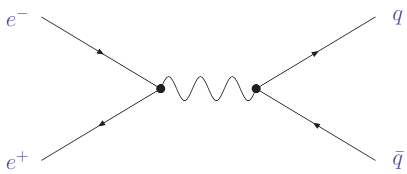
Sven-Olaf Moch
Universität Hamburg & DESY, Zeuthen

Bonn, Sep 23, 2012

Tutorial II

Belgian Dutch German summer school (BND 2012)

The amplitude for electron-positron annihilation and subsequent quark-anti-quark production is given to leading order by the following diagram:



The photon has momentum $q, +q^2 = Q^2 > 0$ and the quark and anti-quark carry the momenta $p, p^2 = 0$ and $p_1, p_1^2 = 0$.

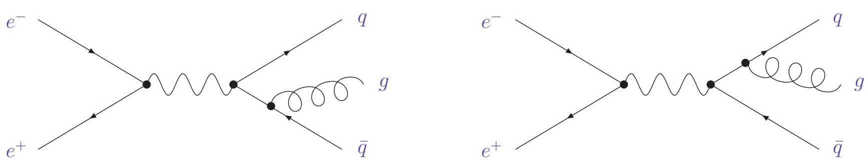
- Calculate the total cross section for this reaction.

$$e^- + e^+ \to \quad \gamma(q) \quad \to \mathrm{q}(p) + \overline{\mathrm{q}}(p_1)\,. \tag{1}$$

Next, we want to consider the radiative corrections in Quantum Chromodynamics to order α_s. The amplitude for electron-positron annihilation and production of a quark-anti-quark pair and additional radiation of a gluon with momentum k,

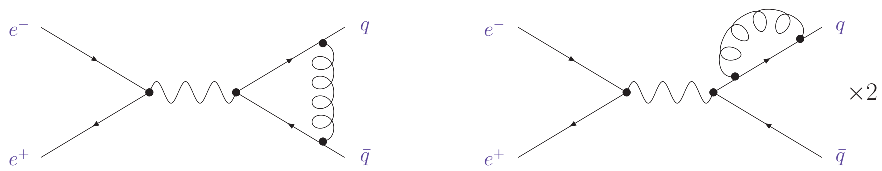
$$e^- + e^+ \to \quad \gamma(q) \quad \to \mathrm{q}(p) + \overline{\mathrm{q}}(p_1) + \mathrm{g}(p_2)\,, \tag{2}$$

is given by the following diagrams:



- This process can be decribed with the help of the scaling variable $x = (2p \cdot q)/Q^2$. What is the physical interpretation of x?
- Calculate the cross section $d\sigma_T$ for this reaction for transversely polarized photons as a function of the scaling variable $x = (2p \cdot q)/Q^2$ and the photon momentum $q^2 = Q^2$.
- Why is the result divergent in the infrared?

- What is the role of the following set of diagrams?



Hint: The squared amplitude $\mathcal{A}$ for the reaction $\gamma(q,\mu) \to \mathrm{q}(p) + \overline{\mathrm{q}}(p_1) + \mathrm{g}(p_2,\mu_1)$ is given by

$$\sum_{spins} |\mathcal{A}|^2 = (e^2 g_s^2 C_F)\, g^{\mu\nu} g^{\mu_1\mu_2} \tag{3}$$
$$\Big\{ \mathbf{tr}(\not{p}_1\gamma_\mu(\not{p}+\not{p}_2)\gamma_{\mu_1}\not{p}\gamma_{\mu_2}(\not{p}+\not{p}_2)\gamma_\nu)\frac{1}{(p+p_2)^4}$$
$$+\mathbf{tr}(\not{p}_1\gamma_{\mu_1}(\not{p}_1+\not{p}_2)\gamma_\mu\not{p}\gamma_\nu(\not{p}_1+\not{p}_2)\gamma_{\mu_2})\frac{1}{(p_1+p_2)^4}$$
$$-\mathbf{tr}(\not{p}_1\gamma_{\mu_1}(\not{p}_1+\not{p}_2)\gamma_\mu\not{p}\gamma_{\mu_2}(\not{p}+\not{p}_2)\gamma_\nu)\frac{1}{(p+p_2)^2(p_1+p_2)^2}$$
$$-\mathbf{tr}(\not{p}_1\gamma_\mu(\not{p}+\not{p}_2)\gamma_{\mu_1}\not{p}\gamma_\nu(\not{p}_1+\not{p}_2)\gamma_{\mu_2})\frac{1}{(p+p_2)^2(p_1+p_2)^2}\Big\}\,.$$

Here, e is the electro-magnetic charge, $C_F = (N_c^2-1)/(2N_c)$ arises from the quark color charge and g_s is the strong coupling constant.
The differential phase space for the reaction in Eq.(2) in x is in D-dimensions given by

$$d\mathrm{PS} = \frac{1}{2}\frac{1}{(4\pi)^{D/2-1}}\frac{(Q^2)^{D/2-2}}{\Gamma(D/2-1)}(1-x)^{D/2-2}\int_0^1 dy\,[y(1-y)]^{D/2-2}\,, \tag{4}$$

where the relation of the scattering angle θ in the center-of-mass system for q is related to y as $y = \frac{1}{2}(1+\cos\theta)$.

It is advantageous to automatize the calculation. A FORM script is available on the web-page of this lecture and under `www-zeuthen.desy.de/~moch`.