Quantum Chromodynamics

lecture V

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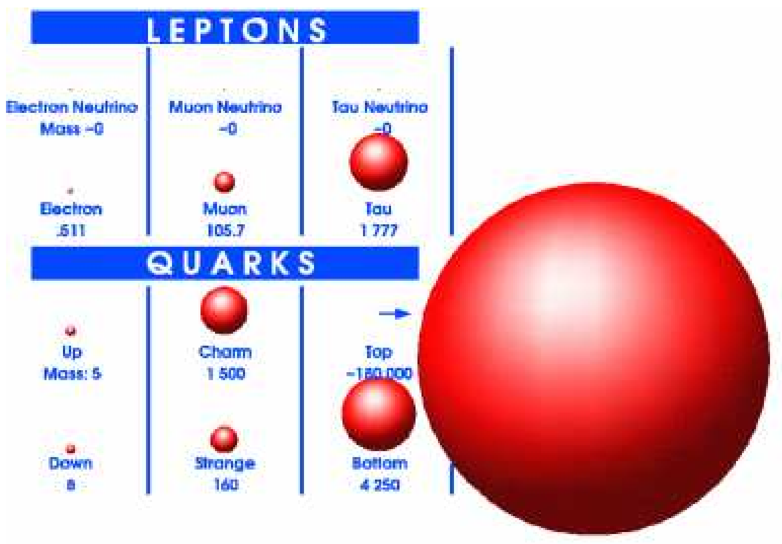
Belgian Dutch German summer school (BND 2012), Bonn, Sep 25, 2012

Plan

- Introduction to QCD
  Friday, September 21, 2012
- QCD at work: infrared safety, factorization and evolution
  Saturday, September 22, 2012
- Higgs boson production
  Sunday, September 23, 2012
- Gauge boson production and QCD jets
  Monday, September 24, 2012
- *Top quark production*
  Tuesday, September 25, 2012

Plan

- Some basics on the heaviest elementary particle



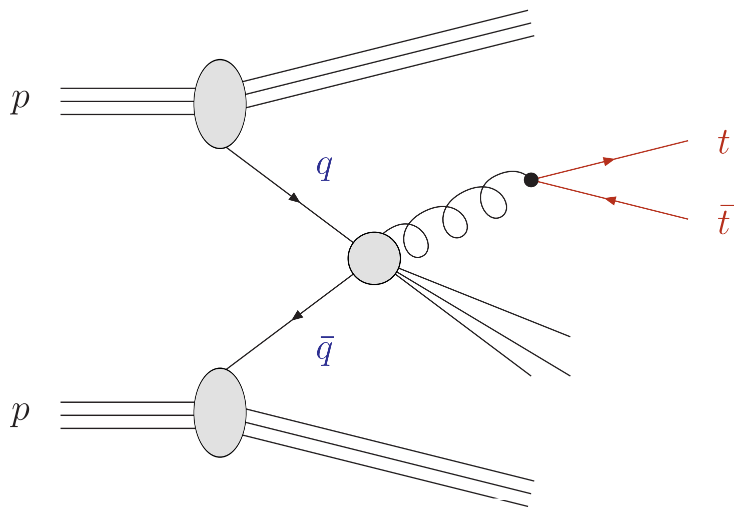
Abundant production of top-quarks

- Orders for top-quarks from www.particlezoo.com

Top-quark decays

Top-quark pair production

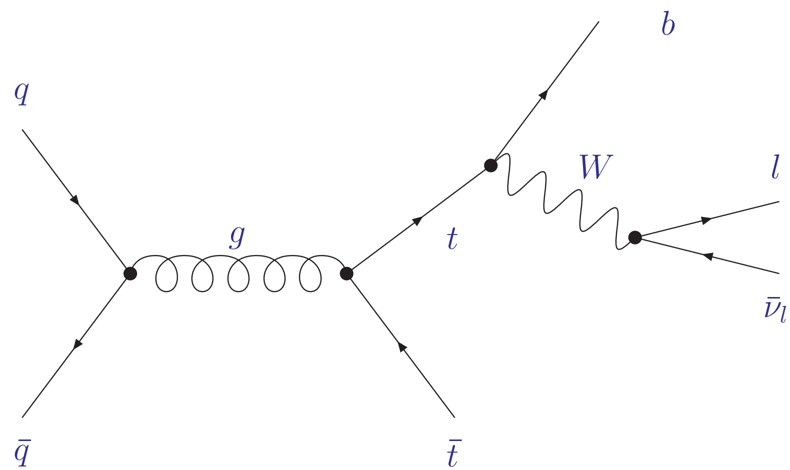
- Hadronic reaction $pp/p\bar{p}$:
  - recall master equation $\sigma_{pp\to t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij\to t\bar{t}}$



- Parton cross section $\hat{\sigma}_{ij\to t\bar{t}}$ known to NLO in QCD Nason, Dawson, Ellis '88; Beenakker, Smith, van Neerven '89; Mangano,Nason, Ridolfi '92; Bernreuther, Brandenburg, Si, Uwer '04; Mitov, Czakon '08; . . .
  - NLO accurate to $\mathcal{O}(15\%)$ at LHC (NNLO around the corner)
- Relevant kinematics:
  - high-energy limit $s \gg m^2$ with BFKL logarithms $\ln s/m^2$
  - partonic threshold $s \simeq 4m^2$ with Sudakov logarithms $\ln \beta$ (velocity of heavy quark $\beta = \sqrt{1 - 4m^2/s}$)

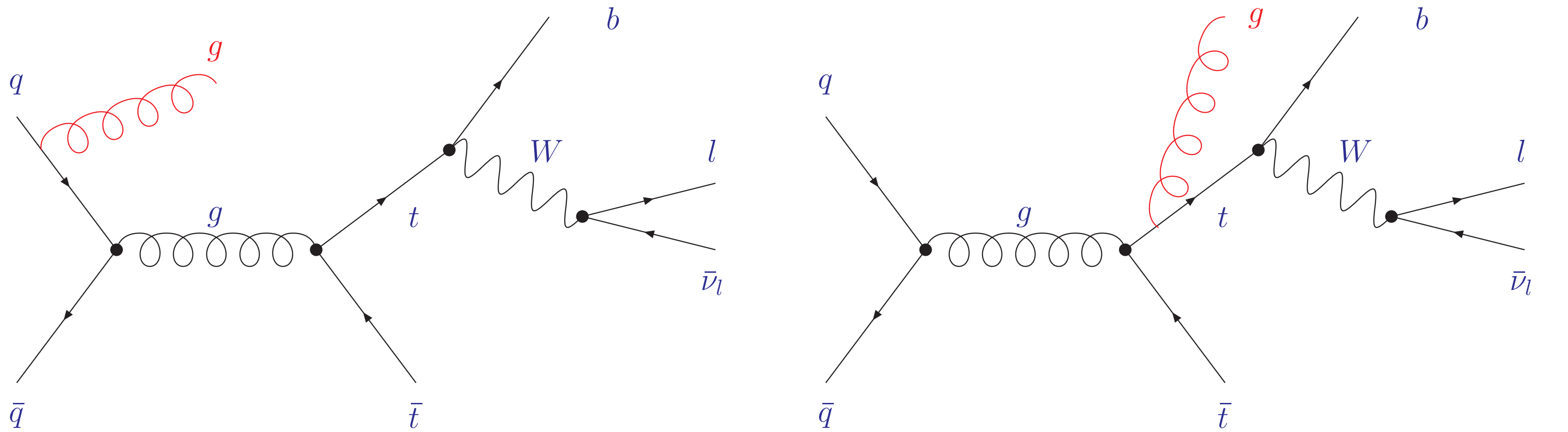
Hard scattering process

- Born process ($q\bar{q}$-channel) with leptonic decay $t \to bl\bar{\nu}_l$



Radiative corrections

- Real corrections (examples): gluon emission
  - phase space integration $\rightarrow$ infrared divergences (soft/collinear singularities)

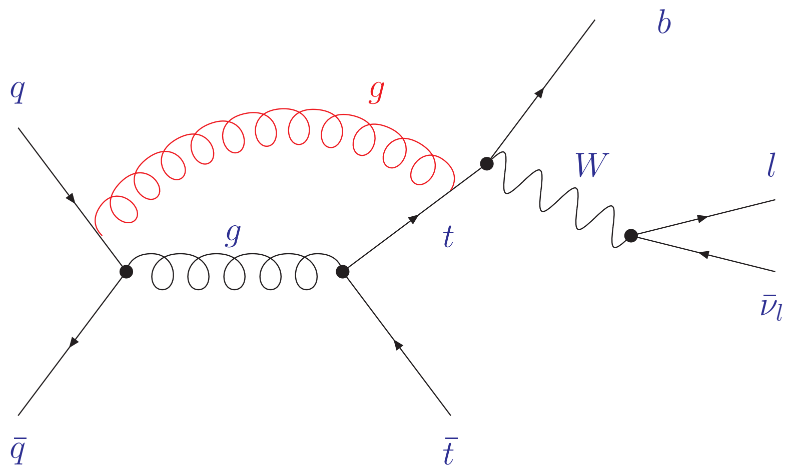


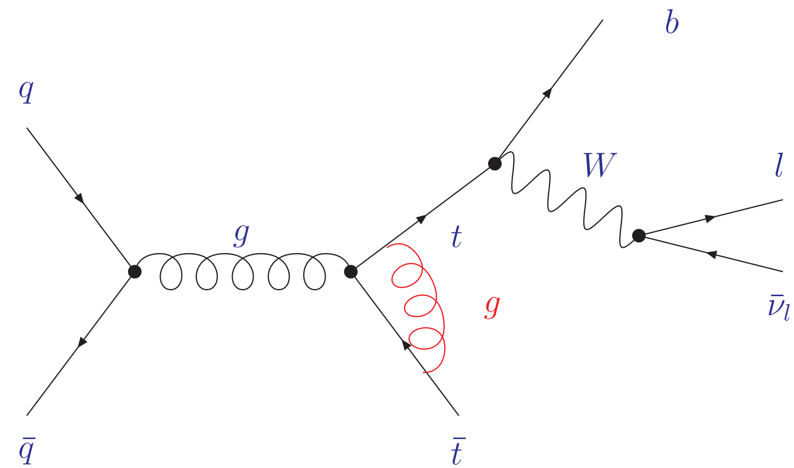
- Parton shower MC
  - emission probability modeled by Sudakov exponential with cut-off Q_0
  - leading logarithmic accuracy

$$\Delta\left(Q^2, Q_0^2\right) = \exp\left(-C_F \frac{\alpha_s}{2\pi} \ln\left(\frac{Q^2}{Q_0^2}\right)\right)$$

Radiative corrections

- Virtual corrections (examples): gluon exchange
  - box diagram (left) and vertex corrections (right)
  - infrared divergences cancel against real emission contributions





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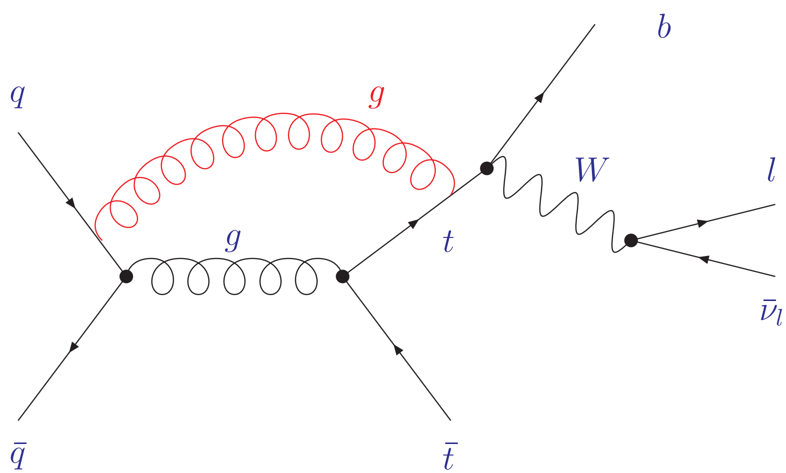
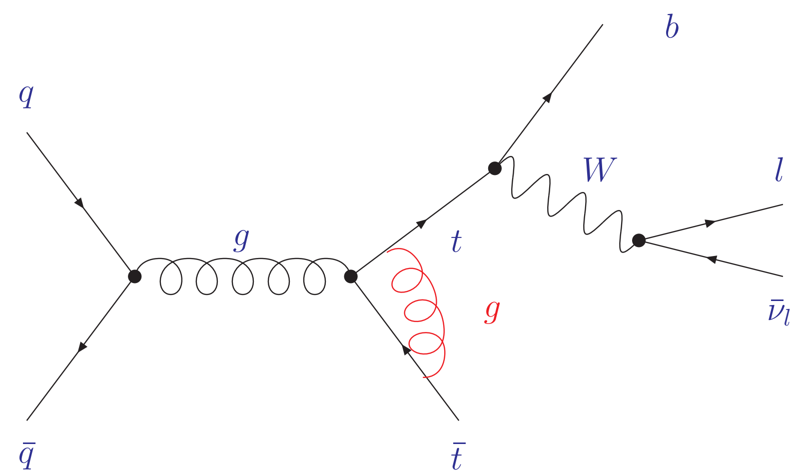
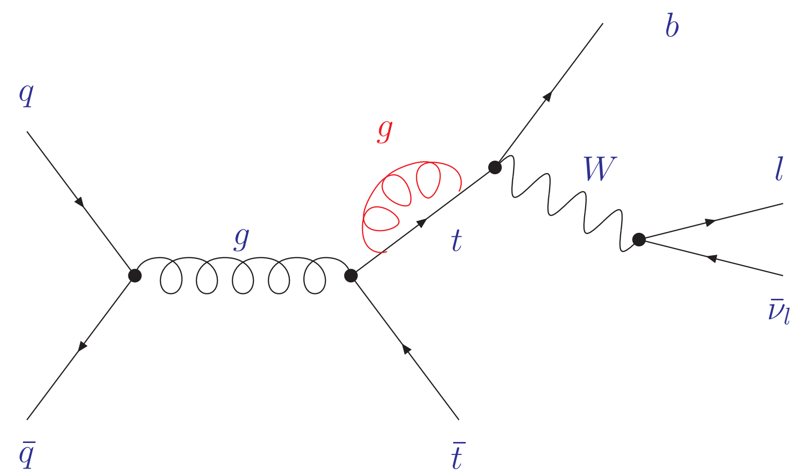


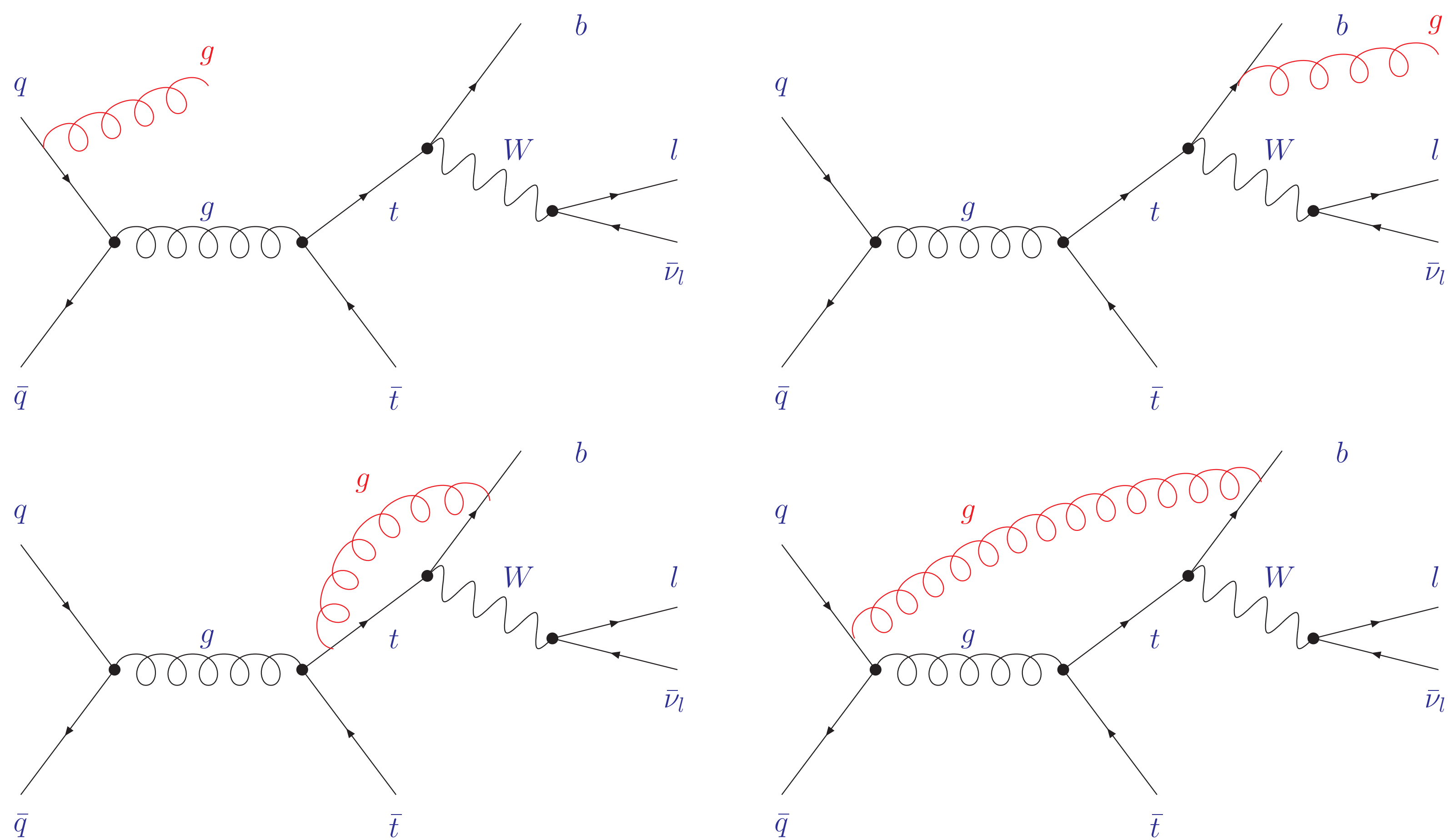


- Mass renormalization from self-energy corrections to top quark



Heavy-to-light corrections

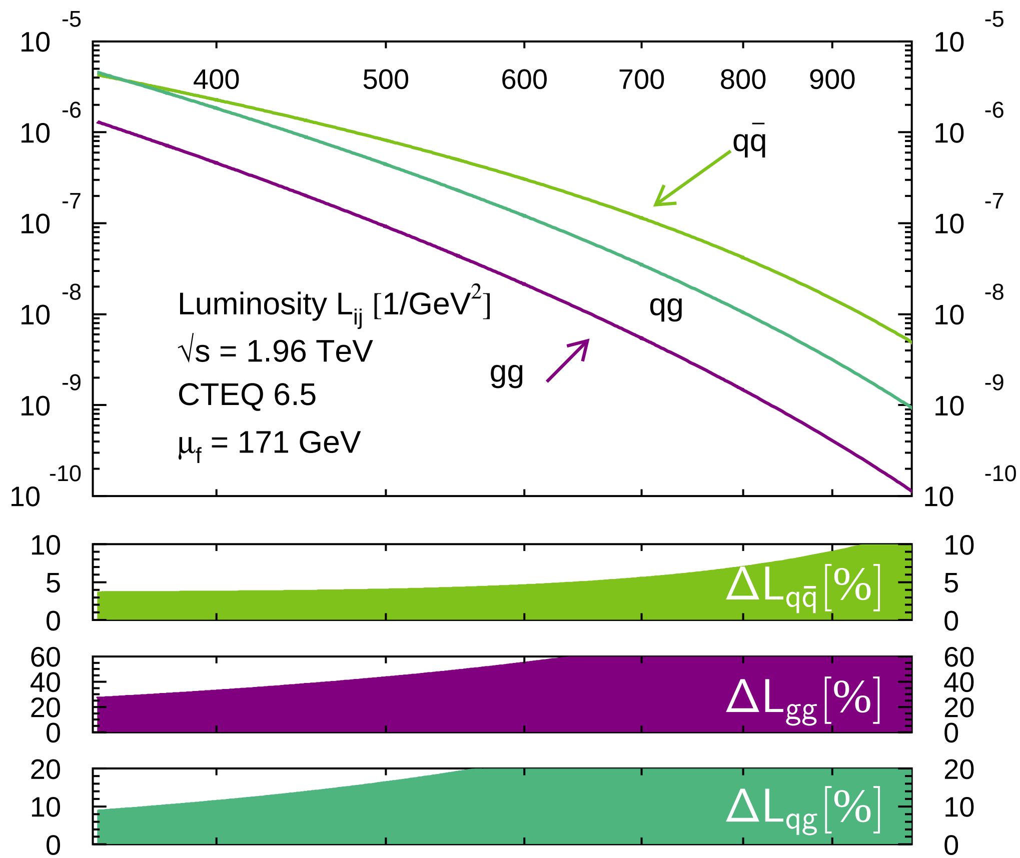
- Interference between top quark and its decay products (b quark)
  - real emission and virtual diagrams
  - complete NLO corrections for top production and decay
    Melnikov, Schulze '09; Bernreuther, Si '10 (contained in MCFM Campbell, Ellis '12)

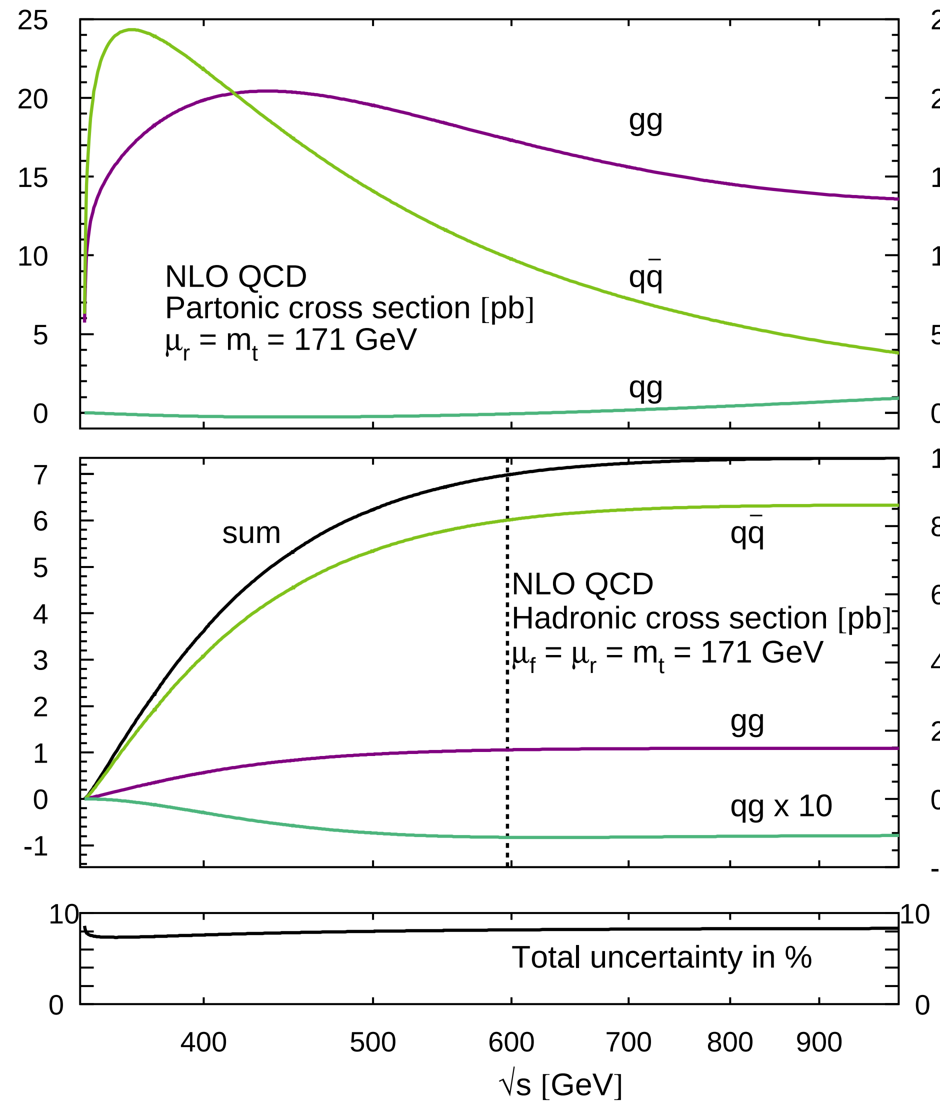


Total cross section

Tevatron

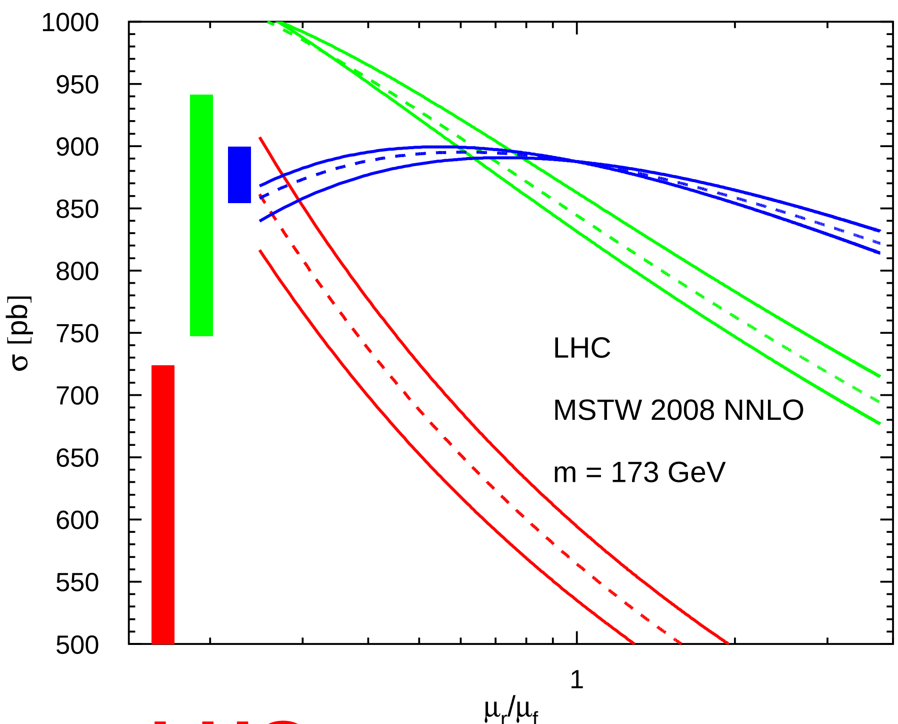
$$\sigma_{pp\to t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij\to t\bar{t}}$$



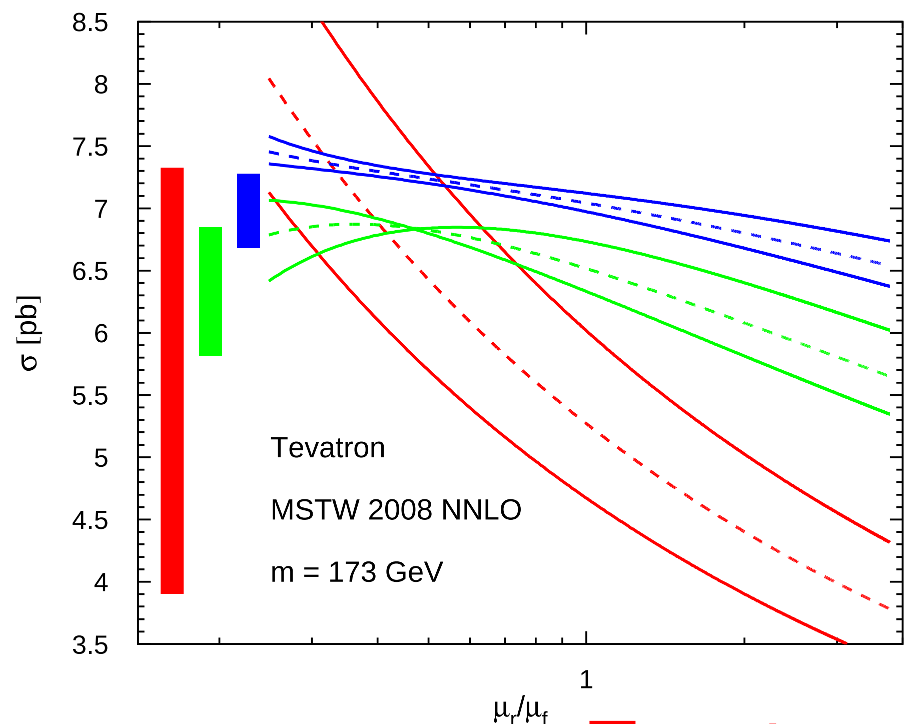


Scale dependence

- Theoretical uncertainty from variation of scales μ_R, μ_F
  - plot with PDF set MSTW 2008 (but largely independent on PDFs)
  - mass $m_t = 173$ GeV
  - stable predictions in range $\mu_R, \mu_F \in [m_t/2, 2m_t]$
    - $\Delta\sigma \simeq 5\%$ at LHC
    - $\Delta\sigma \simeq 4\%$ at Tevatron



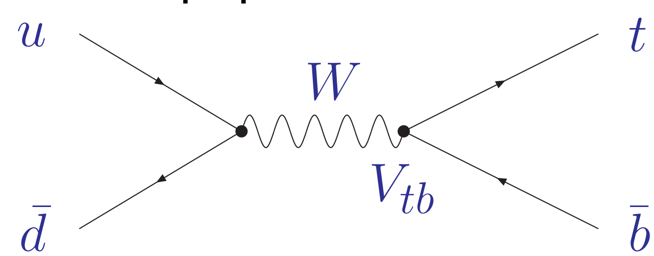
LHC



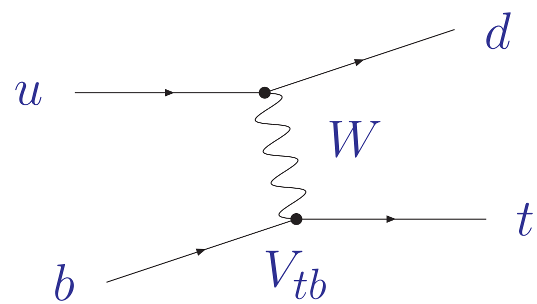
Tevatron

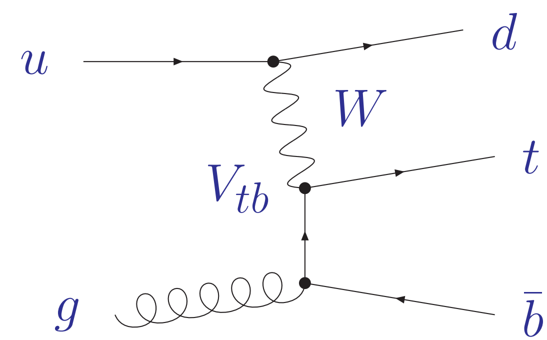
Single top-quark production

- Study of charged-current weak interaction of top quark
- s-channel production

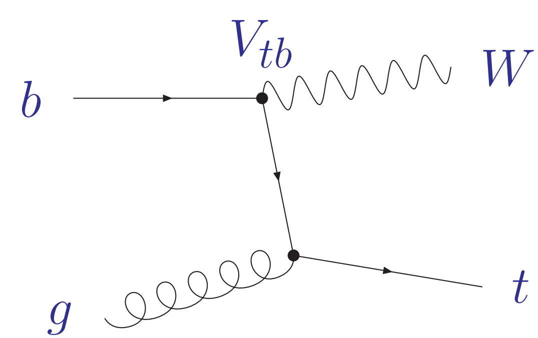


- t-channel production
  - bg-channel at NLO enhanced by gluon luminosity





- Wt-production
  - small at Tevatron, contributes at LHC



Theory description

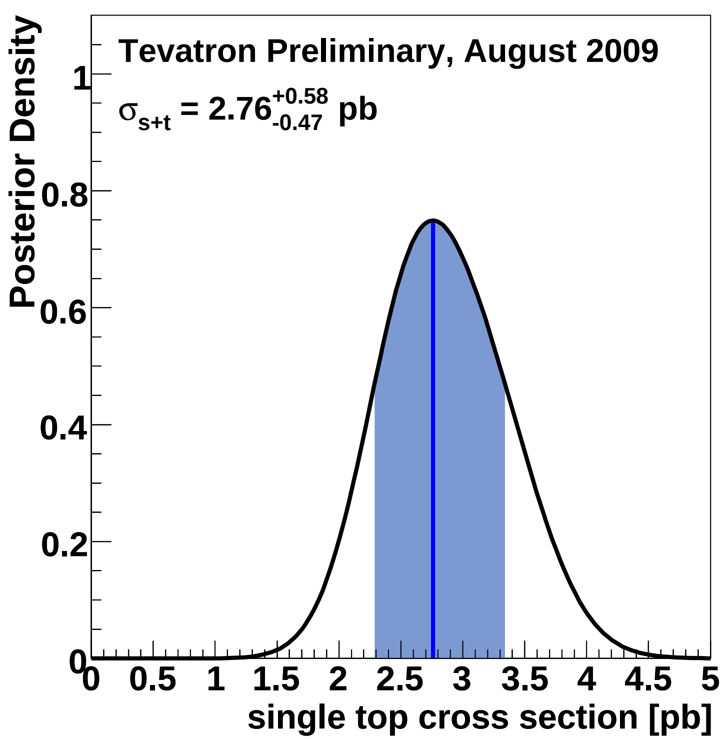
- Large corrections from extensions of Standard Model
  - t-channel: anomalous couplings or flavor changing neutral currents
  - s-channel: charged "top-pion", Kaluza-Klein modes of W or W'-boson
- QCD theory status:
  - NLO corrections known Harris, Laenen, Phaf, Sullivan, Weinzierl '02; Sullivan '04; . . . ; Campbell, Frederix, Maltoni, Tramontano '09
  - implementations in MC merged with PS
    in MCNLO Frixione, Laenen, Motylinski, Webber, White '09
    in POWHEG Aioli, Nason, Oleari, Re '09
  - approximate NNLO corrections Kidonakis '11

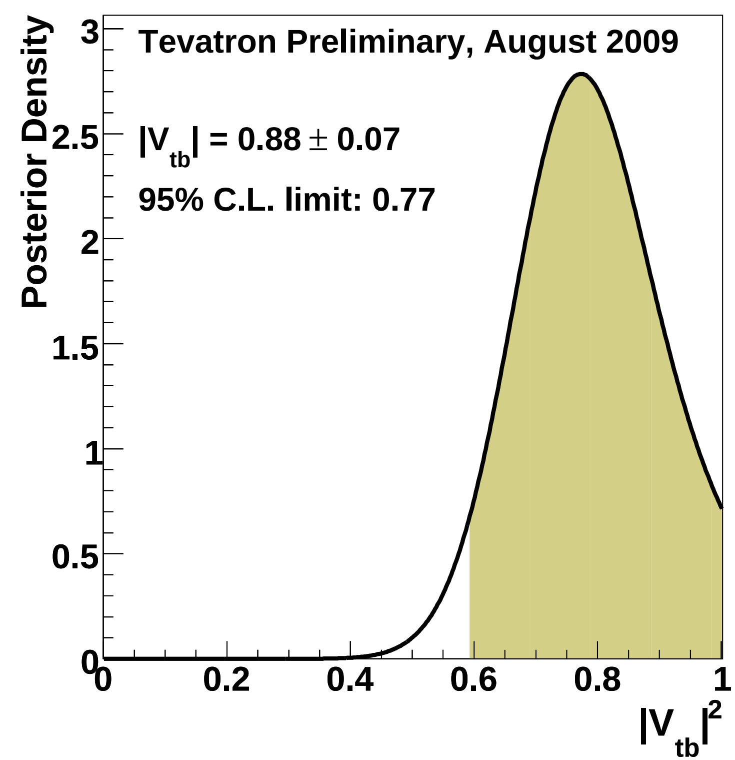
Treatment of heavy quarks

- Scheme with $n_l = 4$ light flavors + heavy quark of mass m_b at low scales
  - no mass singularities for $m_b, m_t \gg \Lambda_{QCD}$, no (evolving) PDFs
- Scheme with $n_l = 5$ light flavors
  - b PDF for $Q \ggg m_b$ generated perturbatively

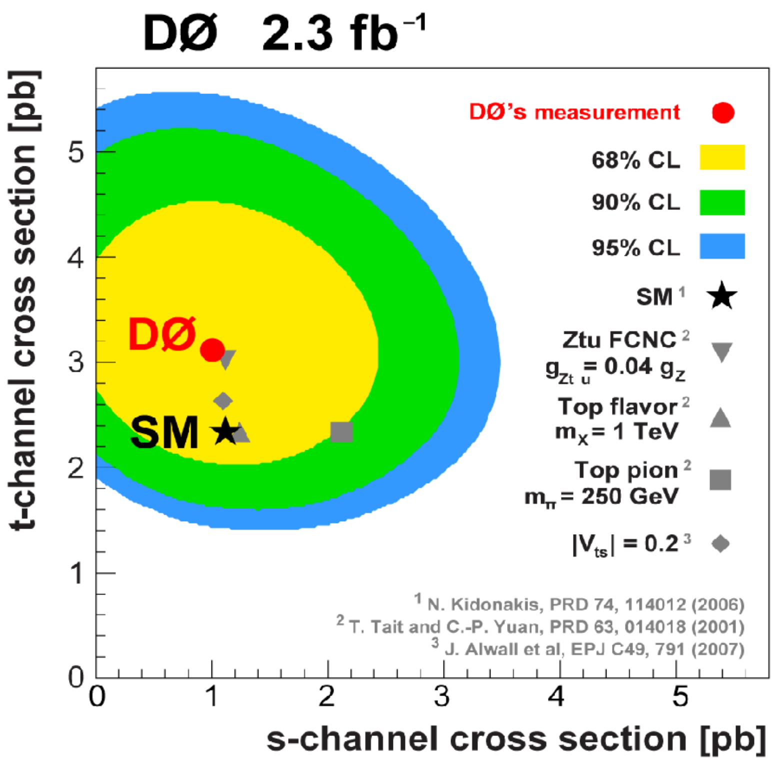
Tevatron measurement

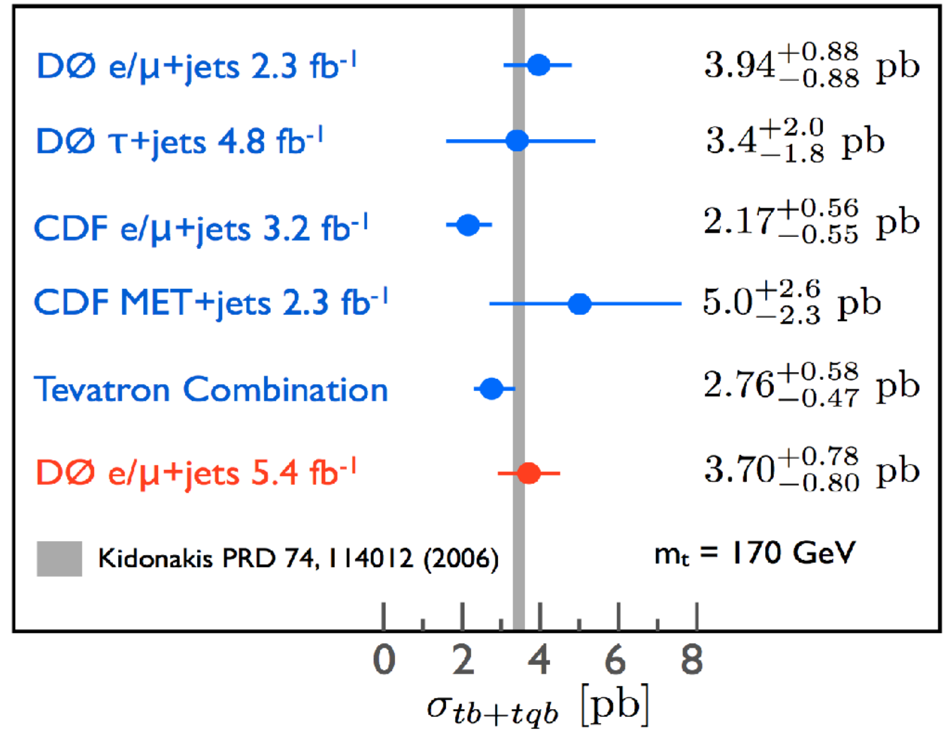
- Flagship measurement of Tevatron run II (control QCD bckgrd !)
- Analysis uses integrated luminosity 3.2 fb^{-1} (CDF), 2.3 fb^{-1} (DZero)
  - measured cross section: $\sigma_{s+t} = 2.76 + 0.58\mathrm{pb} - 0.47\mathrm{pb}$
  - direct extraction of CKM-matrix element V_{tb}: $|V_{tb}| = 0.88 \pm 0.07$





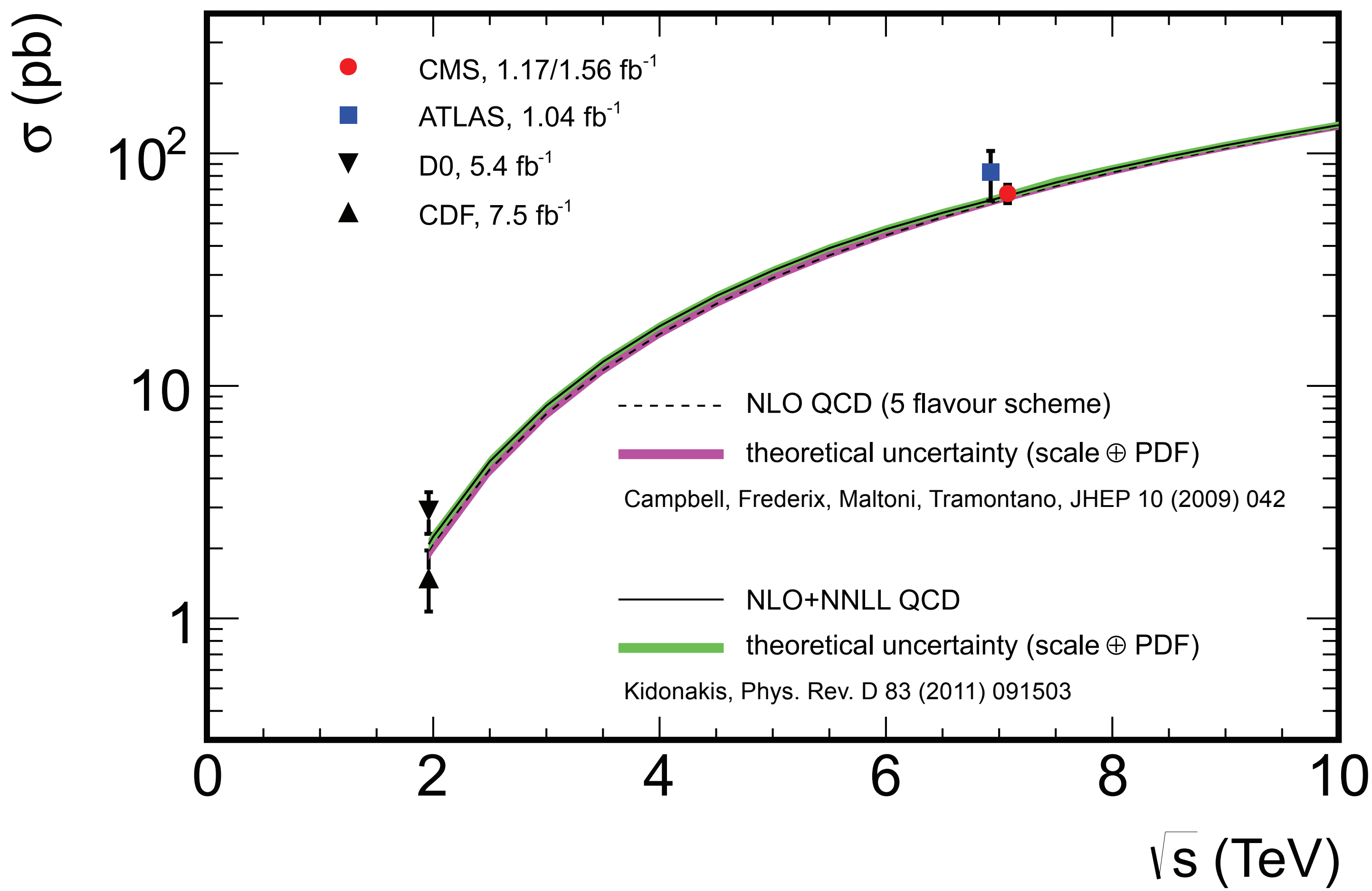
Tevatron measurement





- Latest Tevatron run II results DZero '12

LHC measurement



- First LHC results for t-channel and Wt-production CMS '12
  - control of large background: $W+$ jets, $t\bar{t}$ and QCD jets

Top quark mass

- Recall choices on mass renormalization

Pole mass

- Based on (unphysical) concept of top quark being a free parton
  - m_q^{ren} coincides with pole of propagator at each order

$$\not{p} - m_q - \Sigma(p, m_q)\Big|_{\not{p}=m_q} \to \not{p} - m_q^{\text{pole}}$$

- Definition of pole mass ambiguous up to corrections $\mathcal{O}(\Lambda_{QCD})$
  - heavy-quark self-energy $\Sigma(p, m_q)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{QCD})$

$\overline{MS}$ scheme

- $\overline{MS}$ mass definition
  - one-loop minimal subtraction

$$\delta m_q^{(1)} = m_q \frac{\alpha_s}{4\pi} 3C_F \left(\frac{1}{\epsilon} - \gamma_E + \ln 4\pi \right)$$

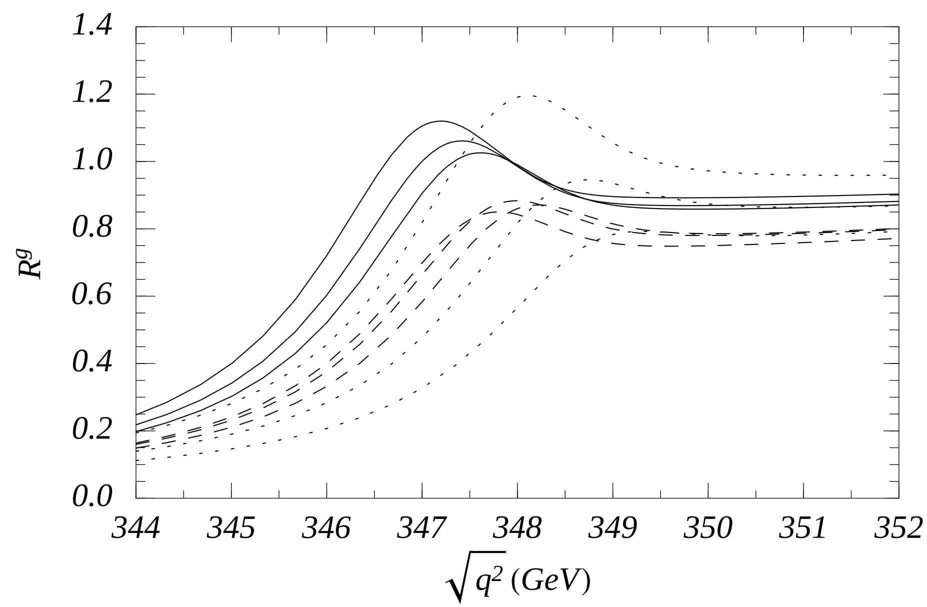
- $\overline{MS}$ scheme induces scale dependence: $m(\mu)$

Top quark pole mass

Illustration for top quark pole mass

ILC

- Pole mass measurements are strongly order-dependent
  - e.g. threshold scan of cross section in e^+e^- collision
    Beneke, Signer, Smirnov '99;
    Hoang, Teubner '99;
    Melnikov, Yelkhovsky '98;
    Penin, Pivovarov '99;
    Yakovlev '99
  - LO (dotted), NLO (dashed), NNLO (solid)

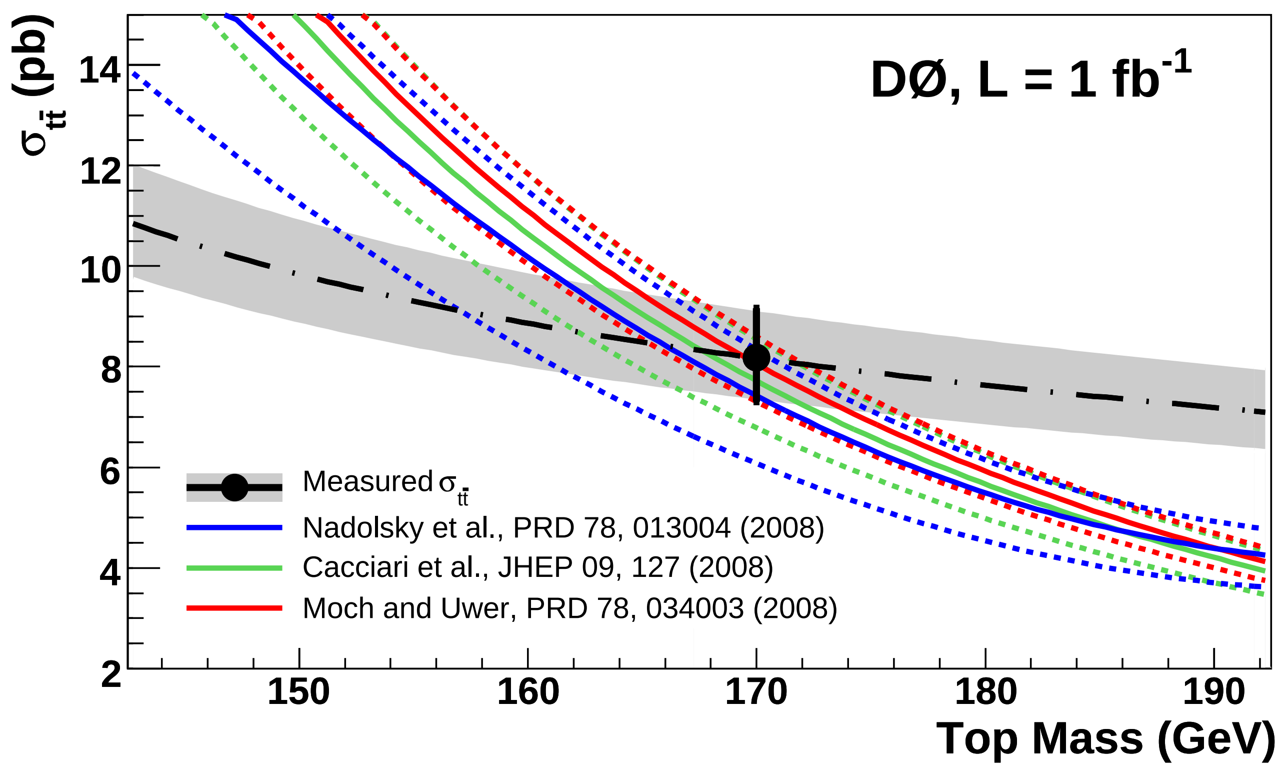


Top quark pole mass

Illustration for top quark pole mass

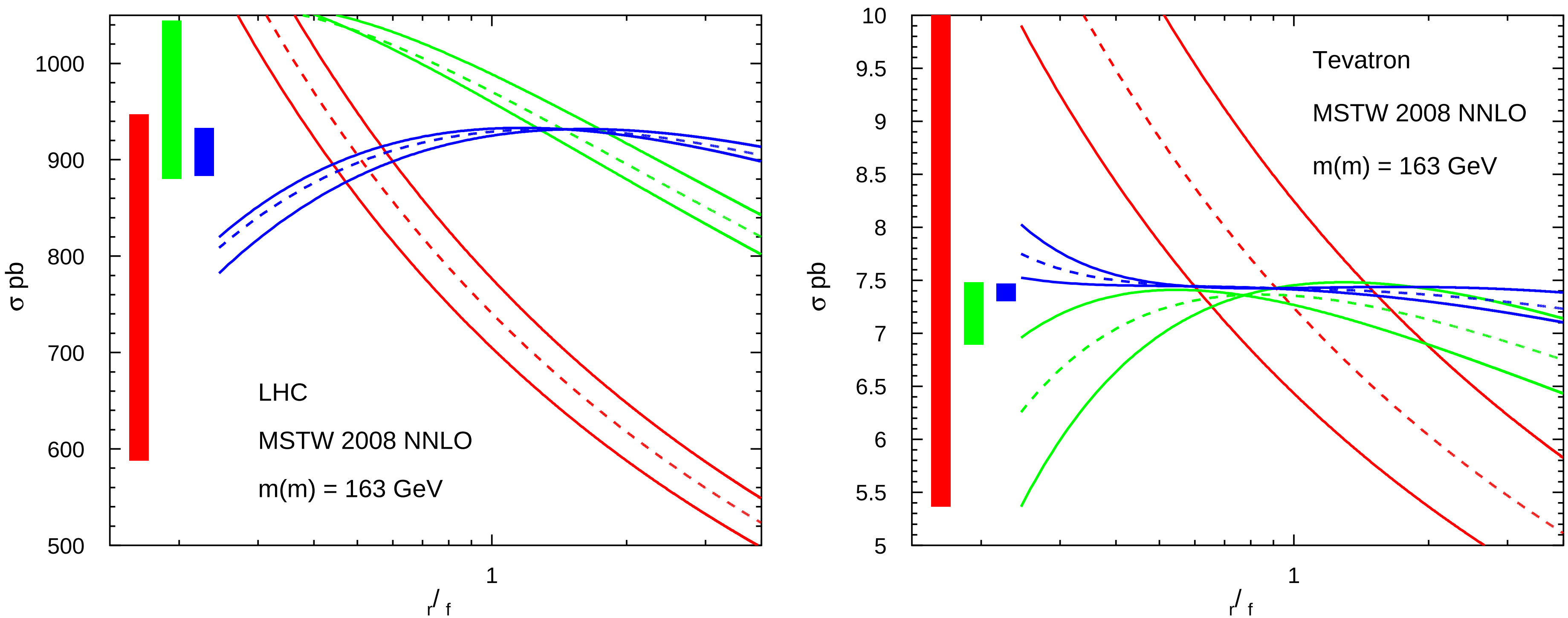
Tevatron

- Total cross section and different channels of Tevatron analyses (theory uncertainty band from scale variation)
- Determination of m_t from total cross section (slope $d\sigma/dm_t$)
  - e.g. DZero '09: NLO $m_t = 165.5^{+6.1}_{-5.9}$; NNLO $m_t = 169.1^{+5.9}_{-5.2}$; ...



Total cross section with $\overline{MS}$ mass

- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
  - short distance mass probes at scale of hard scattering
  - conversion between pole mass and $\overline{MS}$ mass definition in perturbation theory: $m_t = m(\mu_R)\left(1 + a_s(\mu_R) d^{(1)} + a_s(\mu_R)^2 d^{(2)}\right)$
- Scale dependence greatly reduced



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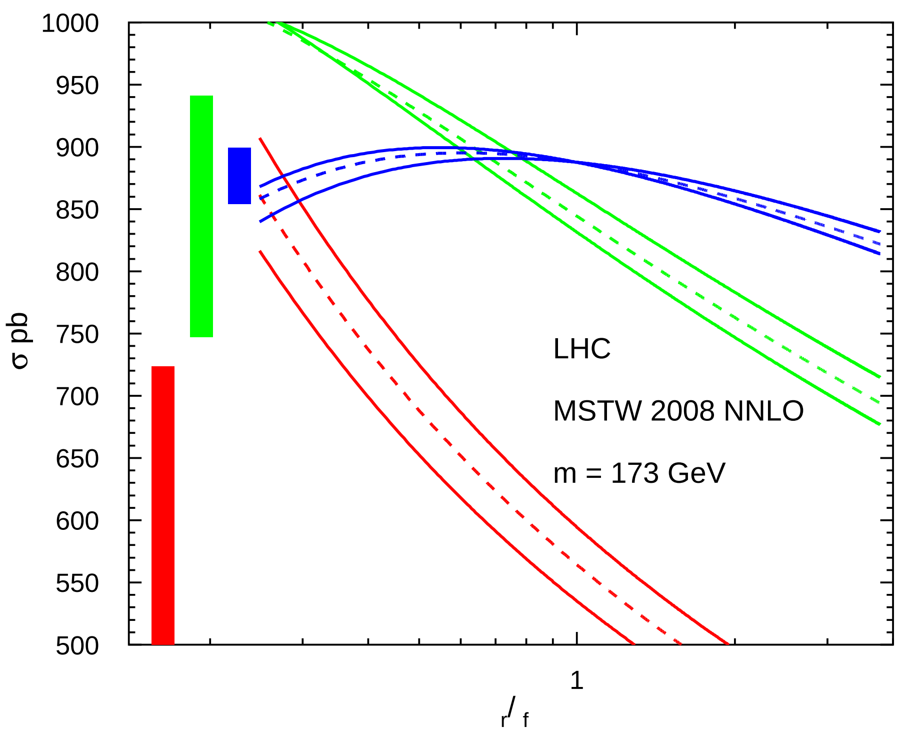
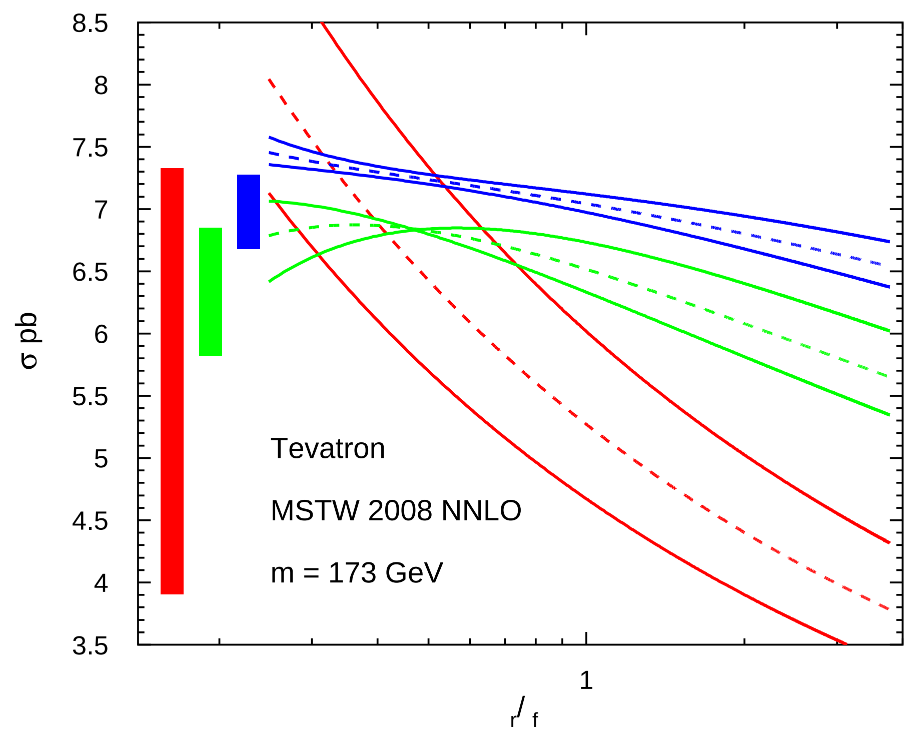
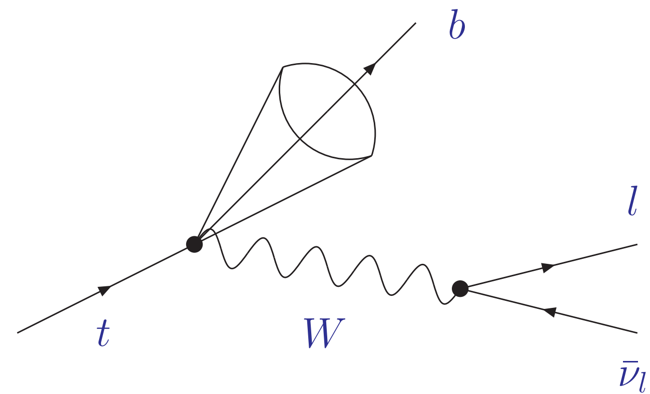
- Pole mass scheme for comparison





Current methods

- Current methods based on reconstructed physics objects
  - jets, identified charged leptons, missing transverse energy
  - $m_t^2 = (p_{W-\mathrm{boson}} + p_{b-\mathrm{jet}})^2$



Template method

- Distributions of kinematically reconstructed top mass values compared to templates for nominal top mass values
  - distributions rely on parton shower predictions
  - no NLO corrections applied
- Future improvements:
  - use of NLO QCD predictions matched to parton shower (MC@NLO, Powheg, . . .)
  - systematic study of distributions sensitive to m_t
  - template overlap method for infrared safe jet observables
    Almeida, Lee, Perez, Sterman, Sung ‘10

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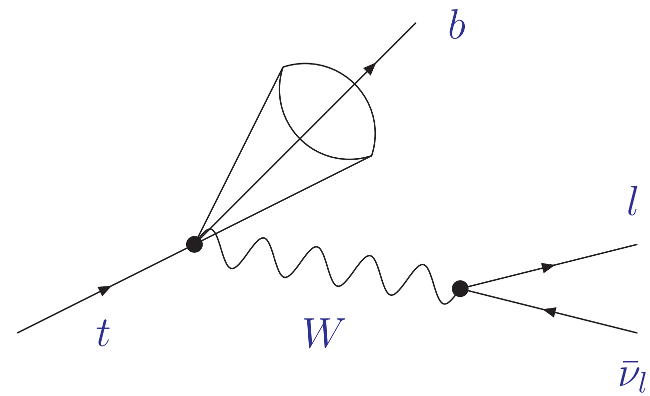
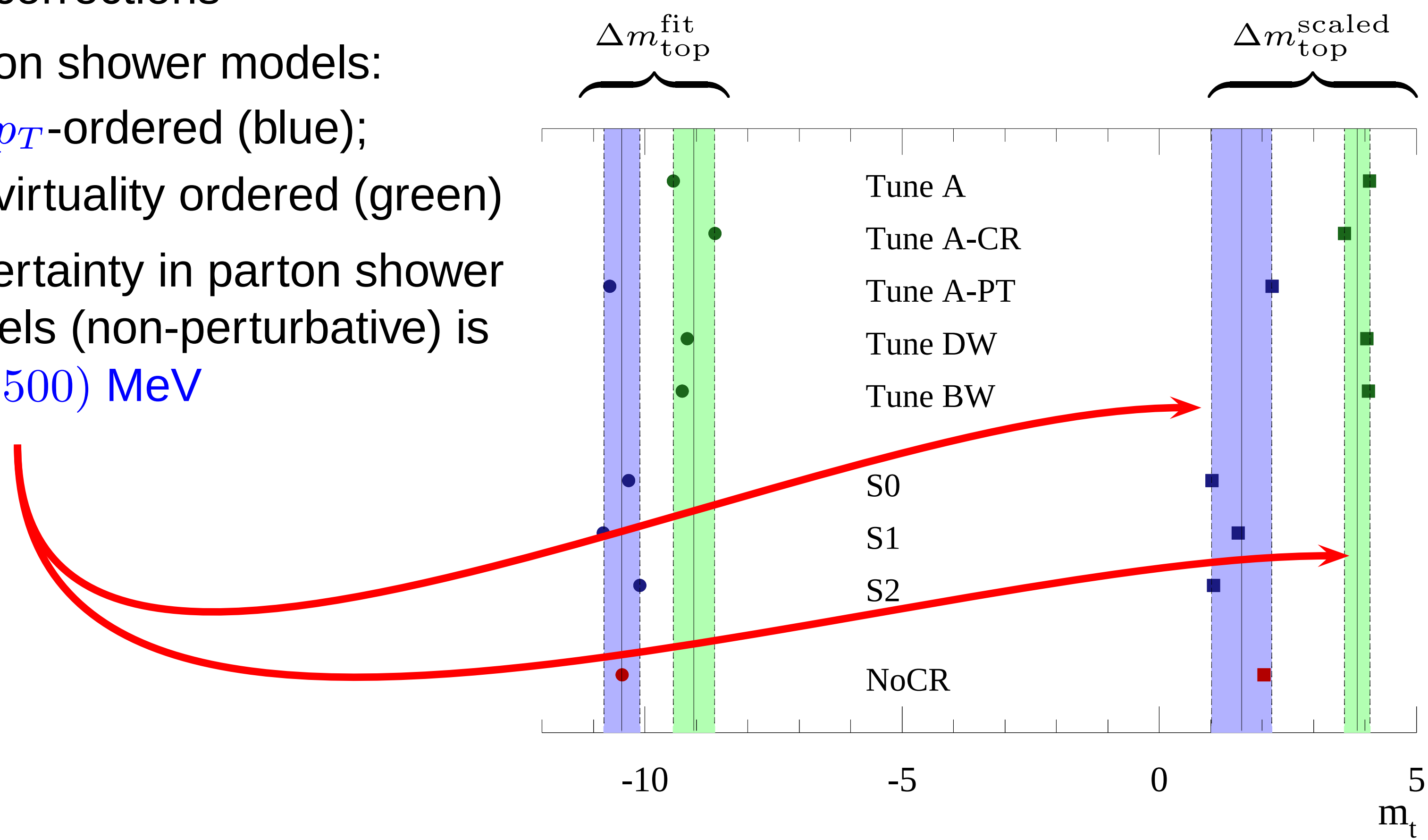


Matrix element method

- Event-by-event likelihood for kinematic configurations arising from events of a given top mass.
  - tree level matrix elements only
  - combinatorics of assignment of jets to top quarks
- Future improvements:
  - advance matrix element method include QCD radiation
    Alwall, Freitas, Mattelaer ‘10
  - computation of NLO weighted events Campbell, Giele, Williams ‘12

Non-perturbative corrections

- Simulation of top mass measurement Skands, Wicke '07
  - test of different Monte Carlo tunes for non-perturbative physics / colour reconnection
  - calibration offsets before/after scaling with jet energy scale corrections
- Parton shower models:
  - p_T-ordered (blue);
  - virtuality ordered (green)
- Uncertainty in parton shower models (non-perturbative) is $\mathcal{O}(\pm 500)$ MeV



Tevatron combination

- Error budget in Tevatron determination
  CDF & D0 coll. 1207.1069
  - lepton+jets channel with matrix element method
- *Modeling signal* encompasses all perturbative uncertainties
  - radiative corrections (initial/final)
  - higher order QCD corrections
  - . . .
- Uncertainties too optimistic
  $\Delta m_t \simeq 150 \dots 250$ MeV
- Contradicts lattice bound
  $\Delta m_t \geq 200$ MeV
  (if interpreted as pole mass)

TABLE VIII: Individual components of uncertainty on CDF and D0 m_t measurements in the lepton+jets channel for Run II data [26, 27].

Systematic Source	Uncertainty [GeV] CDF (5.6 fb^{-1}) $m_t = 173.00$ GeV	D0 (3.6 fb^{-1}) $m_t = 174.94$ GeV
DETECTOR RESPONSE		
Jet energy scale		
Light-jet response (1)	0.41	n/a
Light-jet response (2)	0.01	0.63
Out-of-cone correction	0.27	n/a
Model for b jets	0.23	0.07
Semileptonic b decay	0.16	0.04
b-jet hadronization	0.16	0.06
Response to $b/q/g$ jets	0.13	0.26
In-situ light-jet calibration	0.58	0.46
Jet modeling	0.00	0.36
Jet energy resolution	0.00	0.24
Jet identification	0.00	0.26
Lepton modeling	0.14	0.18
MODELING SIGNAL		
Signal modeling	0.56	0.77
Parton distribution functions	0.14	0.24
Quark annihilation fraction	0.03	n/a
Initial and final-state radiation	0.15	0.26
Higher-order QCD corrections	n/a	0.25
Jet hadronization and underlying event	0.25	0.58
Color reconnection	0.37	0.28
Multiple interactions model	0.10	0.05
MODELING BACKGROUND		
Background from theory	0.27	0.19
Higher-order correction for heavy flavor	0.03	0.07
Factorization scale for W+jets	0.07	0.16
Normalization to predicted cross sections	0.25	0.07
Distribution for background	0.07	0.03
Background based on data	0.06	0.23
Normalization to data	0.00	0.06
Trigger modeling	0.00	0.06
b-tagging modeling	0.00	0.10
Signal fraction for calibration	n/a	0.10
Impact of multijet background on the calibration	n/a	0.14
METHOD OF MASS EXTRACTION		
Calibration method	0.10	0.16
STATISTICAL UNCERTAINTY	0.65	0.83
UNCERTAINTY ON JET ENERGY SCALE	0.80	0.83
OTHER SYSTEMATIC UNCERTAINTIES	0.67	0.94
TOTAL UNCERTAINTY	1.23	1.50

Alternative methods

- Top mass from leptonic decay: m_{lb} distribution
- Top mass from total cross section

Top mass from leptonic decay

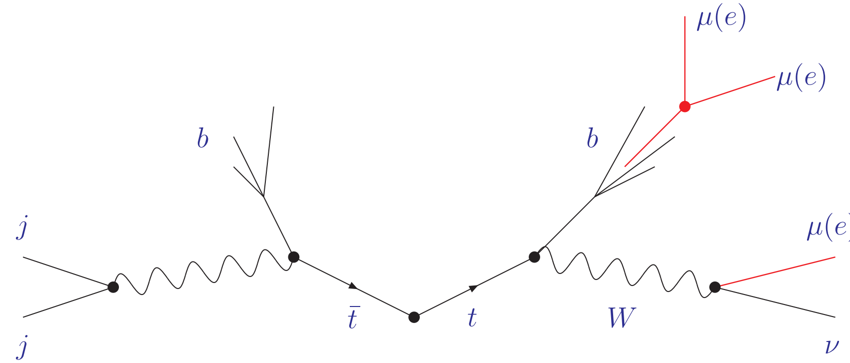
- Top mass from exclusive hadronic states

$$pp \to (t \to W^+ + b \to W^+ + J/\psi) + (\bar{t} \to W^- + \bar{b})$$

  - identification of μ-pair in J/ψ decay; leptonic or hadronic decay of W

    Kharchilava '00

    Chierici, Dierlamm '06

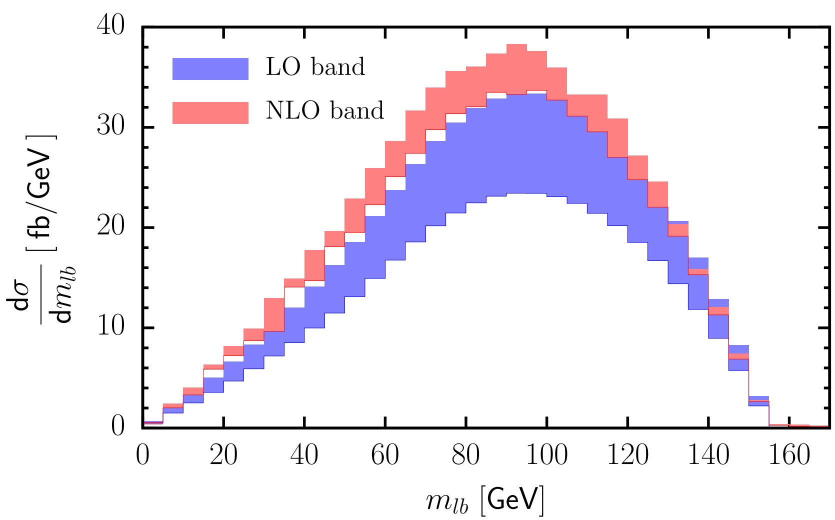


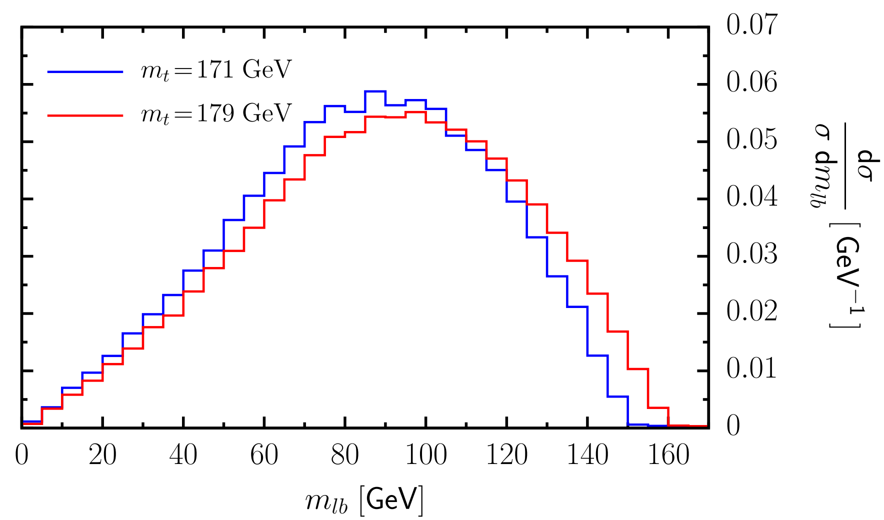
Top mass from leptonic decay

- Top mass from exclusive hadronic states

$$pp \to (t \to W^+ + b \to W^+ + J/\psi) + (\bar{t} \to W^- + \bar{b})$$

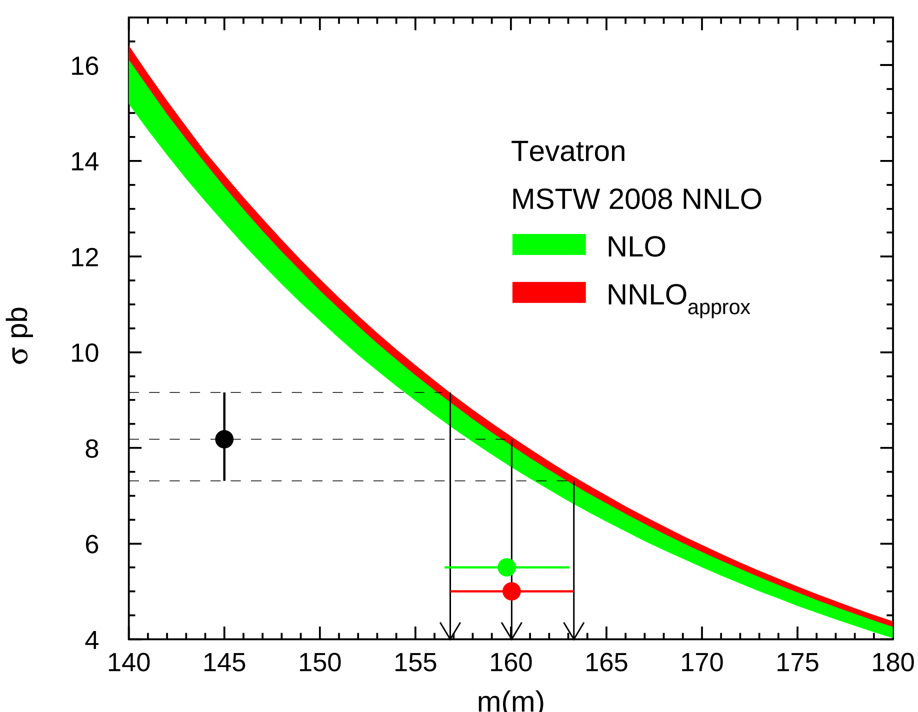
- Study of m_{lb} distribution at NLO in QCD Biswas, Melnikov, Schulze '10
  - NLO QCD corrections to production **and** decay very important for value of m_t (effects of order $\Delta m_t = \mathcal{O}(\text{few})$ GeV
- Invariant mass distribution of lepton and b-jet (LHC14)
  - scale dependence at LO and NLO (left)
  - normalized m_{lb} distributions, $m_t = 171\ \mathrm{GeV}$ and $179\ \mathrm{GeV}$ (right)



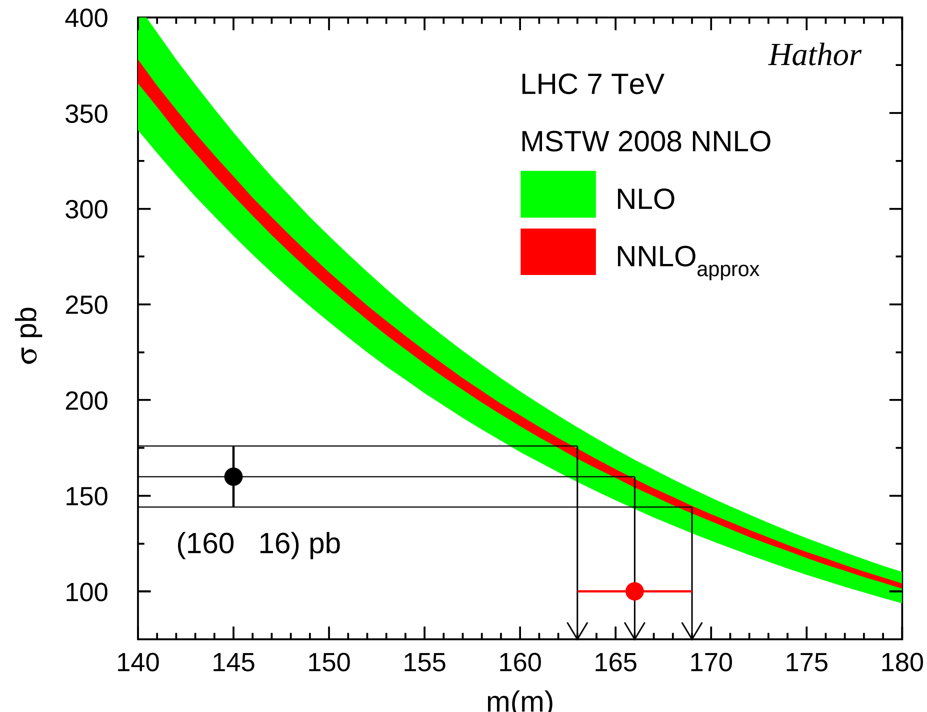


Top mass from total cross section

- Total top quark cross section as function of $\overline{MS}$ mass
  Langenfeld, S.M., Uwer '09
  - good apparent convergence of perturbative expansion
  - small theoretical uncertainity form scale variation



Tevatron



LHC

Tevatron

- Determine top quark mass from Tevatron cross section data
  - $\sigma_{t\bar{t}} = 7.56^{+\,0.63}_{-\,0.56}$ pb D0 coll. arXiv:1105.5384
  - $\sigma_{t\bar{t}} = 7.50\,^{+0.48}_{-0.48}$ pb CDF coll. CDF-note-9913
- Fit of m_t for individual PDFs
  - parton luminosity at Tevatron driven by $q\bar{q}$
  - $\overline{\mathrm{MS}}$-scheme for $m_t^{\overline{\mathrm{MS}}}(m_t)$, then scheme transformation to pole mass m_t^{pole} at NNLO

	ABM11	JR09	MSTW08	NN21
$m_t^{\overline{\mathrm{MS}}}(m_t)$	$162.0\,^{+2.3}_{-2.3}\,^{+0.7}_{-0.6}$	$163.5\,^{+2.2}_{-2.2}\,^{+0.6}_{-0.2}$	$163.2\,^{+2.2}_{-2.2}\,^{+0.7}_{-0.8}$	$164.4\,^{+2.2}_{-2.2}\,^{+0.8}_{-0.2}$
m_t^{pole}	$171.7\,^{+2.4}_{-2.4}\,^{+0.7}_{-0.6}$	$173.3\,^{+2.3}_{-2.3}\,^{+0.7}_{-0.2}$	$173.4\,^{+2.3}_{-2.3}\,^{+0.8}_{-0.8}$	$174.9\,^{+2.3}_{-2.3}\,^{+0.8}_{-0.3}$
(m_t^{pole})	($169.9\,^{+2.4}_{-2.4}\,^{+1.2}_{-1.6}$)	($171.4\,^{+2.3}_{-2.3}\,^{+1.2}_{-1.1}$)	($171.3\,^{+2.3}_{-2.3}\,^{+1.4}_{-1.8}$)	($172.7\,^{+2.3}_{-2.3}\,^{+1.4}_{-1.2}$)

- Good consistency within errors for $m_t^{\mathrm{pole}} = 171.7\ldots174.9$ at NNLO

LHC

- Check predictions at LHC with $\sqrt{s} = 7$ TeV
  - cross section computation with HATHOR (version 1.3)
    Aliev, Lacker, Langenfeld, S.M., Uwer, Wiedermann '10; S.M., Uwer, Vogt '12
- Atlas at $\sqrt{s} = 7$ TeV $\sigma_{t\bar{t}} = 177^{+11}_{-10}$ pb
  Atlas coll. ATLAS-CONF-2012-024
- CMS at $\sqrt{s} = 7$ TeV $\sigma_{t\bar{t}} = 165.8^{+13.3}_{-13.3}$ pb
  CMS coll. CMS-PAS-TOP-11-024

	ABM11	JR09	MSTW08	NN21
$m_t^{\overline{MS}}(m_t)$	159.0 $^{+2.1}_{-2.0}$ $^{+0.7}_{-1.4}$	165.3 $^{+2.3}_{-2.2}$ $^{+0.6}_{-1.2}$	166.0 $^{+2.3}_{-2.2}$ $^{+0.7}_{-1.5}$	166.7 $^{+2.3}_{-2.2}$ $^{+0.8}_{-1.3}$
$m_t^{\rm pole}$	168.6 $^{+2.3}_{-2.2}$ $^{+0.7}_{-1.5}$	175.1 $^{+2.4}_{-2.3}$ $^{+0.6}_{-1.3}$	176.4 $^{+2.4}_{-2.3}$ $^{+0.8}_{-1.6}$	177.4 $^{+2.4}_{-2.3}$ $^{+0.8}_{-1.4}$
$(m_t^{\rm pole})$	(166.1 $^{+2.2}_{-2.1}$ $^{+1.7}_{-2.3}$)	(172.6 $^{+2.4}_{-2.3}$ $^{+1.6}_{-2.1}$)	(173.5 $^{+2.4}_{-2.3}$ $^{+1.8}_{-2.5}$)	(174.5 $^{+2.4}_{-2.3}$ $^{+2.0}_{-2.3}$)

- Large spread $m_t^{\rm pole} = 168.6 \dots 177.4$ at NNLO (marginally consistent)
  - larger gluon and α_s imply larger $m_t^{\rm pole}$

Higgs potential

Renormalization group equation

- Quantum corrections to Higgs potential $V(\Phi) = \lambda \left|\Phi^\dagger \Phi - \frac{v}{2}\right|^2$
- Radiative corrections to Higgs self-coupling λ
  - electro-weak couplings g and g' of $SU(2)$ and $U(1)$
  - top-Yukawa coupling y_t

$$16\pi^2 \frac{d\lambda}{dQ} \;=\; 24\lambda^2 - \left(3g'^2 + 9g^2 - 12y_t^2\right)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 6y_t^4 + \ldots$$

Higgs potential

Triviality

- Large mass implies large λ
  - renormalization group equation dominated by first term

$$16\pi^2 \frac{d\lambda}{dQ} \simeq 24\lambda^2 \qquad \longrightarrow \qquad \lambda(Q) = \frac{m_H^2}{2v^2 - \frac{3}{2\pi^2} m_H^2 \ln(Q/v)}$$

- $\lambda(Q)$ increases with Q
- Landau pole implies cut-off Λ
  - scale of new physics smaller than Λ to restore stability
  - upper bound on m_H for fixed Λ

$$\Lambda \leq v \exp\left(\frac{4\pi^2 v^2}{3 m_H^2}\right)$$

- Triviality for $\Lambda \to \infty$
  - vanishing self-coupling $\lambda \to 0$ (no interaction)

Higgs potential

Vacuum stability

- Small mass
  - renormalization group equation dominated by y_t

$$16\pi^2 \frac{d\lambda}{dQ} \simeq -6y_t^4 \qquad \longrightarrow \qquad \lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)}{1 - \frac{9}{16\pi^2} y_0^2 \ln(Q/Q_0)}$$

- $\lambda(Q)$ decreases with Q
- Higgs potential unbounded from below for $\lambda < 0$
- $\lambda = 0$ for $\lambda_0 \simeq \frac{3}{8\pi^2} y_0^4 \ln(Q/Q_0)$
- Vacuum stability

$$\Lambda \leq v \exp\left(\frac{4\pi^2 m_H^2}{3y_t^4 v^2}\right)$$

  - scale of new physics smaller than Λ to ensure vacuum stability
  - lower bound on m_H for fixed Λ

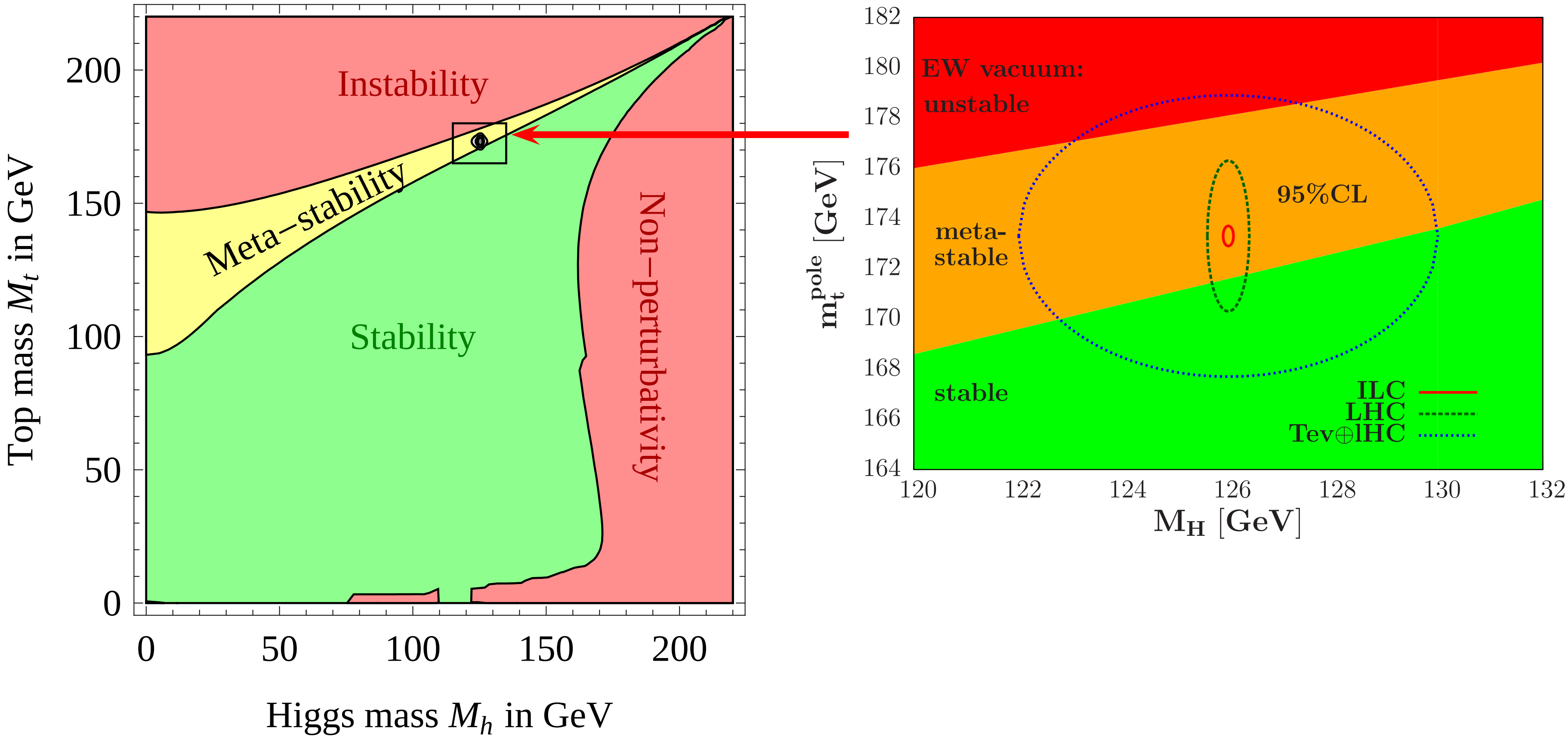
Implications on electroweak vacuum

- Relation between Higgs mass m_H and top quark mass m_t
  - condition of absolute stability of electroweak vacuum $\lambda(\mu) \geq 0$
  - extrapolation of Standard Model up to Planck scale M_P
  - $\lambda(M_P) \geq 0$ implies lower bound on Higgs mass m_H

$$m_H \geq 129.2 + 1.8 \times \left(\frac{m_t^{\text{pole}} - 173.2 \text{ GeV}}{0.9 \text{ GeV}} \right) - 0.5 \times \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0 \text{ GeV}$$

  - recent NNLO analyses Bezrukov, Kalmykov, Kniehl, Shaposhnikov '12; Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12
  - uncertainity in results due to α_s and m_t (pole mass scheme)
- Top quark mass from Tevatron in well-defined scheme
  - $m_t^{\overline{\text{MS}}}(m_t) = 163.3 \pm 2.7 \text{ GeV}$ implies in pole mass scheme $m_t^{\text{pole}} = 173.3 \pm 2.8 \text{ GeV}$
  - good consistency of mass value between different PDF sets

Fate of the universe



Degrassi, Di Vita, Elias-Miro, Espinosa, Giudice et al. '12; Alekhin, Djouadi, S.M. '12; Masina '12

- Uncertainty in Higgs bound due to m_t from in $\overline{MS}$ scheme
  - bound relaxes $m_H \geq 129.4 \pm 5.6\ \text{GeV}$
  - "fate of universe" still undecided

Summary (part V)

- Top quark theory
  - improved understanding of theory and application of new concepts
  - resummation important for Tevatron and LHC phenomenology
- Cross sections
  - NNLO predictions for $t\bar{t}$
  - NLO corrections to $t\bar{t}+$ jet
  - electroweak corrections
- $\overline{MS}$ mass definition
  - greatly reduced scale dependence
  - much improved convergence of perturbation theory

Top quark mass

- Top quark mass is parameter of Standard Model Lagrangian
- Measurements of m_t require careful definition of observable
- Radiative corrections at higher orders mandatory for scheme definition