AutoDipole

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Collaboration with

S. Moch and P. Uwer

`Mini-workshop on fixed order multi-leg automatic NLO calculations'

at Universität Wuppertal, 3 June 2009

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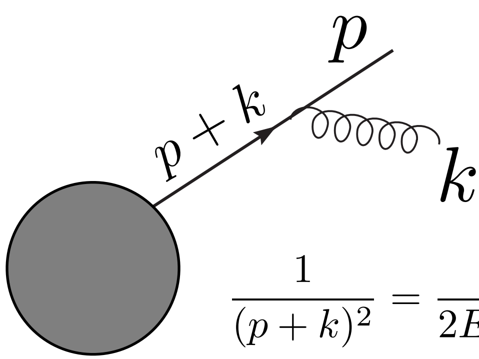
1. Introduction

- LHC at CERN
  - The Standard Model predictions are required to identify new physics
- Perturbative QCD
  - Hard part is calculated by the perturbative expansion of QCD
- Leading order (LO)
  - Well automated
- Next-to-leading order (NLO)
  - Some parts are separately automated
  - Now emerging full automation
  - Automating dipoles subtraction : one part in the automation

    It dealds with the soft/collinear safety at QCD NLO

■ QCD NLO　Real emission correction

- Soft and collinear singularities



$$\frac{1}{(p+k)^2} = \frac{1}{2E_k E_p(1-\beta\cos\theta_{kp})} \rightarrow \infty$$

$\mathrm{E}_k \rightarrow 0$
: Soft divergence
(IR divergence)

$m_q = 0$ and $\theta_{kq} \rightarrow 0$
: Collinear divergence
(mass singulariry)

- Phase space integral of those singularities

$$\int d\Phi_m \left[|\mathrm{M}|^2\right] \supset \begin{cases} \text{Soft region:} & \displaystyle\int \frac{d^3k}{k}\left[\frac{1}{k^2}\right] \simeq \int_{\mu_k} \frac{dk}{k} \simeq \log\mu_k + \cdots \\ \text{Collinear region:} & \displaystyle\int_{-1}^{1} d\cos\theta_{ij}\left[\frac{1}{s_{ij}}\right] \propto \int_{-1}^{1} d\cos\theta_{ij}\left[\frac{1}{1-\cos\theta_{ij}}\right] \simeq \int_{\mu_\theta} \frac{d\theta}{\theta} \simeq \log\mu_\theta + \cdots \end{cases}$$

■ QCD at NLO : Cancellation of soft/collinear divergences

$$\sigma_{\mathrm{NLO}} = \sigma_{\mathrm{real}} + \sigma_{\mathrm{virtual}}$$

σ_{real} → Soft/Collinear

$\sigma_{\mathrm{virtual}}$ → Soft/Collinear

$\sigma_{\mathrm{virtual}}$ → UV → Renormalization → Finite

Soft/Collinear + Soft/Collinear → Finite in observables

:IR safe (more precisely, soft/collinear safe)

Kinoshita-Lee-Nauenberg theorem

■ Dipole subtraction

- A general and practical procedure to treat soft/collinear divergences at QCD NLO

S.Catani and M.H.Seymour, Nucl.Phys.B485(1997)291
S.Catani, S.Dittmaier, M.H.Seymour, Z.Trocsanyi, Nucl.Phys.B627(2002)189

1. Construct the counter terms which cancel all soft/collinear divergences
2. Subtract it from $\sigma_{\rm real}$ and add it to $\sigma_{\rm virtual}$

$$\sigma_{\rm NLO} = \sigma_{\rm real} + \sigma_{\rm virtual}$$

$$= (\sigma_{\rm real} - \sigma_{\rm a}) + (\sigma_{\rm virtual} + \sigma_{\rm a})$$

$$= \int {\rm d}\Phi_{m+1} \left[|{\rm M}_{\rm real}|^2 - \sum_i {\rm D}_i \right] \Big|_{\rm D=4} + \int {\rm d}\Phi_m \left[|{\rm M}_{\rm 1\text{-}loop}|^2 + \int {\rm d}\Phi_1 \sum_i {\rm D}_i \right] \Big|_{\rm D=4}$$

Finite

$$\int_0 k dk \left[\frac{1}{k^2} - \frac{1}{k^2} \right] < \infty$$ Finite

- Calculation of the real correction can be done in 4-dim.

- Dipole term is systematically constructed based on the universal factorization of the singularities

■ Multi-parton processes

- Requires many dipole terms and tedious to calculate by hand ⟶ Automation is required
- The algorithm is in a combinatorial way ⟶ It is possible

■ Our aim

1. Automatize the dipole subtraction
2. Apply it to the signals in LHC and the QCD backgrounds

- Recent work in the same direction
  - T. Gleisberg and F. Krauss, Eur.Phys.J.C53(2008)501, arXiv0709.2881
  - M.H. Seymour and C. Tevlin, arXiv0803.2231
  - R. Frederix and T. Gehrmann and N. Greiner, JHEP0809:122, arXiv0808.2128
  - M. Czakon, C.G. Papadopoulos and M. Worek, arXiv0905.0883

■ Achievement

AutoDipole Version 1.0 beta   K. Hasegawa, S. Moch, and P. Uwer

- The beta version is publicly available at

  https://indico.desy.de/conferenceOtherViews.py?view=standard&confId=1573

  See also the lecture and tutorial:`Real parton emission and automated dipole subtraction'

  at Computer Algebra and Particle Physics 2009 on 1 April 2009

- Publications before the full automatization

  - Nucl.Phys.Proc.Suppl.183(2008)268, arXiv:0807.3701

    Main ingredients are constructed and checked

  - Nucl.Phys.Proc.Suppl.186(2009)86

    Compete agreements for $pp \to t\bar{t} + 1$ jet in

    S. Dittmaier, P. Uwer and S. Weinzierl, Eur. Phys. J. C59 (2009) 625

2. AutoDipole package

2-1. Aim of the package

2-2. Structure

2-3. Algorithm and creation order

2-4. Color linked Born amplitude squared

2-5. Gluon emitter case

2-6. Safety check

2-1. Aim of the package

AutoDipole Version 1.0 beta

Aim:

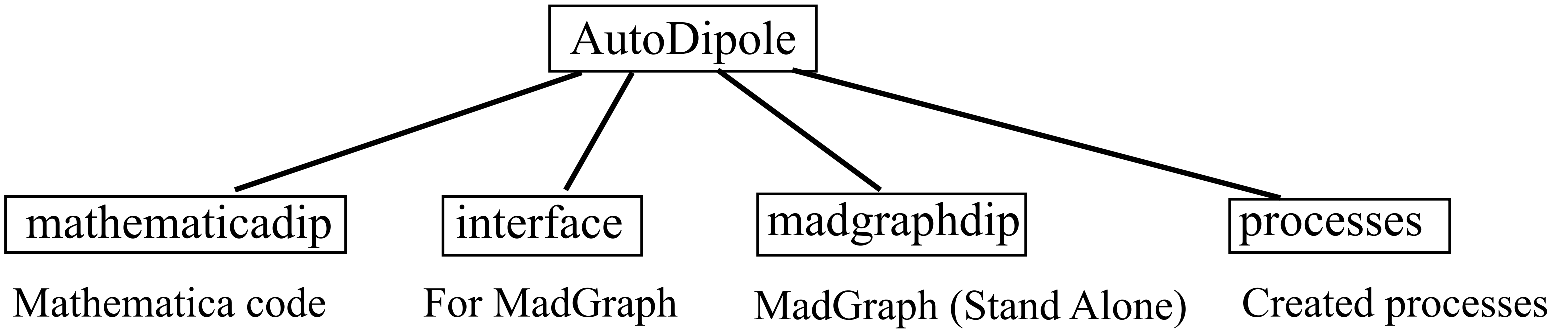
- Automatic generation of the subtracted real part,

$$|\mathrm{M}|^2_{\mathrm{real}} - \sum_i \mathrm{D}_i$$

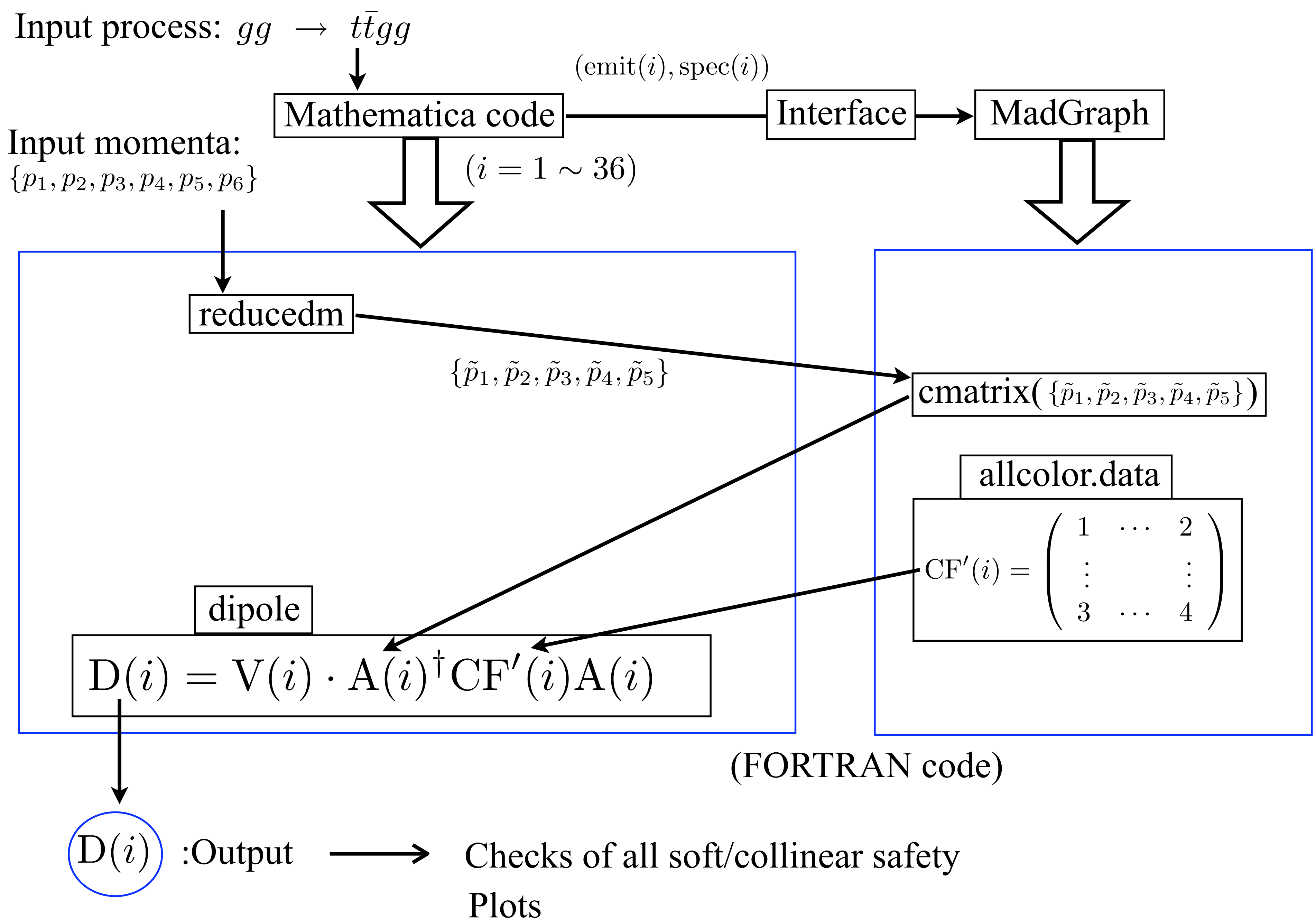
- Automatic checks of all soft/collinear safety

2-2. Structure

- Directory structure

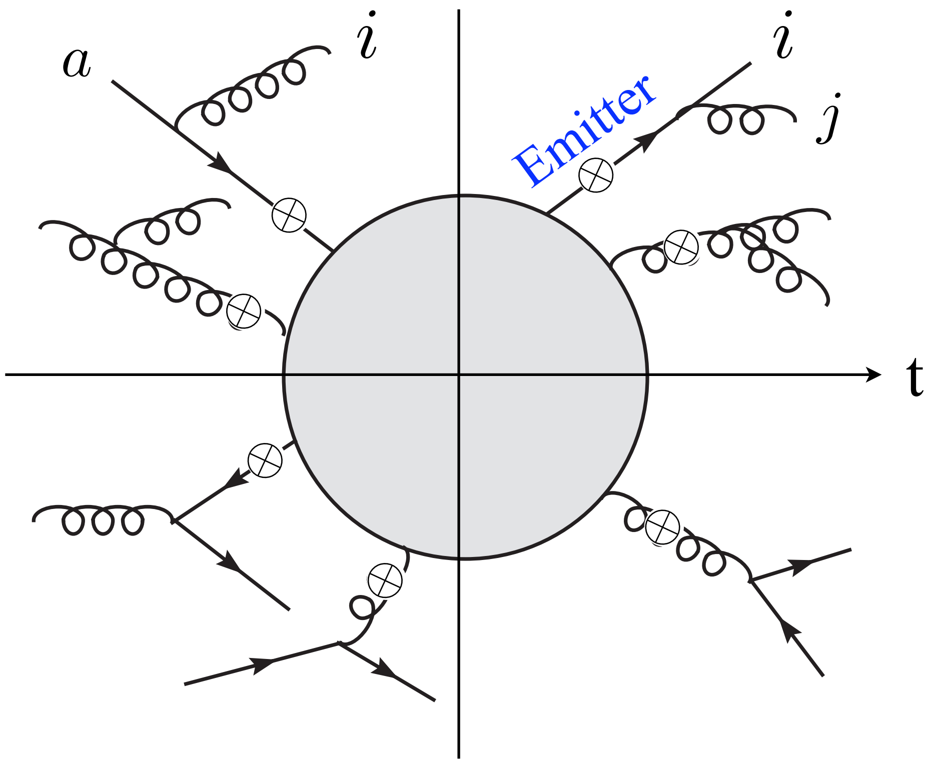


■ Scheme to calculate $\sum_i \mathrm{D}_i$



2-3. Algorithm and creation order

1. Choose emitter pair



Choose all possible leg-pair which matches one of the seven patterns

(a, i) or (i, j)

Initial parton=a,b
Final parton=i,j,k

2. Choose spectator

Choose a different leg from emitter pair

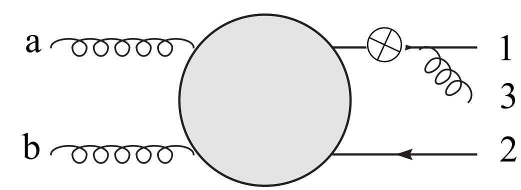
Spectator : $k \neq i, j$ $\quad b \neq a$

emitter pair \ spectator	k	b
(i, j)	$\mathrm{D}_{ij,k}$ $(k \neq i, j)$	$\mathrm{D}_{ij}^{\ b}$
(a, i)	$\mathrm{D}^{ai}_{\ k}$ $(k \neq i)$	$\mathrm{D}^{ai,b}$ $(b \neq a)$

3. Use dipole formulae

$$\mathrm{D}_{ij,k}(p_1,\cdots,p_{m+1}) = -\frac{1}{2p_i\cdot p_j} < 1,\cdots,\tilde{ij},\cdots,\tilde{k},\cdots,m{+}1|\frac{\mathrm{T}_k\cdot \mathrm{T}_{ij}}{\mathrm{T}_{ij}^2}\mathrm{V}_{ij,k}|1,\cdots,\tilde{ij},\cdots,\tilde{k},\cdots,m{+}1>_m$$

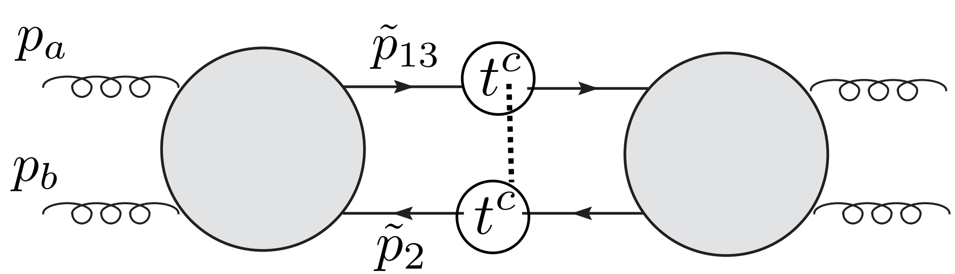
Example : $g(a)g(b) \to u(1)\bar{u}(2)g(3)$



$$\mathrm{D}_{13,2}(p_1,p_2,p_3,p_a,p_b) = -\frac{1}{2p_1\cdot p_3}\langle gg\to \tilde{u}\tilde{\bar{u}}|\frac{\mathrm{T}_{\bar{u}}\cdot \mathrm{T}_{ug}}{\mathrm{T}_{ug}^2}\mathrm{V}_{13,2}|gg\to \tilde{u}\tilde{\bar{u}}\rangle_2$$

- Dipole splitting function : $\mathrm{V}_{13,2}(z,y) = \delta_{ss'}8\pi\alpha C_F\left[\frac{2}{1-z_i(1-y_{ij,k})} - (1+z_i)\right]$

- Color linked Born amplitude squared (CLBS): $\langle gg\to \tilde{u}\tilde{\bar{u}}|\mathrm{T}_{\bar{u}}\cdot \mathrm{T}_{ug}|gg\to \tilde{u}\tilde{\bar{u}}\rangle_2$



- Reduced momenta satisfy the energy-momentum conservation and on-shell condition

$$p_a^\mu + p_b^\mu = \tilde{p}_{13}^\mu + \tilde{p}_2^\mu \qquad \tilde{p}_{13}^2 = \tilde{p}_2^2 = 0$$

⇨ Make it possible to reduce into the physical born amplitude, which can be calculated by well automated LO softwares

■ Mathematica code

Input: $gg \to u\bar{u}g$ (Process at NLO real correction)

↓

Generate dipole terms and write down

↓

Show the contents of all dipoles and all soft/collinear limits

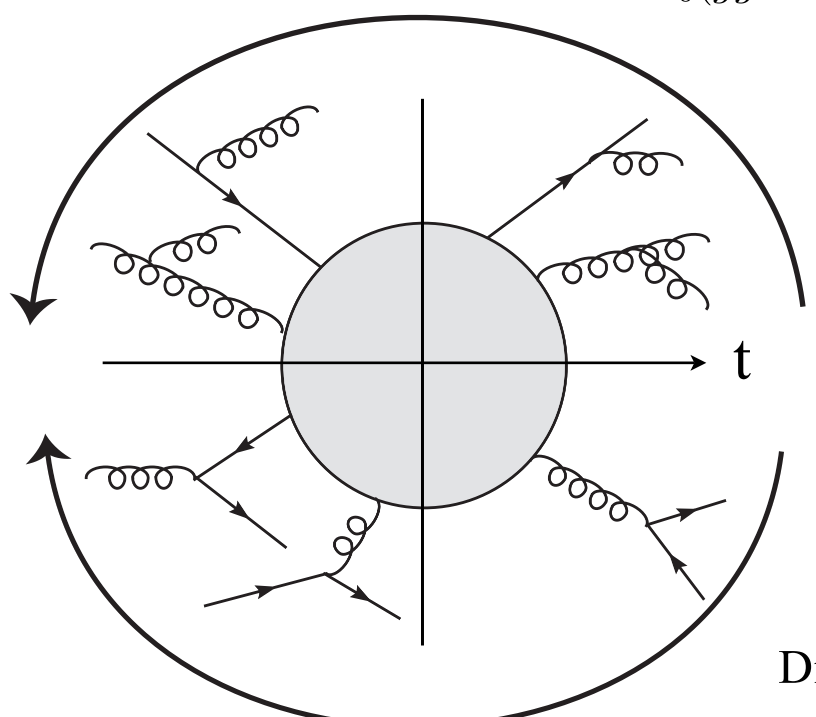
↓

Write Fortran files: dipole.f reducedm.f

- Creation order

Sorted by the splitting
and the reduced Born

Dipole 1 $\mathrm{B1} = \mathrm{M}_0(gg \to u\bar{u})$



Dipole 4 $\mathrm{B4} = \mathrm{M}_0(ug \to ug)$

Dipole 2 $\mathrm{B2} = \mathrm{M}_0(gg \to gg)$

Dipole 3

2-4. Color linked Born amplitude squared

$$ {}_m\!< 1, \cdots, \tilde{ij}, \cdots, \tilde{k}, \cdots, m+1 | \mathrm{T}_k \cdot \mathrm{T}_{ij} | 1, \cdots, \tilde{ij}, \cdots, \tilde{k}, \cdots, m+1 >_m $$

- MadGraph

T. Stelzer and W.F. Long, Comput. Phys.Commun.81(1994) 357, hep-ph/9401258

Johan Alwall et al, JHEP 0709:028,2007, arXiv:0706.2334

- An automated general LO
- In order to evaluate the helicity amplitude, HELAS library is used

K. Hagiwara, H. Murayama, I. Watanabe, Nucl.Phys.B367(1991)257

- Color decomposition

$$ \mathrm{M} = \sum_a C_a J_a $$

$J_1 = A_1 - A_3 + \cdots$: Joint amplitude

$$ \longrightarrow \quad |\mathrm{M}|^2 = \left(\vec{\mathrm{J}}\right)^\dagger \ \mathrm{CF} \ \vec{\mathrm{J}} $$

$(\mathrm{CF})_{ab} = C_a^\star C_b$: Color matrix

■ MadGraph with interface

- Normal Born squared

$< 1, \cdots, m||1, \cdots, m >_m$

$\supset$ CF

→ Color linked Born squared

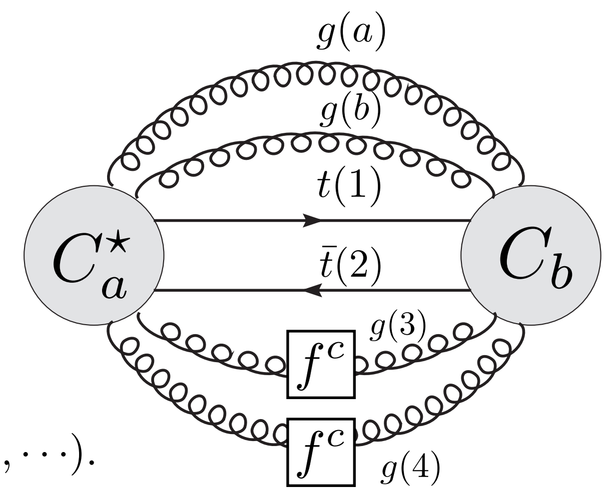
$< 1, \cdots, m|\mathrm{T}_i \cdot \mathrm{T}_k|1, \cdots, m >_m \quad \supset \ \mathrm{CF}' = \mathrm{CF} \otimes t(ij) \otimes f(k)$

→ Different helicity Born squared

$< 1, \cdots, (i, \lambda), \cdots, m||1, \cdots, (i, \lambda'), \cdots, m >_m$

- Example

$$g(a)g(b) \to t(1)\bar{t}(2)g(3)g(4) \otimes \hat{f}(3) \otimes \hat{f}(4)$$



$$(\mathrm{CF})_{1b} = \frac{1}{54}(512, 8, -64, 80, 8, -10, -1, -64, -64, 8, -1, -10, -1, 62, -10, \cdots).$$

⇩

$$(\mathrm{CF}')_{1b} = \frac{1}{4}(8, 0, 8, 16, 0, -2, 0, 8, -1, -1, 1, 2, -8, -7, 1, \cdots).$$

2-5. Gluon emitter case

$$\mathrm{D}_{g_i g_j,k} = -\frac{1}{s_{ij}} \mathrm{V}^{\mu\nu}_{ij,k} \mathrm{T}^{ij,k}_{\mu\nu}$$

where

$$\mathrm{V}^{\mu\nu}_{ij,k} = 16\pi\mu^{2\epsilon}\alpha_s C_A \left[-C_1 g^{\mu\nu} + C_2 L^\mu L^\nu \right],$$

$$\mathrm{T}^{ij,k}_{\mu\nu} = A_\mu T^c T^c A_\nu$$

- MadGraph $\longrightarrow$ HELAS library $\supset$ $\epsilon^{\pm}_{\mu[\mathrm{HELAS}]}(k) = \frac{1}{\sqrt{2}}\left(\mp\epsilon_{\mu(1)} - i\epsilon_{\mu(2)}\right)$

: The polarization vector is in circular base

- Dipole term also should be in the circular base

S. Weinzierl, SACLAY-SPH-T-98-083

$$A_{\mu *} \mathrm{V}^{\mu\nu} A_\nu = \sum_{\lambda',\lambda} A^{\lambda'} \mathrm{V}^{\lambda'\lambda} A^\lambda$$

where

$$\mathrm{V}^{\lambda'\lambda} = \epsilon^{\lambda' *}_{\mu} \mathrm{V}^{\mu\nu} \epsilon^{\lambda}_{\nu}$$

$$A^\lambda = \epsilon^{\lambda *}_{\mu}(\tilde{p}_{ij}) A^\mu$$

$$\longrightarrow \quad \mathrm{D}_{g_i g_j,k} = -\frac{1}{s_{ij}} 16\pi\mu^{2\epsilon}\alpha_s \left[\left(C_1 + C_2 |\mathrm{E}_+|^2 \right) |\mathrm{M}_{col}|^2 + 2C_2 \mathrm{Re}[\mathrm{E}_+^* \mathrm{E}_- \mathrm{N}(+,-)] \right],$$

where

$$|\mathrm{M}_{col}|^2 = \sum_\lambda A^{\lambda *} T \cdot T A^\lambda$$

$$\mathrm{N}(+,-) = A^{(+)*} T \cdot T A^{(-)}$$

$$\mathrm{E}_\pm = \epsilon_\pm \cdot L$$

- The package uses HELAS vector for the CLBS

and

XZC vector for E_+ in the splitting function

$$\epsilon^{+}_{[\mathrm{XZC}]}(k,q) = \frac{< q-|\gamma^{\mu}|k- >}{\sqrt{2} < qk >^{*}}$$

Z. Xu, D.H. Zhang and L. Chang, Nucl. Phys. B291 (1987) 392

$\longrightarrow$
- The quantity is expressed in the term of the spinor product
- We can well separate the remaining part from the CLBS

- E_+ is in the terms of the spinor product

$$E_+ = \frac{z_i < p_j p_i >< \tilde{p}_{ij} p_i >^{\star}}{\sqrt{2} < p_j \tilde{p}_{ij} >}$$

- E_+ with massive cases $p_k^2 = m_k^2$

$$\mathrm{E}_+ \supset < p_i - |\not{p}_k|\tilde{p}_{ai}- > = < p_i - |\not{p}^{\flat}_k|\tilde{p}_{ai}- >$$

$$= < p_i - |p^{\flat}_k + >< p^{\flat}_k + |\tilde{p}_{ai}- >$$

Massless momentum:

$$p^{\flat}_k = p_k - \frac{m_k^2}{2p_k \cdot \tilde{p}_{ai1}} \tilde{p}_{ai}$$

$$(p^{\flat}_k)^2 = 0$$

- All E_+ in our package are calculated in terms of spinor products

-The phase difference between HELAS and XZC vectors must be taken care of

2-6. Consistency check

- We can construct a simple soft/collinear safe quantity

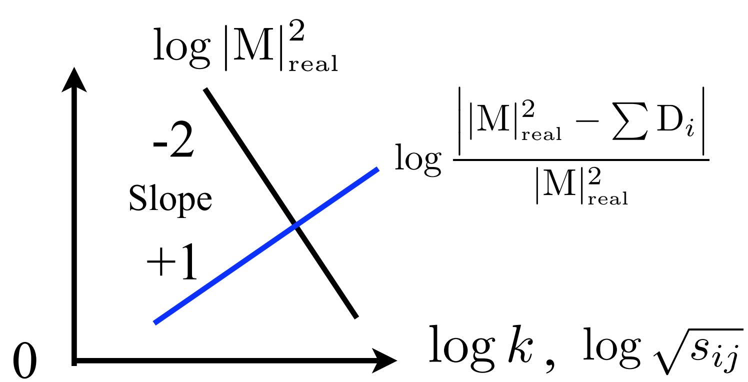
$$\lim_{\mathrm{L}(i)} \left[\, |\mathrm{M}|^2 - \sum_{j \subset \mathrm{S}(i)} \mathrm{D}(j) \right]$$

$\begin{cases} \mathrm{L}(i) & \text{: A single-unresolved soft/collinear limit} \\ \mathrm{S}(i) & \text{: the set of the corresponding dipoles} \end{cases}$

- Limiting behavior

  - We can predict the limiting behavior

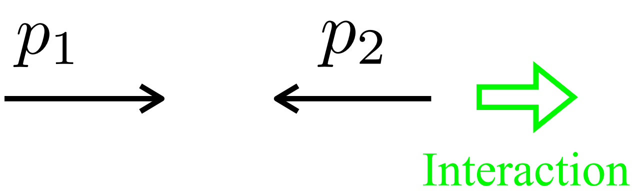
$$|\mathrm{M}|^2 - \sum_{j \subset \mathrm{S}(i)} \mathrm{D}(j) = \begin{cases} \dfrac{1}{k^2}(a_0 + a_1 k + a_2 k^2 + \cdots) - \dfrac{1}{k^2} a_0 = \dfrac{1}{k}(a_1 + a_2 k + \cdots) & \text{Soft } (k \to 0) \\ \dfrac{1}{s_{ij}}(b_0 + b_1 \sqrt{s_{ij}} + b_2 s_{ij} + \cdots) - \dfrac{1}{s_{ij}} b_0 = \dfrac{1}{\sqrt{s_{ij}}}(b_1 + b_2 \sqrt{s_{ij}} + \cdots) & \text{Collinear } (\theta_{ij} \to 0) \end{cases}$$

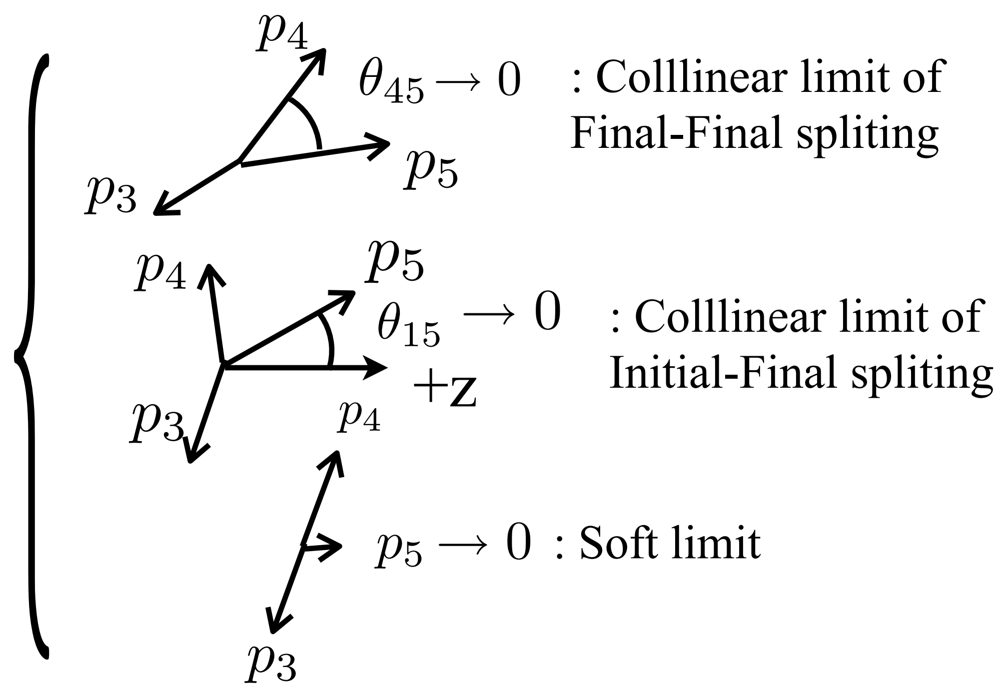


■ Automated check of all soft/collinear safety

- Generate soft/collinear limits

$g(1)g(2) \to u(3)\bar{u}(4)g(5)$





- Check the cancellation of the singularities

- Visualize in the plot

3. Results

3-1. $pp \to t\bar{t} + 1 \text{ jet}$

3-2. $pp \to W^+W^- + 1 \text{ jet}$

3-3. Further checks

3-1. $pp \to t\bar{t} + 1$ jet

- NLO real emission corrections

$$gg \to t\bar{t}gg \qquad 0 \to t\bar{t}gggg \qquad + \text{crossings}$$

$$\left.\begin{array}{l} q\bar{q} \to t\bar{t}gg \\ qg \to t\bar{t}gq \\ \bar{q}g \to t\bar{t}\bar{q}g \\ gg \to t\bar{t}\bar{q}q \end{array}\right\} \quad 0 \to t\bar{t}q\bar{q}gg \qquad + \text{crossings}$$

$$\left.\begin{array}{l} q\bar{q} \to t\bar{t}\bar{q}q \\ \quad\vdots \end{array}\right\} \quad 0 \to t\bar{t}q\bar{q}q\bar{q} \qquad + \text{crossings}$$

$$\left.\begin{array}{l} qq' \to t\bar{t}qq' \\ \quad\vdots \end{array}\right\} \quad 0 \to t\bar{t}q\bar{q}q'\bar{q}' \qquad + \text{crossings}$$

- Run of the package at process : $gg \to t\bar{t}gg$

  - Contents of dipole terms (Output of Mathematica code)

```
Number of dipoles

[Dipole1] : 36

 B1 : 36

{Splitting (1):(i,j)=(f,g)}:  16 (16)

                [1.(ij,k)=(fg,k): Dij,k]  8 (8)

                [2.(ij,a)=(fg,a): Dij^a]  8 (8)

{Splitting (2):(i,j)=(g,g)}:  4 (2)

                [3.(ij,k)=(gg,k): Dij,k]  2 (2)

                [4.(ij,a)=(gg,a): Dij^a]  2 (0)

{Splitting (3):(a,i)=(f,g)}:  0 (0)

                [5.(ai,k)=(fg,k): D^ai,k]  0 (0)

                [6.(ai,b)=(fg,b): D^ai,b]  0 (0)

{Splitting (4):(a,i)=(g,g)}:  16 (8)

                [7.(ai,k)=(gg,k): D^ai,k]  12 (8)

                [8.(ai,b)=(gg,b): D^ai,b]  4 (0)

-------------------
```

of the dipoles

of the dipoles with massive quark

$\mathrm{D}_{g(i)g(j),k}$

- All soft/collinear limits (Output of Mathematica code)

```
The collinear and soft limits and the corresponding dipoles

NLO: {{g, p[1]}, {g, p[2]}} --> {{t, p[3]}, {tbar, p[4]}, {g, p[5]}, {g, p[6]}}

--------------------

Collinear pairs        Corresponding dipoles

1. {3, 5}               {1, 2, 9, 10}

2. {4, 5}               {3, 4, 11, 12}

3. {3, 6}               {5, 6, 13, 14}

4. {4, 6}               {7, 8, 15, 16}

5. {5, 6}               {17, 18, 19, 20}

6. {1, 5}               {21, 22, 23, 33}

7. {2, 5}               {24, 25, 26, 34}

8. {1, 6}               {27, 28, 29, 35}

9. {2, 6}               {30, 31, 32, 36}

--------------------

Soft gluon   Collinear assemble    Corresponding dipoles

1. {5}       {1, 2, 5, 6, 7}
 {1, 2, 3, 4, 9, 10, 11, 12, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 33, 34}

2. {6}       {3, 4, 5, 8, 9}
 {5, 6, 7, 8, 13, 14, 15, 16, 17, 18, 19, 20, 27, 28, 29, 30, 31, 32, 35, 36}

--------------------
```

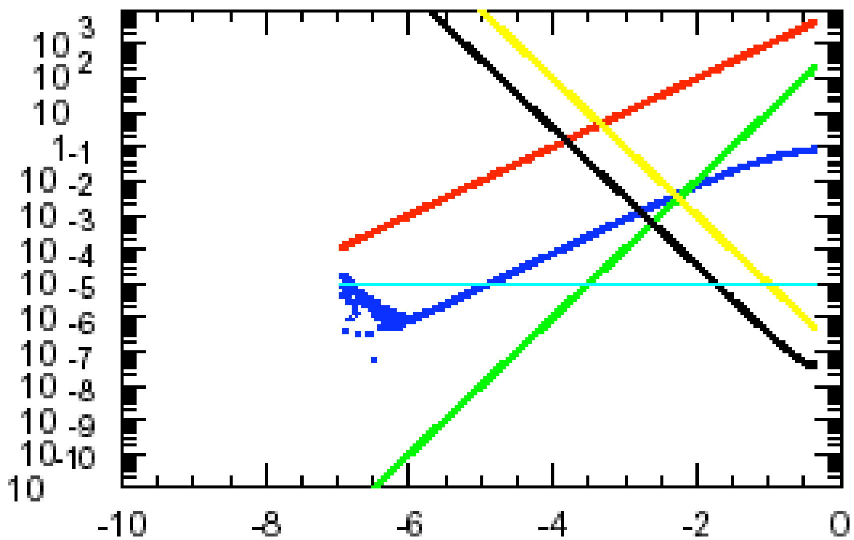
$$\lim_{L(i)} \left[|M|^2 - \sum_{j \subset S(i)} D(j) \right]$$

$$L(5) = \{\theta_{56} \to 0\}$$
$$S(5) = \{D(17), D(18), D(19), D(20)\}$$

$$L(10) = \{E_5 \to 0\}$$
$$S(10) = \{S(1), S(2), S(5), S(6), S(7)\}$$

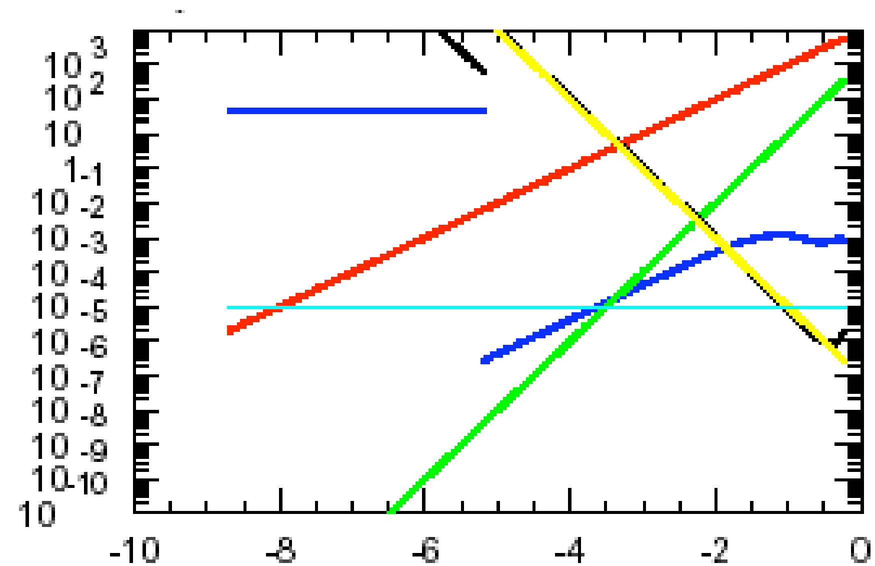
- Plots of all soft/collinear limits

- Collinear limit: $L(6) = \{\theta_{15} \to 0\}$



$$x = \log\left(\sqrt{\frac{s_{15}}{s_{12}}}\right)$$

- Soft limit: $L(10) = \{E_5 \to 0\}$



$$x = \log\left(\frac{2E_5}{\sqrt{s_{12}}}\right)$$

$\log |M(gg \to t\bar{t}gg)|^2$

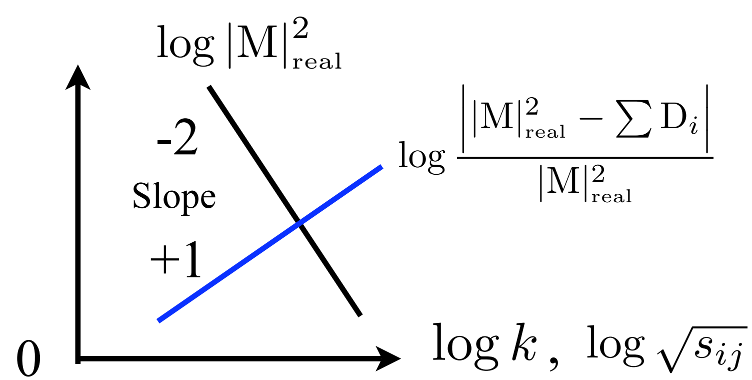
$\log \frac{\left||M|^2 - \sum D_i\right|}{|M|^2}$

$-2x$

x

$2x$

- Prediction



→ We can confirm the cancellation

- Complete agreement (at least 12 digits) with the results in

S. Dittmaier, P. Uwer and S. Weinzierl, Eur. Phys. J. C59 (2009) 625

$$b_0 = |\bar{\mathrm{M}}|^2$$

$$d_0 = \sum_{all} \mathrm{D}(i)$$

	$b_0[\mathrm{GeV}^{-4}]$	$d_0[\mathrm{GeV}^{-4}]$
	$g(p_a)g(p_b) \to t(p_t)\bar{t}(p_{\bar{t}})g(p_c)g(p_d)$	
AutoDipole	$7.820396708696\|13 \cdot 10^{-10}$	$1.02594003852 4\|07 \cdot 10^{-9}$
Version 2	$7.820396708696\|05 \cdot 10^{-10}$	$1.02594003852 4\|05 \cdot 10^{-9}$
	$q(p_a)\bar{q}(p_b) \to t(p_t)\bar{t}(p_{\bar{t}})g(p_c)g(p_d)$	
AutoDipole	$1.120772113616\|20 \cdot 10^{-10}$	$1.22619016939 9\|00 \cdot 10^{-10}$
Version 2	$1.120772113616\|19 \cdot 10^{-10}$	$1.22619016939 9\|08 \cdot 10^{-10}$
	$q(p_a)g(p_b) \to t(p_t)\bar{t}(p_{\bar{t}})g(p_c)q(p_d)$	
AutoDipole	$2.7564127314678\|5 \cdot 10^{-11}$	$4.79768338384\|667 \cdot 10^{-11}$
Version 2	$2.7564127314678\|3 \cdot 10^{-11}$	$4.79768338384\|750 \cdot 10^{-11}$
	$\bar{q}(p_a)g(p_b) \to t(p_t)\bar{t}(p_{\bar{t}})\bar{q}(p_c)g(p_d)$	
AutoDipole	$3.4615016829595\|6 \cdot 10^{-11}$	$8.3455579589494\|2 \cdot 10^{-11}$
Version 2	$3.4615016829595\|3 \cdot 10^{-11}$	$8.3455579589494\|0 \cdot 10^{-11}$
	$g(p_a)g(p_b) \to t(p_t)\bar{t}(p_{\bar{t}})\bar{q}(p_c)q(p_d)$	
AutoDipole	$1.214205201147\|80 \cdot 10^{-11}$	$2.13553289076\|589 \cdot 10^{-11}$
Version 2	$1.214205201147\|79 \cdot 10^{-11}$	$2.13553289076\|600 \cdot 10^{-11}$
	$q(p_a)\bar{q}(p_b) \to t(p_t)\bar{t}(p_{\bar{t}})\bar{q}(p_c)q(p_d)$	
AutoDipole	$5.1371095999006\|8 \cdot 10^{-12}$	$9.0633090240835\|6 \cdot 10^{-12}$
Version 2	$5.1371095999006\|3 \cdot 10^{-12}$	$9.0633090240835\|0 \cdot 10^{-12}$

- The values of the version 2 is used
(Version 1 and 2 agrees at least 14 digits for $|\bar{\mathrm{M}}|^2$ and 12 digits for dipoles)

3-2. $pp \to W^+W^- + 1$ jet

One mode of NLO real emission process $u\bar{u} \to W^+W^-gg$

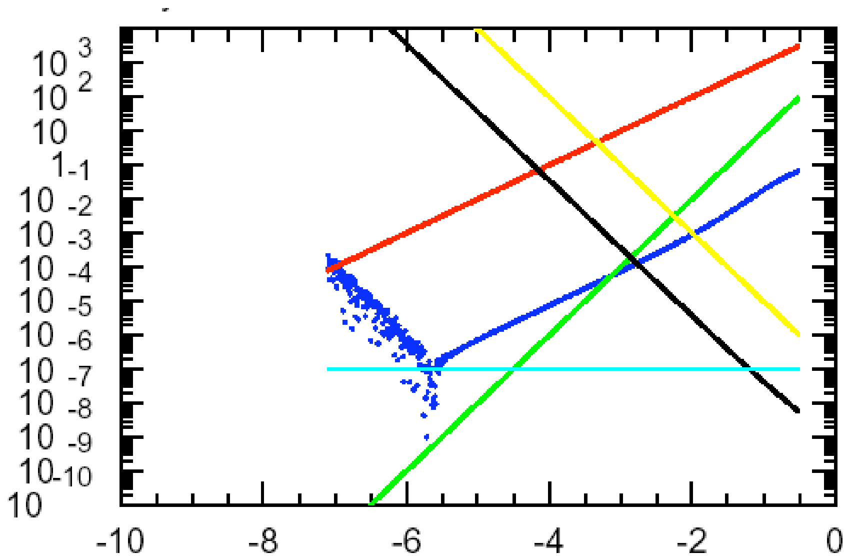
- Contents of dipole terms

```
Number of dipoles
[Dipole1] : 10
 B1 : 10
{Splitting (1):(i,j)=(f,g)}:  0 (0)
            [1.(ij,k)=(fg,k): Dij,k]  0 (0)
            [2.(ij,a)=(fg,a): Dij^a]  0 (0)
{Splitting (2):(i,j)=(g,g)}:  2 (0)
            [3.(ij,k)=(gg,k): Dij,k]  0 (0)
            [4.(ij,a)=(gg,a): Dij^a]  2 (0)
{Splitting (3):(a,i)=(f,g)}:  8 (0)
            [5.(ai,k)=(fg,k): D^ai,k]  4 (0)
            [6.(ai,b)=(fg,b): D^ai,b]  4 (0)
{Splitting (4):(a,i)=(g,g)}:  0 (0)
            [7.(ai,k)=(gg,k): D^ai,k]  0 (0)
            [8.(ai,b)=(gg,b): D^ai,b]  0 (0)
-------------------
[Total] : 10
 (Massive dipoles : 0)
-------------------
END
```

- All soft/collinear limits

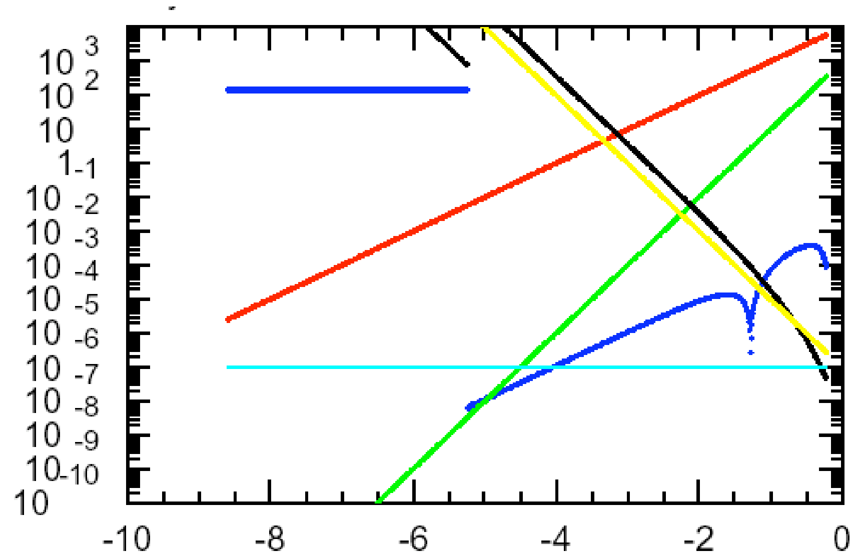
```
**********************************************************************
The collinear and soft limits and the corresponding dipoles
NLO: {{u, p[1]}, {ubar, p[2]}} --> {{Wp, p[3]}, {Wm, p[4]}, {g, p[5]}, {g, p[6]}}
------------------
Collinear pairs      Corresponding dipoles
1. {5, 6}             {1, 2}
2. {1, 5}             {3, 7}
3. {2, 5}             {4, 8}
4. {1, 6}             {5, 9}
5. {2, 6}             {6, 10}
------------------
Soft gluon   Collinear assemble    Corresponding dipoles
1. {5}       {1, 2, 3}          {1, 2, 3, 4, 7, 8}
2. {6}       {1, 4, 5}          {1, 2, 5, 6, 9, 10}
------------------
END
```

- Collinear limit: $\theta_{25} \rightarrow 0$

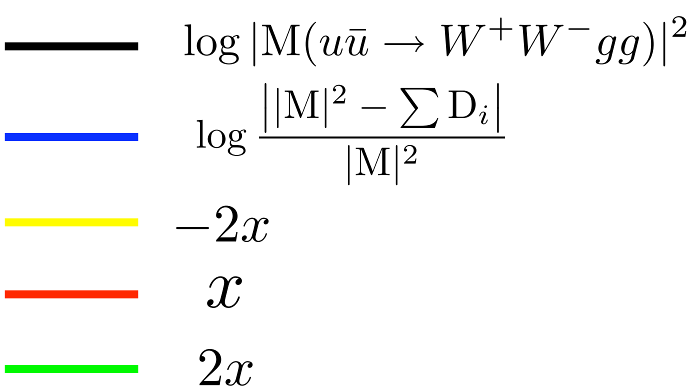


$$x = \log\left(\sqrt{\frac{s_{25}}{s_{12}}}\right)$$

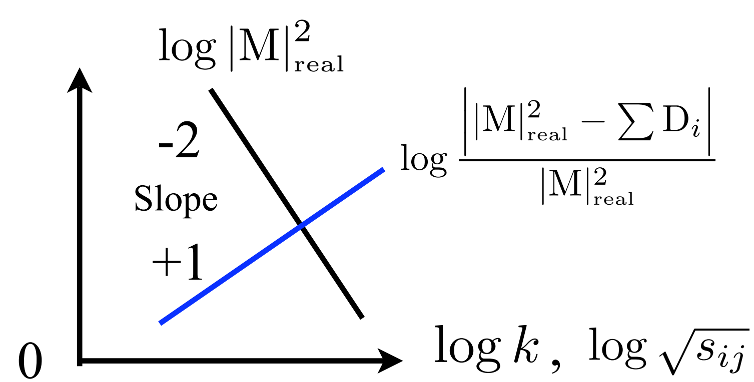
- Soft limit: $\mathrm{E}_5 \rightarrow 0$



$$x = \log\left(\frac{2\mathrm{E}_5}{\sqrt{s_{12}}}\right)$$



- Prediction



- Complete agreement with the results

S. Dittmaier, S. Kallweit, P. Uwer, Phys.Rev.Lett.100(2008)062003, arXiv:0710.1577 [hep-ph]

-Input

$$u(1)\bar{u}(2) \to W^+(3)W^-(4)g(5)g(6) \qquad m_W = 80.425[\mathrm{GeV}]$$

1	0.5000000000000000D+03	0.0000000000000000D+00	0.0000000000000000D+00	0.5000000000000000D+03
2	0.5000000000000000D+03	0.0000000000000000D+00	0.0000000000000000D+00	-0.5000000000000000D+03
3	0.1329741000140751D+03	-0.6363565279102492D+02	-0.6997883706273866D+02	-0.4761718919424017D+02
4	0.2866753417240371D+03	-0.2542950722662938D+02	0.1806402621336436D+03	0.2060024436410712D+03
5	0.2620781822395729D+03	0.2202901744039709D+03	-0.6239572701521099D+02	0.1275303333223518D+03
6	0.3182723760223148D+03	-0.1312250143863166D+03	-0.4826569805569400D+02	-0.2859155877691829D+03

-Results

	$\lvert\bar{\mathrm{M}}\rvert^2$	$\sum_{all} \mathrm{D}(i)$
AutoDipole:	0.62740253709801**2** × 10⁻⁹	0.11414993487832**0** × 10⁻⁹
DKU:	0.62740253709800**7** × 10⁻⁹	0.11414993487831**9** × 10⁻⁹

13 digits agreement

3-3.Further check

- We checked the cancellation of all soft/collinear singularities in the following real emission processes

	$2 \to 3$	$2 \to 4$	$2 \to 5$	$2 \to 6$
Massless				
(Including lepton)	$e^+e^- \to u\bar{u}g$			
	$e^-u \to e^-ug$			
	$e^-g \to e^-u\bar{u}$			
	$u\bar{u} \to e^+e^-g$			
(Parton only)	$gg \to u\bar{u}g$	$gg \to u\bar{u}gg$	$u\bar{u} \to d\bar{d}ggg$	
	$gg \to 3g$	$gg \to 4g$		
	$u\bar{u}g \to d\bar{d}g$			
(Including W/Z boson)		$\bar{u}u \to W^+W^-gg$	$gg \to W^+\bar{u}dgg$	
Massive				
(Including lepton)	$e^+e^- \to t\bar{t}g$			
(Parton only)	$gg \to t\bar{t}g$	$gg \to t\bar{t}gg$	$gg \to t\bar{t}ggg$	$u\bar{u} \to t\bar{t}b\bar{b}gg$
		$u\bar{u} \to t\bar{t}gg$	$gg \to t\bar{t}b\bar{b}g$	
		$ug \to t\bar{t}ug$		
		$\bar{u}g \to t\bar{t}\bar{u}g$		
		$gg \to t\bar{t}u\bar{u}$		
		$u\bar{u} \to t\bar{t}u\bar{u}$		

4. Outlook

- Summary

  - Dipole subtraction is a general and practical procedure at QCD NLO
  - Automated dipole subtraction: AutoDipole (Version 1.0 beta) publicly available
    - Mathematica code and an interface with MadGraph
  - Application to QCD backgrounds in LHC
    - Complete agreement for

      $pp \to t\bar{t} + 1\ \mathrm{jet}\ : gg \to t\bar{t}gg \quad u\bar{u} \to t\bar{t}gg \quad ug \to t\bar{t}ug \ \ldots$

      $pp \to W^+W^- + 1\ \mathrm{jet}\ : \bar{u}d \to W^+W^-gg$

    - Checks of all soft/collinear safeties in many processes

      Complex ones: $gg \to t\bar{t}ggg \quad gg \to t\bar{t}b\bar{b}g \quad gg \to W^+\bar{u}dgg$

- Plan

  - Long write-up paper in preparation
  - Apply to interesting LHC processes
  - Automate the creation of the integrated dipole

Extra Slides

Process: $gg \to t\bar{t}g$

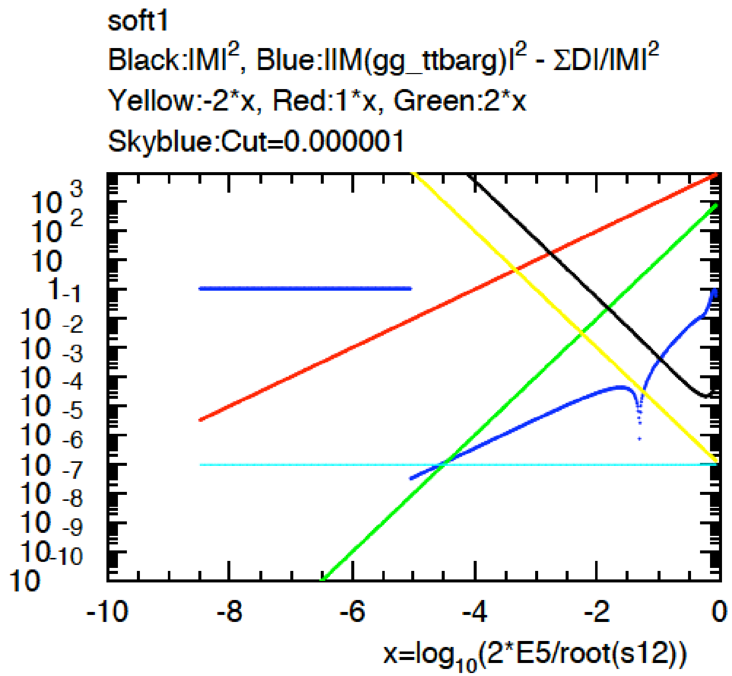
- Contents of dipole terms

```
Number of dipoles

[Dipole1] : 12

 B1 : 12

{Splitting {1}:{i,j}={f,g}}:  6 (6)

               [1.{ij,k}={fg,k}: Dij,k]  2 (2)

               [2.{ij,a}={fg,a}: Dij^a]  4 (4)

{Splitting {2}:{i,j}={g,g}}:  0 (0)

               [3.{ij,k}={gg,k}: Dij,k]  0 (0)

               [4.{ij,a}={gg,a}: Dij^a]  0 (0)

{Splitting {3}:{a,i}={f,g}}:  0 (0)

               [5.{ai,k}={fg,k}: D^ai,k]  0 (0)

               [6.{ai,b}={fg,b}: D^ai,b]  0 (0)

{Splitting {4}:{a,i}={g,g}}:  6 (4)

               [7.{ai,k}={gg,k}: D^ai,k]  4 (4)

               [8.{ai,b}={gg,b}: D^ai,b]  2 (0)
-------------------

-------------------

[Total] : 12

 (Massive dipoles : 10)
-------------------
END
```

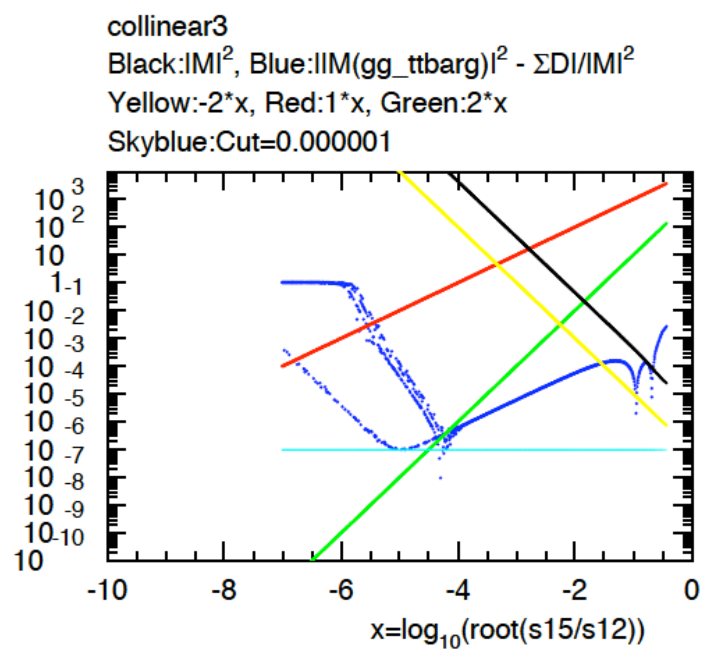
- All soft/collinear limits

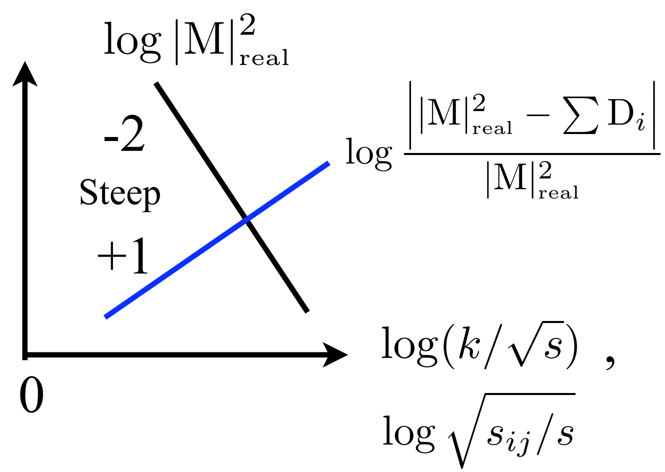
```
*********************************************************************

The collinear and soft limits and the corresponding dipoles

NLO: {{g, p[1]}, {g, p[2]}} --> {{t, p[3]}, {tbar, p[4]}, {g, p[5]}}
-------------------

Collinear pairs       Corresponding dipoles

1. {3, 5}              {1, 3, 4}

2. {4, 5}              {2, 5, 6}

3. {1, 5}              {7, 8, 11}

4. {2, 5}              {9, 10, 12}

Collinear pairs

1.

2.

include a massive quark
-------------------

Soft gluon    Collinear assemble    Corresponding dipoles

1. {5}        {1, 2, 3, 4}          {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
-------------------

END
```

- Soft limit check



- Collinear limit check





■ Process: $gg \to t\bar{t}ggg$

- One mode of NLO real emission process to $pp \to t\bar{t} + 2\text{jets}$
  - Background to the Higgs production mode: $pp \to t\bar{t}\ H$

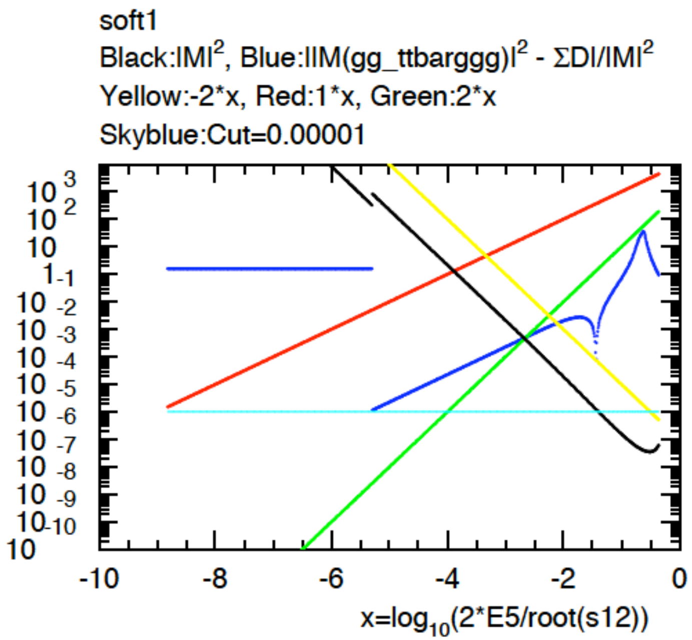
- Contents of dipole terms

```
Number of dipoles

[Dipole1] : 75

 B1 : 75

  {Splitting (1):(i,j)=(f,g)}:  30 (30)

                [1.(ij,k)=(fg,k): Dij,k]  18 (18)

                [2.(ij,a)=(fg,a): Dij^a]  12 (12)

  {Splitting (2):(i,j)=(g,g)}:  15 (6)

                [3.(ij,k)=(gg,k): Dij,k]  9 (6)

                [4.(ij,a)=(gg,a): Dij^a]  6 (0)

  {Splitting (3):(a,i)=(f,g)}:  0 (0)

                [5.(ai,k)=(fg,k): D^ai,k]  0 (0)

                [6.(ai,b)=(fg,b): D^ai,b]  0 (0)

  {Splitting (4):(a,i)=(g,g)}:  30 (12)

                [7.(ai,k)=(gg,k): D^ai,k]  24 (12)

                [8.(ai,b)=(gg,b): D^ai,b]  6 (0)

  -------------------
  --------------------

  [Total] : 75

   (Massive dipoles : 48)

  --------------------

  END
```

- All soft/collinear limits

```
**************************************************************************

The collinear and soft limits and the corresponding dipoles

NLO: {{g, p[1]}, {g, p[2]}} --> {{t, p[3]}, {tbar, p[4]}, {g, p[5]}, {g, p[6]}, {g, p[7]}}

 Collinear pairs        Corresponding dipoles

 1. {3, 5}               {1, 2, 3, 19, 20}

 2. {4, 5}               {4, 5, 6, 21, 22}

 3. {3, 6}               {7, 8, 9, 23, 24}

 4. {4, 6}               {10, 11, 12, 25, 26}

 5. {3, 7}               {13, 14, 15, 27, 28}

 6. {4, 7}               {16, 17, 18, 29, 30}

 7. {5, 6}               {31, 32, 33, 40, 41}

 8. {5, 7}               {34, 35, 36, 42, 43}

 9. {6, 7}               {37, 38, 39, 44, 45}

 10. {1, 5}               {46, 47, 48, 49, 70}

 11. {2, 5}               {50, 51, 52, 53, 71}

 12. {1, 6}               {54, 55, 56, 57, 72}

 13. {2, 6}               {58, 59, 60, 61, 73}

 14. {1, 7}               {62, 63, 64, 65, 74}

 15. {2, 7}               {66, 67, 68, 69, 75}

 --------------------

 Soft gluon   Collinear assemble    Corresponding dipoles

 1. {5}        {1, 2, 7, 8, 10, 11}          {1, 2, 3, 4, 5, 6, 19, 20, 21,
   22, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 46, 47, 48, 49, 50, 51, 52, 53, 70, 71}

 2. {6}        {3, 4, 7, 9, 12, 13}          {7, 8, 9, 10, 11, 12, 23, 24, 25,
   26, 31, 32, 33, 37, 38, 39, 40, 41, 44, 45, 54, 55, 56, 57, 58, 59, 60, 61, 72, 73}

 3. {7}        {5, 6, 8, 9, 14, 15}          {13, 14, 15, 16, 17, 18, 27, 28,
   29, 30, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75}

 --------------------

 END
```

- Soft limit check



- Collinear limit check