ZFITTER

in the Era of the LHC and a Linear Collider or

The Quest for Precision

ZFITTER collaboration [∼1985 - 2012]

Tord Riemann

DESY, Zeuthen, Germany

https://indico.desy.de/conferenceDisplay.py?confId=4980

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Precision

Physics needs Precision at highest level
Just-so

- Galileo Galilei
  Observations, Measurements, Reproducibility
- Planets Movements + Kepler + Newton
  Precision and Interpretation
- Black Body Radiation + Planck
  Quantum Physics
- LEP + Veltman et al.
  Quantum Field Theory and Gauge Boson Self Interactions
- Linear Collider, also GigaZ: ? ? ?

Precision calculations at work

Precision calculations

- $\rightarrow$ time-consuming
- $\rightarrow$ complicated
- their necessity depends on very details of the measuring machinery
  $\rightarrow$ see P. Uwer's talk on m_{top}

My main activities:

- NNLO corrections for Bhabha scattering at meson factories
- New technique for Tensor reduction for NLO Feynman integrals

  - Explore finite sums over signed minors [include Gram-type determinants], weighted with scalar products of external momenta

  - Deal systematically with Integrals in higher dimensions [E. Boos et al.]

- 1-loop or n-loop Feynman integrals: new approaches

  E.g.: Mellin-Barnes representations with the aim of automatic multiple infinite summation

  - resulting class of functions not known

  - beyond the Polylogarithms or Harmonic Polylogarithms or Generalized Harmonic Polylogarithms

Loops and Legs 2012

Precision calculations not in a nutshell, but in detail:
"Loops and Legs 2012", 15-20 April 2012, Wernigerode
http://indico.desy.de/conferenceDisplay.py?confId=4362

Foto: Koglin

Workshop on Precision Calculations at LEP in 1995

At a certain moment, the community has to set benchmarks

They have to be documented with great care, because they are valid longer than one expects at that moment

Setting the stage in 1995, till now relevant:

"Electroweak Working Group Report" e-Print: hep-ph/9709229

It is a part of:
D. Bardin, W. Hollik, G. Passarino (eds.)
"Reports of the working group on precision calculations for the Z resonance"
CERN 95-03 (31 March 1995)

This work is one of the basics for the successful work of the LEP Electroweak Working Group
D. Schaile et al., M. Gruenewald et al.

ZFITTER as a project 1985 - 2030 (?)

How to retain a software project over decades?
one condition: interactions with users

Software support comprises:

- code
- description
- availability
- updating/debugging
- long-term people !!
- licencing - including ”conditions of proper use”

Needs according to competitors:

- Support
- Numerical excellence
- Modern Computer Language
- Modularity

`http://zfitter.com`
Also crucial . . .

- **Reliability**
- **Precision**
- **Documentation → open-source package!**

ZFITTER code - in a nutshell

Evaluates: Observables + Pseudo-Observables
Uses language of: QED + effective Born cross section

ZFITTER Approach:

- Analytical formulae with cuts for cross sections in $e^+e^- \to f^+f^-(+n\gamma)$:

$$\frac{1}{|s - M_Z^2 + iM_Z\gamma_Z|^2} \sim \frac{i}{2M_Z\Gamma_Z}\left(\frac{1}{s - M_Z^2 + iM_Z\Gamma_Z} - \frac{1}{s - M_Z^2 - iM_Z\Gamma_Z}\right)$$

- Effective Born Cross sections with 4 form factors:
  $\rho_{ef}(s,t), \kappa_e(s,t), \kappa_f(s,t), \kappa_{ef}(s,t)$

Theory contents developed:

- 1986: Electroweak Standard Model - 1-loop, later: + higher orders
- Later: Electroweak + QCD = Standard Model
- **Interfaces** for New Physics and/or Model-Independent Approaches
- **Add-ons:**
  - W boson width [nothing to add since 1986]
  - atomic parity violation

ZFITTER v.6.21 (1999) $\to$ v.6.42 (2005)

ZFITTER authors:

A. Akhundov, A. Arbuzov, M. Awramik, D. Bardin, M. Bilenky, A. Chizhov, P. Christova, M. Czakon, O. Fedorenko, A. Freitas, M. Grnewald, M. Jack, L. Kalinovskaya, A. Olshevsky, S. Riemann, T. Riemann, M. Sachwitz, A. Sazonov, Yu. Sedykh, I. Sheer, L. Vertogradov, H. Vogt

Pre-history $\sim$ 1977-1982

- D. Bardin, P. Christova, O. Fedorenko
  electroweak complete 1-loop "Bible I" and "Bible II" $\to$ NPB

ZFITTER begin in 1985/1986

- A. Akhundov, D. Bardin, T. Riemann
  Δ_r with $m_{top} \neq 0 \to$ PLB
- A. Akhundov, D. Bardin, T. Riemann
  Z-decay width with $m_{top} \neq 0 \to$ NPB
- D. Bardin, S. Riemann, T. Riemann
  W-decay width with $m_{top} \neq 0 \to$ ZfPC [EPJC]

LEP1 . . . LEP2 . . . ZFITTER Support Group since 2005

Founded in 2004/2005 after D. Bardin finished active support of the project until 2010

Spokesperson 2004 – 2012: T.R.

Hunting the Hidden Standard Higgs

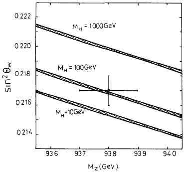
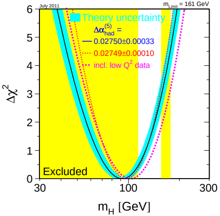


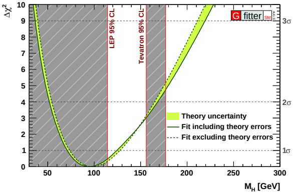
Fig 1 Graph of $\sin^2\theta_W$ versus M_Z, influenced by M_H through radiative corrections The thickness corresponds to the range 30 GeV $\leqslant m_t \leqslant$ 40 GeV, the error bars indicate the accuracy expected at Z boson factories.



2011 LEPEWWG

$\longleftarrow$

Akhundov, Bardin, T.R.
JINR-E2-85-454. Jun 1985. 4 pp. $\rightarrow$
Phys.Lett. B166 (1986) 111
Quotations: 1 in 1986, 1 in 2012



EPJC 60 (2009),
CPC 133(2001), CPC 174(2006)

ZFITTER v.6.42 (2005) → v.6.43 (2008) → v.6.44 (2012)

There is some update history, see http://zfitter.com

But after the inclusion of the
Hollik-Freitas-Weiglein & Awramik-Czakon **2-loop electroweak corrections**
...

...not much happened

Most interesting: corrections to the Z decay width:
QCD 3-loop corrections - in ZFITTER v.6.42
QCD 4-loop corrections - **now: update in ZFITTER v.6.44, being released**
Baikov, Chetyrkin, Kühn – Phys.Rev.Lett. 101 (2008)
Baikov, Chetyrkin, Kühn, Rittinger – subm. to Phys.Rev.Lett. 101 (2008)

$$
\begin{aligned}
\Gamma_Z &\rightarrow \Gamma_Z \times R_{nc} \\
R_{nc} &= \left(1+\frac{\alpha_s}{\pi}\right) + A_2\left[\frac{\alpha_s}{\pi}\right]^2 + A_3\left[\frac{\alpha_s}{\pi}\right]^3 + A_4\left[\frac{\alpha_s}{\pi}\right]^4
\end{aligned}
$$

α_s^4-corrections to the hadronic Z decay rate

"Complete QCD Corrections to Hadronic Z–Decays in Order ${}_s^4$"
P. A. Baikov, K. G. Chetyrkin, J. H. Khn, J. Rittinger
(Submitted on 27 Jan 2012)

Corrections of order α_s^4 for the axial singlet contributions for the decay rates of the Z-boson into hadrons are evaluated in the limit of the heavy top quark mass. Combined with recently finished $\mathcal{O}(\alpha_s^4)$ calculations of the non-signlet corrections, the new results directly lead us to the first complete *$\mathcal{O}(\alpha_s^4)$ prediction for the total hadronic decay rate of the Z-boson.*

The new $\mathcal{O}(\alpha_s^4)$ term in Z-decay rate
lead to a significant stabilization of the perturbative series,
to a reduction of the theory uncertainty in the strong coupling constant α_s, as extracted from these measurements,
and to a small shift of the central value.

$$\Gamma_Z = R_0 \; R_{nc} = \frac{G_F}{\sqrt{2}} \frac{M_Z^3}{24\pi} \; \times \; R_{nc}$$

R_{nc} to order α_s^4

$$
\begin{aligned}
R_{nc} \quad = \quad & 20.1945 \left(1 + \frac{\alpha_s}{\pi}\right) + (28.4587 - 13.0575 + 0) \left[\frac{\alpha_s}{\pi}\right]^2 \\
& + (-257.825 - 52.8736 - 2.12068) \left[\frac{\alpha_s}{\pi}\right]^3 \\
& + (-1615.17 + 263.388 - 25.5814) \left[\frac{\alpha_s}{\pi}\right]^4
\end{aligned}
$$

From ZFITTER v.6.42 to ZFITTER v.6.44

Look at experimental numbers:
Experiment: The Z decay width is experimentally known to be (PDG 2010):

$$\Gamma_Z = (2.4952 \pm 0.0023)\ \mathrm{GeV}$$

so that

$$\Delta\Gamma_Z/\Gamma_Z = 0.00092177 = 9 \times 10^{-4}$$

- The influence of the α_s^4 term from QCD is [see next slide] -2×10^{-4} i.e. about 1/4 of the experimental error
- If one wants to be truly sensitive to the NNNLO Baikov terms, -2×10^{-4}, one has to control the rest of the corrections to about 5×10^{-5} or better.

This is what I want to ”derive”:

The Fortran accuracy has to be controlled with at least 7 digits in order to say it is satisfactory as ”technical precision”.

It is good to have the Baikov NNNLO term at the disposal, but it should not be too influential.
Further one may conclude that a technical accuracy of a code of 5 digits is "completely satisfactory", but it is not "excellent".
The influence of the α_s^4 terms is, for $n_f = 5$, proportional to [PRL 2008]:

$$\begin{aligned} A_4 &= -156.61 + 18.77 n_f - 0.7974 n_f^2 + 0.021 * n_f^3 \\ &= -80.0075 \end{aligned}$$

This corresponds to the term above:

$$A_4 \left[\frac{\alpha_s}{\pi}\right]^4 \approx \frac{1}{20.1945}(-1615.17 + 263.388 - 25.5814)\left[\frac{\alpha_s}{\pi}\right]^4 \approx -2 \times 10^{-4}$$

and the correction to the total Z decay rate becomes then for $\alpha_s = 0.12$

$$\frac{\delta\Gamma_Z}{\Gamma_Z} = A_4 (0.12/\pi)^4 \approx -2 \times 10^{-4}$$

ZFITTER v.6.42: a Lesson

We have the α_s^4 terms of Baikov et al.
Do we control the α_s^3 terms and all that sufficiently?
Let us look into some actually promoted code from a competitor:

$$A_3 \sim m_{CA3} = -4544045/864.0 + 1340\zeta(2) + 118915/36.0\zeta(3) - 1270\zeta(5) + (71621/162.0 - 209/2.0\zeta(2) - 216\zeta(3) + 5\zeta(4) + 55\zeta(5) m_{nf1} + (-13171/1944.0 + 16/9.0\zeta(2) + 26/9.0\zeta(3) m_{nf1}^2);$$

In the **description of the competitor** [in fact a diploma thesis] we find:

$$A_3 \sim C_{23}^A = -\frac{4544045}{864} + 1340\,\zeta(2) + \frac{118915}{36}\zeta(3) - 127\,\zeta(5) + \left[\frac{71621}{162} - \frac{209}{\zeta}(2) - 216\,\zeta(3) + 5\,\zeta(4) + 55\,\zeta(5)\right] m_{nf1} + \left[-\frac{13171}{1944} + \frac{16}{9}\zeta(2) + \frac{26}{9}\zeta(3)\right] m_{nf1}^2 \,.$$

These NNLO terms do not agree.
Look into the **original reference given**:
"QCD corrections to the e+e cross-section and the Z boson decay rate"
Chetyrkin, Kühn, Kwiatkowski, Dec 1994, 87 pp.
In: "Reports of the working group on precision calculations for the Z resonance", pp. 175-263, e-Print: hep-ph/9503396
There is no formula of this kind at all . . .

ZFITTER v.6.42: A Lesson

Then we look into:
Bardin, Christova, Jack, Kalinovskaya, Olchevski, Riemann, Riemann
ZFITTER, CPC133 (2001):

$$
\begin{aligned}
A_3 \sim C_{23}^{A} \quad = \quad & \texttt{COEFA3} = -\frac{4544045}{864} + 1340\zeta(2) + \frac{118915}{36}\zeta(3) - 127\zeta(5) \\
& + \left[\frac{71621}{162} - \frac{209}{\zeta}(2) - 216\zeta(3) + 5\zeta(4) + 55\zeta(5)\right] m_{nf1} \\
& + \left[-\frac{13171}{1944} + \frac{16}{9}\zeta(2) + \frac{26}{9}\zeta(3)\right] m_{nf1}^2;
\end{aligned}
$$

Here we see the $-127\zeta(5)$ of the description and not the $-1270\zeta(5)$ found in the C++ code.
The CPC133 is the only place where to find the expression ...
And finally ($D5 = \zeta(5)$):
In the Fortran program **ZFITTER/dizet6_42.f**:

$$
\begin{aligned}
COEFA3 \quad = \quad & -4544045D0/864 + 1340 * D2 + 118915D0/36 * D3 - 1270D0 * D5 \\
& +(71621D0/162 - 209D0/2 * D2 - 216D0 * D3 + 5D0 * D4 + 55D0 * D5)ANF \\
& +(-13171D0/1944 + 16D0/9 * D2 + 26D0/9 * D3)ANF^2 *
\end{aligned}
$$

This number agrees with the C++ code of the competitor of 2009 ..., but not with its description ...
Explanation: private communications, copy-paste, wrong and incomplete referencing and all that.

ZFITTER v.6.42: Not a Lesson

In ZFITTER file bkqcdl5_14.f we find the Fortran function XRMQCD, which is authored by ZFITTER:

```
      FUNCTION XRMQCD(AMZ2,AMW2,AMT2,S)
 ...
       XRMQCD=1D0/(12D0*SW2)
     &          *(3D0/4D0/CW2*(1D0+VB2)
     &          +AMT2/4D0/AMW2*(VT2*(XDVFTZ/ALTZ-XPVFTZ)
     &          +XDAFTZ/ALTZ-XPAFTZ)
     &          -AMT2/AMW2*(3D0*D2+105D0/8D0))
      ELSE
       XRMQCD=1D0/4D0/SW2/CW2
     &          *(AMT2/AMZ2*((1D0-4D0*QTM*SW2)**2*XV1r+XA1r)
     &          +AMT2/(AMZ2-S)*((1-4*QTM*SW2)**2*(XV1rs-XV1r)+XA1rs-XA1r)
     &          +2D0*AMT2/AMZ2*(-23D0/8D0+D2+3D0*D3)
     &      -1D0/4D0*(1D0+(1D0-4D0*QBM*SW2)**2)*S/(AMZ2-S)*LOG(S/AMZ2))
```

In a **C++ code of a competitor** we see similar but not identical coding:

```
rmqcd = 1.0/(4.0*SW2*CW2)*( mt2/MZ2*(VT2*XV1r + XA1r)
                                        + mt2/(MZ2-S)*(VT2*(XV1rs - XV1r) + XA1rs - XA1r)
                                        + 2.0*mt2/MZ2*(-23.0/8.0*D2+3.0*D3)
                                        - 1.0/4.0*(1.0 + QBM)*S/(MZ2-S)*TMath::Log(S/MZ2) );
```

One may **observe two mistakes**, resulting from copy-paste with a loss, in C++/rmqcd compared to ZFITTER, where we have:

$$\begin{aligned} \ldots \quad = & \\ & +2D0*AMT2/AMZ2*(-23D0/8D0+D2+3D0*D3) \qquad (1)\\ & -1D0/4D0*(1D0+(1D0-4D0*QBM*SW2)**2)*S/(AMZ2-S)*LOG(S/AMZ2)) \end{aligned}$$

In both cases: the $D3 = \zeta(3)$ comes from a QCD 2-loop Formula (Kniehl 1990).

Summary

- **Precision**
  - is a crucial item besides **"Planned" Discovery** for a Linear Collider
- Should place the issue of **Precision** properly in Workshop Proceedings and Documents?
- ZFITTER project 1985 - 2012 $\rightarrow$ future
- Last update ZFITTER v.6.44 in February 2012