Sven-Olaf Moch
Universität Hamburg & DESY, Zeuthen

Bonn, Sep 24, 2012

Tutorial III

Belgian Dutch German summer school (BND 2012)

Show that the total partonic cross section $\hat{\sigma}_{q\bar{q}\to l^+l^-}$ for the process

$$q\bar{q} \to l^+l^- , \tag{1}$$

factorizes in the narrow width approximation for the Z-boson. Use this result to calculate the cross section for Z-boson production at LHC ($\sqrt{s} = 7$ TeV) according to the factoriaztion theorem as

$$\sigma_{pp\to Z\to l^+l^-} = \sum_{ij} \int dx_1\, dx_2\, f_i(x_1)\, f_j(x_2)\, \hat{\sigma}_{ij\to Z\to l^+l^-}\,(x_1, x_2, s)\,, \tag{2}$$

For the parton distribution function you can assume for simplicity the following functional form

$$f_u(x) = 2 \cdot f_d(x) = \frac{0.4}{x}\,, \qquad f_{\bar{u}}(x) = f_{\bar{d}}(x) = f_u(x)\,. \tag{3}$$

Hint: You can take all necessary ingredients and constants from *Particle Data Booklet*, `http://pdg.lbl.gov`. Chapter 44 of the PDG provides you with a formula for the parton cross section.
Alternatively, start off from

$$\hat{\sigma}_{q\bar{q}\to l^+l^-} = \frac{4\pi\alpha^2}{3s}\frac{1}{N_c}\left\{e_q^2 - 2e_q v_l v_q \chi_1(s) + \left(a_l^2 + v_l^2\right)\left(a_q^2 + v_q^2\right)\chi_2(s)\right\} \tag{4}$$

where e_q is the electric charge and a_i, v_i are the weak vector and axial-vector couplings. The functions χ_i are given in terms of Z-boson mass M_Z and total decay width Γ_Z as

$$\chi_1(s) = \left(\frac{\sqrt{2}G_F M_Z^2}{16\pi\alpha}\right)\frac{s(s-M_Z^2)}{(s-M_Z^2)^2 + \Gamma_Z^2 M_Z^2}\,, \tag{5}$$

$$\chi_2(s) = \left(\frac{\sqrt{2}G_F M_Z^2}{16\pi\alpha}\right)^2\frac{s^2}{(s-M_Z^2)^2 + \Gamma_Z^2 M_Z^2}\,. \tag{6}$$

In the narrow width approximation

$$\frac{1}{(s-M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \simeq \frac{\pi}{M_Z\Gamma_Z}\,\delta(s-M_Z^2)\,. \tag{7}$$

Then integrate the differential cross section in the invariant mass M^2 around M_Z in an interval $|M - M_Z| \leq \Delta$ (assume $\Gamma_Z \ll \Delta \ll M_Z$),

$$\int_{(M_Z-\Delta)^2}^{(M_Z+\Delta)^2} dM^2 \, \frac{d\hat{\sigma}}{dM^2} \simeq \frac{4\pi\alpha^2}{3s} \, \frac{1}{N_c} \left(a_l^2 + v_l^2\right) \left(a_q^2 + v_q^2\right) \underbrace{\left(\frac{\sqrt{2} G_F M_Z^2}{16\pi\alpha}\right)^2 \frac{\pi s^2}{M_Z \Gamma_Z} \, \delta(s - M_Z^2)}_{= \chi_2(s)} \,.$$

Sorting the factors and using the expression for the branching ratio $BR_{Z\to l^+l^-} = \Gamma_{Z\to l^+l^-}/\Gamma_Z$

$$\hat{\sigma}_{q\bar{q}\to Z} = \frac{\pi}{3} \sqrt{2} G_F M_Z^2 \left(a_q^2 + v_q^2\right) \delta(s - M_Z^2) \,, \tag{8}$$

$$\Gamma_{Z\to l^+l^-} = \frac{M_Z}{12\pi} \sqrt{2} G_F M_Z^2 \left(a_l^2 + v_l^2\right) \,, \tag{9}$$

one arrives at

$$\hat{\sigma}_{q\bar{q}\to Z\to l^+l^-} \simeq BR_{Z\to l^+l^-} \, \hat{\sigma}_{q\bar{q}\to Z} \,. \tag{10}$$