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Tutorial I

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In Quantum Chromodynamics one defines the renomalized coupling as follows

$$g_0 \to Z_g \, \mu^{\varepsilon} \, g(\mu)\,, \tag{1}$$

where g_0 denotes the bare (unrenormalized) coupling, g is the renormalized coupling, Z_g the renormalization constant and μ an arbitrary scale.
We apply dimensional regularization and ε is the parameter for the shift from 4 dimensions:

$$D = 4 - 2\varepsilon\,. \tag{2}$$

- Calculate the QCD β-function (recall $\alpha_s(\mu) = \dfrac{g(\mu)^2}{4\pi}$)

  $$\beta(\alpha_s) = \mu^2 \frac{d\alpha_s(\mu)}{d\mu^2} \tag{3}$$

  by using the renormalization constant

  $$Z_g = 1 - \frac{1}{2}\frac{\alpha_s(\mu)}{4\pi}\left(\frac{1}{\varepsilon} - \gamma_E + \ln(4\pi)\right)\left\{\frac{11N_c - 2n_f}{3}\right\} \tag{4}$$

  where N_c is the number of colors and n_f the numbers of quark flavors. γ_E denotes Euler's constant.

- Solve the QCD β-function at leading order. What choices do you have for the initial conditions? How do you fix them?

- What are the limits on the matter content of QCD in order to remain an asymptotically free gauge theory?

- Compute $\alpha_s^{LO}(M_Z)$ (leading order) and $\alpha_s^{NLO}(M_Z)$ (next-to-leading order) from input $\alpha_s^{NNLO}(M_Z)$ (next-to-next-to-leading order). Choose some order of perturbation theory and compute $\alpha_s^{(n_f=4)}(M_Z)$ and $\alpha_s^{(n_f=5)}(M_Z)$ from input $\alpha_s^{(n_f=3)}(M_Z)$.

Hint: This task can be easily solved with the Mathematica package `RunDec.m`, cf. `hep-ph/0004189`.

- Compute in Quantum Electrodymancs the divergent part of the fermionic contribution to the photon self-energy.

Hint:

– Feynman rules in QED:

photon propagator $\dfrac{-g_{\mu\nu}}{p^2+\mathrm{i}0}$

fermion propagator $\dfrac{i}{\not{p}-m+\mathrm{i}0}$

fermion-photon vertex $ie\gamma_\mu$

– one-loop integral:

$$\int \frac{d^D\ell}{(2\pi)^D}\frac{1}{(\ell^2+2q\ell+a+i\varepsilon)^n} = (-1)^n \frac{i}{(4\pi)^{2-\varepsilon}}\frac{\Gamma(n-2+\varepsilon)}{\Gamma(n)}\left(q^2-a-i\varepsilon\right)^{2-\varepsilon-n}$$

– Feynman parametrization:

$$\frac{1}{AB} = \int_0^1 \frac{1}{[xA+(1-x)B]^2}dx$$

– Gamma function:

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon}+\dots$$