Lecture 1 Plan:

1) An intro to the world of non-thermal particles (ie. "Cosmic Rays")

2) Shocks- what are they? What do they do to the gas passing through them?

3) Cosmic Ray Acceleration at Shocks

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The World of Non-Thermal Particles

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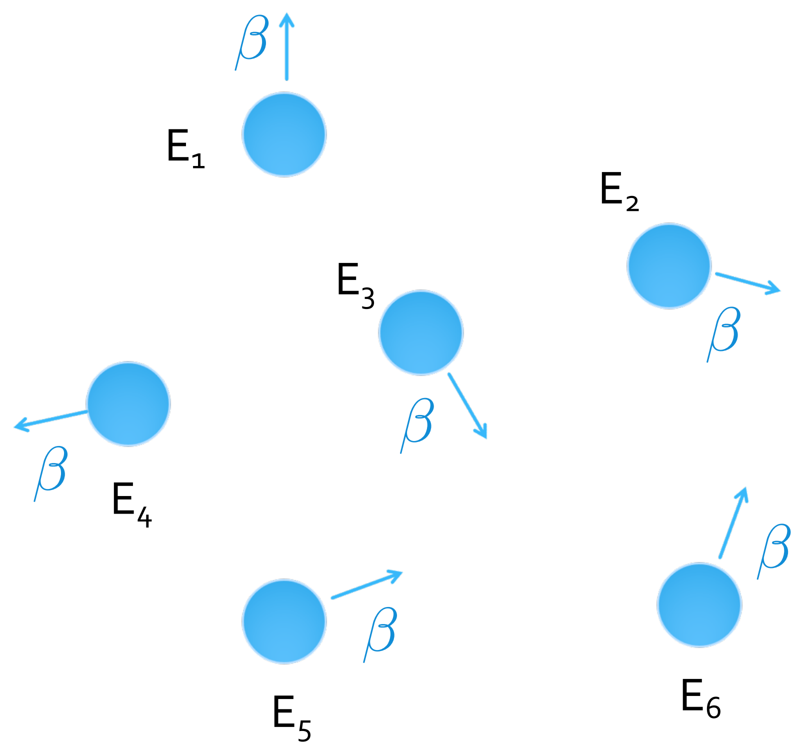
Thermal Particles

Thought experiment- imagine an ensemble of particles all with the same energy bouncing around in a box.....



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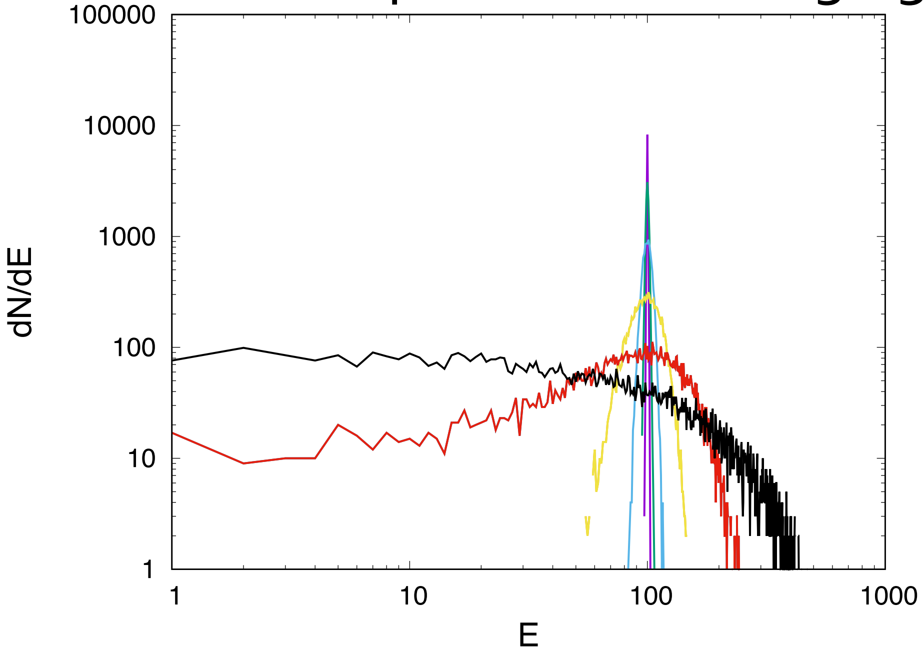
Origin of Thermalised Particle Distribution Function

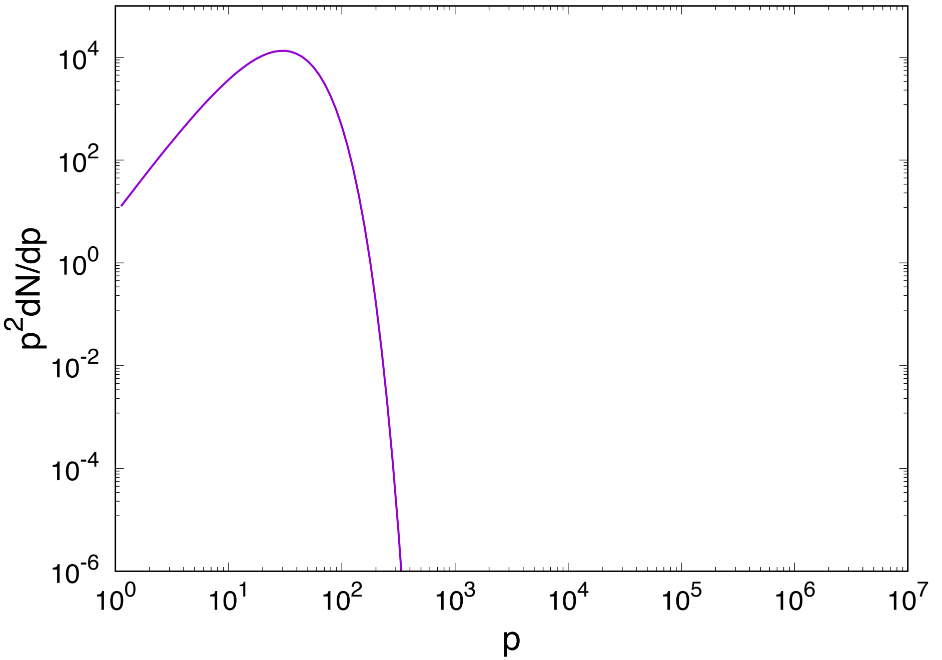
Ensemble of particles exchanging energies:

E_1	E_2	E_3	E_4	E_5	E_6
100	100	100	100	100	100
101	99	100	100	100	100
101	99	100	100	99	101
100	99	101	100	99	101
100	98	101	100	99	102
99	99	101	100	99	102

Relaxing to a Thermal Distribution

Ensemble of particles exchanging energies:





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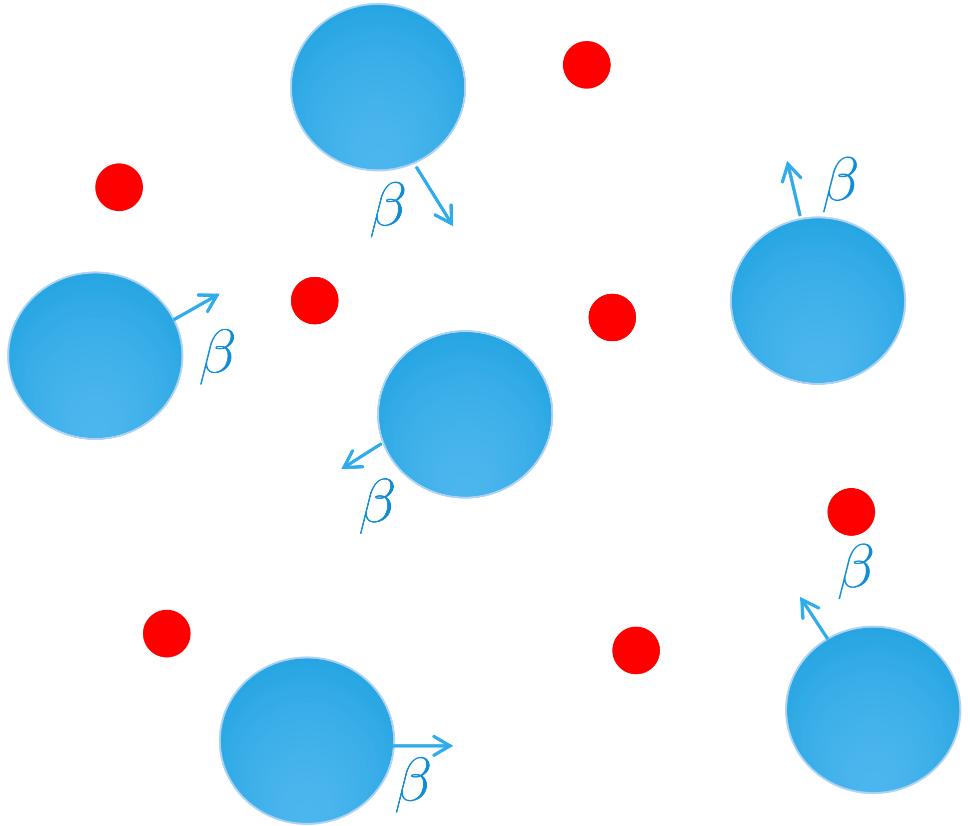
Microscopic Particles Thermalising off Macroscopic Objects

Another thought experiment- imagine an ensemble of thermalized macro particles (blue) with a cold set of micro particles injected (red).....



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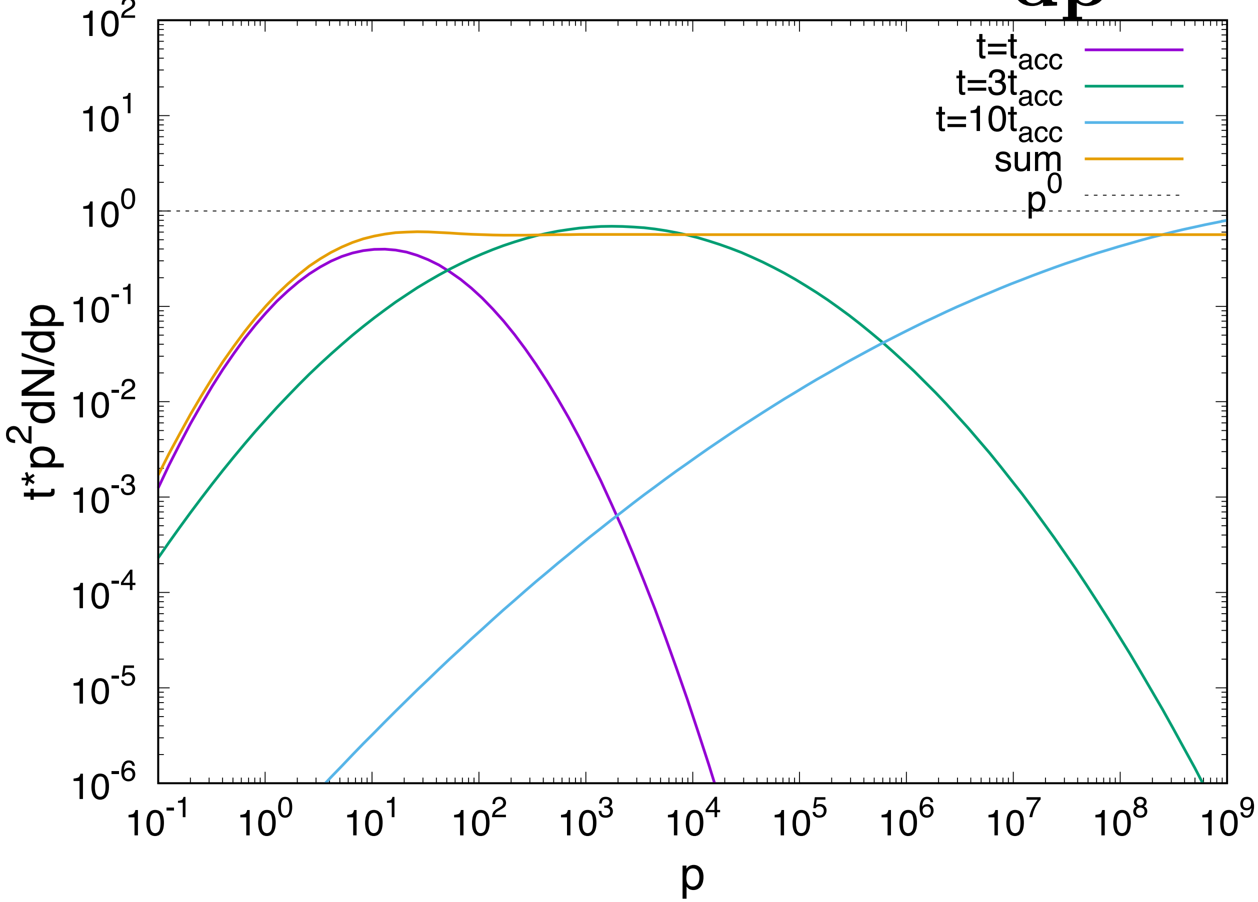
Green's Function for Stochastic Acceleration

$$\frac{\mathbf{dN}}{\mathbf{dp}} \propto \mathbf{e}^{-(\ln \mathbf{p})^2/4(\mathbf{t}/\mathbf{t_{acc}})}$$



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Shocks……..a Surprise!

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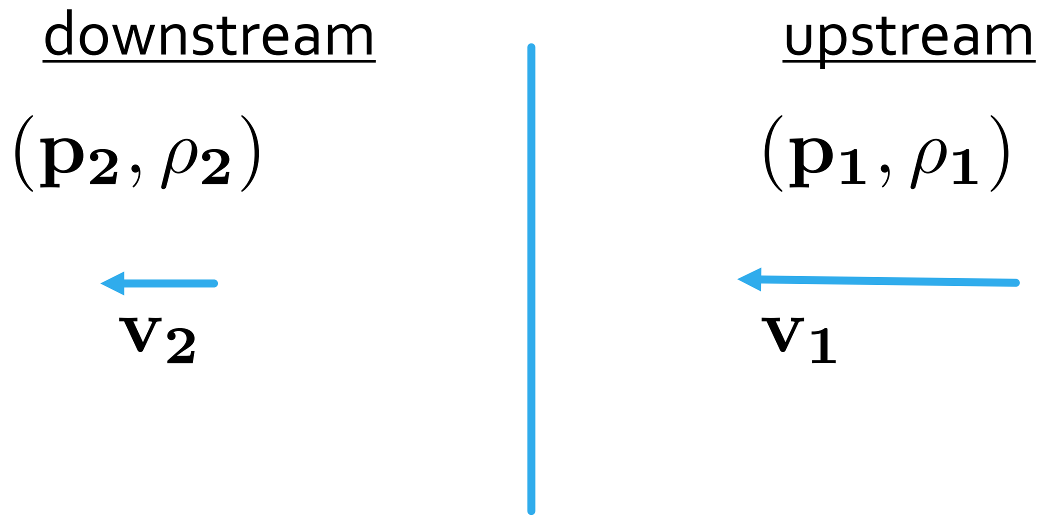
Collisional Shock- Conservation Conditions



Number Flux: $$\rho_1 \mathbf{v_1} = \rho_2 \mathbf{v_2}$$

Momentum Flux: $$\mathbf{p_1} + \rho_1 \mathbf{v_1^2} = \mathbf{p_2} + \rho_2 \mathbf{v_2^2}$$

Energy Flux: $$\frac{\gamma}{\gamma - 1}\mathbf{p_1 v_1} + \frac{1}{2}\rho_1 \mathbf{v_1^3} = \frac{\gamma}{\gamma - 1}\mathbf{p_2 v_2} + \frac{1}{2}\rho_2 \mathbf{v_2^3}$$

Collisional Shock- Cold Shock Case

Momentum Flux:

$$\rho_1 \mathbf{v_1^2} = \mathbf{p_2} + \rho_2 \mathbf{v_2^2}$$

$$\frac{\mathbf{p_2}}{\rho_1 \mathbf{v_1^2}} = \left(\mathbf{1} - \frac{\mathbf{v_2}}{\mathbf{v_1}}\right)$$

Energy Flux:

$$\frac{\mathbf{1}}{\mathbf{2}} \rho_1 \mathbf{v_1^3} = \left(\frac{\gamma}{\gamma - \mathbf{1}}\right) \mathbf{p_2 v_2} + \frac{\mathbf{1}}{\mathbf{2}} \rho_2 \mathbf{v_2^3}$$

$$\frac{\mathbf{2}\gamma}{\gamma - \mathbf{1}} \frac{\mathbf{p_2 v_2}}{\rho_1 \mathbf{v_1^3}} = \left(\mathbf{1} - \left(\frac{\mathbf{v_2}}{\mathbf{v_1}}\right)^{\mathbf{2}}\right) = \left(\mathbf{1} - \frac{\mathbf{v_2}}{\mathbf{v_1}}\right)\left(\mathbf{1} + \frac{\mathbf{v_2}}{\mathbf{v_1}}\right)$$

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Collisional Shock- Cold Shock Case

$$\frac{\mathbf{v_2}}{\mathbf{v_1}}\left(\mathbf{1}-\frac{\mathbf{v_2}}{\mathbf{v_1}}\right)=\left(\frac{\gamma-\mathbf{1}}{\mathbf{2}\gamma}\right)\left(\mathbf{1}-\left(\frac{\mathbf{v_2}}{\mathbf{v_1}}\right)^{\mathbf{2}}\right)$$

$$\left(\frac{\mathbf{v_2}}{\mathbf{v_1}}-\mathbf{1}\right)\left(\frac{\mathbf{v_2}}{\mathbf{v_1}}-\left(\frac{\gamma-\mathbf{1}}{\gamma+\mathbf{1}}\right)\right)=\mathbf{0}$$

So what are collisional shocks good for?
Stimulating the unstimulated degrees of freedom in the system where energy can be stored

Collisional Shock- Partition of Momentum and Energy

Downstream Momentum Partition:

$$\mathbf{p_2} = \frac{\mathbf{3}}{\mathbf{4}}\rho_{\mathbf{1}}\mathbf{v_1^2}$$

Downstream Energy Partition:

$$\frac{\gamma}{\gamma - \mathbf{1}}\mathbf{p_2 v_2} = \frac{\mathbf{15}}{\mathbf{16}}\left[\frac{\mathbf{1}}{\mathbf{2}}\rho_{\mathbf{1}}\mathbf{v_1^3}\right]$$

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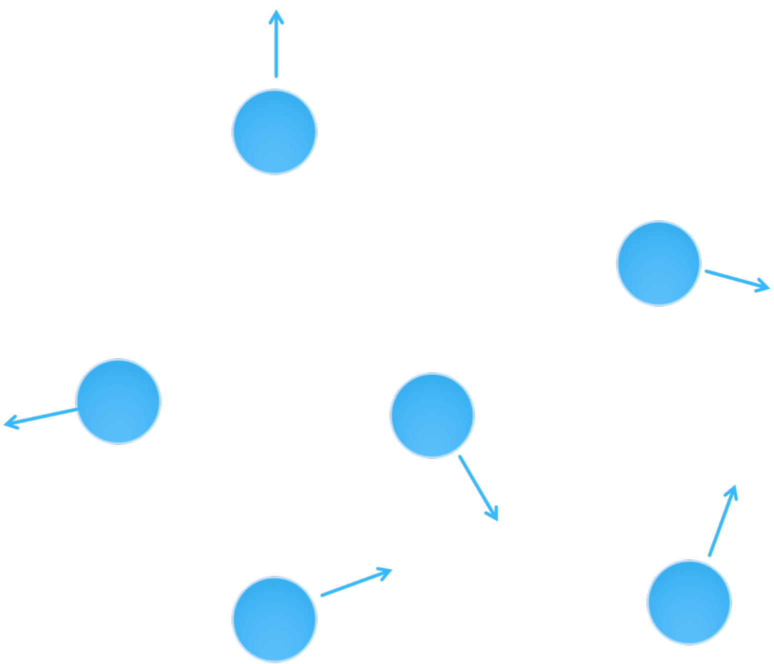
Collision Time

$$\mathbf{t} = \frac{\mathbf{1}}{\mathbf{n_e}\sigma_\mathbf{T}\mathbf{c}}$$

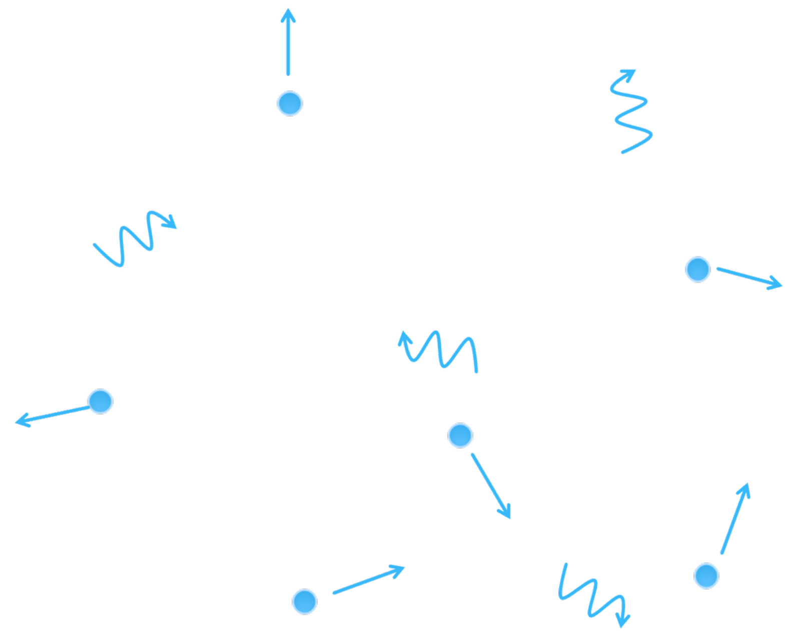
$$\approx \left(\frac{\mathbf{1\ cm^{-3}}}{\mathbf{n_e}}\right)\ \mathbf{Myr}$$

Energy Exchange at Shocks

Collisional Shock



Collisionless Shock



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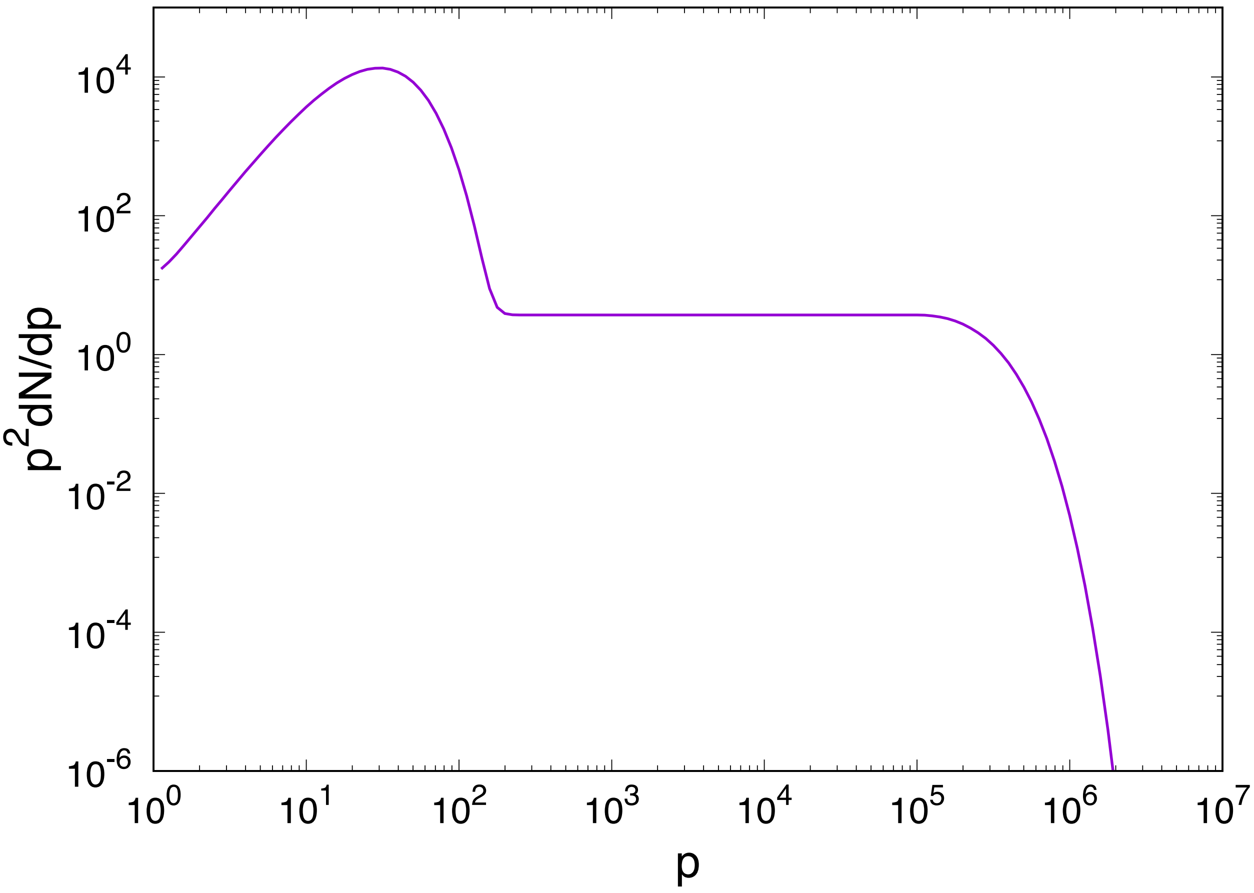
Collisionless Shock- the Injection Problem



$$\mathbf{V = \frac{1}{2} m \Delta v^2}$$

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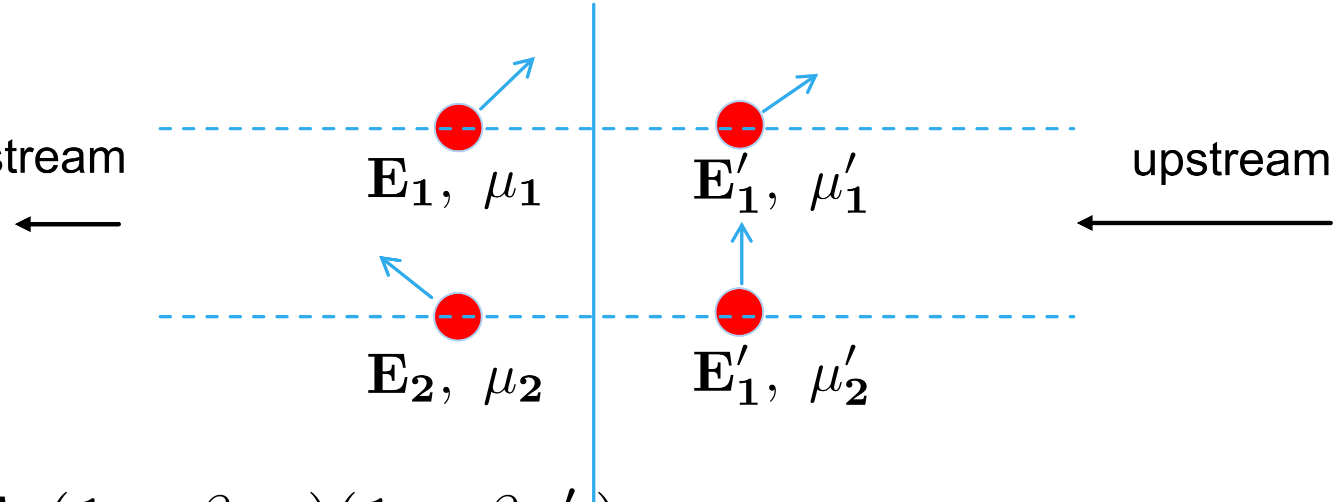
Particle Acceleration at Collisionless Shocks



$$\mathbf{E_2} = \mathbf{\Gamma^2 E_1}(\mathbf{1} - \beta\mu_1)(\mathbf{1} + \beta\mu'_2)$$

$$\mu' = \frac{\mu - \beta}{\mathbf{1} - \beta\mu}$$

$$\mathbf{E_2} = \mathbf{\Gamma^2 E_1}(\mathbf{1} - \beta\mu_1)\left(\mathbf{1} + \beta\left(\frac{\mu_2 - \beta}{\mathbf{1} - \beta\mu_2}\right)\right)$$

$$\mathbf{E_2} = \mathbf{E_1}\left(\frac{\mathbf{1} + \beta\mu_1}{\mathbf{1} + \beta\mu_2}\right)$$

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Fermi Shock Acceleration

<u>Energy</u>

$$\frac{\Delta E}{E} = \frac{4v}{3c} = \frac{4}{3}\beta \text{ (energy gain)}$$

$$E_1 = \left(1 + \frac{4}{3}\beta\right) E_0$$

$$E_n = \left(1 + \frac{4}{3}\beta\right)^n E_0$$

<u>Number</u>

$$\frac{\Delta N}{N} = -\frac{4v}{3c} = -\frac{4}{3}\beta \text{ (advection downstream)}$$

$$N_1 = \left(1 - \frac{4}{3}\beta\right) N_0$$

$$N_n = \left(1 - \frac{4}{3}\beta\right)^n N_0$$

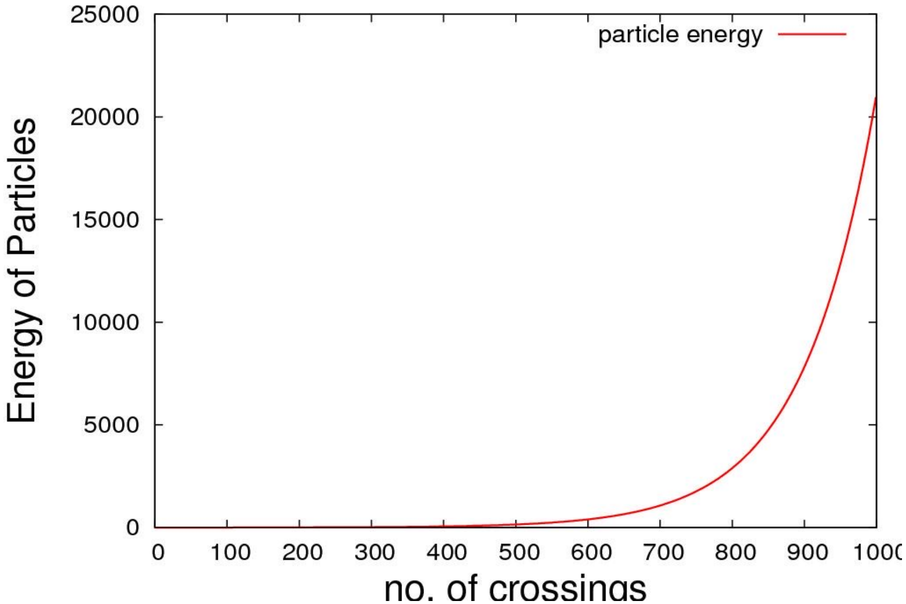
So $n \sim 1/\beta$ crossings are needed before the particle population is significantly altered → SNRs have $v_{sh} \sim 10^3$ km s^{-1} so $\beta \sim 10^{-2}$

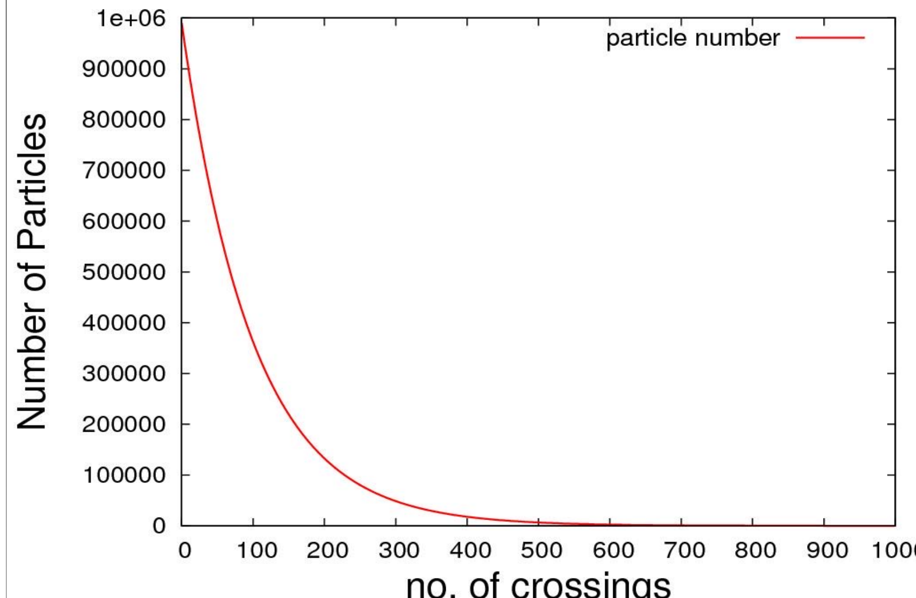
Fermi Shock Acceleration

Energy

Number

$\beta \sim 10^{-2}$





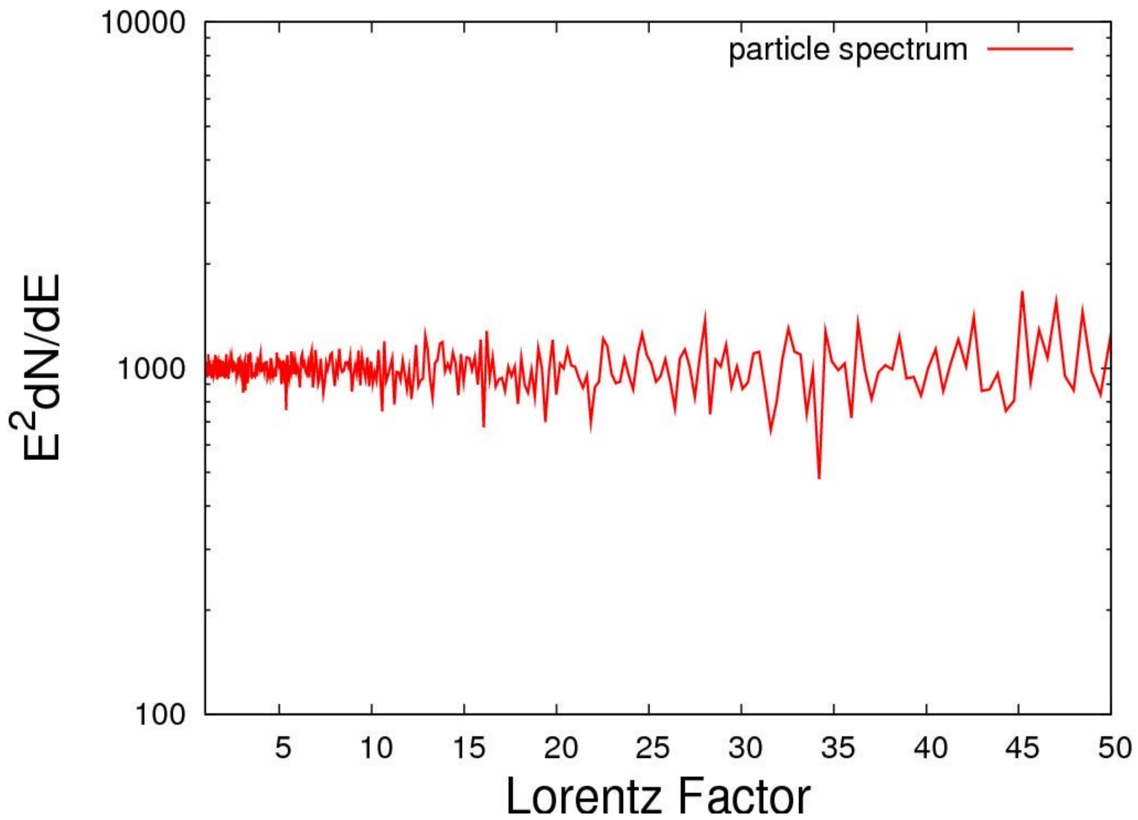
Fermi Shock Acceleration

So,

$$\frac{\Delta N}{\Delta E} = \frac{N_0}{E_0}\left(\frac{1-4\beta/3}{1+4\beta/3}\right)^{n}$$

$$\approx \frac{N_0}{E_0}\left(1+4\beta/3\right)^{-2n}$$

$$\approx N_0 E_0 E^{-2}$$



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Stochastic Acceleration (Fermi Second Order)

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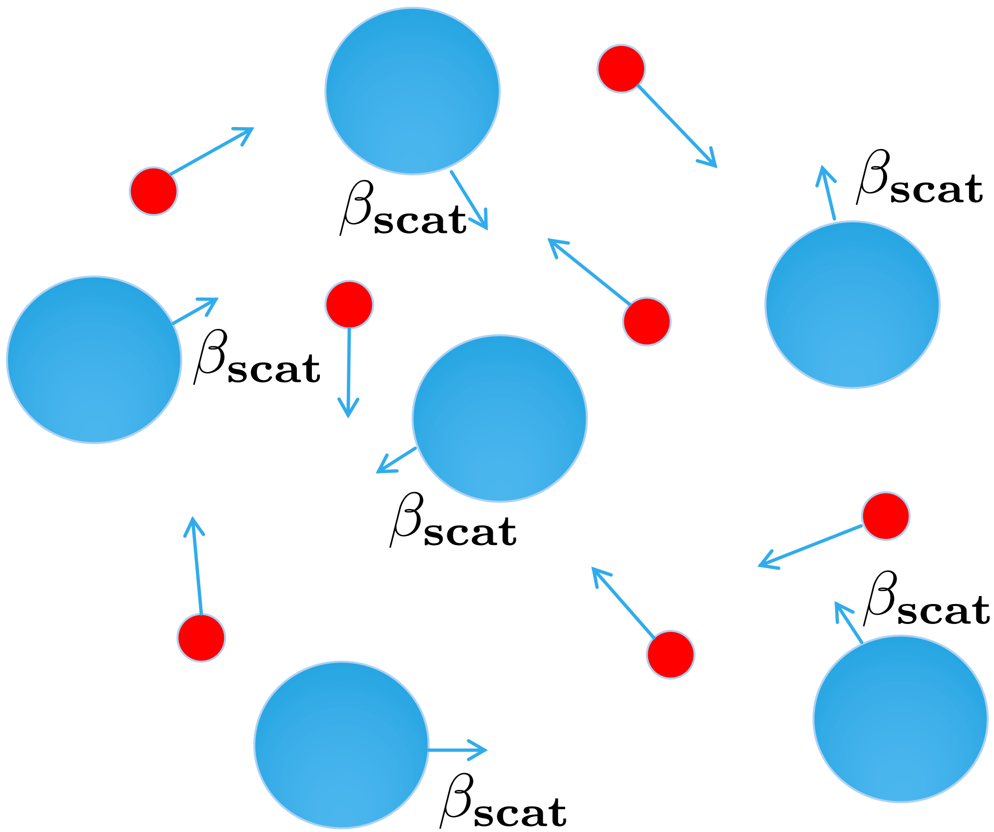
Stochastic Acceleration/Propagation

$$\mathbf{D_{xx}D_{pp}} \approx \beta^{\mathbf{2}}_{\mathbf{scat}}\mathbf{p}^{\mathbf{2}}$$



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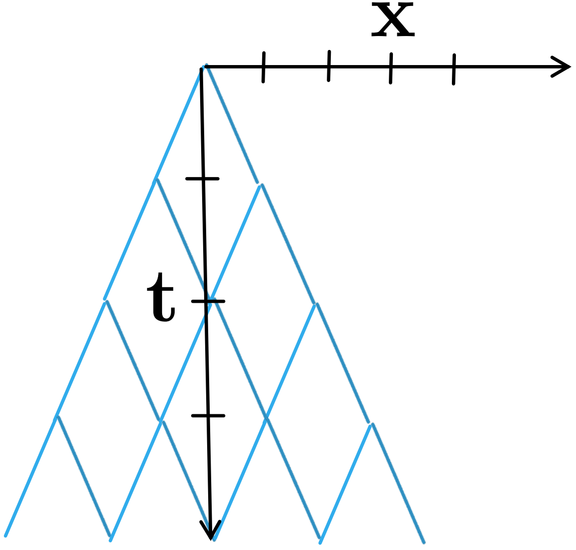
Random Walks



$$\gamma(\mathbf{t}+\mathbf{1}) = \mathbf{t}!$$

$$\gamma(\mathbf{t}+\mathbf{1}) = \int_{\mathbf{0}}^{\infty} \mathbf{x}^{\mathbf{t}} \mathbf{e}^{-\mathbf{x}} \mathbf{dx}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) = \frac{\gamma(\mathbf{t}+\mathbf{1})}{[\gamma([\mathbf{t}-\mathbf{x}]/\mathbf{2}+\mathbf{1})\gamma([\mathbf{x}+\mathbf{t}]/\mathbf{2}+\mathbf{1})](\mathbf{2}^{\mathbf{t}})}$$

$$\mathbf{f}(\mathbf{x},\mathbf{t}) \approx \frac{\mathbf{e}^{-\mathbf{x}^{\mathbf{2}}/(\mathbf{2t})}}{[\pi/(\mathbf{t}/\mathbf{2})]^{\mathbf{1}/\mathbf{2}}}$$

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Random Walks in Physical Space and Momentum Space

Spatial spread:

$$\mathbf{\Delta x = D_{xx}/c} \qquad \frac{\mathbf{dN}}{\mathbf{dx}} \propto \mathbf{e^{-x^2/4D_{xx}t}}$$

$$\frac{\mathbf{dN}}{\mathbf{dx}} \propto \mathbf{e^{-x^2/4c^2 t_{scat} t}}$$

Momentum spread:

$$\frac{\mathbf{\Delta E}}{\mathbf{E}} \propto \beta$$

$$\frac{\mathbf{dN}}{\mathbf{dp}} \propto \mathbf{e^{-(\ln p)^2/4(D_{pp}/p^2)t}}$$

$$\frac{\mathbf{dN}}{\mathbf{dp}} \propto \mathbf{e^{-(\ln p)^2/4(t/t_{acc})}}$$

Momentum Continuity Equation (Boltzman Equation)

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \nabla_{\mathbf{p}} \cdot \left[(\mathbf{D_{pp}} \nabla_{\mathbf{p}} \mathbf{f}) - \frac{\mathbf{p}}{\tau_{\mathbf{loss}}(\mathbf{p})} \mathbf{f} \right] - \frac{\mathbf{f}}{\tau_{\mathbf{esc}}(\mathbf{p})} + \frac{\mathbf{Q}}{\mathbf{p^2}}$$

Acceleration

Radiative Losses

Escape

Source term

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Stochastic Particle Acceleration-Random Walk Result (Momentum)

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \nabla_{\mathbf{p}} \cdot \left[(\mathbf{D_{pp}} \nabla_{\mathbf{p}} \mathbf{f}) - \frac{\mathbf{p}}{\tau_{\mathbf{loss}}(\mathbf{p})} \mathbf{f} \right] - \frac{\mathbf{f}}{\tau_{\mathbf{esc}}(\mathbf{p})} + \frac{\mathbf{Q}}{\mathbf{p^2}}$$

Steady state

No losses

Delta injection

$$\mathbf{D_{pp}} \propto \mathbf{p^q}$$

$$\frac{\partial^2 \mathbf{f}}{\partial \mathbf{p^2}} + \frac{(\mathbf{2+q})}{\mathbf{p}} \frac{\partial \mathbf{f}}{\partial \mathbf{p}} - \frac{\mathbf{4}\tau_{\mathbf{acc}}}{\tau_{\mathbf{esc}}} \frac{\mathbf{f}}{\mathbf{p^2}} = \delta(\mathbf{p})$$

For $\mathbf{f} = \mathbf{p}^{-\alpha}$ and $\mathbf{q} = \mathbf{2}$

$$\alpha^{\mathbf{2}} - \mathbf{3}\alpha - \frac{\mathbf{4}\tau_{\mathbf{acc}}}{\tau_{\mathbf{esc}}} = \mathbf{0}$$

Particle Acceleration- When Are E^{-2} Type Spectra Expected?

$$\alpha^2 - 3\alpha - \frac{4\tau_{\mathbf{acc}}}{\tau_{\mathbf{esc}}} = 0$$

$$\alpha = \frac{3}{2} \pm \left(\frac{4\tau_{\mathbf{acc}}}{\tau_{\mathbf{esc}}} + \frac{9}{4}\right)^{1/2}$$

$$\frac{\tau_{\mathbf{acc}}}{\tau_{\mathbf{esc}}} = 1$$

$$\mathbf{f} = \frac{\mathbf{dN}}{\mathbf{d^3p}} = \mathbf{p}^{-4}$$

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Fermi (First Order) Acceleration Time

$$\mathbf{t_{acc} = E\frac{\Delta t_{cycle}}{\Delta E_{cycle}}}$$

Transport of particles in each region is dictated by competition between diffusion and advection

downstream | upstream

$$\mathbf{t_{diff} = \frac{R^2}{D_{xx}}} \qquad \mathbf{t_{adv} = \frac{R}{v_{adv}}}$$

Balancing these timescales

$$\mathbf{t_{resid} = \frac{D_{xx}}{(c\beta_{sh})^2}}$$

Fermi (First Order) Acceleration Time

$$\mathbf{t_{acc} = E\frac{\Delta t_{cycle}}{\Delta E_{cycle}}}$$

$$\mathbf{t_{resid} = \frac{D_{xx}}{(c\beta_{sh})^2}}$$

However, during the time it takes advection to dominate over diffusion, the particle will have crossed the shock $\mathbf{1}/\beta$ times

$$\mathbf{\Delta t_{cycle} = \frac{D_{xx}}{(c^2\beta_{sh})}}$$

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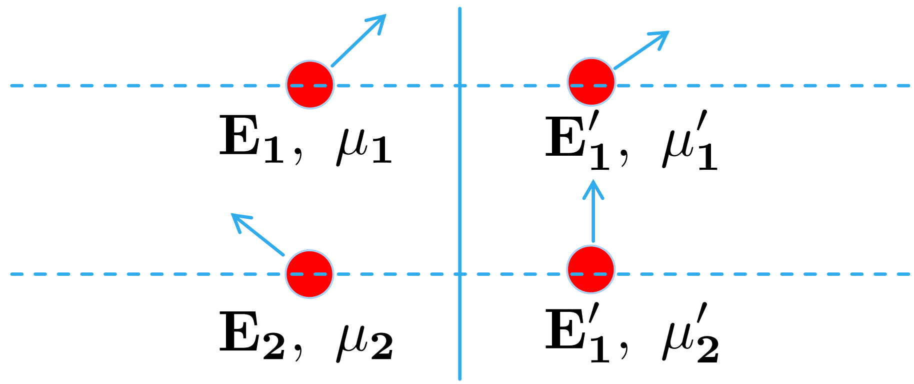
Fermi (First Order) Acceleration Time

$$\mathbf{t_{acc} = E\frac{\Delta t_{cycle}}{\Delta E_{cycle}}}$$

$$\mathbf{\Delta t_{cycle} = \frac{D_{xx}}{(c^2\beta_{sh})}}$$

$$\mathbf{\Delta E_{cycle} = E\beta_{sh}}$$

$$\mathbf{t_{acc} = \frac{D_{xx}}{(c\beta_{sh})^2} = \frac{t_{scat}}{\beta_{sh}^2}}$$



$$\mathbf{E_2 = E_1\left(\frac{1+\beta\mu_1}{1+\beta\mu_2}\right)}$$

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Fermi (Second Order) Acceleration Time

$$\mathbf{t_{acc}} = \mathbf{E}\frac{\mathbf{\Delta t_{scat}}}{\mathbf{\Delta E_{scat}}}$$

$$\mathbf{\Delta E_{scat}} = \mathbf{E}\beta^{\mathbf{2}}_{\mathbf{scat}}$$

$$\mathbf{t_{acc}} = \frac{\mathbf{t_{scat}}}{\beta^{\mathbf{2}}_{\mathbf{scat}}}$$

The Need for Efficient Accelerators
….what means efficient?

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Particle Acceleration in AGN

$$\mathbf{t_{acc}} = \eta \frac{\mathbf{R_{lar}}}{\mathbf{c}\beta^{\mathbf{2}}}$$

$$\mathbf{t_{esc.}} = \frac{\mathbf{R^2}}{\eta \mathbf{c} \mathbf{R_{lar}}}$$

Maximum energy
(Hillas criterion)

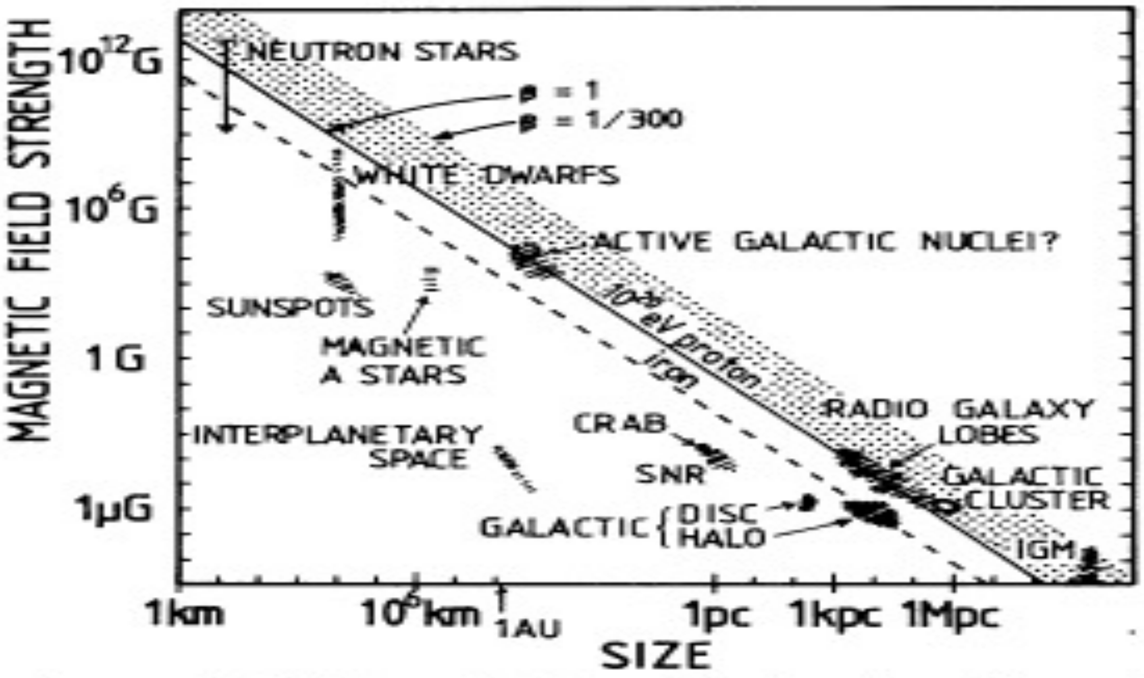
$$\mathbf{R_{lar}} = \frac{\beta}{\eta}\mathbf{R}$$

AM Hillas (1984)

$$\mathbf{R_{lar}(E,B)} = \left(\frac{\mathbf{E}}{\mathbf{10\ EeV}}\right)\left(\frac{\mathbf{1\ mG}}{\mathbf{B}}\right)\mathbf{10\ pc}$$

The Hillas Criterion (Implicitly Assumes Accelerator is Efficient)

AM Hillas (1984)



$\eta \approx 1$ assumed in above plot

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The End of the Accelerated Spectra: Cutoffs

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Particle Transport Equation

- Cut-offs arise naturally in the general solution of the transport equation for particles

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \nabla_{\mathbf{p}} \cdot \left[(\mathbf{D_{pp}} \nabla_{\mathbf{p}} \mathbf{f}) - \frac{\mathbf{p}}{\tau_{\mathbf{loss}}(\mathbf{p})} \mathbf{f} \right] - \frac{\mathbf{f}}{\tau_{\mathbf{esc}}(\mathbf{p})} + \frac{\mathbf{Q}}{\mathbf{p^2}}$$

Acceleration

Radiative Losses

Escape

Source term

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Cut-off Shape

- Interplay of acceleration and cooling defines the value of the cut-off of the primary particles:

$$\frac{\mathbf{dN}}{\mathbf{dE_e}} \propto \mathbf{E_e^{-\Gamma} e^{-(E_e/E_{\max})^{\beta_e}}} \qquad \beta_\mathbf{e} = \mathbf{2 - q - r}$$

- In the following, demonstrations for this result will be shown for the case of stochastic acceleration scenarios. However, in reality, this result is more general, holding also for shock acceleration scenarios.

[see Schlickeisser et al. 1985, Zirakashvili et al. 2007, Stawarz et al. 2008]

A Simple Case- q=1, only escape

- Bohm diffusion (**q=1**) + only escape results in simple exponential cutoff.
- Some simplifications to the transport equation:

$$\frac{\partial \mathbf{f}}{\partial \mathbf{t}} = \nabla_{\mathbf{p}} \cdot \left[(\mathbf{D_{pp}} \nabla_{\mathbf{p}} \mathbf{f}) - \frac{\mathbf{p}}{\tau_{\mathbf{loss}}(\mathbf{p})} \mathbf{f} \right] - \frac{\mathbf{f}}{\tau_{\mathbf{esc}}(\mathbf{p})} + \frac{\mathbf{Q}}{\mathbf{p^2}}$$

Steady state

No losses

Delta injection

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A Simple Case (II)- q=1, only escape

- Rearranging the terms (and explicitly stating the dependences from p of the parameters):

$$\frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_0 \frac{p}{p_0}\frac{\partial f}{\partial p}\right) - \frac{f}{\tau_{esc}(p)} = \delta(p), \qquad \tau_{esc}(p) \propto p^{-1}$$

$$\boxed{\frac{\partial^2 f}{\partial p^2} + \frac{3}{p}\frac{\partial f}{\partial p} - \left(\frac{1}{D_0 \tau_0}\right) f = \delta(p)}$$

Cutoff comes from balancing 1st and 3rd term → $f \propto A e^{-p/p_\tau}$

Recall generally, $\beta_e = 2 - q - r$

$$q = 1,\ r = 0,\ \rightarrow\ \beta_e = 1$$

(Note- energy losses for the $r = 0$ case will not alter this result)

Particle Acceleration with Cooling

$$\mathbf{\frac{dE_e}{cdt} = \frac{4}{3}\Gamma_e^2 \sigma_T U_B}$$

$$\mathbf{t_{cool} = \frac{9}{8\pi\alpha}\left(\frac{m_e}{E_\gamma^{sync}}\right) t_{lar}}$$

$$\mathbf{t_{cool} = E_e \frac{dt}{dE_e}}$$

$$\mathbf{\sigma_T U_{Bcrit} \frac{hc}{(m_e c^2)^2} = (2\pi/3)\alpha}$$

$$\mathbf{t_{cool} = \frac{9}{8\pi\alpha}\frac{h}{E_e}\frac{U_{B_{crit.}}}{U_B}}$$

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Particle Acceleration with Cooling

$$\mathbf{t_{cool}} = \frac{\mathbf{9}}{\mathbf{8}\pi\alpha}\frac{\mathbf{h}}{\mathbf{E_e}}\frac{\mathbf{U_{B_{crit.}}}}{\mathbf{U_B}}$$

$$\mathbf{t_{cool}} = \frac{\mathbf{9}}{\mathbf{8}\pi\alpha}\left(\frac{\mathbf{m_e}}{\mathbf{E_\gamma^{sync}}}\right)\mathbf{t_{lar}}$$

$$\mathbf{t_{lar}} = \frac{\mathbf{2}\pi\mathbf{E_e}}{\mathbf{eBc}} = \Gamma_{\mathbf{e}}\left(\frac{\mathbf{B_{crit}}}{\mathbf{B}}\right)\frac{\mathbf{h}}{\mathbf{m_e}}$$

$$\mathbf{E_\gamma^{sync}} = \Gamma_{\mathbf{e}}^{\mathbf{2}}\left(\frac{\mathbf{B}}{\mathbf{B_{crit}}}\right)\mathbf{m_e}$$

Particle Acceleration with Cooling

$$\mathbf{t_{acc}} = \eta \frac{\mathbf{R_{lar}}}{\mathbf{c}\beta^{\mathbf{2}}}$$

$$\mathbf{t_{cool}} = \frac{\mathbf{9}}{\mathbf{8}\pi\alpha} \left(\frac{\mathbf{m_e}}{\mathbf{E_\gamma^{sync}}} \right) \mathbf{t_{lar}}$$

$$\mathbf{E_\gamma^{sync}} \approx \frac{\mathbf{9}}{\mathbf{4}} \eta^{-\mathbf{1}} \beta^{\mathbf{2}} \frac{\mathbf{m_e}}{\alpha}$$

Maximum synchrotron energy tells us how efficient accelerator is!

Where is $\mathbf{E_\gamma^{sync}}$?

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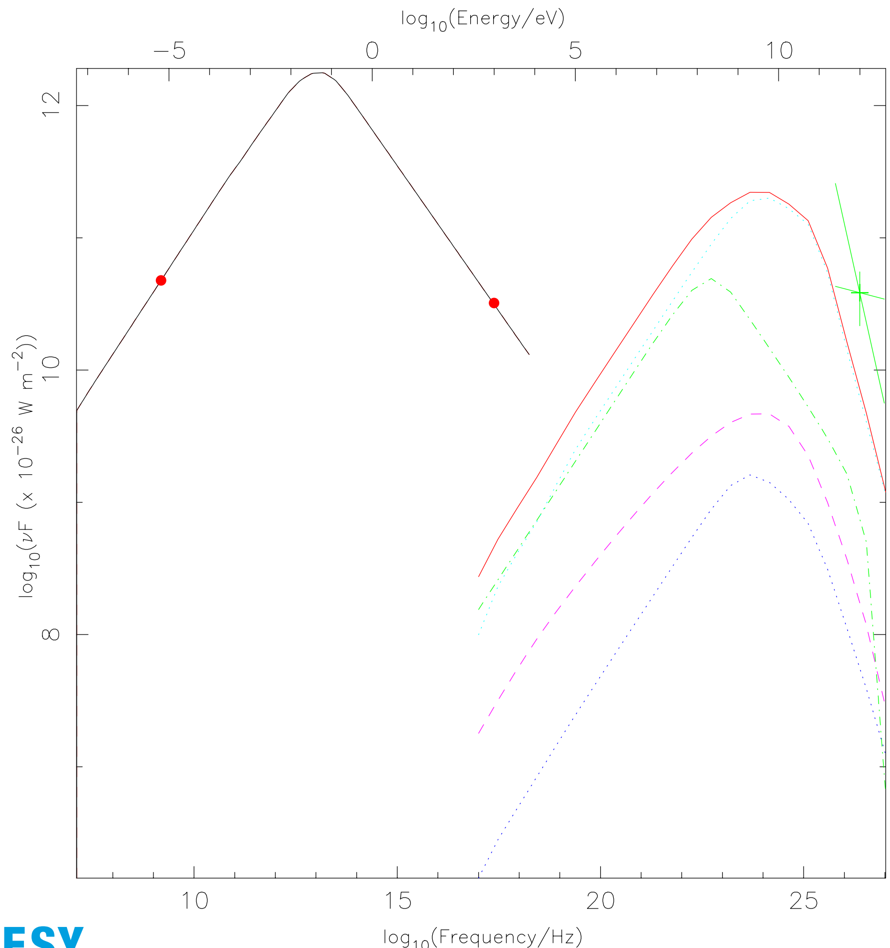
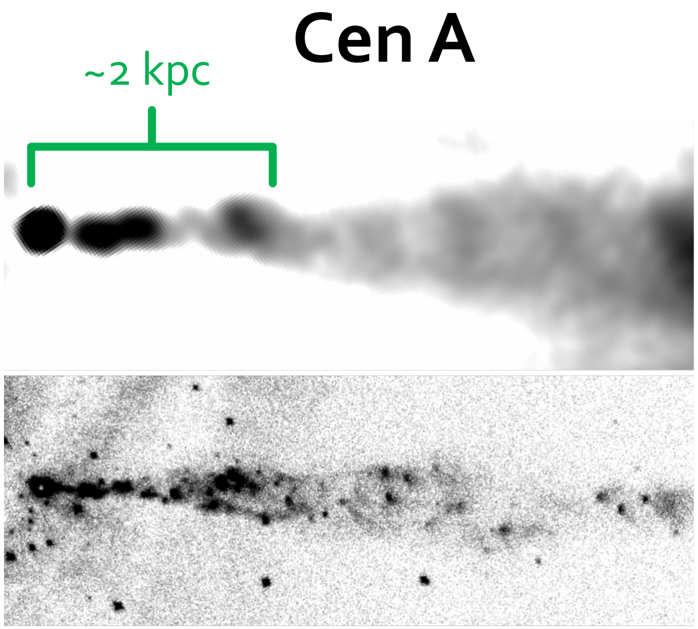
End of First Lecture

Synchroton Cutoff for AGN?

Evidence of a candidate source (Cen A) operating as an efficient accelerator?

Hardcastle et al. (1103.1744)





$$\eta < 10^4$$

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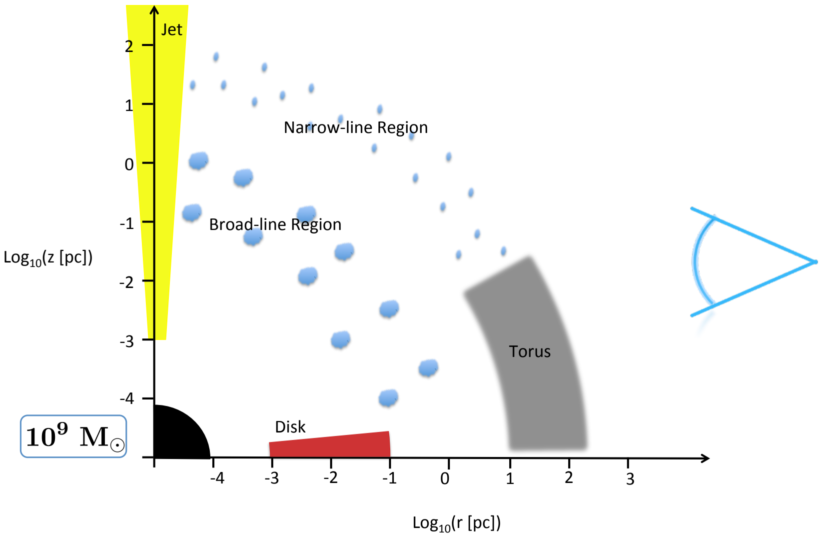
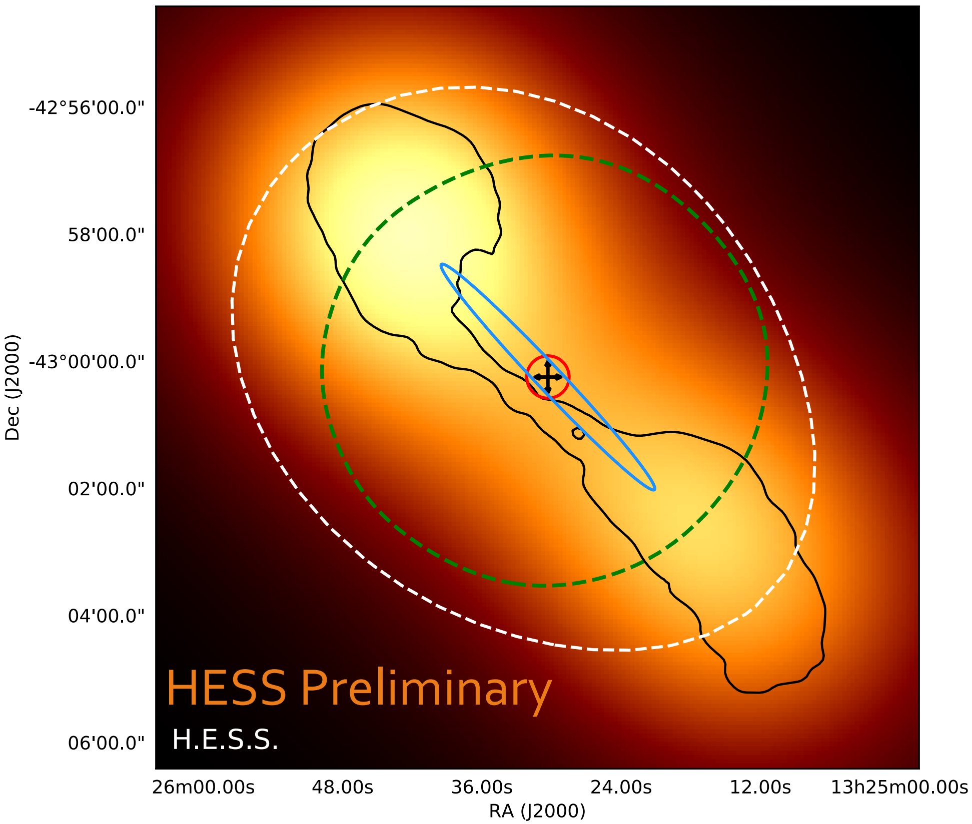
Centaurus A - VHE Extension



HESS Detected Extension on ~2kpc scale



[HESS- F. Rieger, A. Taylor, et al., Nature- accepted today!]

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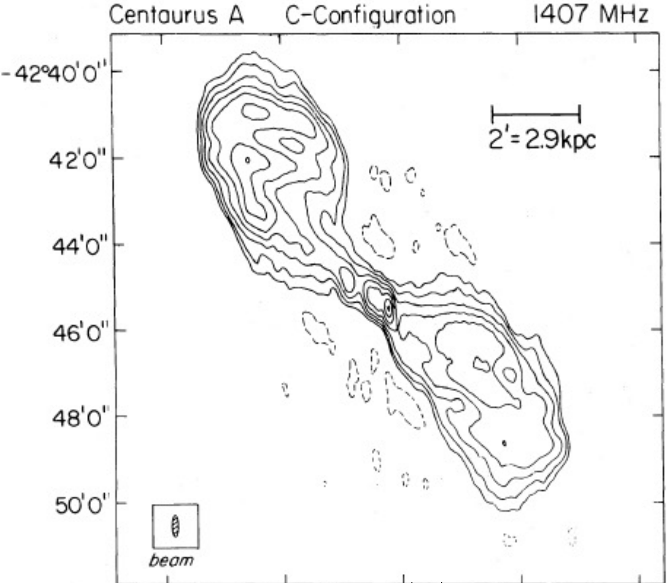
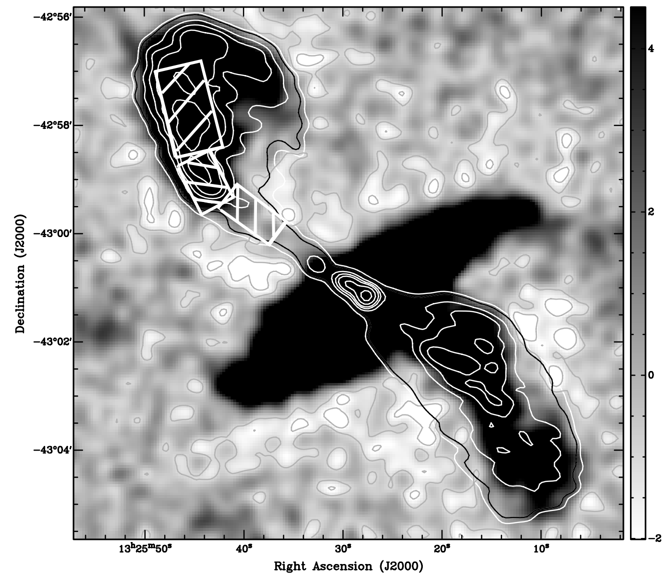
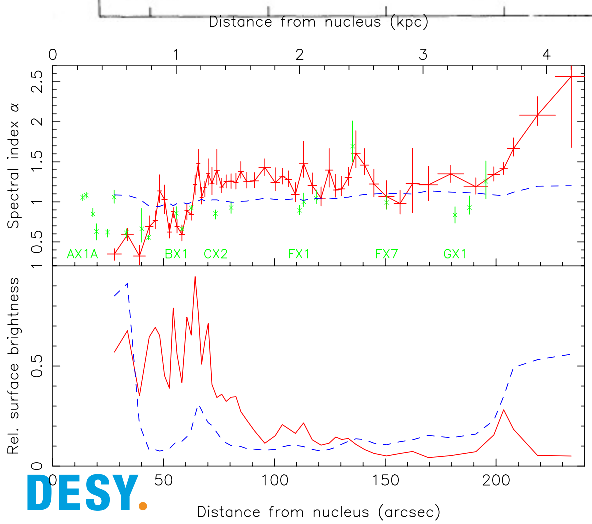
Centaurus A's Inner Jet- A Cosmic Lab

[J. Burns et al., ApJ (1983)]



[A. Weiss et al., A&A (2008)]

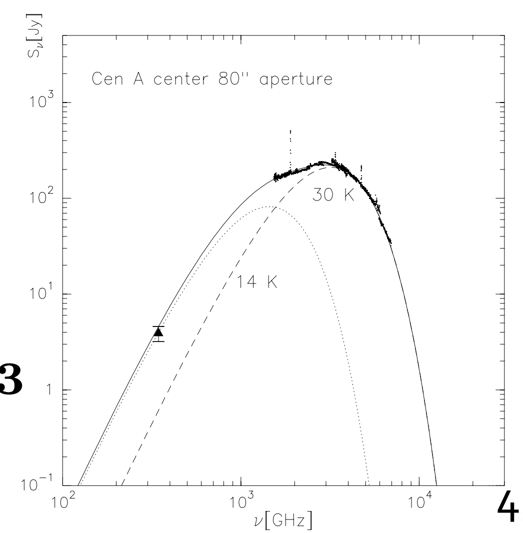




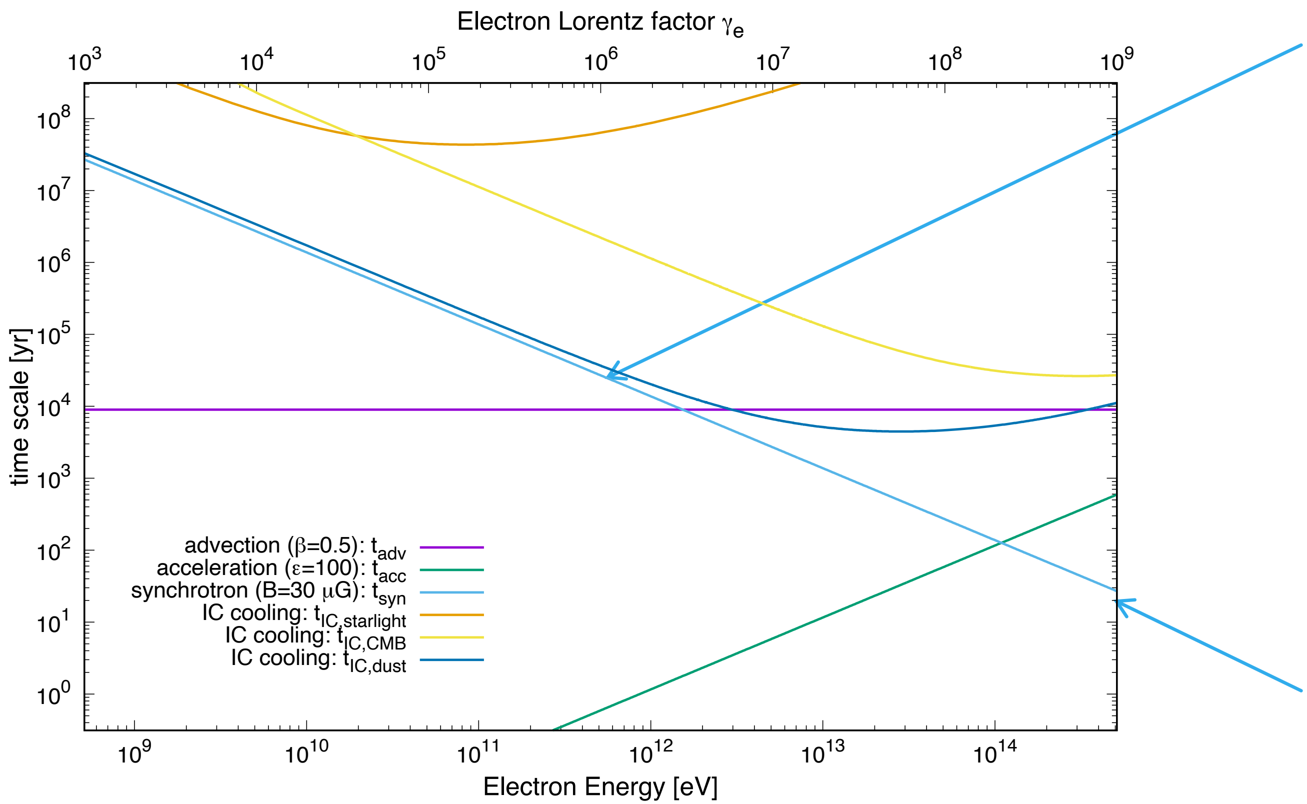
$$B_{eq} = 60\ \mu G$$

$$U_B \approx 10\ eV\ cm^{-3}$$

$$U_{IR} \approx 10\ eV\ cm^{-3}$$



Transport & Cooling Times of Electrons in Cen A's Jets



Synchrotron cooling dominates

Cooling time becomes shorter than advection time

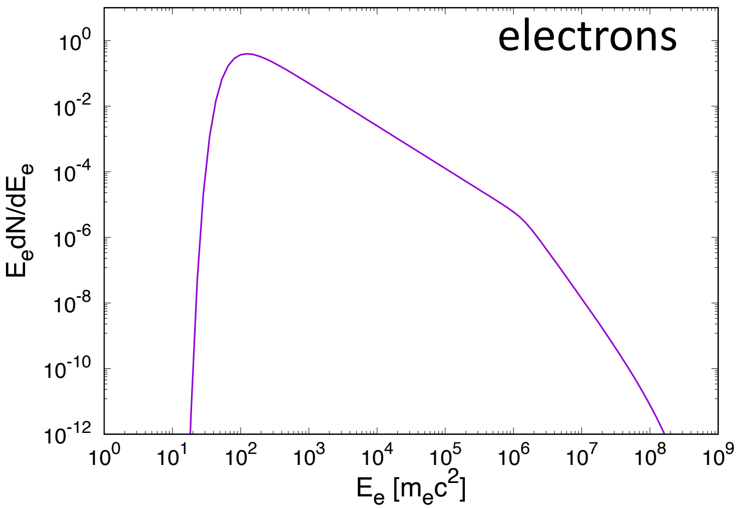
Electrons accelerated to 0.1 PeV energies

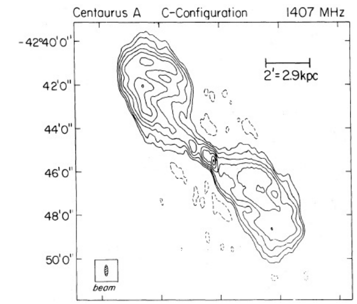
$$\cancel{\frac{\partial n}{\partial t}} = -\nabla_p \cdot \left[\frac{p}{\tau_{acc}(p)} n - \frac{p}{\tau_{loss}(p)} n\right] - \frac{n}{\tau_{esc}(p)} + Q$$

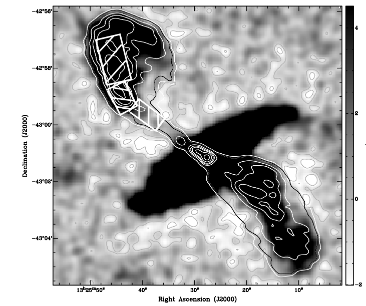
drey

DESY. H.E.S.S.

Distinguishing Cen A's Nucleus and Inner Jet SED

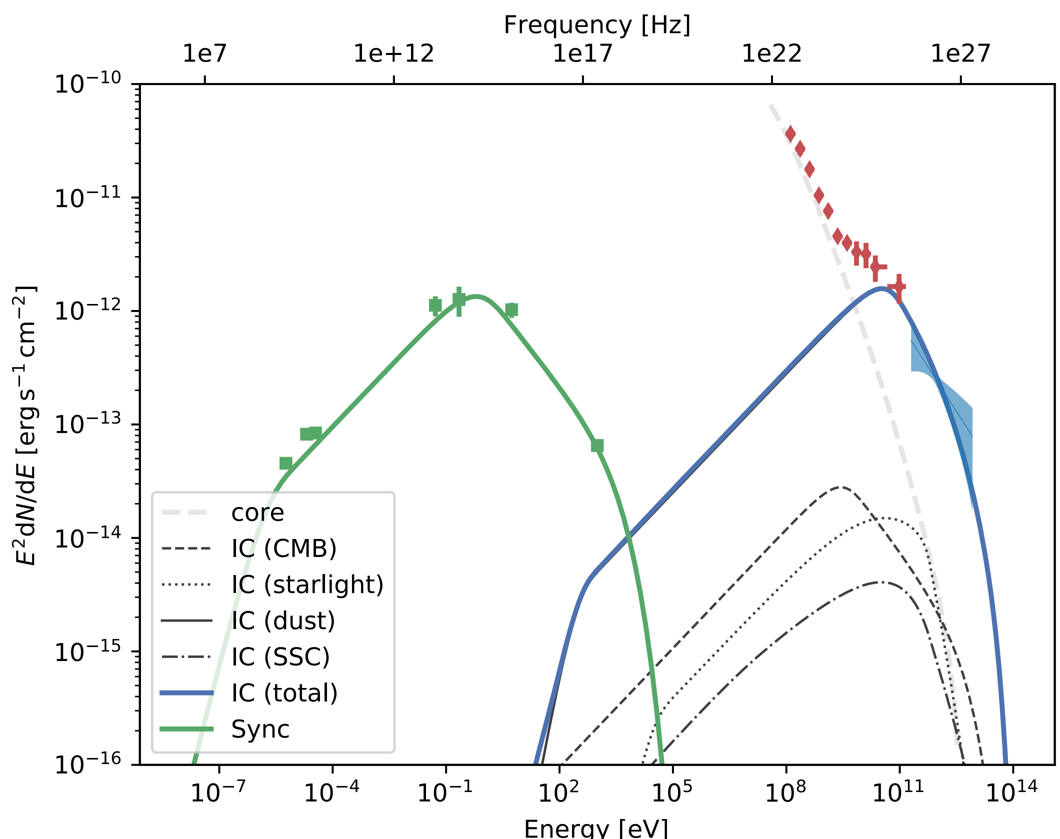






$$\eta = \frac{1}{{\Gamma_e^{max}}^2 (B/B_{crit})\alpha}$$

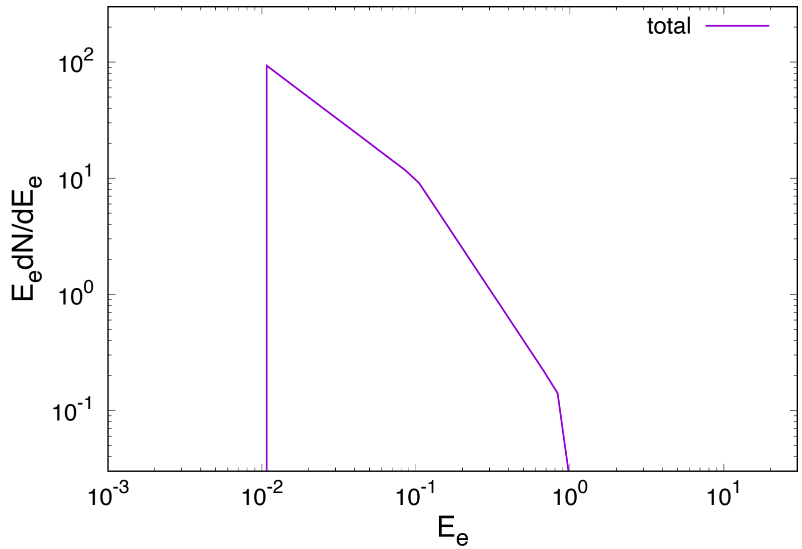
$$\approx 10^4 \left(\frac{10^8}{\Gamma_e^{max}}\right)^2 \left(\frac{20\mu G}{B}\right)$$

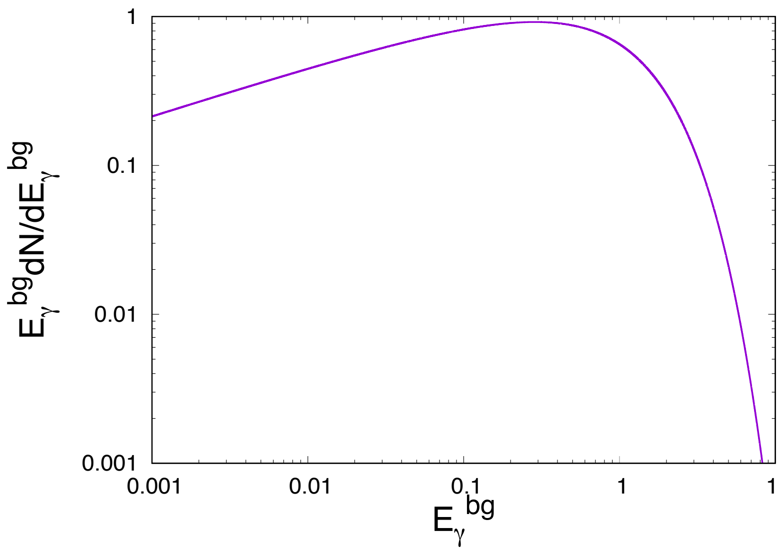


[HESS- F. Rieger, A. Taylor, et al., Nature-accepted today!]

DESY.

Future Probes- Cutoff Region in Synchrotron Spectrum

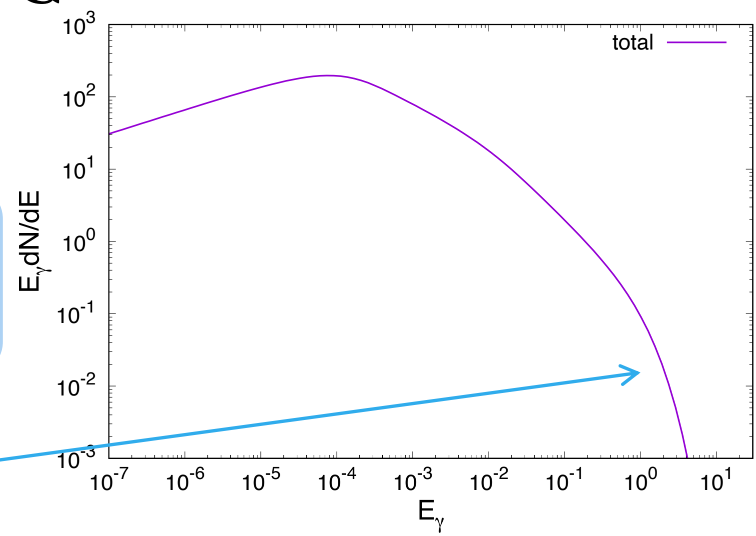




$$\mathbf{B_{crit} = 4 \times 10^{13}\ G}$$

$$\mathbf{E_\gamma^{sync} = \Gamma_e^2 \left(\frac{B}{B_{crit}}\right) m_e}$$

$$E_\gamma \frac{dN}{dE_\gamma}_{tot} = \int \left(\frac{E_\gamma}{E_e^2}\right) \frac{dN}{dE_\gamma} \left(\frac{E_\gamma}{E_e^2}\right) E_e \frac{dN}{dE_e} dE_e$$



Possibility to probe cutoff region

Collisional Shock- Enthalpy

$$\gamma = \frac{\mathbf{w_{nonrel.}}}{\mathbf{e}}$$

$$= \frac{\mathbf{e} + \mathbf{p}}{\mathbf{e}}$$

$$\mathbf{e} = \frac{\mathbf{p}}{\gamma - \mathbf{1}}$$

$$\mathbf{w_{nonrel.}} = \frac{\gamma}{\gamma - \mathbf{1}}\mathbf{p}$$

$$\mathbf{w_{rel.}} = \frac{\gamma}{\gamma - \mathbf{1}}\mathbf{p} + \rho$$

$$= \mathbf{w_{nonrel.}} + \rho$$

Andrew Taylor

Intuitive Insights into Cut-off Shape Origin

Consider the steady-state case of diffusion (constant diffusion coefficient) of particles into an absorbing medium

$$\nabla \cdot (\mathbf{D_{xx}} \nabla \mathbf{f}) - \frac{\mathbf{f}}{\tau(\mathbf{x})} = \delta(\mathbf{r})$$

For $\tau(\mathbf{x}) = \tau_*(\mathbf{x}/\mathbf{x}_*)^{\mathbf{2}}$ $\mathbf{f} \propto \mathrm{const.}$

For $\tau(\mathbf{x}) = \tau_*$ $\mathbf{f} \propto \mathbf{e}^{-\mathbf{x}/\mathbf{x}_\tau}$

For $\tau(\mathbf{x}) = \tau_*(\mathbf{x}/\mathbf{x}_*)^{-\mathbf{2}}$ $\mathbf{f} \propto \mathbf{e}^{-(\mathbf{x}/\mathbf{x}_\tau)^{\mathbf{2}}}$

Andrew Taylor