

Quantum Chromodynamics

lecture III

Sven-Olaf Moch

Universität Hamburg & DESY, Zeuthen

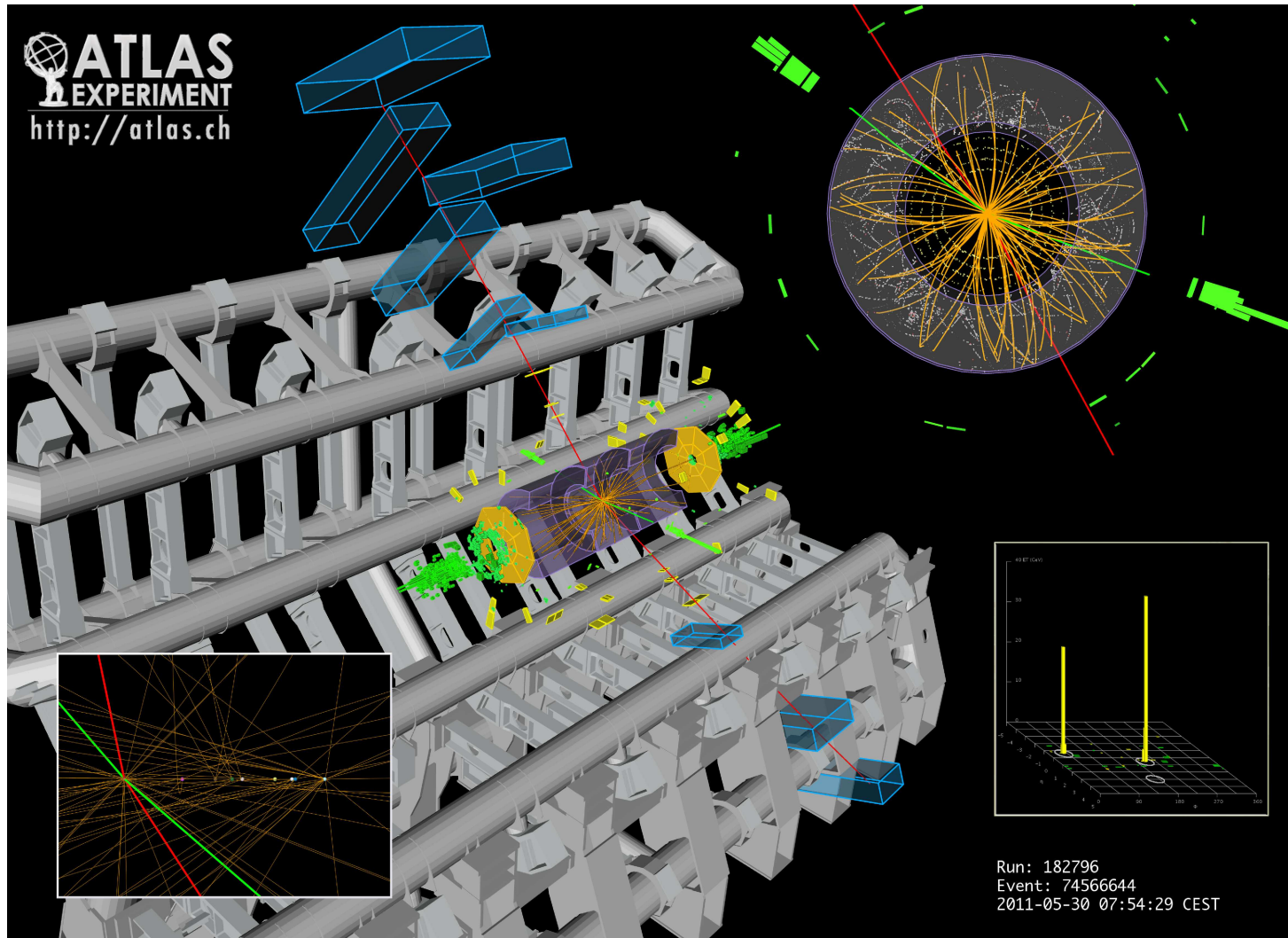
Belgian Dutch German summer school (BND 2012), Bonn, Sep 23, 2012

Plan

- Introduction to QCD
Friday, September 21, 2012
- QCD at work: infrared safety, factorization and evolution
Saturday, September 22, 2012
- *Higgs boson production*
Sunday, September 23, 2012
- Gauge boson production and QCD jets
Monday, September 24, 2012
- Top quark production
Tuesday, September 25, 2012

Hunt for the Higgs

- Higgs candidate event ($2e2\mu$ final state) in LHC run at $\sqrt{s} = 7$ TeV



Challenges

- Solve master equation

new physics = data – Standard Model

- LHC explores the energy frontier
 - searches require understanding of SM background
 - theory has to match or exceed accuracy of LHC data

Challenges

- Solve master equation

new physics = data – Standard Model

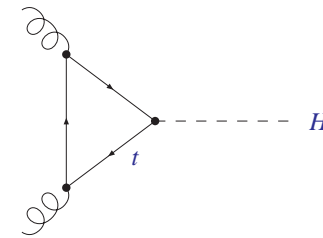
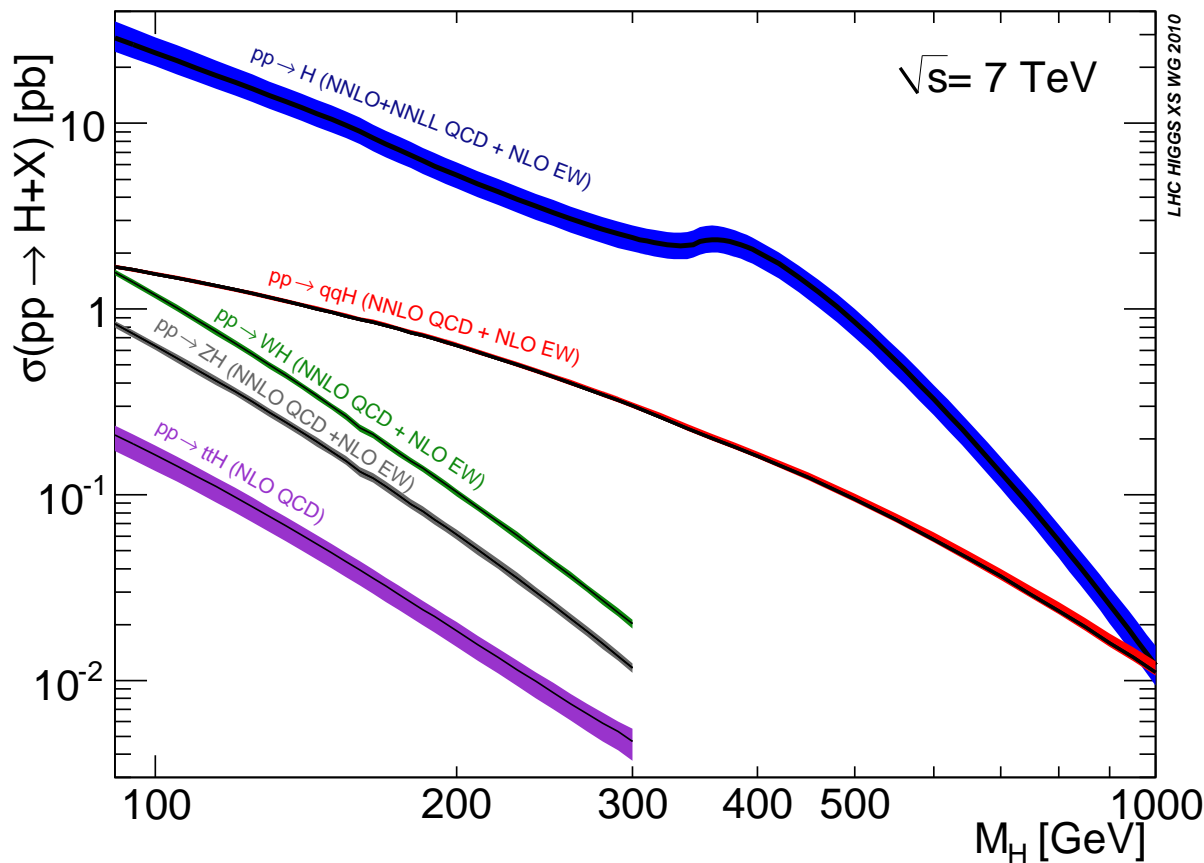
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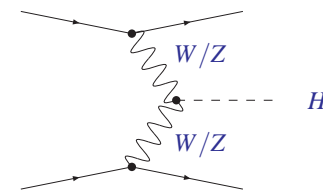
Higgs cross section

Cross section for Higgs production at the LHC

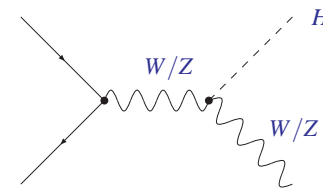
- Dominant channels for Higgs boson production **LHC Higgs XS WG '10**



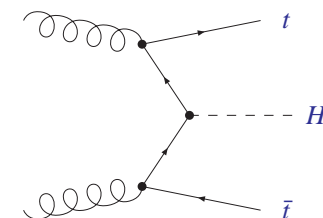
gg-fusion



weak boson fusio

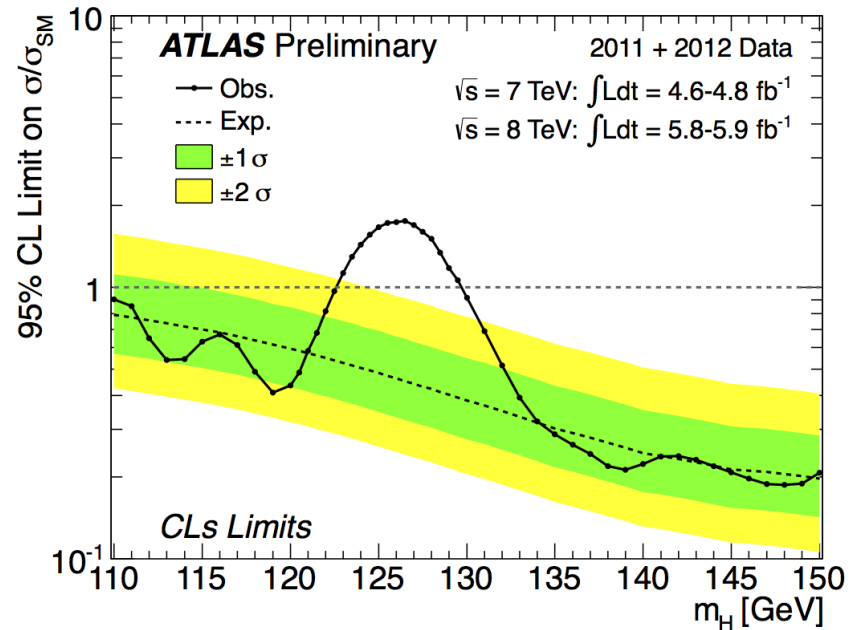
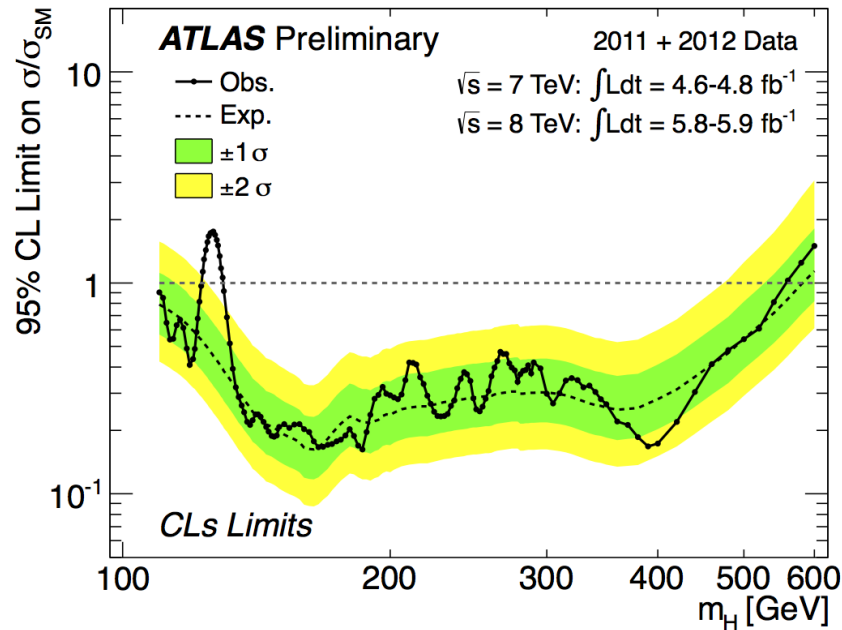


Higgs strahlung



t \bar{t} *H*-channel

Higgs discovery at LHC

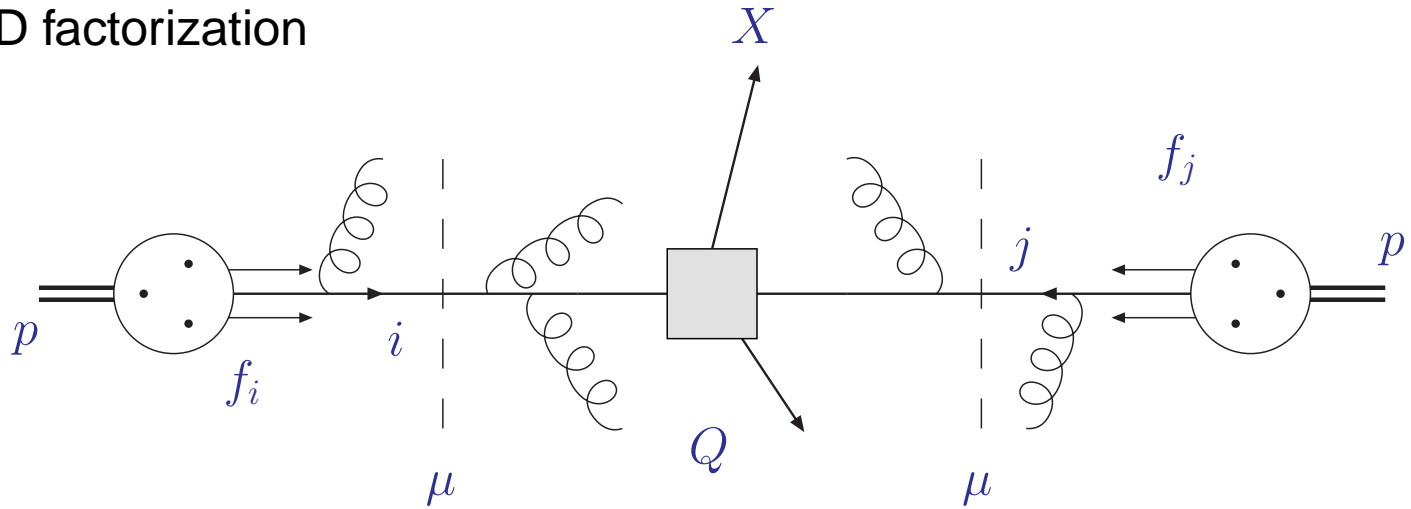


Atlas coll. July 2012

- Higgs mass in the range $m_H = 125 \text{ GeV}$
 - Higgs search driven predominantly by $gg \rightarrow H$
 - current range of excluded Higgs masses at Tevatron optimistic and consequences for LHC interesting

QCD factorization

- QCD factorization

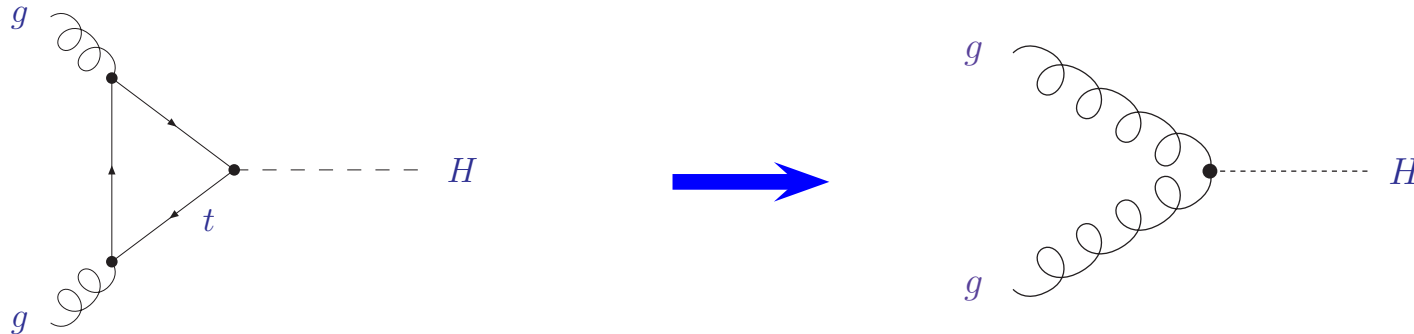


$$\sigma_{pp \rightarrow X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \rightarrow X}(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2)$$

- Hard parton cross section $\hat{\sigma}_{ij \rightarrow X}$ calculable in perturbation theory
 - known to NLO, NNLO, ... ($\mathcal{O}(\text{few}\%)$ theory uncertainty)
- Non-perturbative parameters: parton distribution functions f_i , strong coupling α_s , particle masses m_X
 - known from global fits to exp. data, lattice computations, ...

gg-fusion

Effective theory

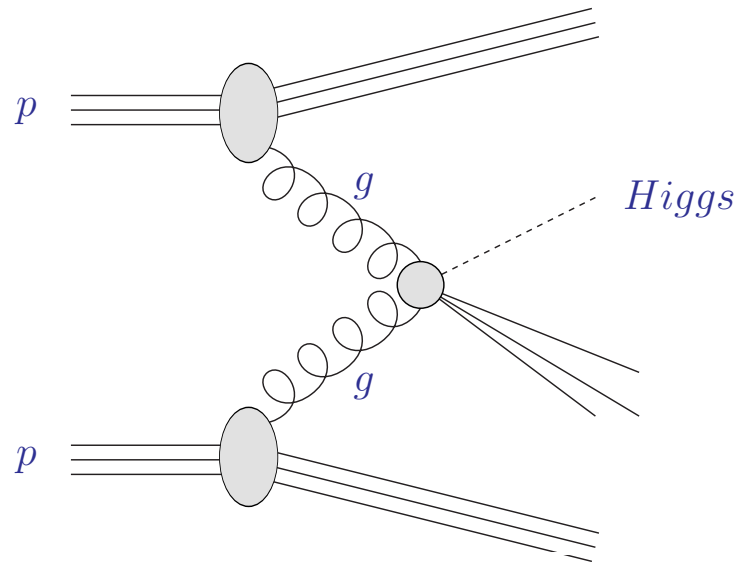


- Integration of top-quark loop (finite result)
 - decay width $H \rightarrow gg$ ($m_q = 0$ for light quarks, m_t heavy)

$$\Gamma_{H \rightarrow gg} = \frac{G_\mu m_H^3}{64 \sqrt{2} \pi^3} \alpha_s^2 f\left(\frac{m_H^2}{4m_t^2}\right)$$

- Effective theory in limit $m_t \rightarrow \infty$; Lagrangian $\mathcal{L} = -\frac{1}{4} \frac{H}{v} C_H G^{\mu\nu a} G_{\mu\nu}^a$
 - operator $H G^{\mu\nu a} G_{\mu\nu}^a$ relates to stress-energy tensor
 - additional renormalization proportional to QCD β -function required
Kluberg-Stern, Zuber '75; Collins, Duncan, Joglekar '77

QCD corrections to ggF



- Hadronic cross section $\sigma_{pp \rightarrow H}$ with $\tau = m_H^2/S$
 - renormalization/factorization (hard) scale $\mu = \mathcal{O}(m_H)$

$$\sigma_{pp \rightarrow H} = \sum_{ij} \int_{\tau}^1 \frac{dx_1}{x_1} \int_{x_1}^1 \frac{dx_2}{x_2} f_i \left(\frac{x_1}{x_2}, \mu^2 \right) f_j (x_2, \mu^2) \hat{\sigma}_{ij \rightarrow H} \left(\frac{\tau}{x_1}, \frac{\mu^2}{m_H^2}, \alpha_s(\mu^2) \right)$$

- Partonic cross section $\hat{\sigma}_{ij \rightarrow H}$

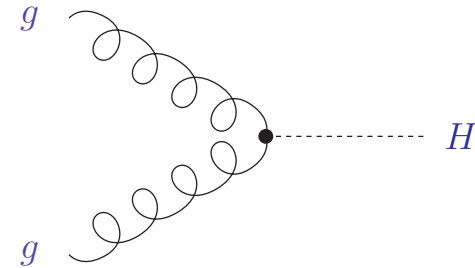
$$\hat{\sigma}_{ij \rightarrow H} = \underbrace{\alpha_s^2 \left[\hat{\sigma}_{ij \rightarrow H}^{(0)} + \alpha_s \hat{\sigma}_{ij \rightarrow H}^{(1)} \right]}_{\text{NLO: standard approximation (large uncertainties)}} + \alpha_s^2 \hat{\sigma}_{ij \rightarrow H}^{(2)} + \dots$$

NLO: standard approximation (large uncertainties)

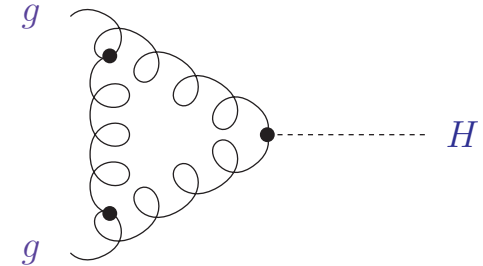
Radiative corrections in a nutshell

- Leading order
 - partonic cross section $x = \tau/x_1$

$$\hat{\sigma}_{gg \rightarrow H}^{(0)} = \delta(1-x)$$



- Next-to-leading order
 - virtual correction (time-like kinematics) (infrared divergent; proportional to Born)
 - dimensional regularization $D = 4 - 2\epsilon$

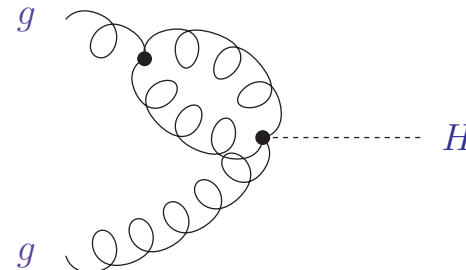


$$\hat{\sigma}_{gg \rightarrow H}^{(1),v} = C_A \frac{\alpha_s}{4\pi} \delta(1-x) \left(\frac{\mu^2}{m_H^2} \right)^\epsilon \left(-\frac{2}{\epsilon^2} + 7\zeta_2 + \mathcal{O}(\epsilon) \right)$$

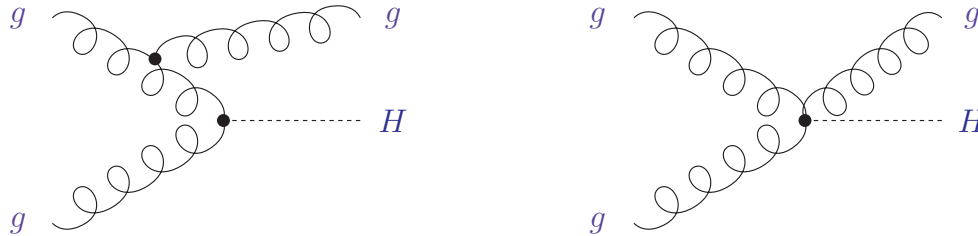
- additional contribution from renormalization of effective operator

$$\alpha_s^{\text{bare}} = \alpha_s^{\text{ren}} \left\{ 1 - \frac{\beta_0}{\epsilon} \frac{\alpha_s^{\text{ren}}}{4\pi} + \mathcal{O}(\alpha_s^2) \right\}$$

- massless tadpoles vanish in dimensional regularization



- Next-to-leading order



- add real and virtual corrections $\hat{\sigma}_{gg \rightarrow H}^{(1)} = \hat{\sigma}_{gg \rightarrow H}^{(1),r} + \hat{\sigma}_{gg \rightarrow H}^{(1),v}$
- collinear divergence remains **splitting functions** $P_{gg}^{(0)}$

$$\hat{\sigma}_{gg \rightarrow H}^{(1)} = \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{m_H^2} \right)^\epsilon \left\{ \begin{aligned} & \frac{1}{\epsilon} C_A \left(\frac{8}{1-x} + \frac{8}{x} - 8(2-x+x^2) + \frac{22}{3} \delta(1-x) \right) - \frac{1}{\epsilon} n_f \frac{4}{3} \delta(1-x) \\ & + C_A \left(16 \frac{\ln(1-x)}{1-x} + \left(\frac{22}{3} + 8\zeta_2 \right) \delta(1-x) - 16x(2-x+x^2) \ln(1-x) \right. \\ & \quad \left. - 8 \frac{(1-x+x^2)^2}{1-x} \ln(x) - \frac{22}{3} (1-x)^3 \right) \\ & \left. + \mathcal{O}(\epsilon) \right\} \end{aligned} \right.$$

- Structure of NLO correction
 - absorb collinear divergence $P_{gg}^{(0)}$ in renormalized parton distributions

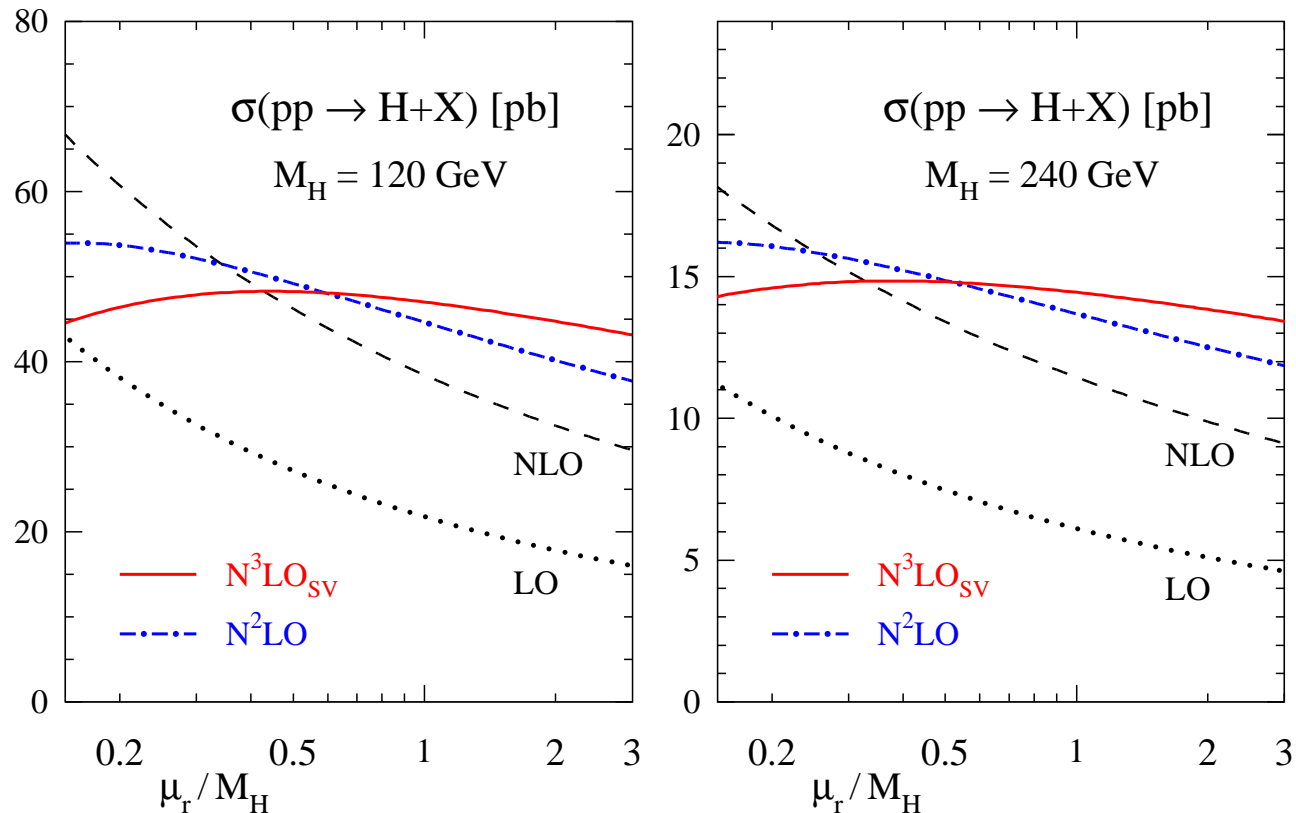
$$\hat{\sigma}_{gg \rightarrow H}^{(1), \text{bare}} = \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{m_H^2} \right)^\epsilon \left\{ \frac{1}{\epsilon} 2 P_{gg}^{(0)}(x) + \hat{\sigma}_{gg \rightarrow H}^{(1)}(x) + \mathcal{O}(\epsilon) \right\}$$

$$g^{\text{ren}}(\mu_F^2) = g^{\text{bare}} - \frac{\alpha_s}{4\pi} \frac{1}{\epsilon} P_{gg}^{(0)}(x) \left(\frac{\mu^2}{\mu_F^2} \right)^\epsilon$$

- partonic (physical) structure function at factorization scale μ_F

$$\hat{\sigma}_{gg \rightarrow H} = \delta(1-x) + \frac{\alpha_s}{4\pi} \left\{ \hat{\sigma}_{gg \rightarrow H}^{(1)}(x) - \ln \left(\frac{m_H^2}{\mu_F^2} \right) 2 P_{gg}^{(0)}(x) \right\}$$

Inclusive cross section



- Apparent convergence of perturbative expansion
 - NNLO corrections still large
Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03
 - improvement through complete soft N^3LO corrections S.M., Vogt '05
or NNLL resummation Catani, de Florian, Grazzini, Nason '03, Ahrens et al. '10
- Perturbative stability under renormalization scale variation

Resummation

- Higgs cross section $\hat{\sigma}_{gg \rightarrow H}$ with $x = \frac{M_H^2}{s}$ ($x \simeq 1$ close to threshold)

$$\alpha_s^n \left(\frac{\ln^{2n-1}(1-x)}{1-x} \right)_+ \longleftrightarrow \alpha_s^n \ln^{2n}(N)$$

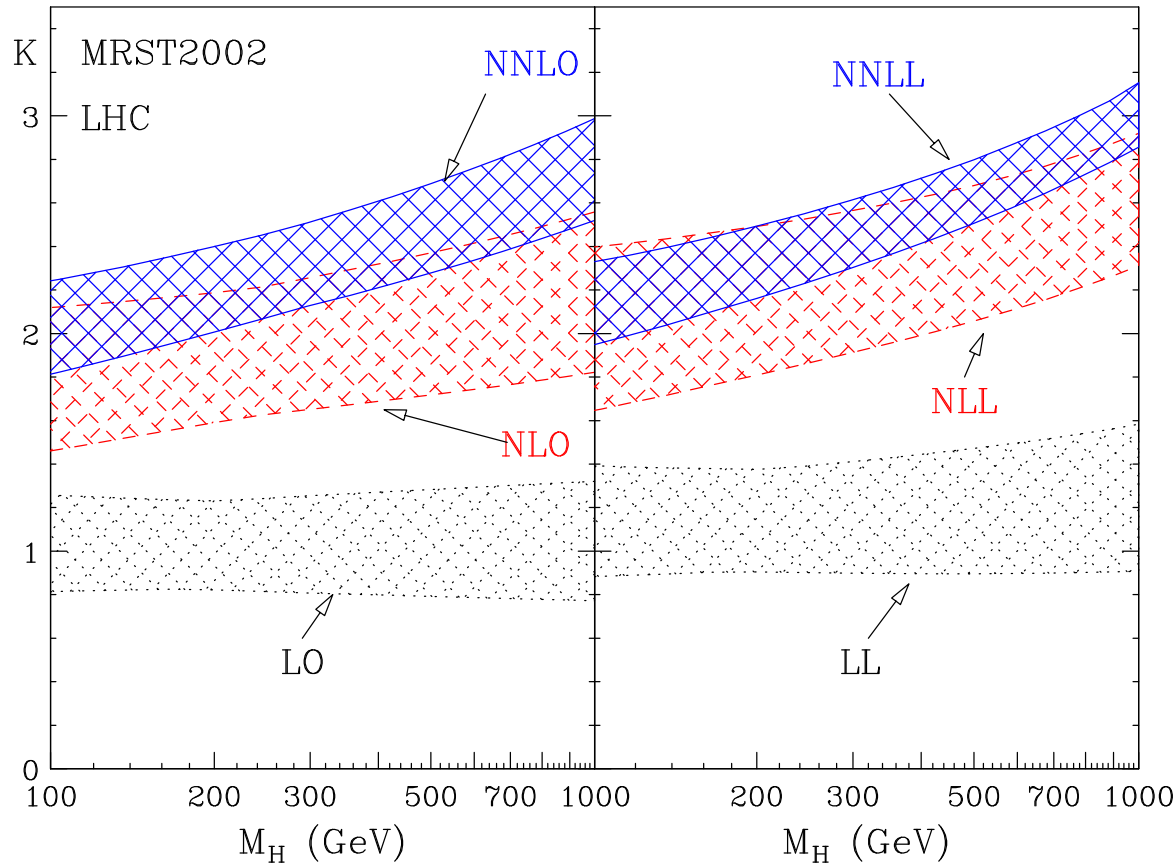
- Recall large double logarithms at NLO

$$\hat{\sigma}_{gg \rightarrow H}^{(1)} = \frac{\alpha_s}{4\pi} \left(\frac{\mu^2}{m_H^2} \right)^\epsilon \left\{ \begin{aligned} & \frac{1}{\epsilon} C_A \left(\frac{8}{1-x} + \frac{8}{x} - 8(2-x+x^2) + \frac{22}{3} \delta(1-x) \right) - \frac{1}{\epsilon} n_f \frac{4}{3} \delta(1-x) \\ & + C_A \left(16 \frac{\ln(1-x)}{1-x} + \left(\frac{22}{3} + 8\zeta_2 \right) \delta(1-x) - 16x(2-x+x^2) \ln(1-x) \right. \\ & \left. - 8 \frac{(1-x+x^2)^2}{1-x} \ln(x) - \frac{22}{3} (1-x)^3 \right) + \mathcal{O}(\epsilon) \end{aligned} \right\}$$

- Threshold resummation to all orders with standard technology

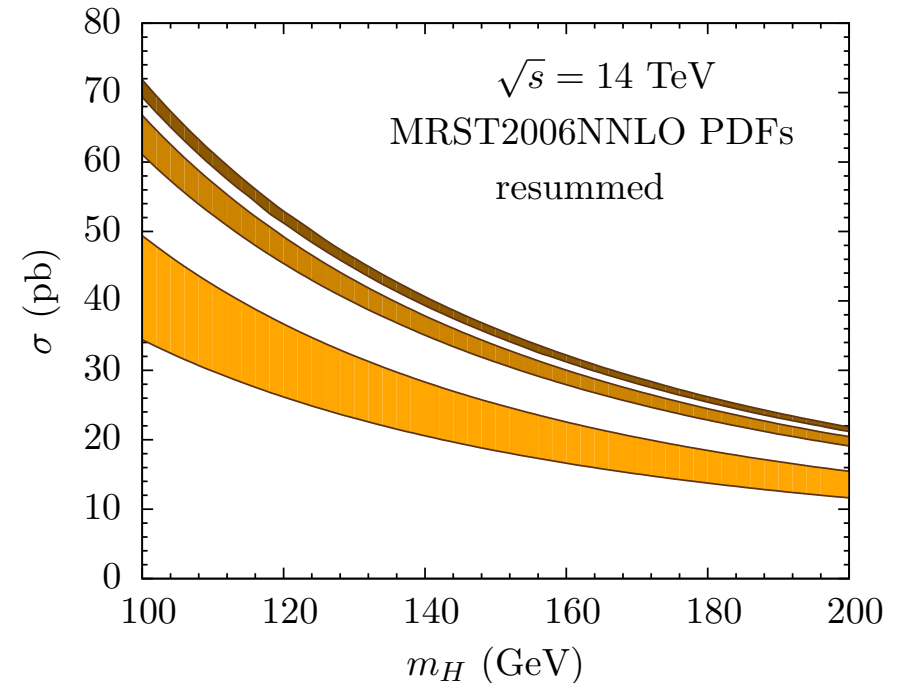
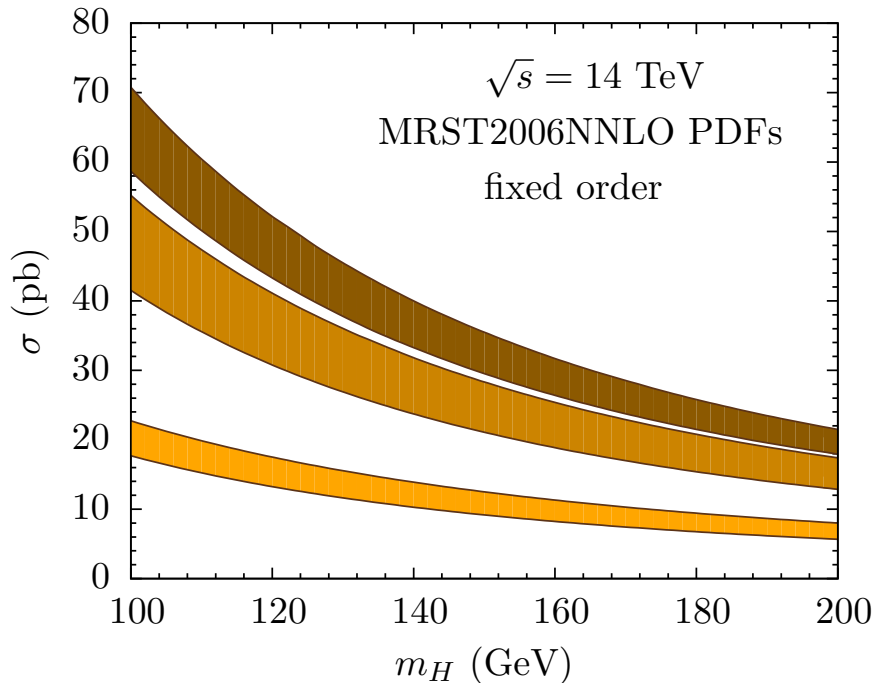
$$\hat{\sigma}_{gg \rightarrow H}^N = (1 + \alpha_s g_{01} + \alpha_s^2 g_{02} + \dots) \cdot \exp(G^N) + \mathcal{O}(N^{-1} \ln^n N)$$

Total Higgs cross section and resummation



- Cross section at LHC with scale variation:
fixed order predictions (left) and resummed perturbation series (right)
 - **NNLO** corrections
Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03
 - **NNLL** resummation
Catani, Grazzini, de Florian, Nason '03, Ahrens et al. '10

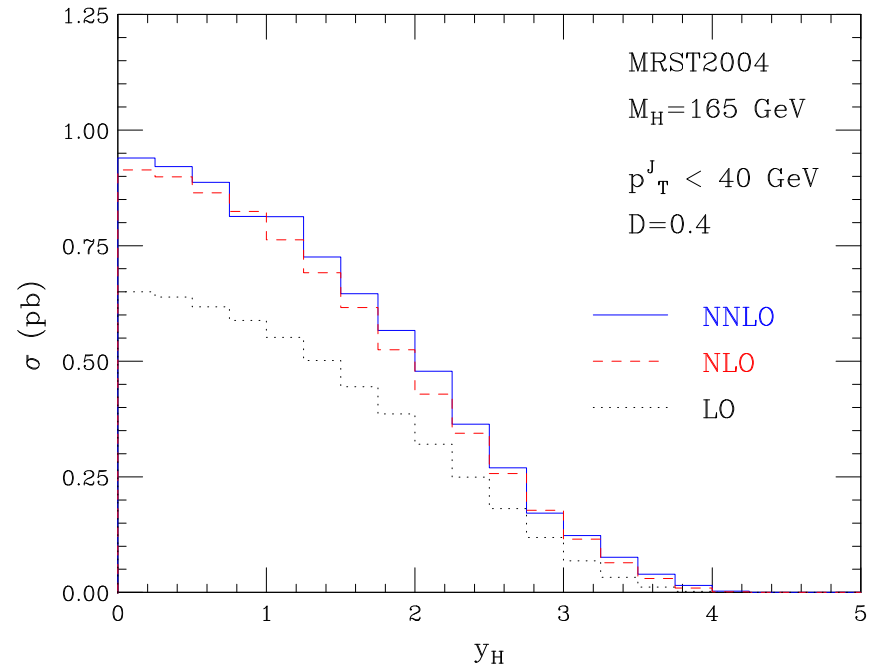
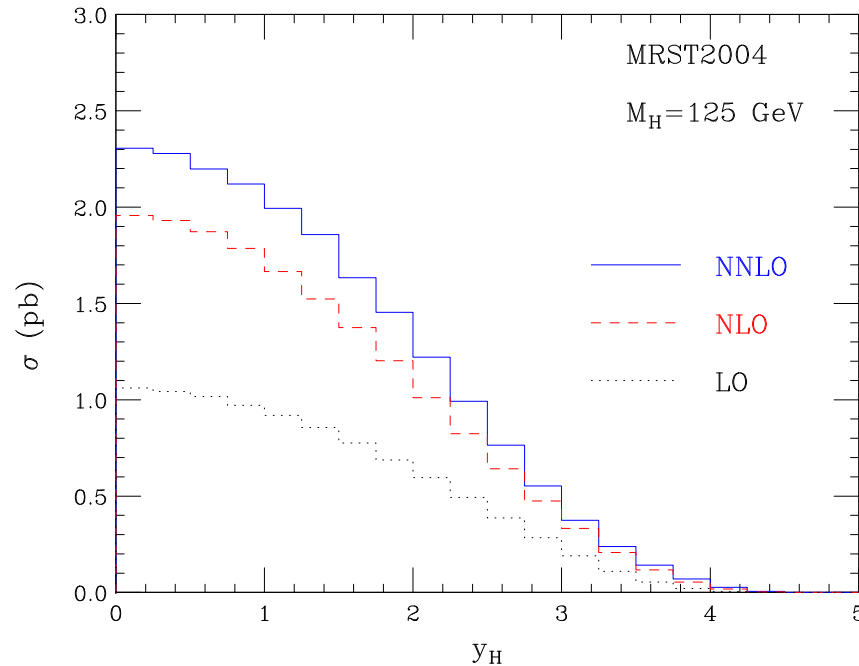
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 - **NNLL** resummation (lots of activity in the last years)
Catani, Grazzini, de Florian, Nason '03, Ahrens et al. '10

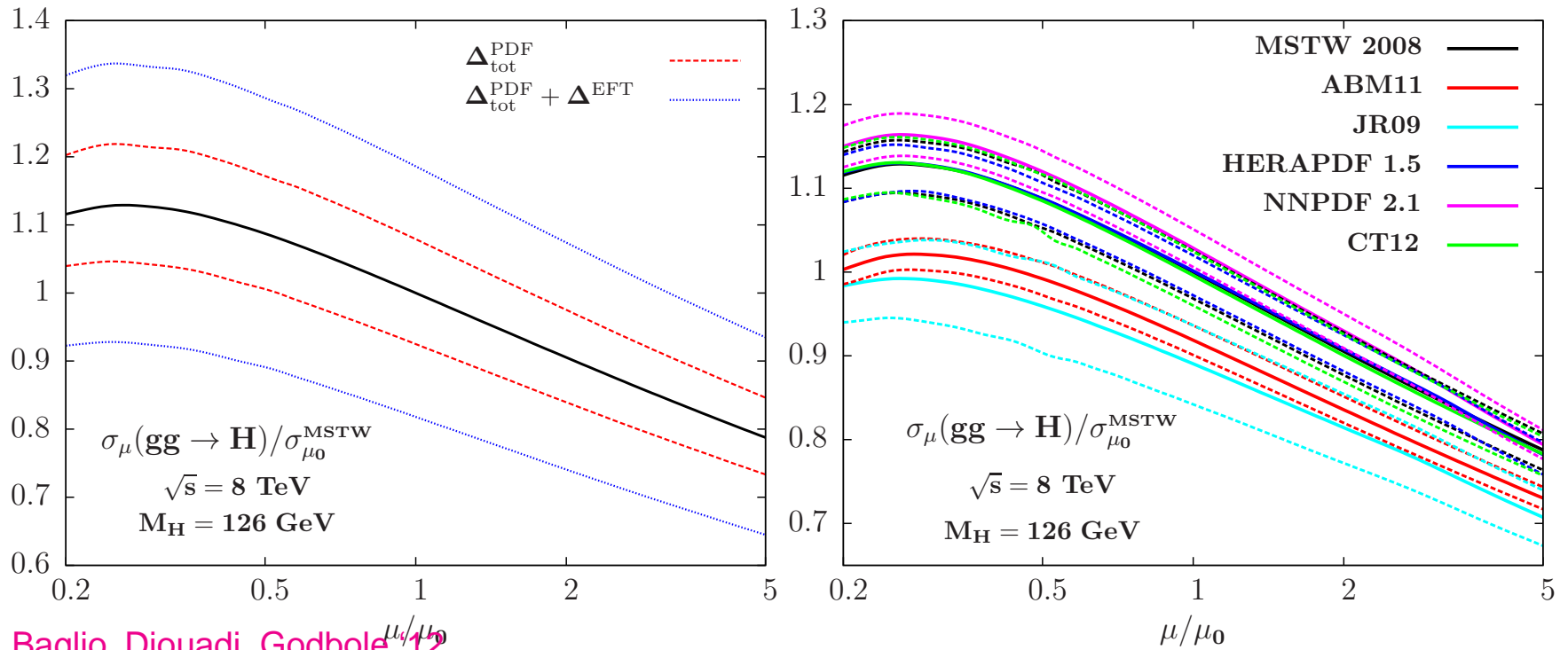
gg fusion (fully exclusive)

- Bin-integrated Higgs rapidity distribution including decay $H \rightarrow \gamma\gamma$
 - QCD corrections up to NNLO Anastasiou, Melnikov, Petriello '05
 - fast parton level Monte Carlo HNNLO Catani, Grazzini '07



- Impact of kinematical cuts on higher order corrections (LHC $\sqrt{s} = 14 \text{ TeV}$)
 - left: Higgs mass $M_h = 125 \text{ GeV}$, no cuts on p_t of jets
 - right: Higgs mass $M_h = 165 \text{ GeV}$ and veto on jets with $p_t > 40 \text{ GeV}$ (k_t algorithm for jet reconstruction with jet size $D = 0.4$)

PDF dependence of gg -fusion cross section at LHC



Baglio, Djouadi, Godbole ^{μ_s/μ_0} 12

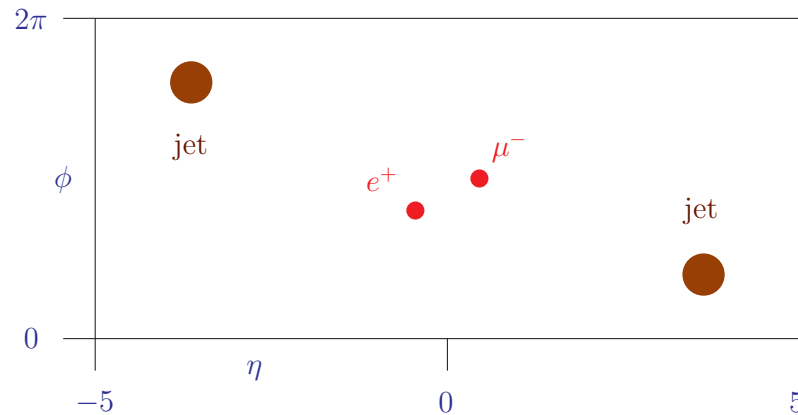
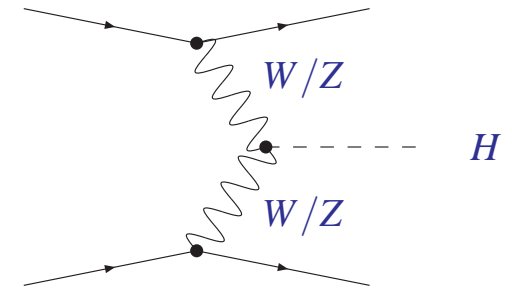
- PDFs uncertainty
 - PDFs (gluon at large x) largest single source of uncertainty
 - PDF uncertainty estimates by LHC Higgs XS WG too optimistic
- Linear addition of errors
 - PDF uncertainty and error due to effective theory:

$$\Delta\sigma = \Delta\sigma_{\text{PDF}} + \Delta\sigma_{\text{EFT}}$$

Vector-boson fusion

- Second largest rate at LHC (WWH coupling)

Signatures

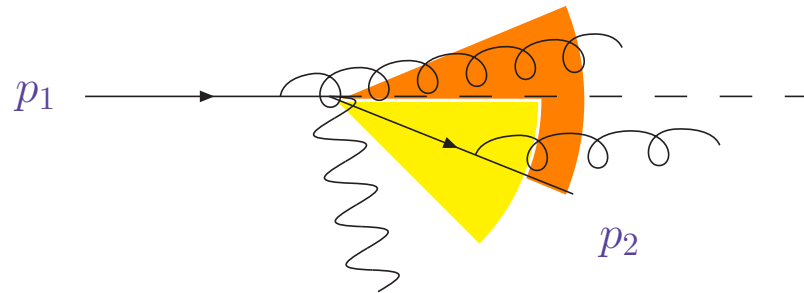


- WW, ZZ fusion \rightarrow Higgs is color singlet
 - two hard (forward) tagging jets (visible in detector)
 - no (or small) hadronic activity between tagging jets
 - color connection between forward jet and proton remnant
 - Higgs decay in the central rapidity region

Perturbative QCD corrections

Where is the hadronic activity ?

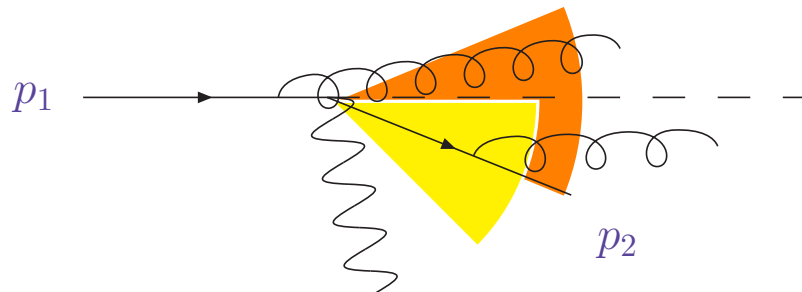
- QCD radiation predominantly in direction of incoming partons (angular ordering)



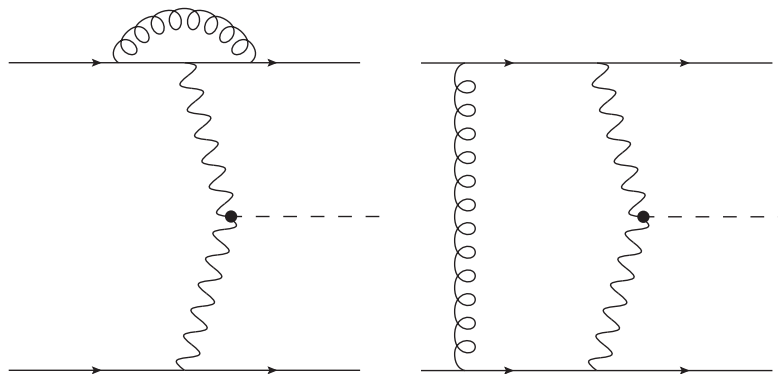
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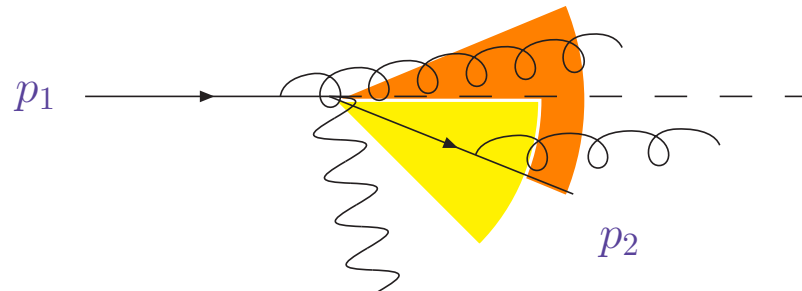
- NLO QCD radiative corrections



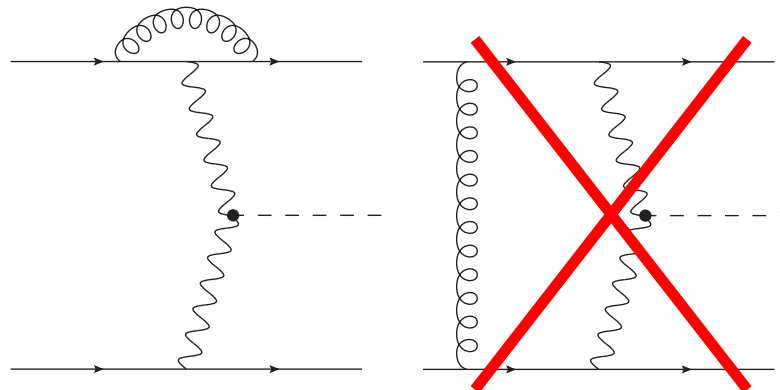
Perturbative QCD corrections

Where is the hadronic activity ?

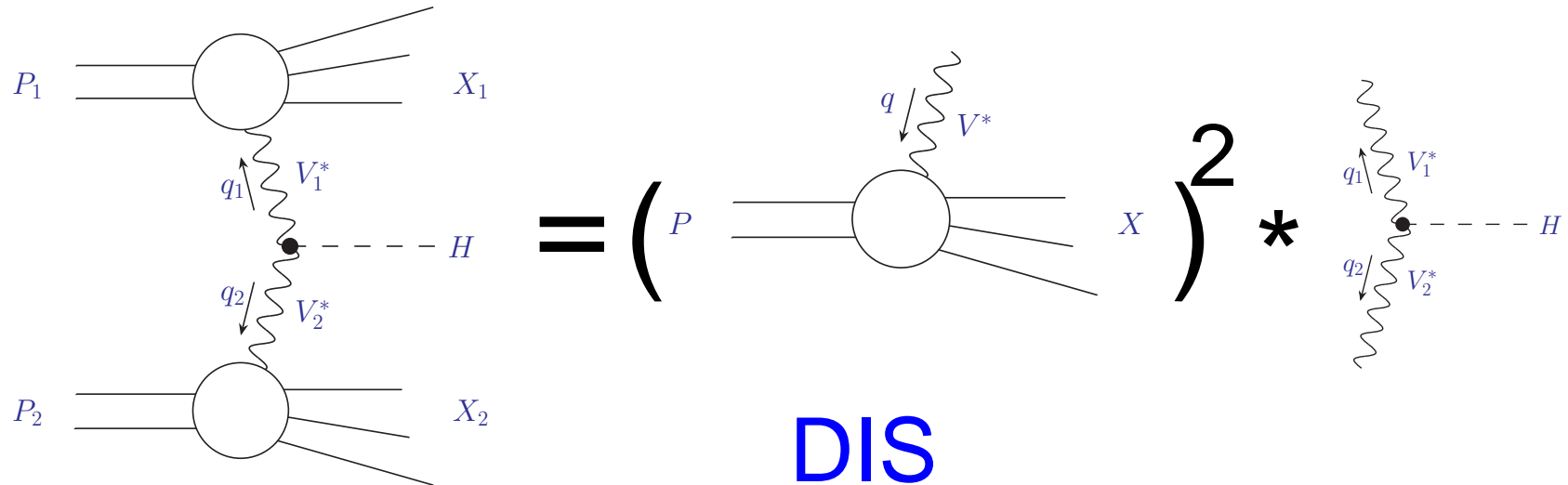
- QCD radiation predominantly in direction of incoming partons (angular ordering)



- NLO QCD corrections factorize (color conservation eliminates t -channel gluon in squared ME)

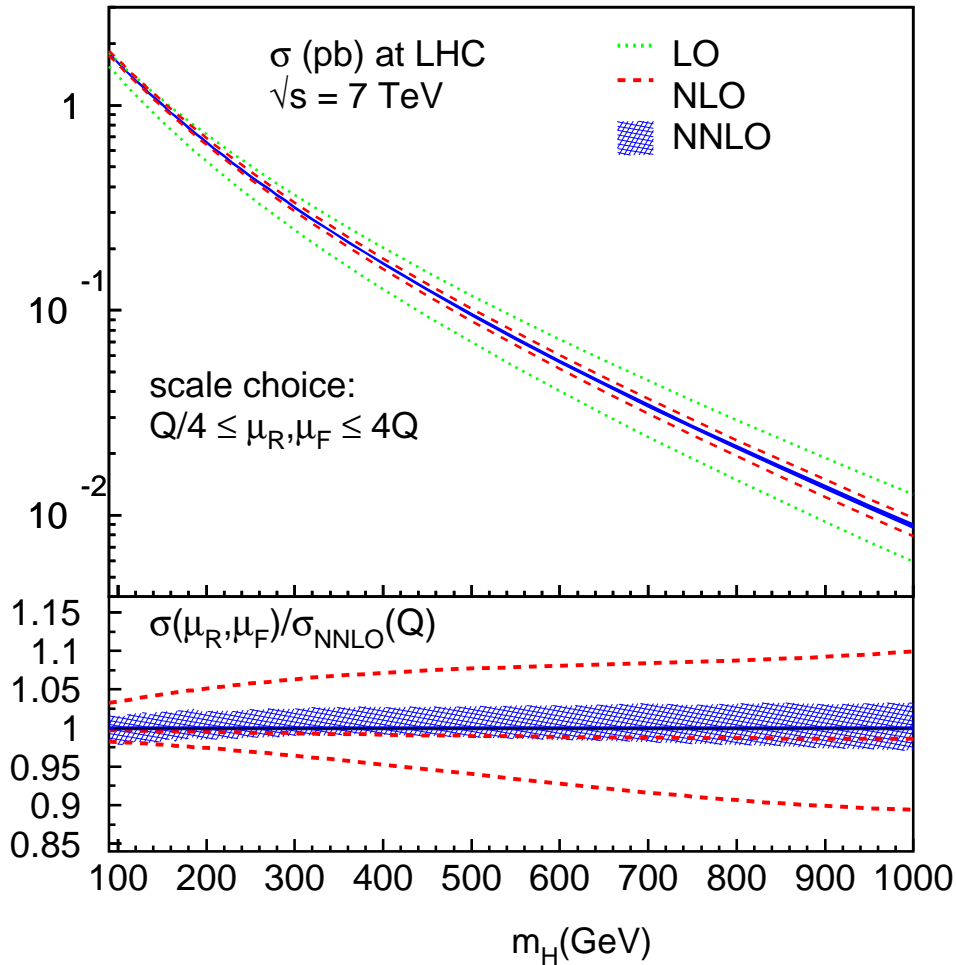


Exact factorization



- Deep-inelastic scattering building block of cross section with structure functions F_1 , F_2 and F_3
- Exact factorization at NLO: so-called structure function approach
Han, Valencia, Willenbrock '92
- Structure function approach is NOT exact at NNLO in QCD
 - but can be still considered a good approximation, holds to $\mathcal{O}(1\%)$
 - NNLO QCD corrections to F_1 , F_2 and F_3 long known
Kazakov, Kotikov '88; Zijlstra, van Neerven '92; S.M., Vermaseren '99

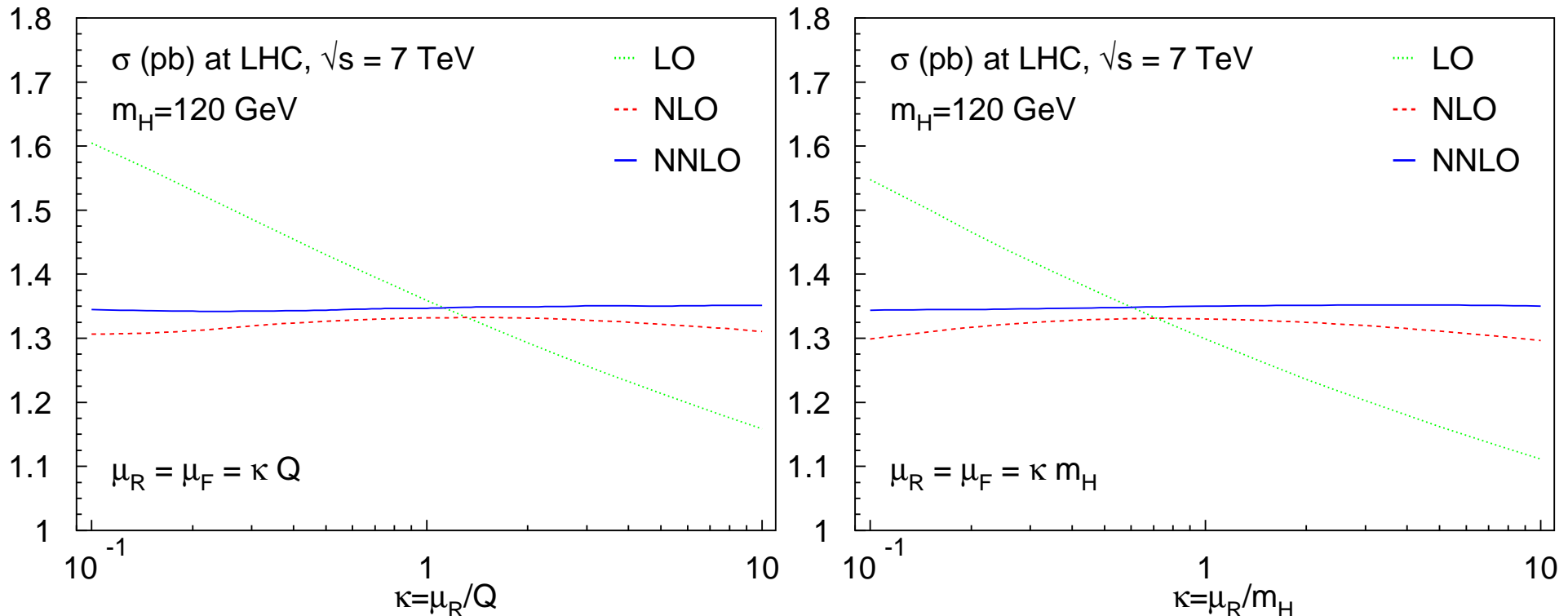
VBF cross section at LHC



- VBF at NNLO
- QCD corrections at second order small
 - apparent convergence
- NNLO results very stable at 2% against QCD scales variation (uniformly over the full mass range)
- Significant reduction of theoretical uncertainty

Bolzoni, Maltoni, S.M., Zaro '11

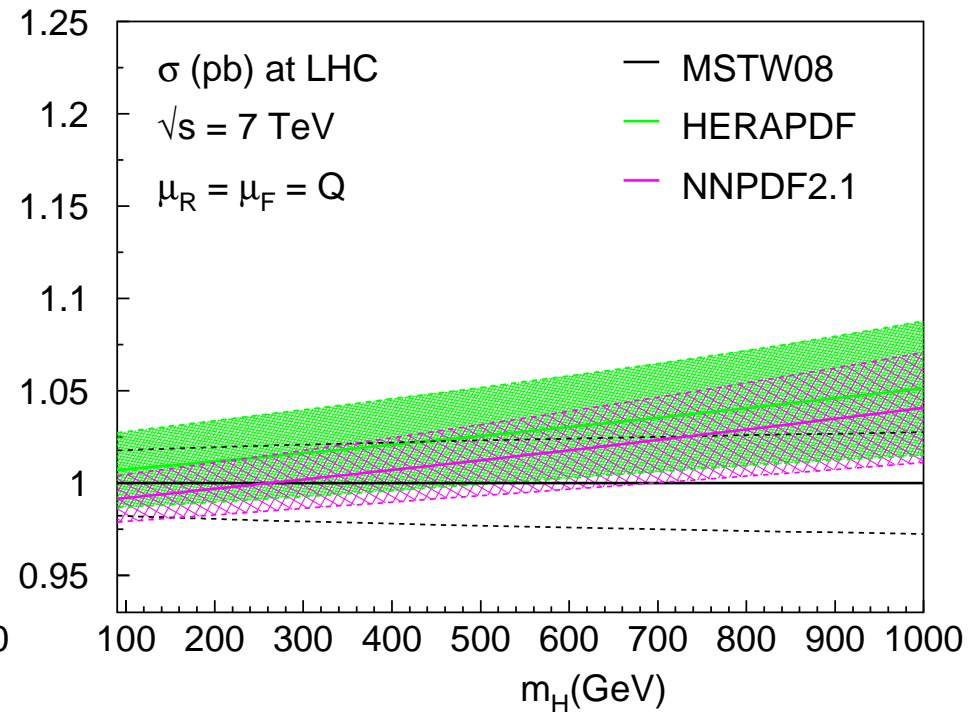
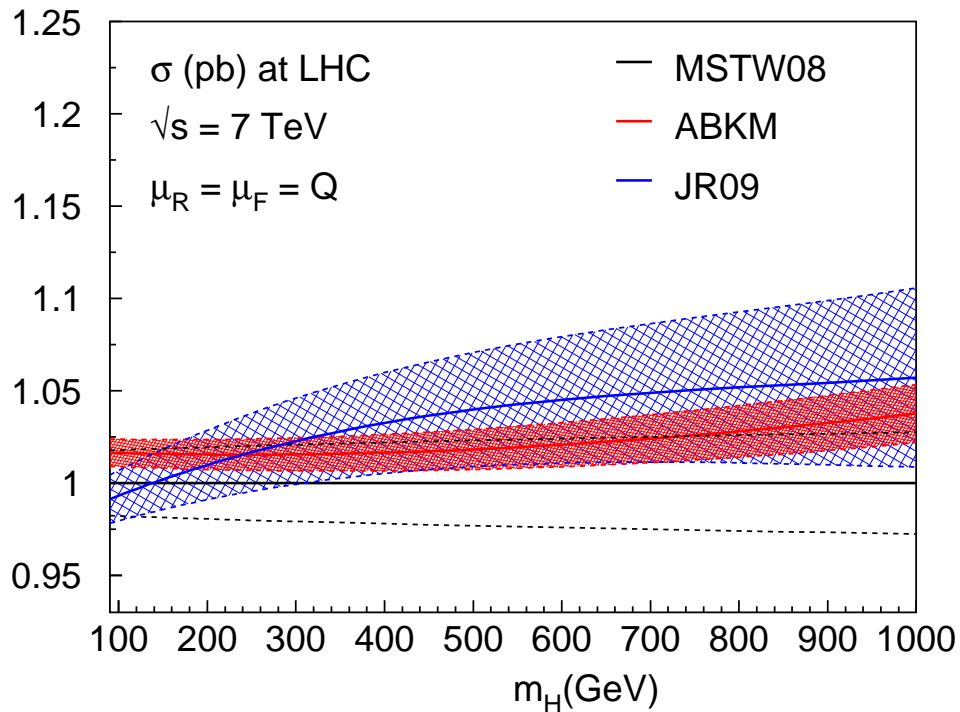
Scale stability at NNLO



Bolzoni, Maltoni, S.M., Zaro '11

- VBF cross sections displays very good scale stability at NNLO over large range for $\mu_R = \mu_F$ preferred (minimal sensitivity)
- Scale choice $\mu_R = \mu_F \simeq Q$ preferred (minimal sensitivity)

PDF dependence of VBF cross section at LHC

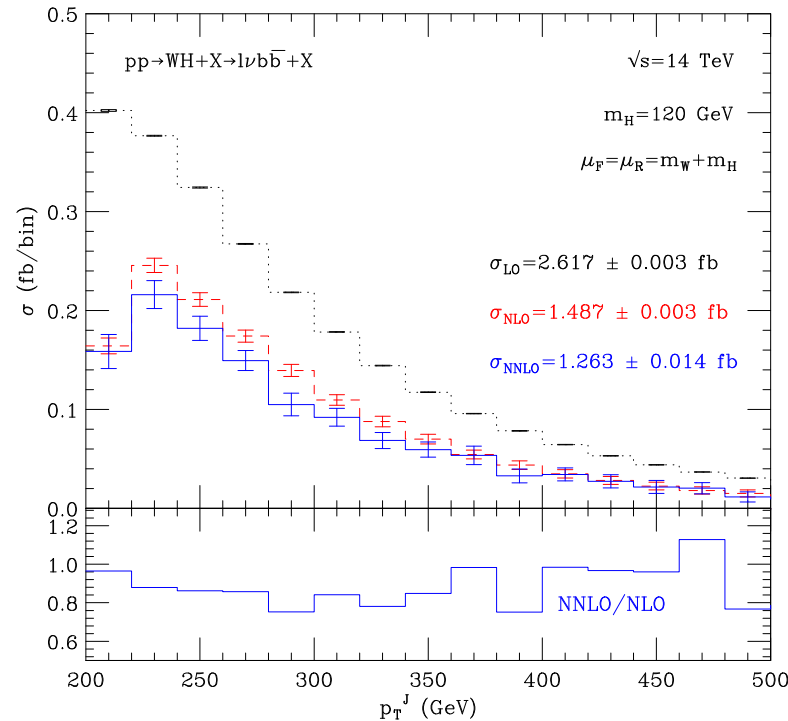


Bolzoni, Maltoni, S.M., Zaro '11

- PDF uncertainty
 - moderate for small Higgs masses $\mathcal{O}(\pm 2\%)$
 - increasingly larger for heavy Higgs bosons up to $\mathcal{O}(\pm 10\%)$

Higgs strahlung

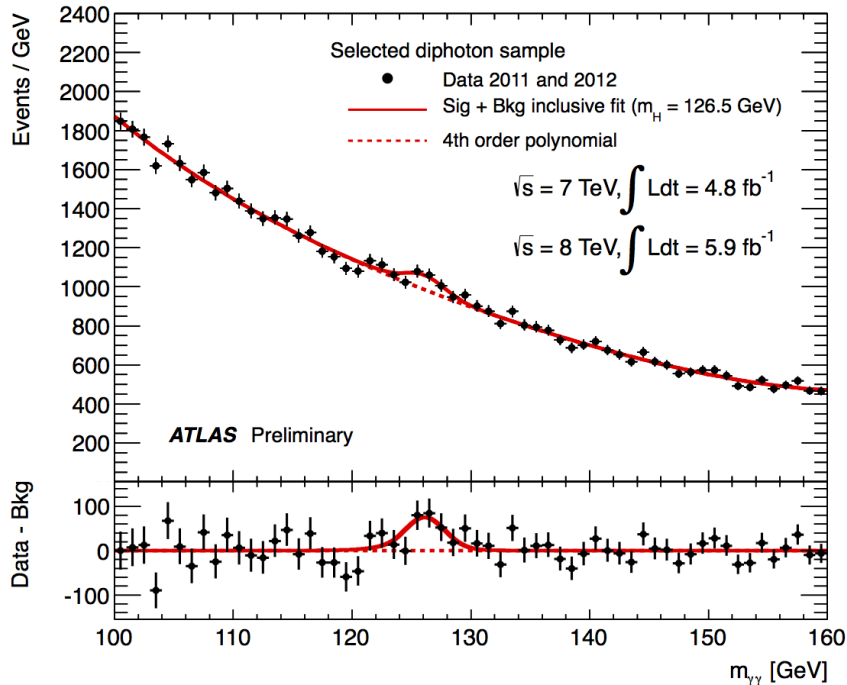
WH production (fully exclusive) Ferrara, Tramontana, Grazzini '11



- Scale dependence at the 1% level both at NLO and NNLO
- LHC $\sqrt{s} = 14$ TeV: lepton $p_t > 30$ GeV, $|y| < 2.5$ and $p_t^{\text{miss}} > 30$ GeV; require $p_t^W > 200$ GeV; (cone alg. with $R = 1.2$)
 - one fat jet with $p_t > 200$ GeV (and $b\bar{b}$ -pair), $|y| < 2.5$;
no other jet with $p_t > 20$ GeV and $|y| < 5$

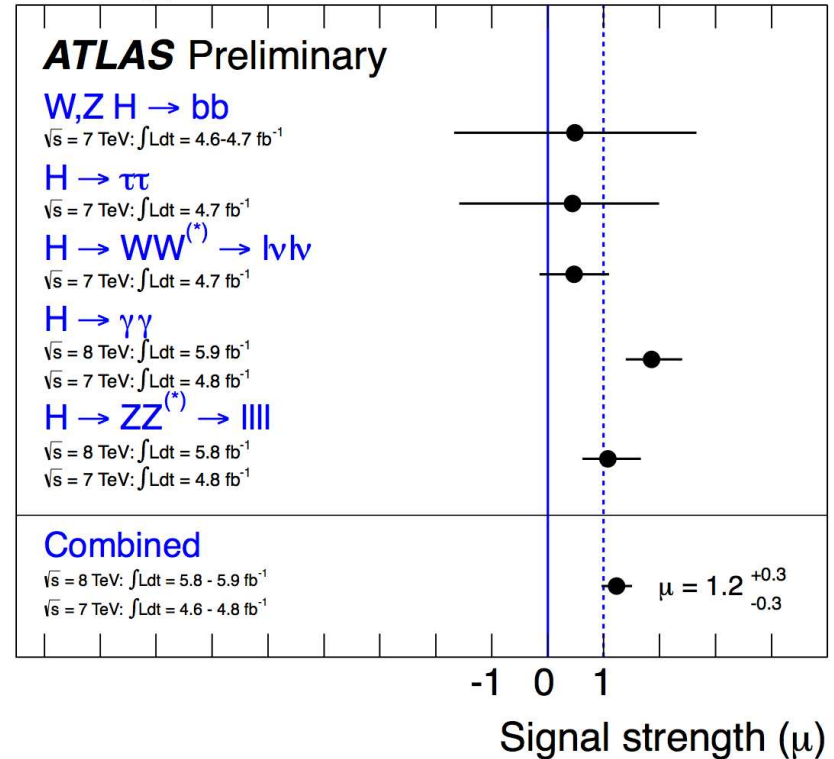
LHC measurements

Atlas coll. July 2012



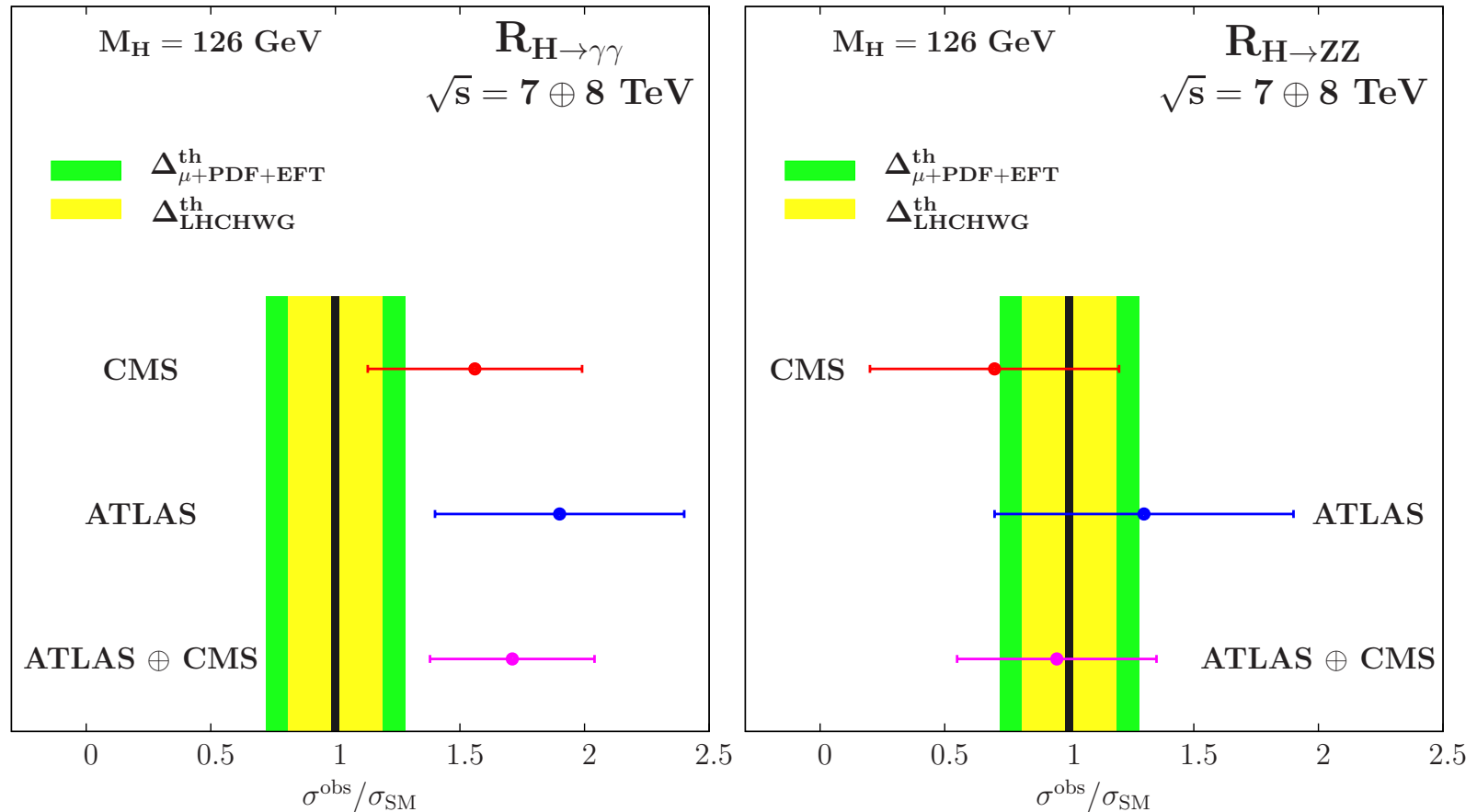
$-2\ln\lambda(\mu) < 1$ Intervals

2011 - 2012 Data



- Measured $H \rightarrow \gamma\gamma$ decay mode (left)
- Signal strength of all analyzed decay modes normalized to SM expectation (right)
- Agreement with SM for $H \rightarrow ZZ$; excess of $H \rightarrow \gamma\gamma$ (new physics ?)

Theory uncertainty



- Theory uncertainty of SM expectations revisited [Baglio, Djouadi, Godbole '12](#)
 - ratios $R_{XX} = \sigma_{H \rightarrow XX}^{\text{obs}} / \sigma_{H \rightarrow XX}^{\text{SM}}$
 - larger PDF uncertainties and linear addition of errors
- Excess of $H \rightarrow \gamma\gamma$ due to optimistic error estimates by Atlas and CMS

Summary (part III)

Standard Model

- Successful experimental program at LHC relies crucially on detailed understanding of Standard Model processes
- QCD at work
 - illustration of factorization, infrared safety and evolution for $gg \rightarrow H$
 - resummation of large logarithms near threshold

Higgs measurements

- Precision predictions for Higgs production at LHC available
 - radiative corrections (higher orders) important
 - essential to control theory uncertainties
 - non-perturbative parameters currently source of largest differences for Higgs cross section predictions