Quantum Chromodynamics

lecture IV

Sven-Olaf Moch

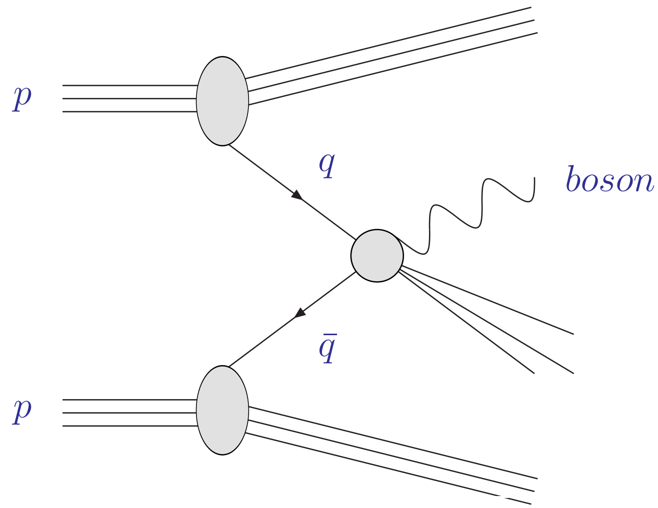
Universität Hamburg & DESY, Zeuthen

Belgian Dutch German summer school (BND 2012), Bonn, Sep 24, 2012

Plan

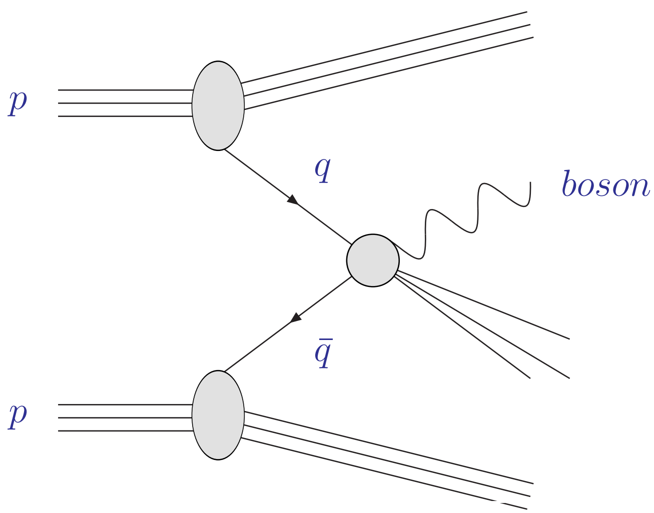
- Introduction to QCD
  Friday, September 21, 2012
- QCD at work: infrared safety, factorization and evolution
  Saturday, September 22, 2012
- Higgs boson production
  Sunday, September 23, 2012
- *Gauge boson production and QCD jets*
  Monday, September 24, 2012
- Top quark production
  Tuesday, September 25, 2012

Vector boson production



- Kinematical variables (inclusive)
  - energy (cms) $s = Q^2$ (time-like)
  - scaling variable $x = M^2_{W^\pm/Z}/s$

Vector boson production



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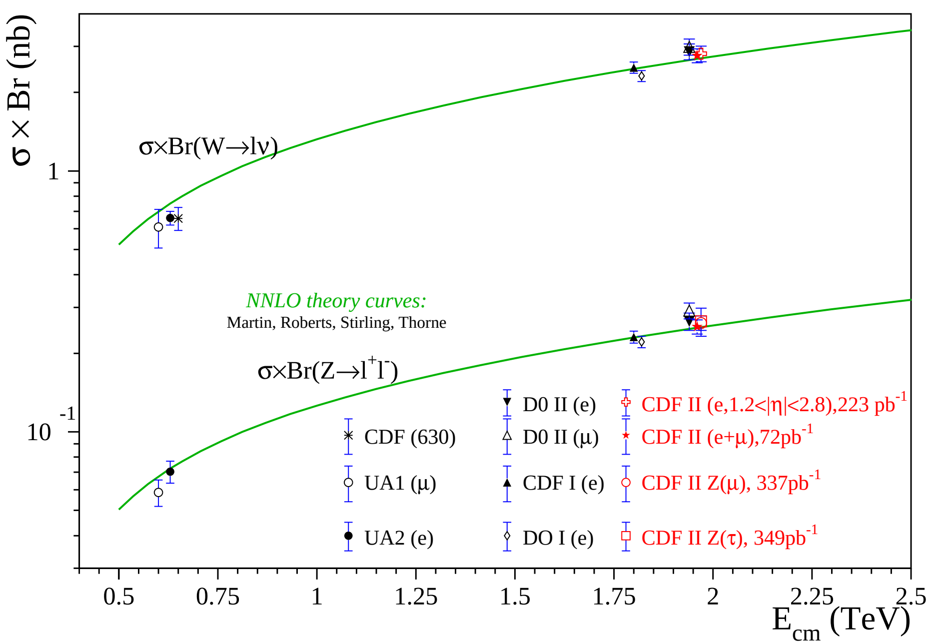
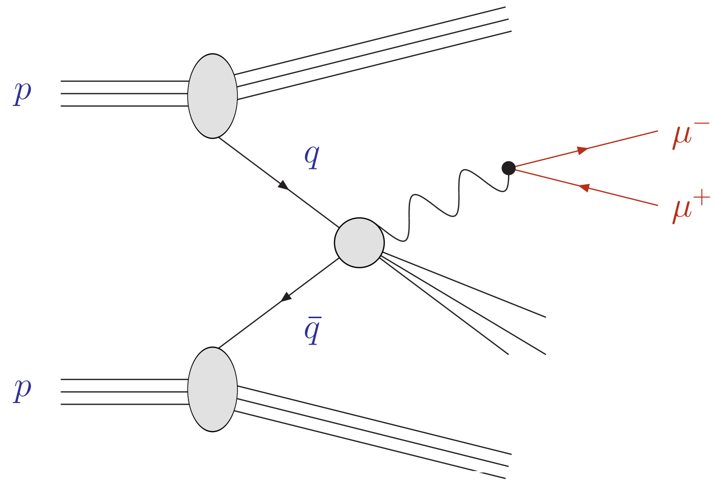


- 20 years of measurements of $W^\pm$ and Z cross sections at hadron colliders

QCD corrections to W/Z production



- Hadronic cross section $\sigma_{pp\to V}$ with $\tau = M_V^2/s$ and $V = \gamma^*/W^\pm/Z$
  - renormalization/factorization (hard) scale $\mu = \mathcal{O}(M_V)$

$$\sigma_{pp\to V} = \sum_{ij} \int_\tau^1 \frac{dx_1}{x_1} \int_{x_1}^1 \frac{dx_2}{x_2} f_i\left(\frac{x_1}{x_2}, \mu^2\right) f_j\left(x_2, \mu^2\right) \hat{\sigma}_{ij\to V}\left(\frac{\tau}{x_1}, \frac{\mu^2}{M_V^2}, \alpha_s(\mu^2)\right)$$

- Partonic cross section $\hat{\sigma}_{ij\to V}$

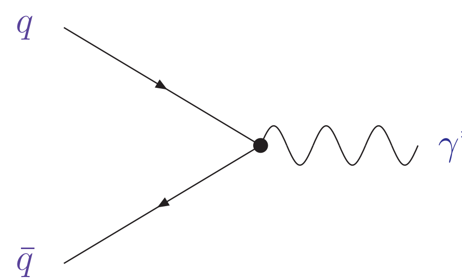
$$\hat{\sigma}_{ij\to V} = \underbrace{\alpha_s^2 \Big[\hat{\sigma}^{(0)}_{ij\to V} + \alpha_s\, \hat{\sigma}^{(1)}_{ij\to V}}_{} + \alpha_s^2\, \hat{\sigma}^{(2)}_{ij\to V} + \ldots \Big]$$

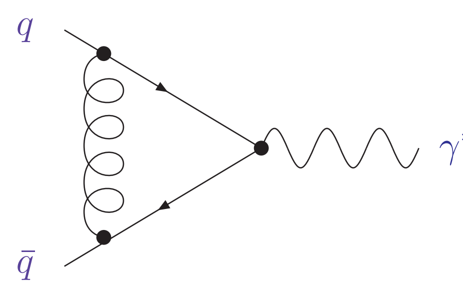
NLO: standard approximation (large uncertainties)

Radiative corrections in a nutshell

- Leading order
  - partonic cross section $x = \tau/x_1$

    $\hat{\sigma}^{(0)}_{q\bar{q}\to V} \;=\; \delta(1-x)$

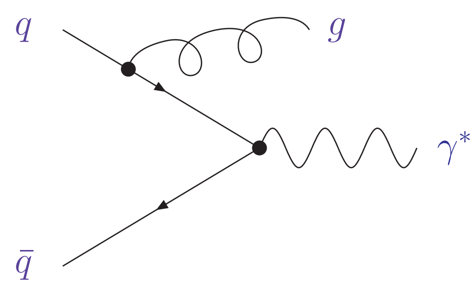




- Next-to-leading order
  - virtual correction
    (infrared divergent; proportional to Born)
  - dimensional regularization $D = 4 - 2\epsilon$

$$\hat{\sigma}^{(1),v}_{q\bar{q}\to V} \quad = \quad C_F\,\frac{\alpha_s}{4\pi}\,\delta(1-x)\left(\frac{\mu^2}{M_V^2}\right)^{\epsilon}\left(-\frac{4}{\epsilon^2}-\frac{6}{\epsilon}-16+2\zeta_2+\mathcal{O}(\epsilon)\right)$$

- Next-to-leading order



  - add real and virtual corrections $\hat{\sigma}^{(1)}_{q\bar{q}\to V} = \hat{\sigma}^{(1),r}_{q\bar{q}\to V} + \hat{\sigma}^{(1),v}_{q\bar{q}\to V}$
  - collinear divergence remains splitting functions $P^{(0)}_{qq}$

$$\hat{\sigma}^{(1)}_{q\bar{q}\to V} = \frac{\alpha_s}{4\pi} C_F \left(\frac{\mu^2}{M_V^2}\right)^{\epsilon} \left\{ \frac{1}{\epsilon}\left(\frac{8}{1-x} - 4 - 4x + 6\delta(1-x)\right) \right.$$
$$+ \left(16\frac{\ln(1-x)}{1-x} + (-16+8\zeta_2)\,\delta(1-x) \right.$$
$$\left. -4\frac{1+x^2}{1-x}\ln(x) + 8\frac{1+x^2}{1-x}\ln(1-x)\right)$$
$$\left. +\mathcal{O}(\epsilon)\right\}$$

- Structure of NLO correction
  - absorb collinear divergence $P_{qq}^{(0)}$ in renormalized parton distributions

$$\hat{\sigma}_{q\bar{q}\to V}^{(1),\text{bare}} = \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{M_V^2}\right)^{\epsilon} \left\{ \frac{1}{\epsilon}\, 2\, P_{qq}^{(0)}(x) + \hat{\sigma}_{q\bar{q}\to V}^{(1)}(x) + \mathcal{O}(\epsilon) \right\}$$

$$q^{\text{ren}}(\mu_F^2) = q^{\text{bare}} - \frac{\alpha_s}{4\pi}\, \frac{1}{\epsilon}\, P_{qq}^{(0)}(x) \left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}$$

  - partonic (physical) structure function at factorization scale μ_F

$$\hat{\sigma}_{q\bar{q}\to V} = \delta(1-x) + \frac{\alpha_s}{4\pi} \left\{ \hat{\sigma}_{q\bar{q}\to V}^{(1)}(x) - \ln\left(\frac{M_V^2}{\mu_F^2}\right) 2\, P_{qq}^{(0)}(x) \right\}$$

Kinematics (differential)

- Proton-proton scattering (two broad-band beams of incoming partons)
  - cms of parton-parton scattering boosted wrt incoming protons
- Final state variables (simple transformations under longitud. boosts)

$$p^\mu = (E, p_x, p_y, p_z) = (m_t \cosh\, y, p_t \sin\, \phi, p_t \cos\, \phi, m_t \sinh\, y)$$

  - rapidity $y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right)$
  - transverse momentum p_t and mass $m_t = \sqrt{p_t^2 + m^2}$
  - azimuthal angle ϕ

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  - transverse momentum p_t and mass $m_t = \sqrt{p_t^2 + m^2}$
  - azimuthal angle ϕ
- Differences in rapidity Δy and azimuthal angle $\Delta\phi$ invariant under boosts
- In practice (for $E \gg m_p$)
  - pseudo-rapidity $\eta = -\ln \tan \left(\frac{\theta}{2} \right)$ with angle from beam axis

Differential distributions (I)

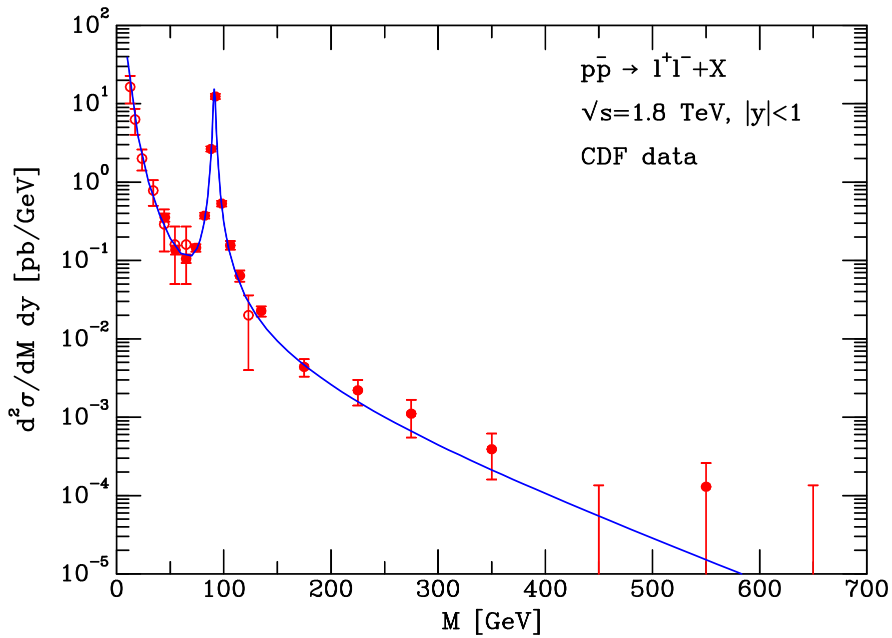
- Cross section for $\hat{\sigma}_{q\bar{q}\to e^+e^-}$
  - Born cross section
    $$\hat{\sigma}_{q\bar{q}\to e^+e^-} = \frac{4\pi\alpha^2}{3s}\,\frac{e_q^2}{N_c} = \sigma^{(0)}\,\frac{e_q^2}{N_c}$$
- Born result for invariant mass distribution $\dfrac{d\hat{\sigma}}{dM^2}$
  - use of $s = M_V^2$ implies $\delta(s - M^2)$

$$M^2\,\frac{d\hat{\sigma}}{dM^2} \quad = \quad \sigma^{(0)}\,\frac{e_q^2}{N_c}\,\delta(s - M^2)$$

- Hadronic cross section from convolution with parton distributions
  - Born result

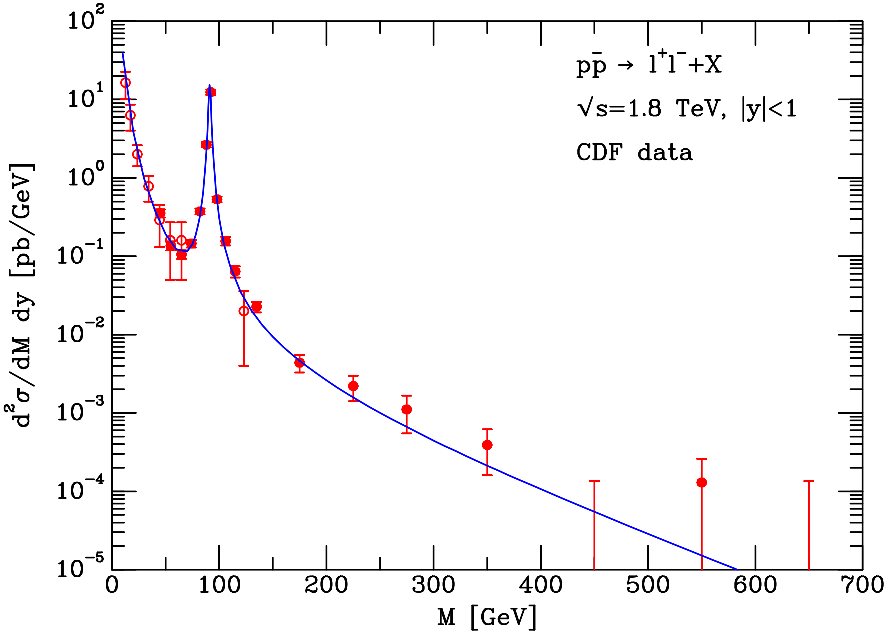
$$M^4\,\frac{d\sigma}{dM^2} \quad = \quad \sigma^{(0)}\,\frac{1}{N_c}\,\frac{M^2}{s}\,\times$$
$$\int_0^1 dx_1\,dx_2\,\delta\left(x_1x_2 - \frac{M^2}{s}\right)\sum_q e_q^2\,\left\{f_q(x_1)\,f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1)\,f_q(x_2)\right\}$$

Differential distributions (II)



- Invariant mass distribution $\frac{d\sigma}{dM^2}$ of lepton pair for Z-production in $p\bar{p}$-collisions
  - CDF data at $\sqrt{s} = 1.8$ TeV and NLO QCD prediction

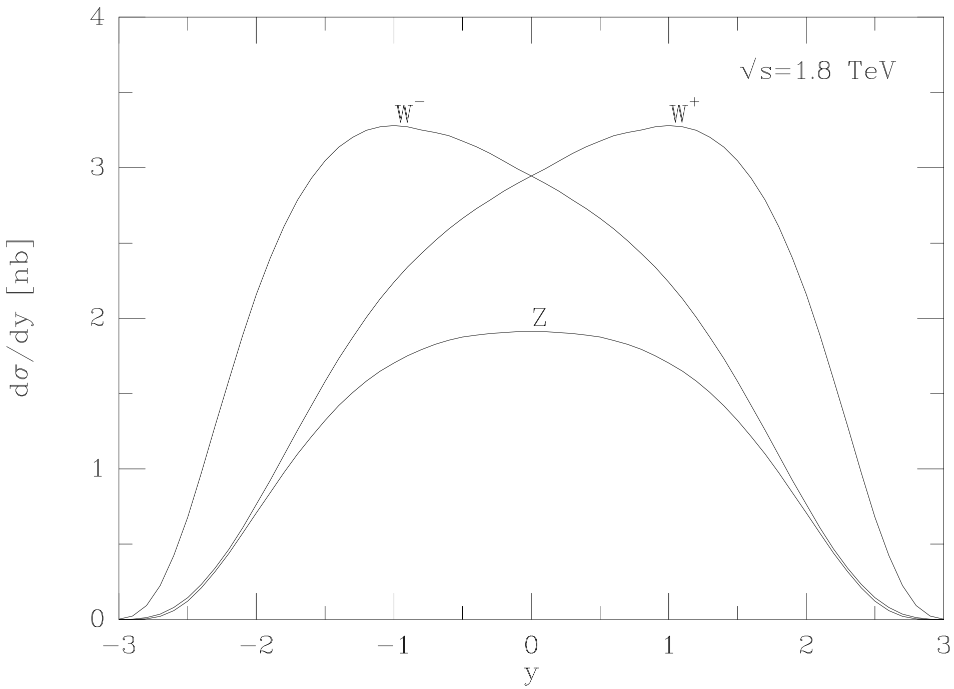
Differential distributions (II)



- Invariant mass distribution $\dfrac{d\sigma}{dM^2}$ of lepton pair for Z-production in $p\bar{p}$-collisions
  - CDF data at $\sqrt{s} = 1.8$ TeV and NLO QCD prediction

- Double-differential cross section $\dfrac{d\sigma}{dM^2 dy}$ local in PDFs
  - $y = \dfrac{1}{2} \ln \left(\dfrac{x_1}{x_2} \right)$ lepton-pair rapidity

$W^{\pm}$ *asymmetry*



- Rapidity distributions for $W^{\pm}$- and Z-production in $p\bar{p}$-collisions
- CP invariance $\longrightarrow$ $\dfrac{d\sigma}{dy}$ for Z-production symmetric around $y = 0$

- $W^{\pm}$ rapidity asymmetry sensitive to flavor decomposition of proton

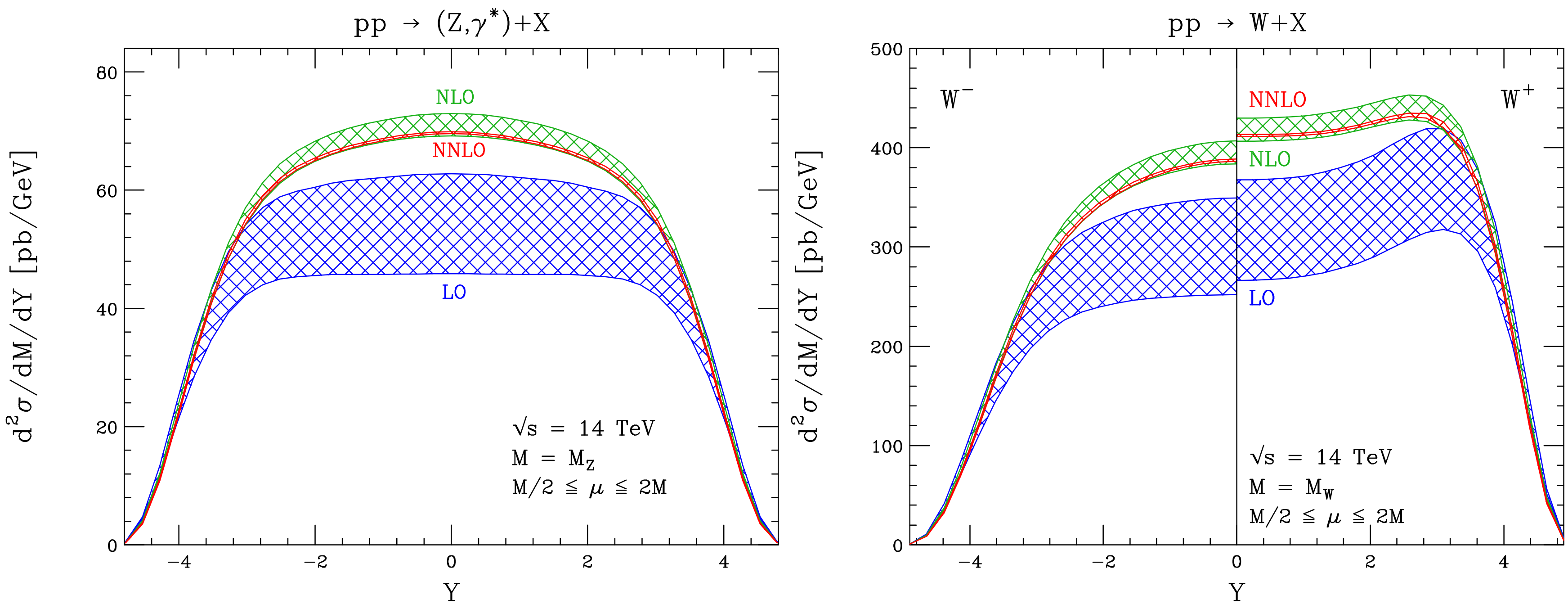
$$A_W(y) = \frac{d\sigma(W^+)/dy - d\sigma(W^-)/dy}{d\sigma(W^+)/dy + d\sigma(W^-)/dy}$$

$$\simeq \frac{u(x_1)d(x_2) - d(x_1)u(x_2)}{u(x_1)d(x_2) + d(x_1)u(x_2)}$$

Gauge boson production at NNLO

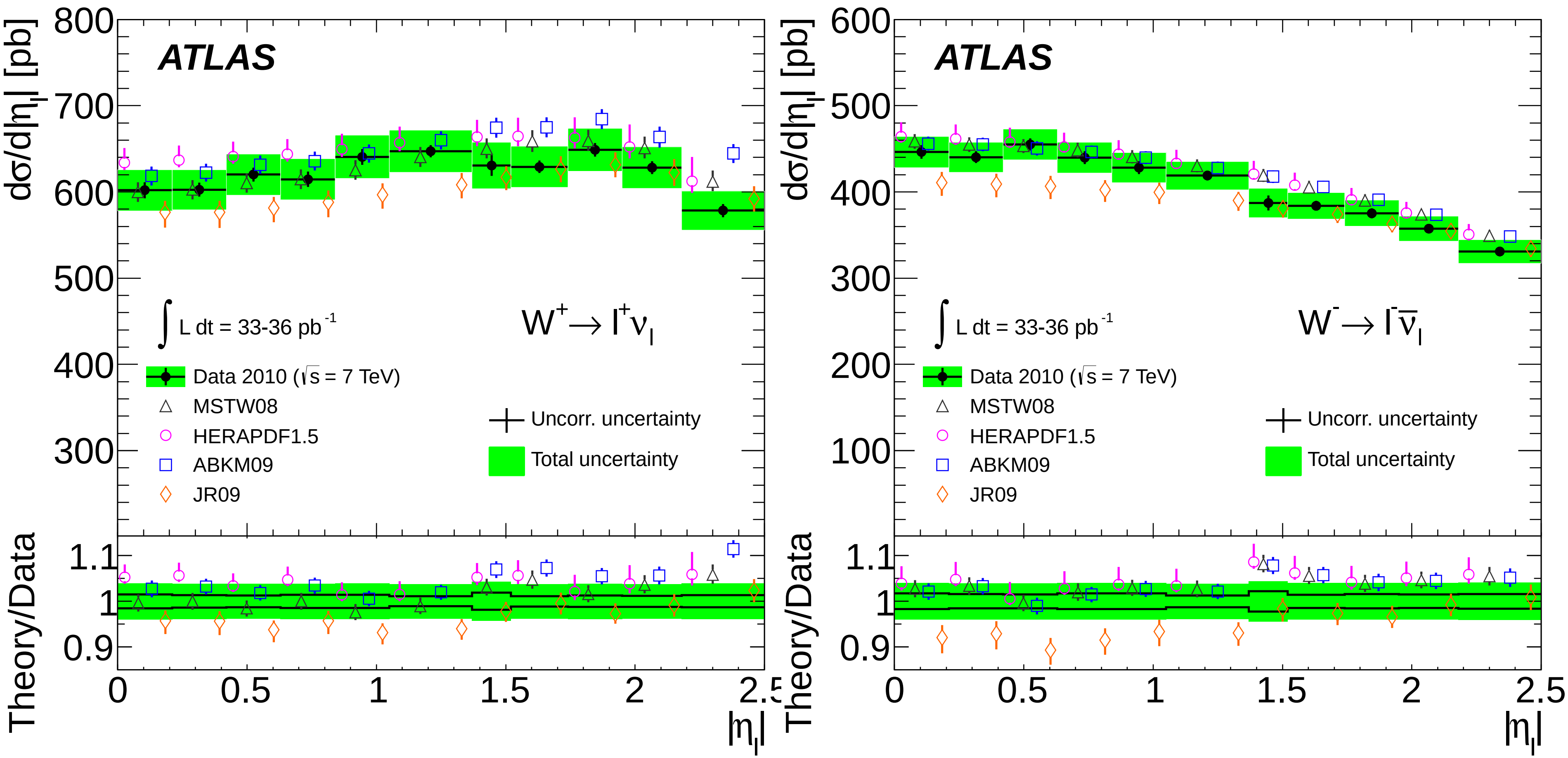
- $W^{\pm}, Z$-boson rapidity distribution (scale variation $\frac{m_{W,Z}}{2} \leq \mu \leq 2m_{W,Z}$)

  Anastasiou, Petriello, Melnikov '05



- NNLO QCD theoretical uncertainties (renormalization / factorization scale) at level of 1% Dissertori et al. '05
- "Standard candle" process for parton luminosity
  - large statistics even in early LHC data

LHC data (ATLAS) for $W^{\pm}$-boson production

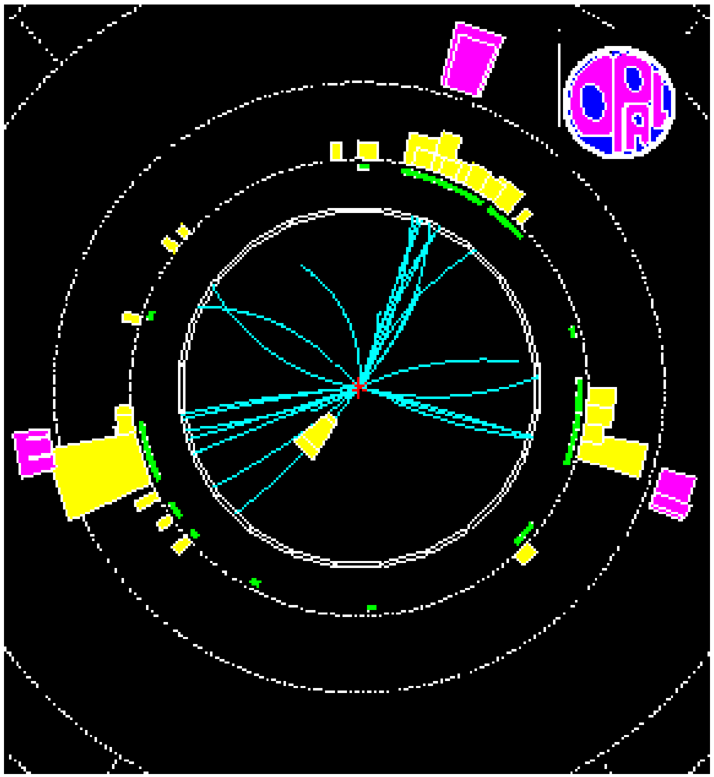


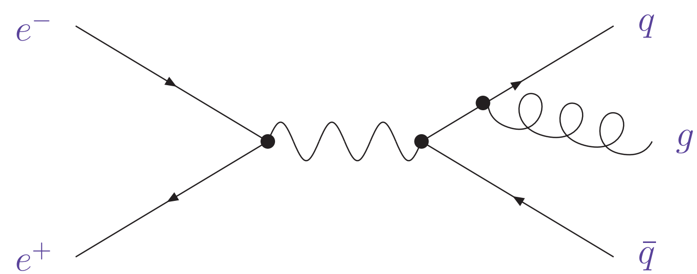
- LHC data for charged lepton rapidity distribution in $W^{\pm}$-boson productions and comparison of NNLO PDF sets
  - kinematic requirements: $p_T > 20$ GeV, $p_{T,\nu} > 25$ GeV and $m_T > 40$ GeV

Jets in QCD

Notion of a jet

- High energy event with collimated bunch of hadrons flying roughly in same direction is called a jet
(hundreds of hadrons; contains a lot of information)





- Jets related to underlying QCD dynamics (quarks and gluons)

Jet algorithms

- Reduce complexity of final state
  (combine many hadrons to simpler objects)
- Connects parton picture to experimental signature
  (precise and quantitative)
- Mapping of particle 4-momenta $\{p_i\}$ to set of jets $\{j_k\}$

$$\left\{ p_i \right\} \quad \longrightarrow \quad \left\{ j_k \right\}$$

Properties of jet definitions

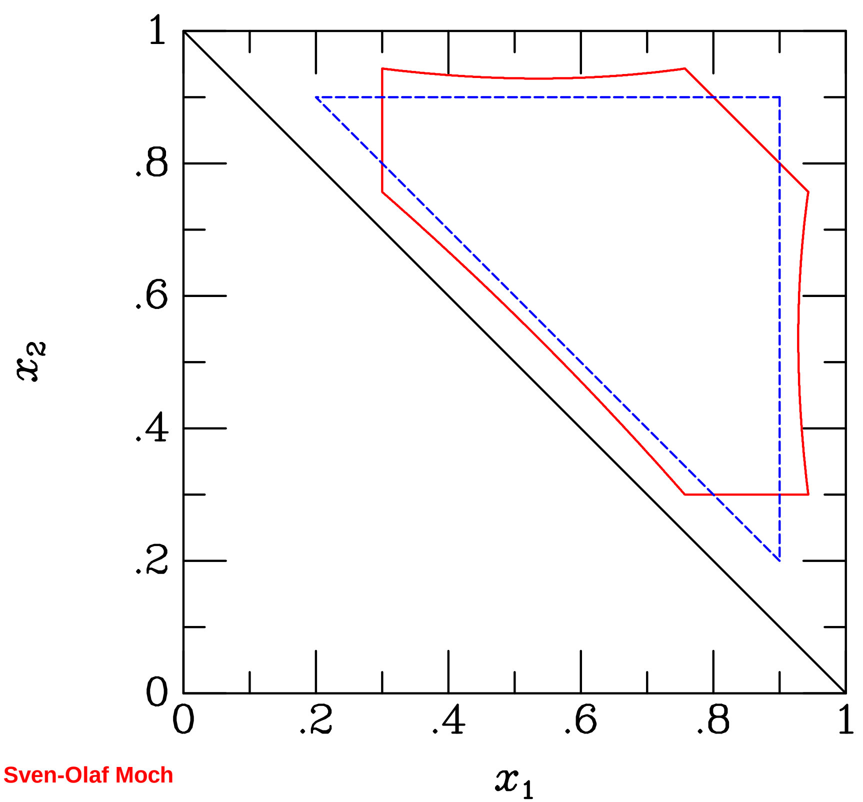
" Toward a standardization of jet definitions" FERMILAB-CONF-90-249-E

1. Simple to implement in an experimental analysis;
2. Simple to implement in a theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order in perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.

Historical definitions

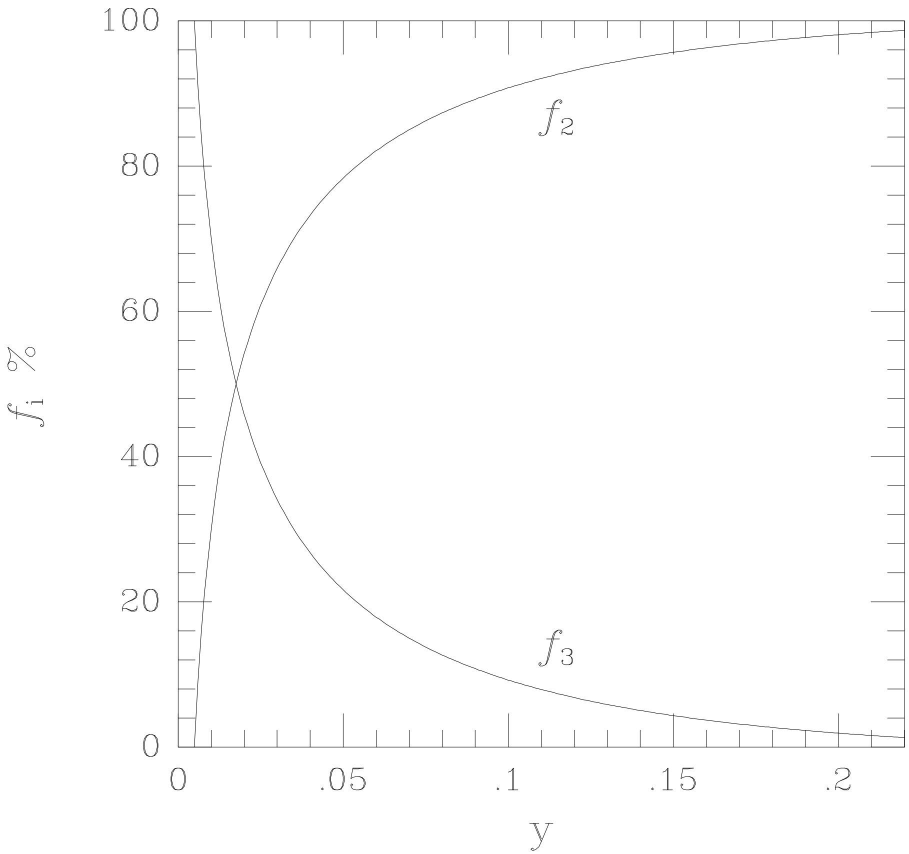
- Historically: Sterman-Weinberg criterium for two-jet event
  - energy fraction $1-\epsilon$ in cone of half angle δ
  - not practical for multi-particle events
- JADE algorithm: $\min\,(p_i+p_j)^2 = \min 2E_iE_j\,(1-\cos\theta_{ij}) > y_{\rm cut}\,s$
  - disadvantage: combines also soft gluons at large relative k_t
    e.g. potential three-jet event

Di-jet phase space in e^+e^- annihilation



- phase space boundaries for region with two and three jets
  - Sterman-Weinberg with $(\epsilon,\delta)=(0.3,30)$ (solid lines)
  - JADE algorithm with $y_{\rm cut}=0.1$ (dashed lines)

Jet rates in e^+e^- annihilation



- Ratio of rates
  $f_i = \frac{\sigma_{i-\text{jet}}}{\sigma}$
  for two and three jets
  - JADE algorithm with $y_{\text{cut}} \leq 0.3$

- Recall: three-jet cross section $\sigma^{e^+e^- \to 3\text{jets}}$

$$\frac{d^2\sigma^{e^+e^- \to 3\text{jets}}}{dx_1 dx_2} = \sigma^{(0)} \, 3 \sum_q e_q^2 \, C_F \, \frac{\alpha_s}{2\pi} \, \frac{x_1^2 + x_2^2}{(1-x_1)\,(1-x_2)}$$

Sudakov form factor

- Differential expression in scaled energies $x_1 = 2\frac{E_q}{\sqrt{s}}$ and $x_2 = 2\frac{E_{\bar{q}}}{\sqrt{s}}$

$$\frac{d^2\sigma^{e^+e^-\to 3\text{jets}}}{dx_1 dx_2} = \sigma^{e^+e^-\to 2\text{jets}} \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

- Transformation of variables to $x_3 = 2\frac{E_g}{\sqrt{s}}$ and $\cos\theta_{qg}$

$$\frac{d^2\sigma^{e^+e^-\to 3\text{jets}}}{d\cos\theta_{qg} dx_3} = \sigma^{e^+e^-\to 2\text{jets}} \frac{\alpha_s}{2\pi} C_F \left(\frac{2}{\sin^2\theta_{qg}} \frac{1+(1-x_3)^2}{x_3} - x_3 \right)$$

  - small angle approximation

$$\frac{2d\cos\theta_{qg}}{\sin^2\theta_{qg}} \simeq \frac{d\theta_{qg}^2}{\theta_{qg}^2} + \frac{d\theta_{\bar{q}g}^2}{\theta_{\bar{q}g}^2}$$

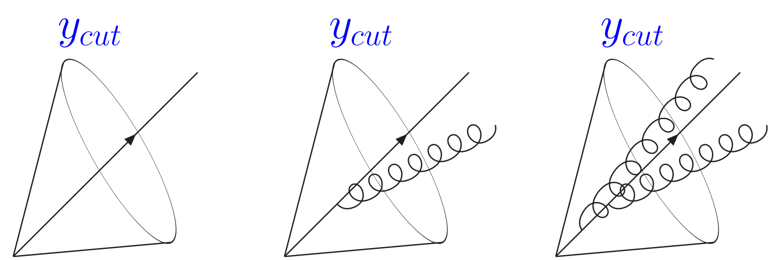
- Independent evolution of the two jets with splitting function

$$P(z) \equiv P_{qq}(1-z) = \frac{1+(1-z)^2}{z}$$

$$d\sigma^{e^+e^-\to 3\text{jets}} \simeq \sigma^{e^+e^-\to 2\text{jets}} \sum_j \frac{\alpha_s}{2\pi} C_F \frac{d\theta_{qg}^2}{\theta_{qg}^2} P(z)$$

Modern jet definitions

- Two main classes of jet algorithms
- Sequential recombination algorithms (bottom-up approach)
  - combine particles starting from closest ones
  - choose distance measure
  - iterate recombination until few objects left, call them jets
  - e.g. k_t-clustering algorithm: $2\min\left(E_i^2, E_j^2\right)\left(1-\cos\theta_{ij}\right) > y_{\rm cut}\, s$

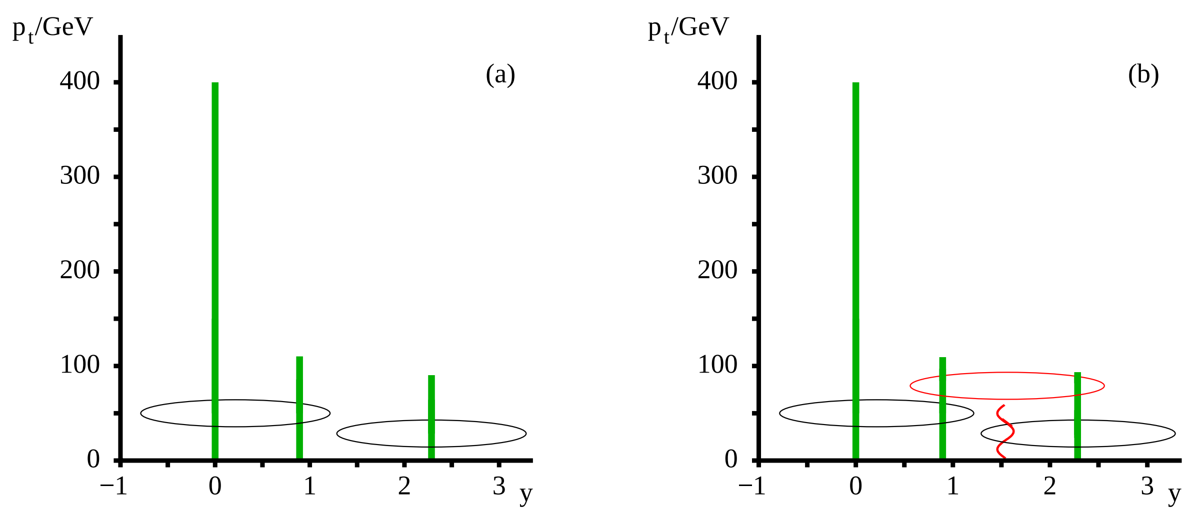


Jets in hadronic collisions

- Metric of η, ϕ
  - define cone of radius R in η, ϕ for $R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$

Cone algorithm

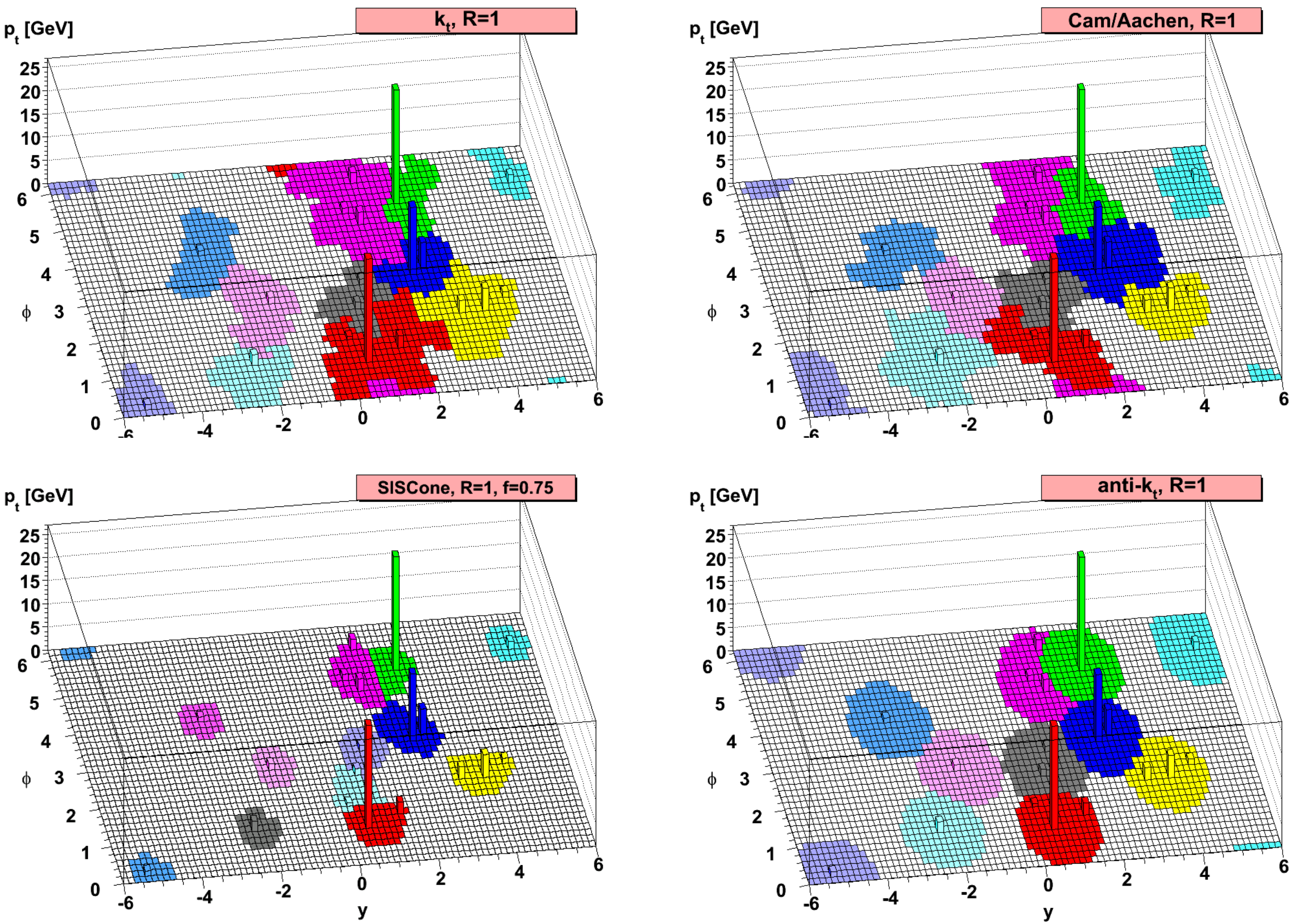
- Top-down approach: find coarse regions of energy flow
  - find stable cones
    (i.e. their axis coincides with sum of momenta of particles in it)
  - e.g. JetClu, MidPoint, ATLAS cone, CMS cone, ...
- Problem
  - infrared unsafe beyond NLO in QCD
  - e.g. midpoint cone-algorithm:
    soft seed gives rise to extra hard jet (fixed for Tevatron run II)



Cone algorithms (II)

- Clustering of parton-level event from `Herwig` and random soft radiation with different jets algorithms
  - k_t algorithm
  - Cambridge/Aachen
  - SISCone
  - anti-k_t algorithm
- Illustration of “active” catchment areas of resulting hard jets

Cone algorithms (II)



(Some) uses of hadronic di-jets

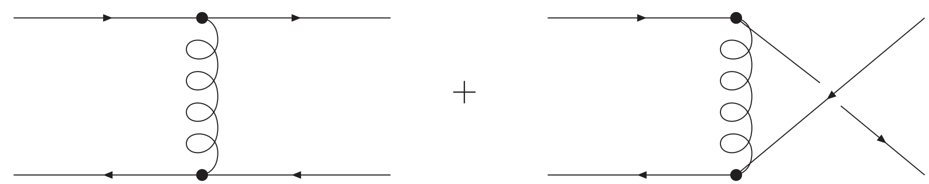
- Hadronic di-jets: large statistics even with high-p_t cuts
  - experimental calibration (HCAL uniformity, establish missing E_t)
  - gluon jets constrain gluon PDF at medium/large x
  - searches for quark sub-structure (di-jet angular correlations)

Hadronic di-jets

- Di-jet differential cross section for scattering
  $\text{parton}_i(k_1) + \text{parton}_j(k_2) \to \text{parton}_k(k_3) + \text{parton}_l(k_4)$

$$\frac{d^3\sigma}{dy_3 dy_4 dp_t^2} = \frac{1}{16\pi s^2} \sum_{i,j,k,l=q,\bar{q},g} \frac{f_i(x_1)}{x_1} \frac{f_j(x_2)}{x_2} \overline{\sum} \frac{1}{1+\delta_{kl}} |\mathcal{A}(ij \to kl)|^2$$

- Example: $\hat{\sigma}^{ud}$ with
  $\overline{\sum} |\mathcal{A}|^2 = (4\pi\alpha_s)^2 \frac{4}{9} \frac{s^2+u^2}{t^2}$



- Kinematics in di-jet cms
  - di-jet rapidity $y^* = \frac{y_3 - y_4}{2}$ determines cms scattering angle
    $\cos\theta^* = \frac{p_z^*}{E^*} = \frac{\sinh y^*}{\cosh y^*} = \tanh\left(\frac{y_3 - y_4}{2}\right)$
  - di-jet invariant mass M_{JJ}^2

$$dy_3 dy_4 dp_t^2 = \frac{1}{2} dx_1 dx_2 d\cos\theta^*$$

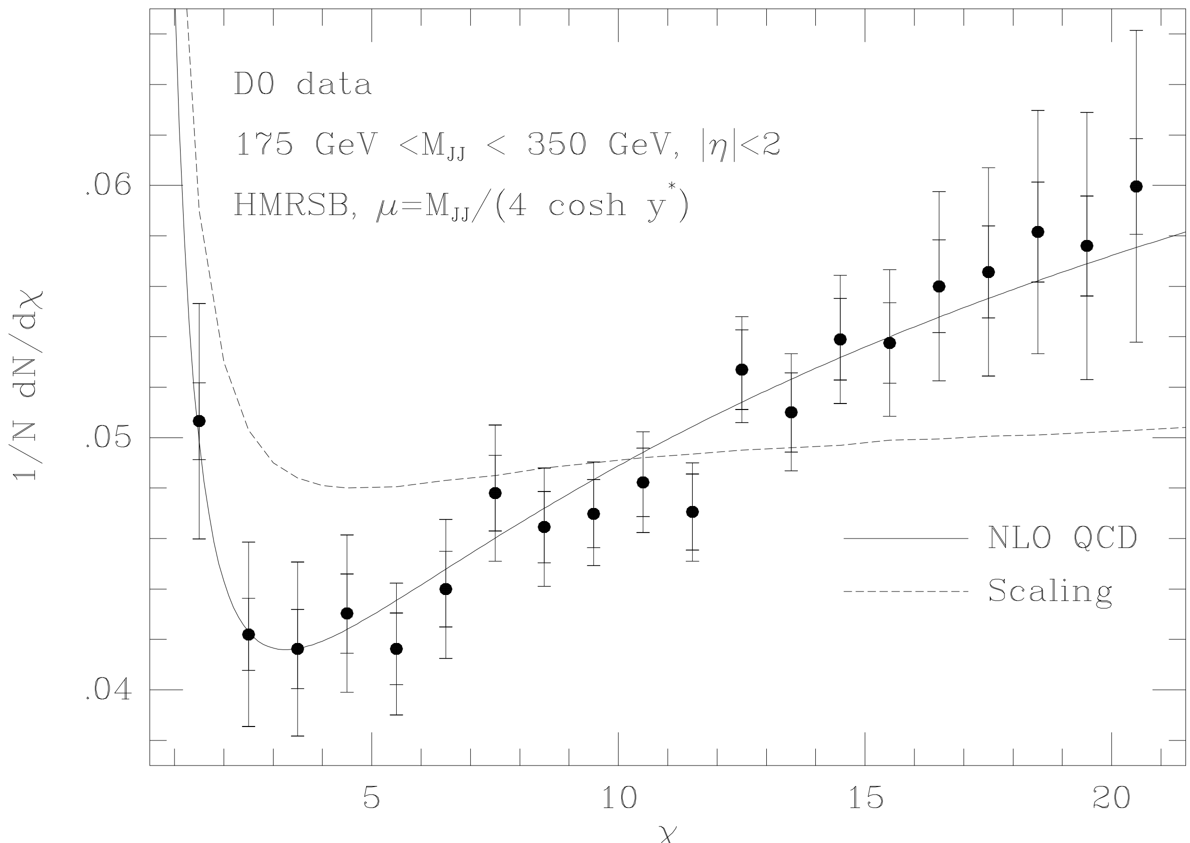
- Cross section σ^{ud} in di-jet cms kinematics

$$\frac{d\hat{\sigma}^{ud}}{d\cos\theta^*} = \frac{\pi\alpha_s^2}{2M_{JJ}^2}\frac{4}{9}\left[\frac{4+(1+\cos\theta^*)^2}{(1-\cos\theta^*)^2} + \frac{4+(1-\cos\theta^*)^2}{(1+\cos\theta^*)^2}\right]$$

- Small angles $\frac{d\hat{\sigma}^{ud}}{d\cos\theta^*} \sim \frac{1}{\sin^4(\theta^*/2)}$ (Rutherford)
  - transform to $\chi = \frac{1+\cos\theta^*}{1-\cos\theta^*}$
  - $\frac{d\hat{\sigma}^{ud}}{d\chi} \sim \mathrm{const}$
- Scalar colored particle (e.g. scalar gluon)
  - $\frac{d\hat{\sigma}^{ud}}{d\cos\theta^*} \sim \mathrm{const}$ transforms to $\frac{d\hat{\sigma}^{ud}}{d\chi} \sim \frac{1}{(1+\chi)^2}$

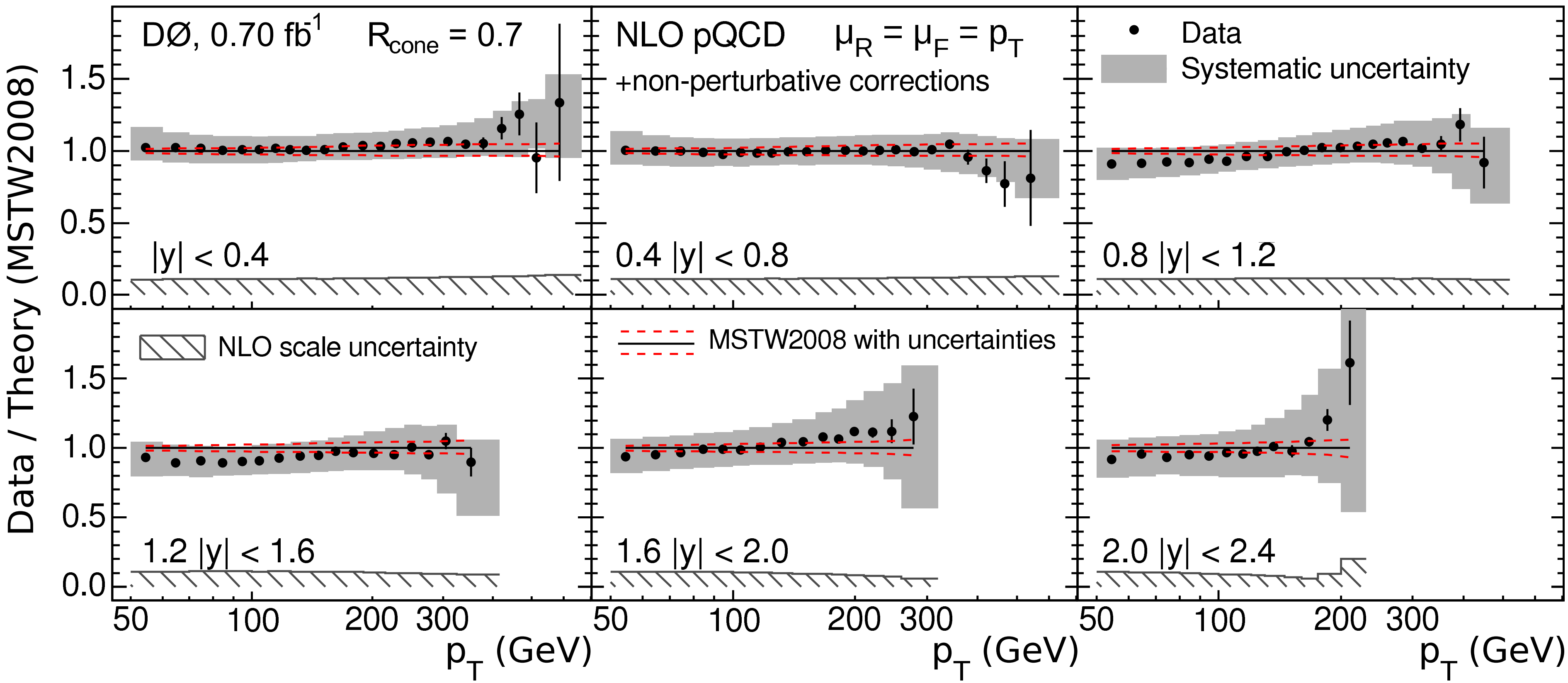
Quark substructure

- Searches for quark sub-structure in di-jet angular correlations



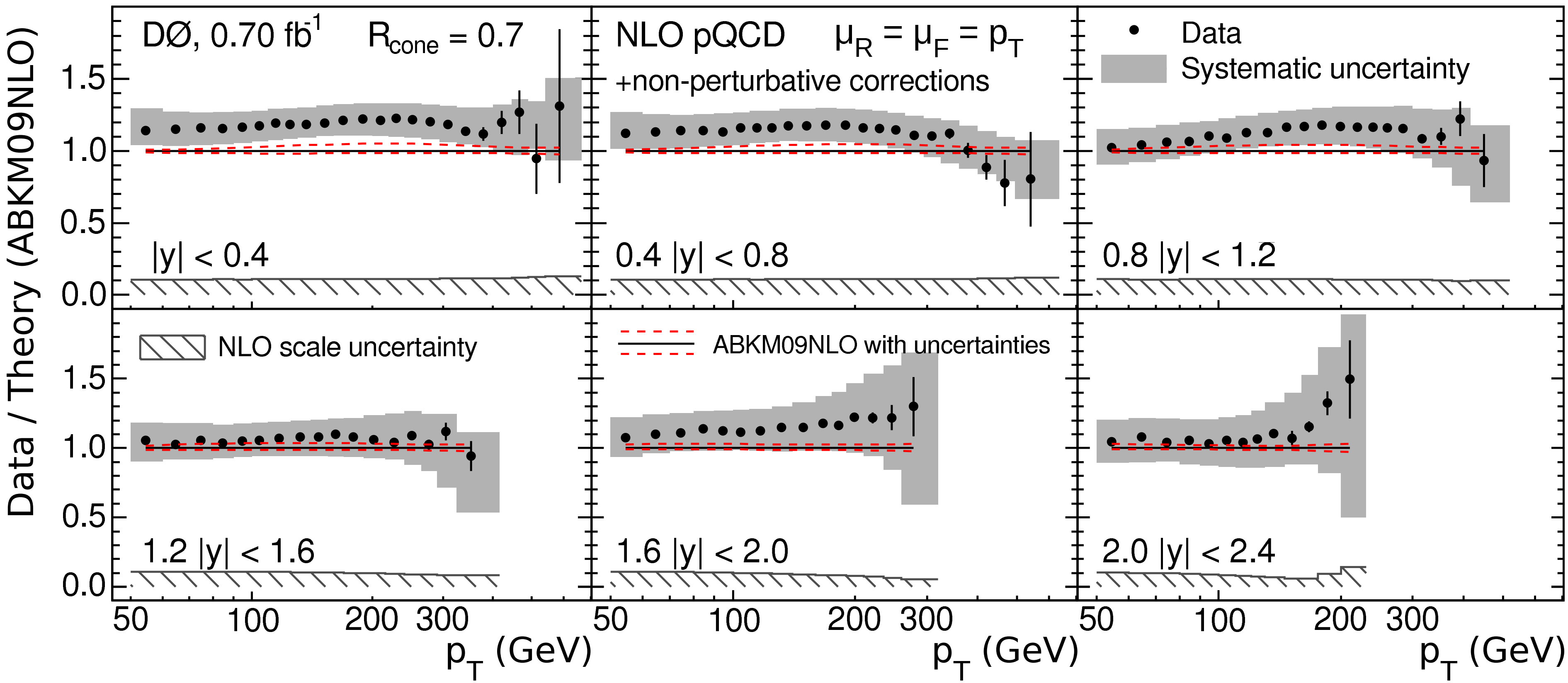
$$\frac{d^2\sigma}{dM_{JJ}^2 d\cos\theta^*} = \sum_{i,j=q,\bar{q},g} \int_0^1 f_i(x_1)\, f_j(x_2)\, \delta(x_1 x_2 s - M_{JJ}^2)\, \frac{d\hat{\sigma}^{ij}}{d\cos\theta^*}$$

Tevatron jets (D0) – 1-jet inclusive



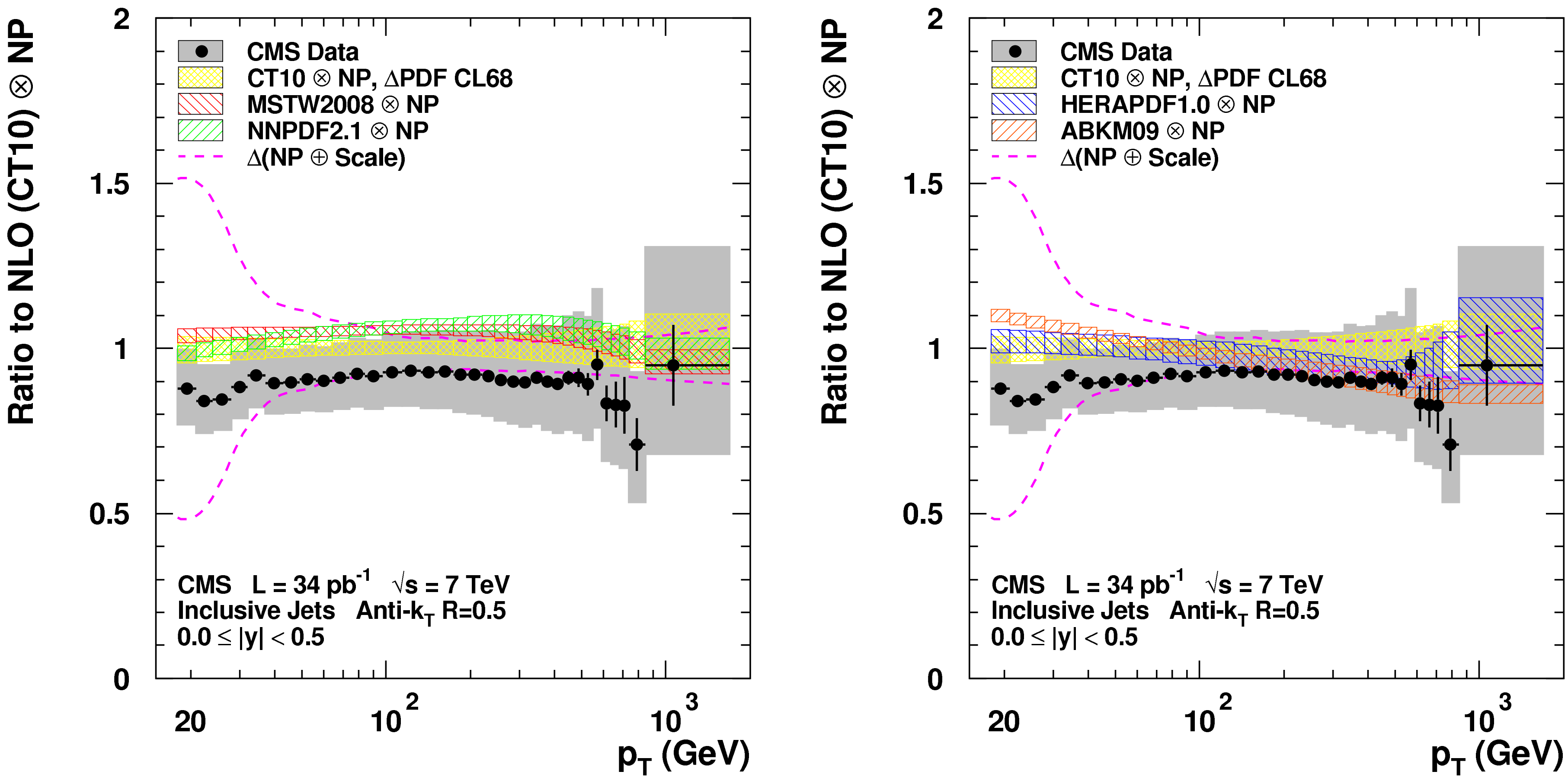
- New analysis of 1-jet inclusive data D0 coll. `arXiv:1110.3771`
  - MSTW PDF set with PDF (red) and theory (shaded) uncertainty

Tevatron jets (D0) – 1-jet inclusive



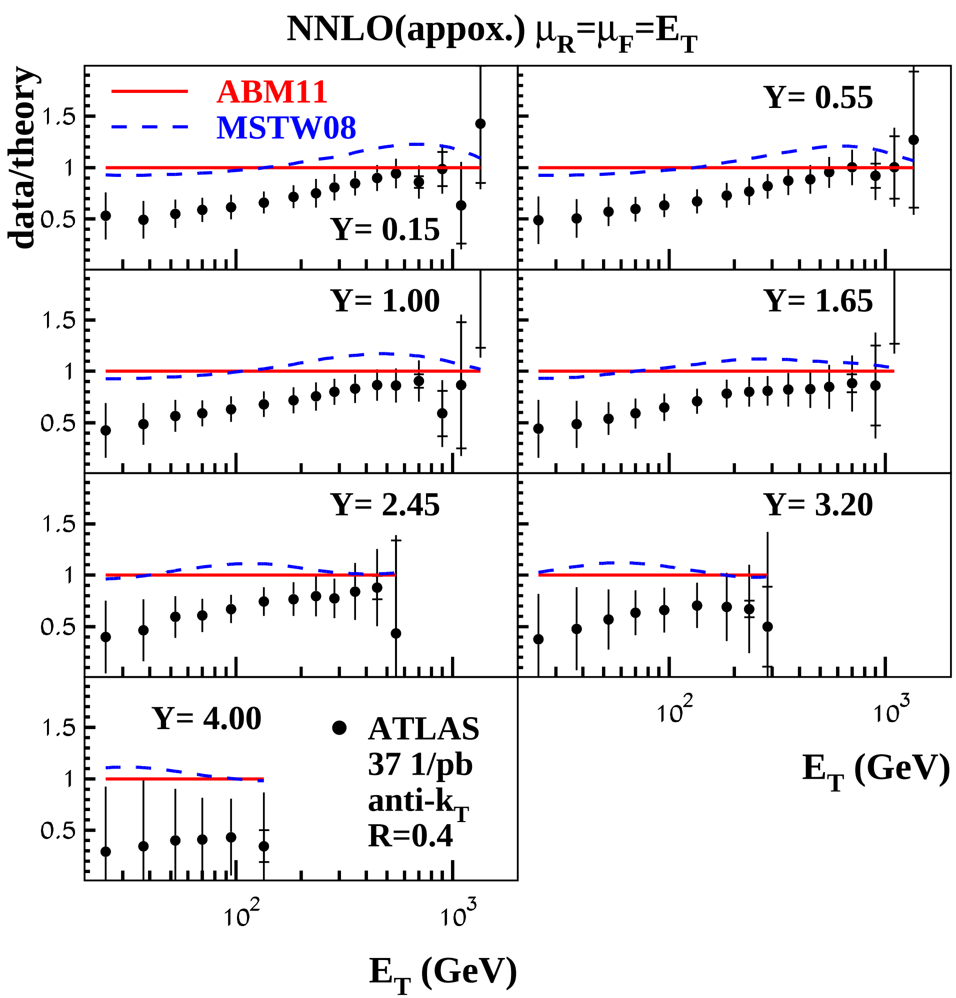
- New analysis of 1-jet inclusive data D0 coll. `arXiv:1110.3771`
  - ABKM PDF set with PDF (red) and theory (shaded) uncertainty

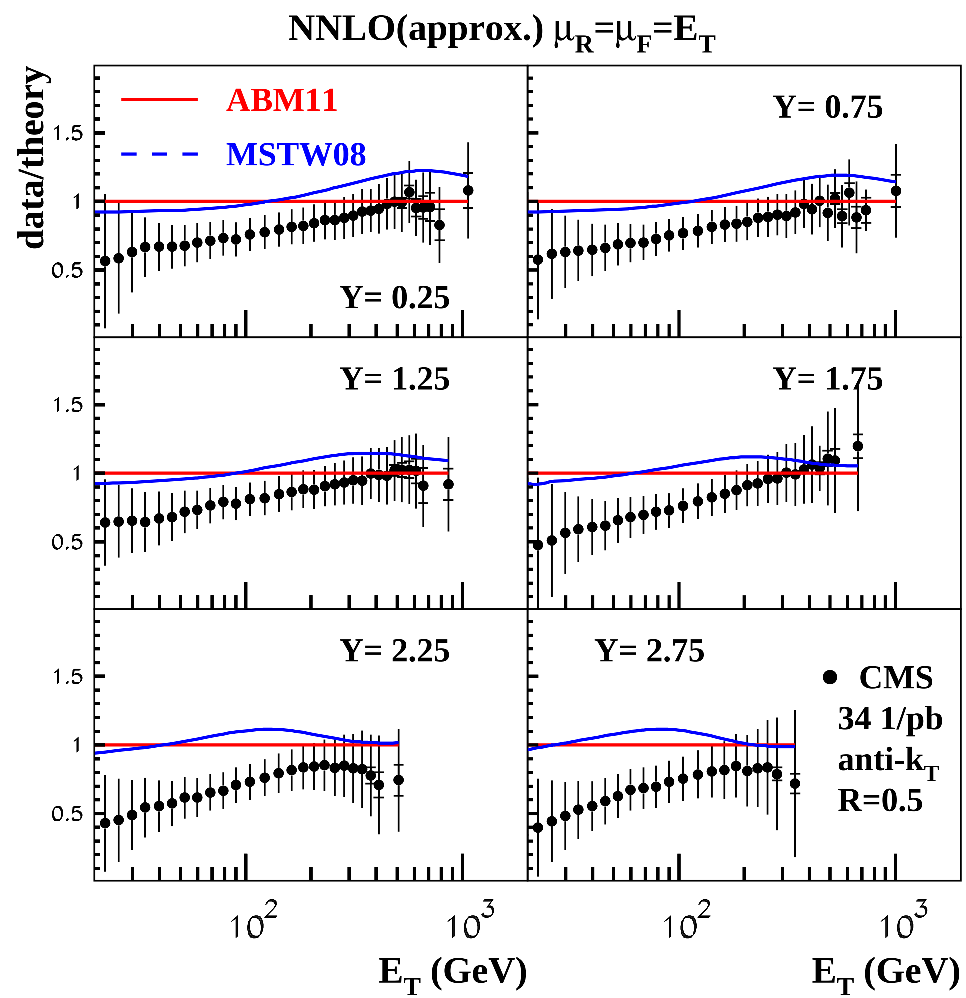
LHC jets (CMS) – 1-jet inclusive



- Analysis of 1-jet inclusive data CMS coll. `CMS NOTE 2011/004`
  - Comparisions of various PDF sets courtesy K. Rabbertz

LHC jet data





- Comparision to LHC data: ATLAS coll. (left) and CMS coll. (right) in good agreement
- LHC jet data prefers small gluon PDF at large x

Summary (part IV)

$W^\pm/Z$-boson production

- $W^\pm/Z$-boson production at hadron colliders
- NLO QCD corrections
  - illustration of factorization, infrared safety and evolution for $q\bar{q} \to V$
- $W^\pm/Z$-boson production at LHC to constrain PDFs
  - flavor PDFs from $W^\pm$ rapidity asymmetry

Jets

- Jet algortihms
  - infrared saftey to all orders crucial
- Jets at the LHC
  - searches for new physics at high E_T
  - constrains on gluon PDF and $\alpha_s(M_Z)$