe of the Local Source Distribution

(if you know the source composition)



Assumptions on Source Population

Spatial Distribution

motivated by star formation rate evolution

$$\frac{dN}{dV_C} \propto (1+z)^3 \qquad z < 1.9$$

$$\frac{dN}{dV_C} \propto (1+1.9)^3 \qquad 1.9 < z < 2.7$$

$$\frac{dN}{dV_C} \propto (1+1.9)^3 e^{-z/1.7} \qquad z > 2.7$$

Energy Distribution

motivated by Fermi acceleration theory

$$\frac{dN}{dE} \propto E^{-\alpha} \exp[-E/E_{Z,max}]$$

$$E_{Z,max} = (Z/26) \times E_{Fe,max}$$

Note- magnetic field horizon effects are neglected in the following.
This amounts to assuming: $d_s < (ct_H \lambda_{scat})^{1/2}$
ie. the source distribution may be approximated to be spatially continuous (also note, presence of t_H term comes from temporally continuous assumption)