Meet 10^{-4} in Bhabha Calculations – A Status Report

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A project in collaboration with
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See also: • **T.R., talk at TESLA Forward Detector WS, 2002, at .../∼riemann/**
• **http://www-zeuthen.desy.de/theory/research/bhabha/**

- **What do we need?**
- **What has been done?**
- **What is being done here?**
- **Prospects**

The Needs

Need a cross-section prediction with 5 significant digits.

Perturbative orders:

$$\left(\frac{\alpha}{\pi}\right) = 2 \times 10^{-3}$$

$$\left(\frac{\alpha}{\pi}\right)^2 = 0.6 \times 10^{-5}$$

Kinematics:

$$\begin{aligned} \sqrt{s} &= 90 \ldots 1000 \text{ GeV} \\ \vartheta &= 26 \ldots 82 \text{ mrad} \\ \cos\vartheta &\sim 0.999\,66 \ldots 0.996\,64 \\ T &= \frac{s}{2}(1-\beta^2\cos\vartheta) > 1.36 \text{ GeV}|_{GigaZ},\ 42.2 \text{ GeV}|_{ILC500} \end{aligned}$$

Conclude:

- t**-channel exchange of** γ **dominates everything else**
- $m_e^2/s \; < \; m_e^2/T \; \leq \; 10^{-5} \ldots 10^{-7}$
- **Calculate: 1-loop EWRC + 2-loop QED + corresp. bremsstrahlung**

Here the table with Bhabha one-loop electroweak predictions from aitalc and from Feyncalc:

A. Lorca, T. Riemann, talk held by A. Lorca at Loops and Legs 2004, Zinnowitz, Germany, transparency 11

zinn04-p11.pdf

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2-loop Bhabha scattering: What to be done?

- **Calculate:**

$$\begin{aligned}\sigma \quad = \quad & |\text{Born} + \text{1-loop} + \text{2-loop}|^2 \\ & + |(\text{Born} + 1\text{-}\gamma) + (\text{1-loop} + 1\text{-}\gamma)|^2 \\ & + |(\text{Born} + 2\text{-}\gamma)|^2\end{aligned}$$

- **Do not include:** $|\text{2-loop}|^2$
  $|(\text{1-loop} + 1\text{-}\gamma)|^2$

We will see in a minute that the following contributions are not completely known:

- **virtual 2-loop contributions ('double boxes')**
- **virtual 1-loop corrections to single bremsstrahlung (5-point functions)**

What has been established: 10^{-3} MC programs for LEP, ILC

Introduction to NLLBHA by Trentadue and to BHLUMI by Jadach in:
Proc. of Loops and Legs, Rheinsberg, Germany, 1996

Recent mini-review: Jadach, "Theoretical error of luminosity cross section at LEP", hep-ph/0306083 [1]

- **BHLUMI v.4.04: Jadach, Placzek, Richter-Was, Was: CPC 1997**
- **see also: Jadach, Melles, Ward, Yost: PLB 1996, thesis Melles 1996 [2]**
- **NLLBHA: Arbuzov, Fadin, Kuraev, Lipatov, Merenkov, Trentadue: NPB 1997, CERN 96-01**
- **SAMBHA: Arbuzov, Haidt, Matteuzzi, Paganoni, Trentadue: hep-ph/0402211**

See e.g.: Table 1 of [1] and Figure 3.1 of [2]

Conclude: The pure, nonlogarithmic $O(\alpha^2)$ terms, originating from pure QED radiative 1-loop and from 2-loop diagrams are not completely covered.

They have to be calulated and integrated into the MC programs.

Beware:

$$m_e, m_\gamma, (d-4), E_\gamma$$

Here the table with 'canonical coefficients determining the size of the photonic corrections':

S. Jadach, talk at workshop on "electroweak physics data and the Higgs mass", DESY, Zeuthen, Germany, Feb./March 2003, hep-ph/0306083, table 1

zeujad-0306083-p5.ps

Here the figure 3.1 with some of the QED virtual corrections to single bremsstrahlung diagrams:

M. Melles, thesis, APP B28 1997 1159, taken from preprint, hep-ph/9612348

melles-9612348-fig31.ps

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Double Boxes etc.: What is being done in our group?

We are interested in an calculation of the virtual second order corrections to

$$\frac{d\sigma}{d\cos\vartheta}(e^+e^- \to e^+e^-)$$

We are using a scheme with

(1) $m_e \neq 0$ **(good with the MC's)**

(2) $m_\gamma = 0$ **(bad with the MC's)**

(3) dim.reg. for UV and IR divergences

To be solved later:
Due to (2), we will have a problem with the compensation of the IR-divergences from our contributions and from bremsstrahlung.

Also active:
Argeri, Bonciani, Feroglia, Mastrolia, Remiddi, van der Bij: same approach
Heinrich, Smirnov: calculation of complicated Feynman integrals

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What has to be calculated?

Not too many QED diagrams:

- Born diagrams: 2 →
- 1-loop diagrams: 14 →, interfere with Born and 1-loop
- 2-loop diagrams: 178 →, interfere with Born only

There are few topologies only:

- 1-loop: 6 →
- 2-loop self energies: 4 →
- 2-loop vertices: 5 →
- 2-loop boxes: 6 →

The many Feynman integrals may be reduced to 'few' master integrals by sophisticated methods.

Here figures with Bhabha Feynman diagrams:

Born: file bha0L.ps

Here figures with Bhabha Feynman diagrams:

1-Loops: file Bha1L.ps

Here figures with Bhabha Feynman diagrams:

2-Loops: files bhabha2Lp1.ps ... bhabha2Lp8.ps, example to be shown: bhabha2Lp7.ps

Here figures with Bhabha prototypes:

1-Loop: file LL1LoopMast.jpg

Here figures with Bhabha prototypes:

2-Loop self energies: file LL04SEProt.jpg

Here figures with Bhabha prototypes:

2-Loop vertices: file BhabhaVerticesA.jpg

Here figures with Bhabha prototypes:

2-Loop boxes: file BhabhaBoxesA.jpg

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The first complete table of master integrals:

So far, we use the Laporta/Remiddi algorithm:

Derive with integration-by-parts identities a system of differential equations for the Feynman integrals and solve the system.

MC,JG,TR, hep-ph/0406203 (→ tables) , with IdSolver (M. Czakon, unpublished):

- **1-loop: 6 masters (all known)**
- **2-loop self energies: 6 masters (all known)**
- **2-loop vertices: 19 masters (all known)**
- **2-loop boxes: 33 masters → (5 published, maybe more known)**

The calculation of the master integrals is done with two methods:

- **derive and solve (systems of) differential equations (with boundary conditions)**
- **derive and solve (up to 8-dimensional) Mellin-Barnes integral representations for single Feynman integrals (Heinrich, Smirnov)**

Here 1/2 tables with Bhabha master integrals:

Master integrals contributing to vertices and boxes: file 2LVB.pdf

Here 2/2 tables with Bhabha master integrals:

Box master integrals contributing to boxes: file 2LB.jpg

Here 1/4 figures with Bhabha 2-loop box master integrals:

Nine double boxes with 7 internal lines:

file BhabhaBox7IMastFinal.jpg

Here 2/4 figures with Bhabha 2-loop box master integrals:

Ten double boxes with 6 internal lines:

file BhabhaBox6lMastFinal..jpg

Here 3/4 figures with Bhabha 2-loop box master integrals:

Eight of fourteen double boxes with 5 internal lines:

file BhabhaBox5lMastAFinal..jpg

Here 4/4 figures with Bhabha 2-loop box master integrals:

Six of fourteen double boxes with 5 internal lines:

file BhabhaBox5lMastBFinal..jpg

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Prospects

Recent essential progress:

- **The complete list of masters**
- **Determination of many masters in a systematic approach (use Harmonic Polylogarithms, introduced by Remiddi, Vermaseren, plus . . .)**

So far:
Not for all technical problems it is completely understood how to solve them in an operational manner.

It seems to be possible to do the calculation with present computers and human resources.
But we go really to the limits.

Finally: maybe the approach with $m_e = m_\gamma = 0$ is successful (Bern, Dixon, Ghinculov, Glover, Tausk, van der Bij).

Offer by drinking-a-beer Frits Berends, NL, in 1989

Okey, I need

One million Swiss francs

and

five – or ten? – Russians

and I will do this

Since then (i.e. in 15 years), there are about a dozen articles in spires with "Bhabha ... two loop" in the title.

And Frits Berends retired quite recently.