Constraining the Higgs boson mass: a non-perturbatice lattice study

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in collaboration with

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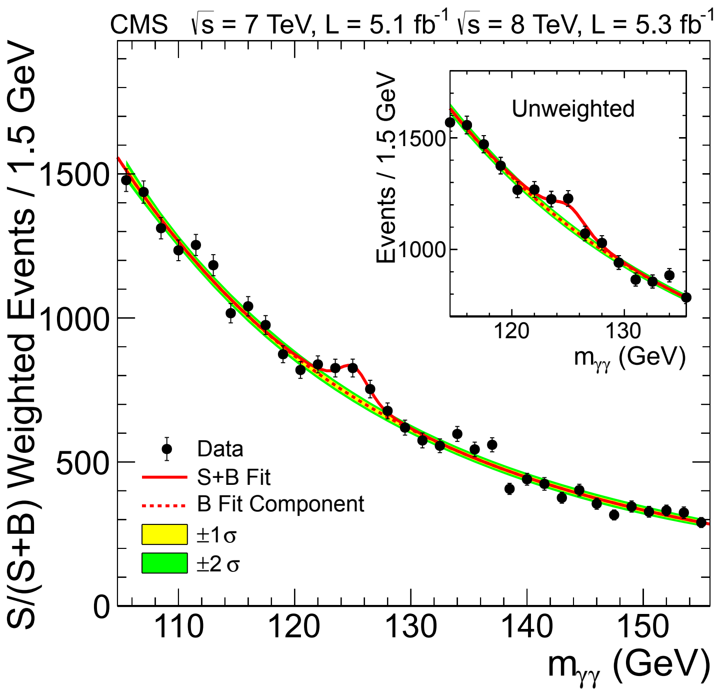
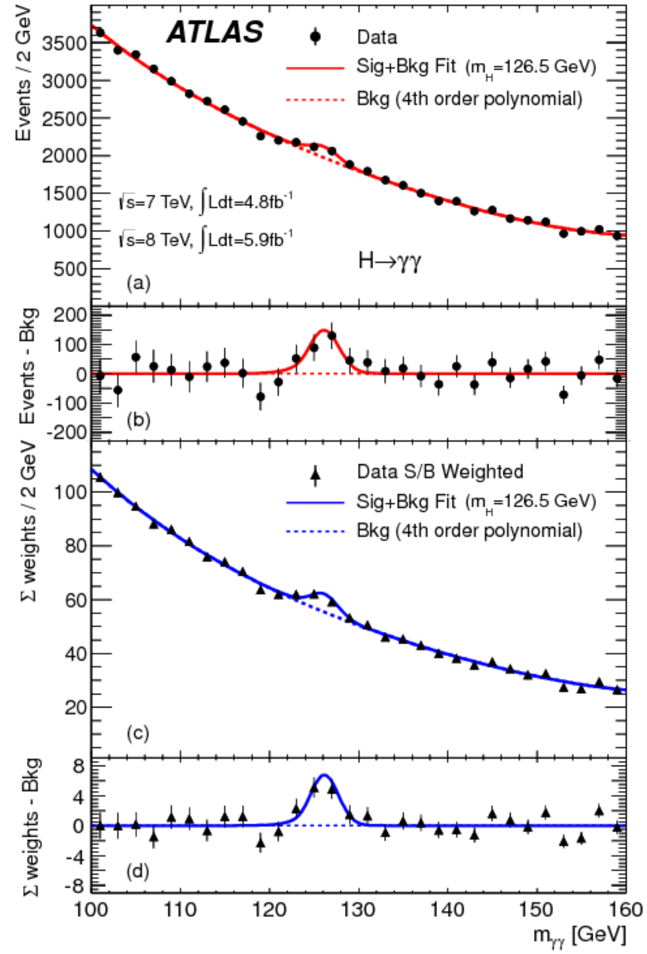
- **Why Higgs-Yukaw model on the lattice?**
- **Higgs-Yukawa sector at physical values of the top quark mass**
  - Non-perturbative lower and upper Higgs boson mass bounds
  - Higgs boson resonance parameters
- **Higgs-Yukawa sector at very heavy fermion masses**
  - Non-perturbative lower and upper Higgs boson mass bounds
  - non-zero temperature
- **Conclusion**

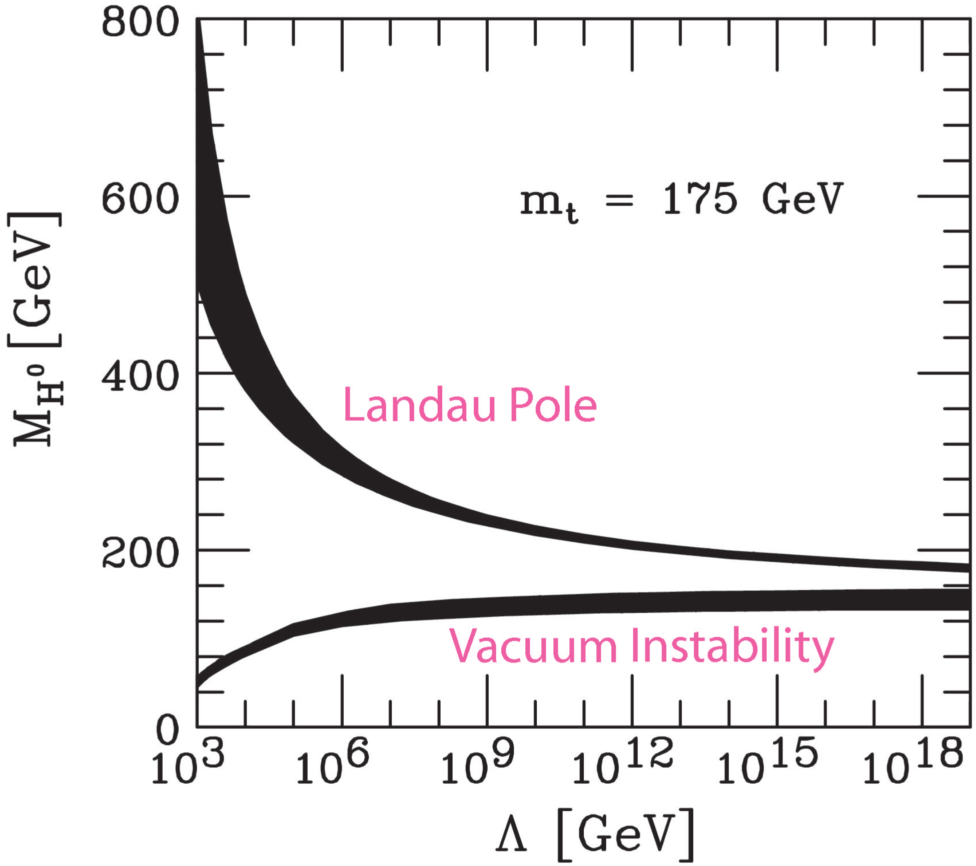
The evidence for a scalar particle at the LHC





CMS **ATLAS**

Theoretical bounds on the Higgs boson mass



- upper bound from triviality
- lower bound from vacuum instability

Why a lattice calculation?

- upper Higgs boson mass bound:
  $\longrightarrow$ coupling can become strong,
  a priori unclear whether perturbation theory is valid

- lower bound:
  $\longrightarrow$ is vacuum instability an artefact of perturbation theory?

- 4th generation: also Yukawa coupling can become strong
  $\longrightarrow$ what are the effects on the mass bounds?

The scalar lattice action

- continuum action

$$S_\varphi[\varphi] = \sum_{x,\mu} \tfrac{1}{2}\partial_\mu\varphi_x^\dagger\partial_\mu\varphi_x + \sum_x \tfrac{1}{2}m_0^2\varphi_x^\dagger\varphi_x + \sum_x \lambda\left(\varphi_x^\dagger\varphi_x\right)^2,$$

with $\partial_\mu\varphi(x) \to \nabla_{\mathrm{latt}}\varphi(x) = (\varphi(x+a\mu) - \varphi(x))/a$

and a rescaling $\Phi(x) = \sqrt{2\kappa}\varphi(x)$, $\lambda = \frac{\hat\lambda}{4\kappa^2}$, $m_0^2 = \frac{1-2\hat\lambda-8\kappa}{\hat\kappa}$

- lattice scalar action (setting lattice spacing $a = 1$)

$$S_\Phi = -\kappa\sum_{x,\mu}\Phi_x^\dagger\left[\Phi_{x+\hat\mu} + \Phi_{x-\hat\mu}\right] + \sum_x \Phi_x^\dagger\Phi_x + \hat\lambda\sum_x\left(\Phi_x^\dagger\Phi_x - 1\right)^2$$

Chiral invariant Higgs-Yukawa lattice action (Lüscher)

- the lattice fermionic and Yukawa parts

$$(L_F + L_Y)[\bar{\psi}, \psi] = \bar{\psi} D_{\text{ov}} \psi + y_b \left(\bar{t}, \bar{b}\right)_L \varphi b_R + y_t \left(\bar{t}, \bar{b}\right)_L \tilde{\varphi} t_R + c.c.$$

- change from continuum:
  - $i\gamma_\mu \partial_\mu \to D_{\text{ov}}$
  - $P_\pm = \frac{1\pm\gamma_5}{2} \to \hat{P}_\pm = \frac{1\pm\hat{\gamma}_5}{2}, \hat{\gamma}_5 = \gamma_5 (1 - aD_{\text{ov}})$

- exact *lattice* $\mathrm{SU}(2)_L$ chiral symmetry: $\gamma_5 D_{\text{ov}} + D_{\text{ov}} \gamma_5 = a D_{\text{ov}} \gamma_5 D_{\text{ov}}$

  Ginsparg-Wilson relation overlap operator D_{ov} Neuberger

$$\psi \to \hat{P}_+ \psi + \Omega_L \hat{P}_- \psi, \bar{\psi} \to \bar{\psi} P_+ \Omega_L^\dagger + \bar{\psi} P_-$$

$$\phi \to \phi \Omega_L^\dagger, \phi^\dagger \to \Omega_L \phi^\dagger \;, \;\; \Omega_L \in \mathrm{SU}(2)$$

- fully emulates Higgs-Yukawa sector of the standard model

The algorithm

Usage of Polynomial Hybrid Monte Carlo Algorithm (Frezzotti, K.J.)

improvements (Gerhold):

- special preconditioning techniques for fermion matrix:
  $\rightarrow$ factors of O(10)-O(100) improvement for condition number

- Fourier acceleration

	FACC	traLength	Nconf	ACtime	cost
$\kappa = 0.12313$	No	2.0	2020	132.1 ± 6.4	2662 ± 129
$\kappa = 0.12313$	Yes	2.0	21780	1.1 ± 0.1	37 ± 1
$\kappa = 0.30400$	No	1.0	2580	34.9 ± 2.1	450 ± 28
$\kappa = 0.30400$	Yes	1.0	22360	3.8 ± 0.2	171 ± 8

- exact Krylow space reweighting

- multiple time scale integrators

Physical setup

- physical input Higgs boson expectation value $v_r/a = 246$ GeV
  top and bottom quark masses: $m_t/a \approx 175$ GeV, $m_b/a \approx 4.2$ GeV

- renormalized quartic coupling: $\lambda = \frac{m_H^2}{v_r^2}$

- renormalized Yukawa couplings: $y_{t,b} = \frac{m_{t,b}}{v_r}$

- setting the value of the lattice spacing 246 GeV $= \frac{v_r}{a} \equiv \frac{v}{\sqrt{Z_G} \cdot a}, \Lambda = a^{-1}$

- renormalization constant from Goldstone propagator $\left[\tilde{G}_G(\hat{p}^2)\right]^{-1} = \frac{\hat{p}^2 + m_{Gp}^2}{Z_G}$

Present setup

- gauge fields are neglected

- mass degenerate quark doublet
  (check of effect by lattice perturbation theory)

Lower bound

very useful guidance and theoretical control from lattice effective potential

$$U[v] = \tfrac{1}{2}m^2v^2 + \lambda v^4 + U_{\text{impr}}[v] + U_F[v]$$

$$U_{\text{impr}}[v] = \lambda v^2 \tfrac{1}{L_s^3 \cdot L_t} \left[\sum_p \tfrac{6}{\hat{p}^2 + m_{Hp}^2} + \sum_{0 \neq p} \tfrac{6}{\hat{p}^2 + m_{Gp}^2} \right]$$

$$U_F[v] = \tfrac{-2N_f}{L_s^3 \cdot L_t} \cdot \sum_p \log \left| \nu^+(p) + yv \left(1 - \tfrac{1}{2}\nu^+(p)\right) \right|^2$$

$$\nu^{\pm}(p) = 1 + \frac{\pm i \sqrt{\tilde{p}^2} + \frac{1}{2}\hat{p}^2 - 1}{\sqrt{\tilde{p}^2 + (\frac{1}{2}\hat{p}^2 - 1)^2}}$$

$$\tilde{p}^2 = \sum_{\mu=0}^{3} \sin^2(p_\mu), \quad \hat{p}^2 = 4 \sum_{\mu=0}^{3} \sin^2\left(\tfrac{p_\mu}{2}\right)$$

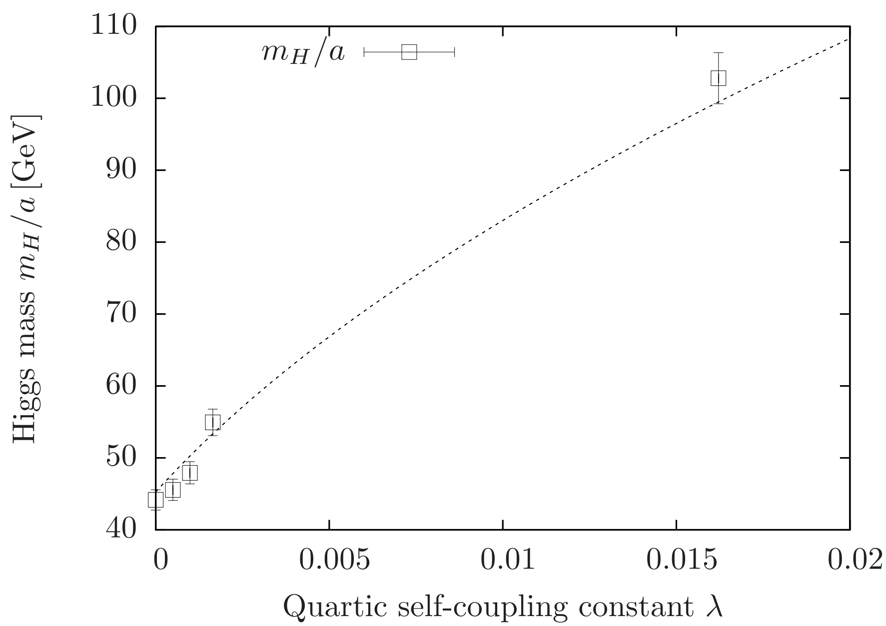
Lower bound

self-consistent determination of vacuum expectation value

$$0 = dU_{\text{eff}}/dv = -m^2 v - 4\lambda v^3 - \frac{\mathsf{d}}{\mathsf{d}v}(U_{\text{impr}}[v] + U_F[v])$$

and Higgs boson mass

$$m_{Hp}^2 = 12\lambda v^2 + \frac{\mathsf{d}^2}{\mathsf{d}v^2}(U_{\text{impr}}[v] + U_F[v])$$



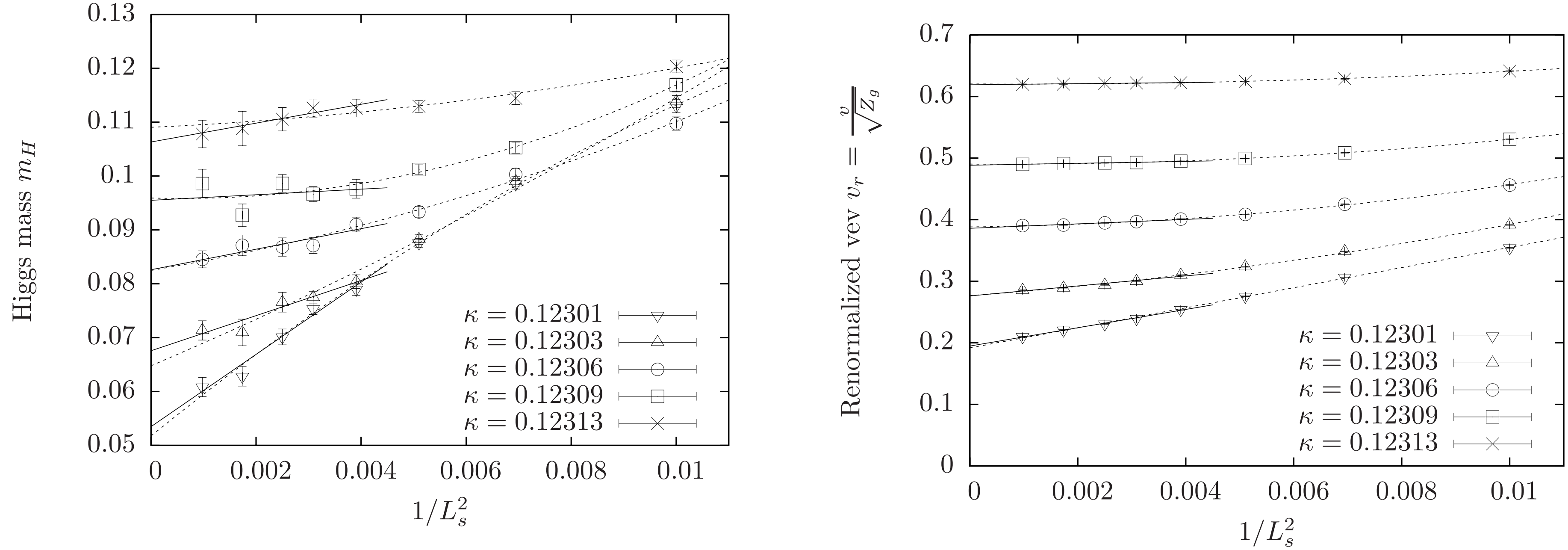
- fixed cut-off $\Lambda = 1/a$
- lower bound reached at $\lambda = 0$ (accordance with expectation from P.T.)
- agreement with lattice effective potential

Finite size effects

Goldstone bosons induce significant finite size effects of the form

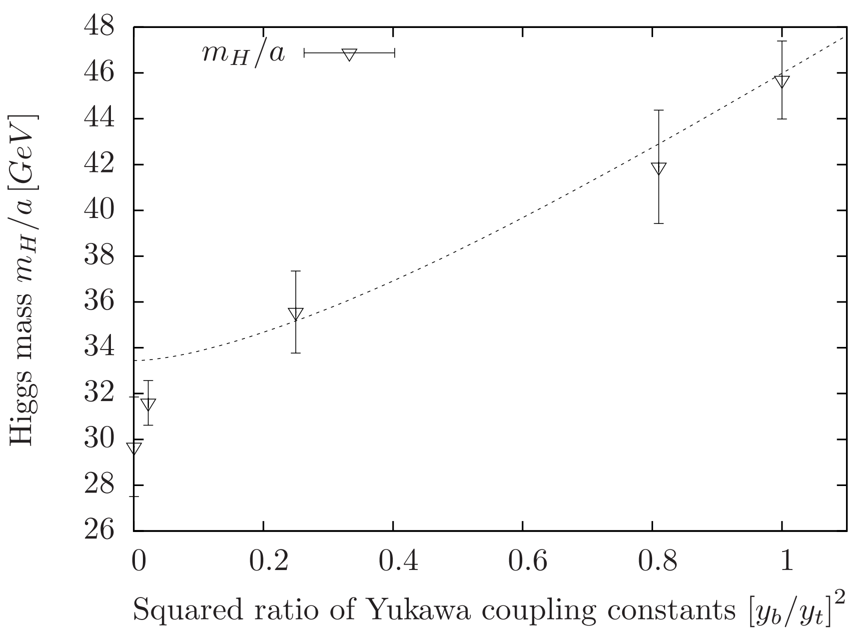
$$f_{v,m}^{(p)}(L_s^{-2}) = A_{v,m}^{(p)} + B_{v,m}^{(p)} \cdot L_s^{-2} + C_{v,m}^{(p)} \cdot L_s^{-4}$$

- data are well described by theoretical expectation (but had to go to lattices of size 40^4)
- allows infinite volume extrapolation
- use difference of only $1/L^2$ and combined $1/L^2 + 1/L^4$ fits as systematic errors



Effect of top-bottom mass splitting

- most simulations are for $y_t/y_b = 1$
  $\leftarrow$ less time consuming simulations
- see effects of mass-splitting

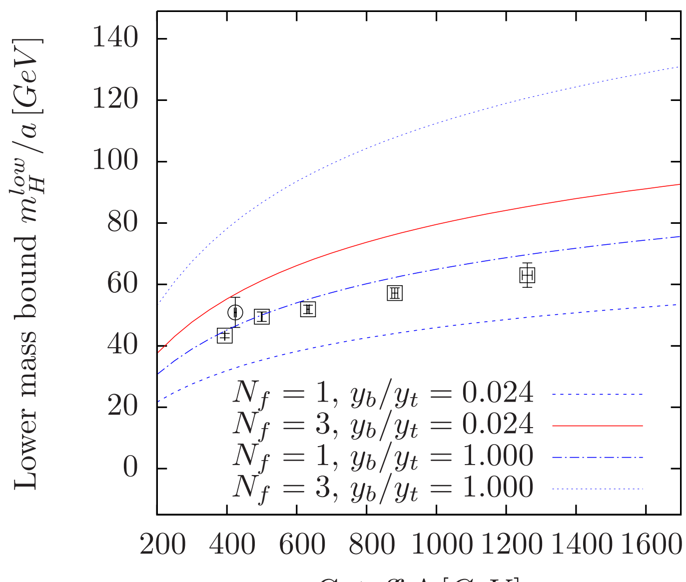


dashed line from

lattice effective potential

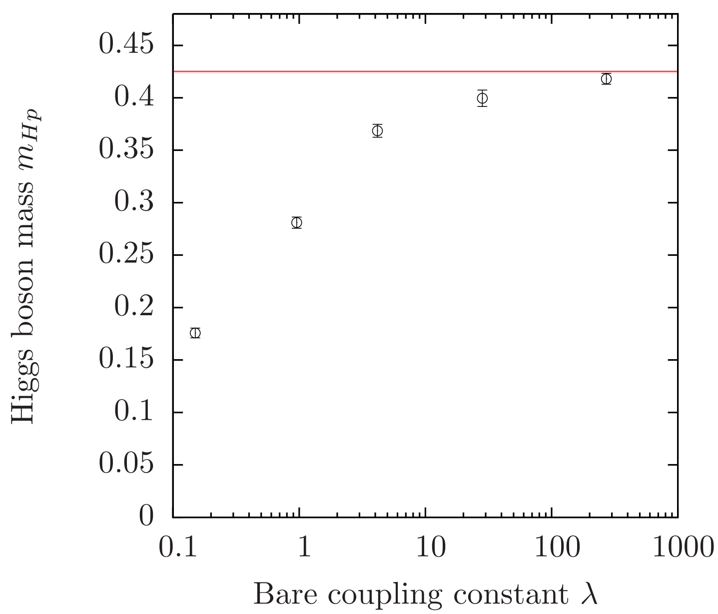
Result for lower Higgs boson mass bound

- data in infinite volume limit
- reliable description from effective potential
- most realistic: $N_f = 3, y_b/y_t = 0.024$ (circle in graph)



Largest Higgs boson mass at $\lambda = \infty$

- at a fixed cut-off Λ
- largest Higgs boson mass obtained at $\lambda = \infty$

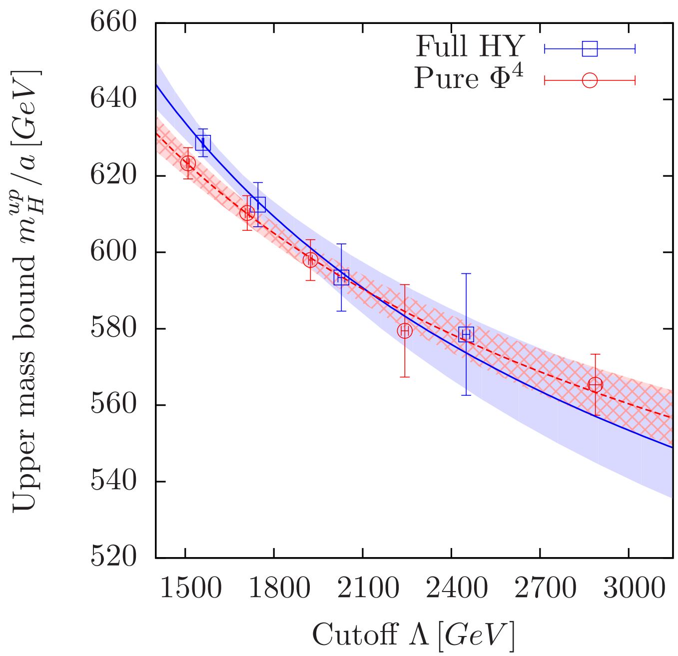


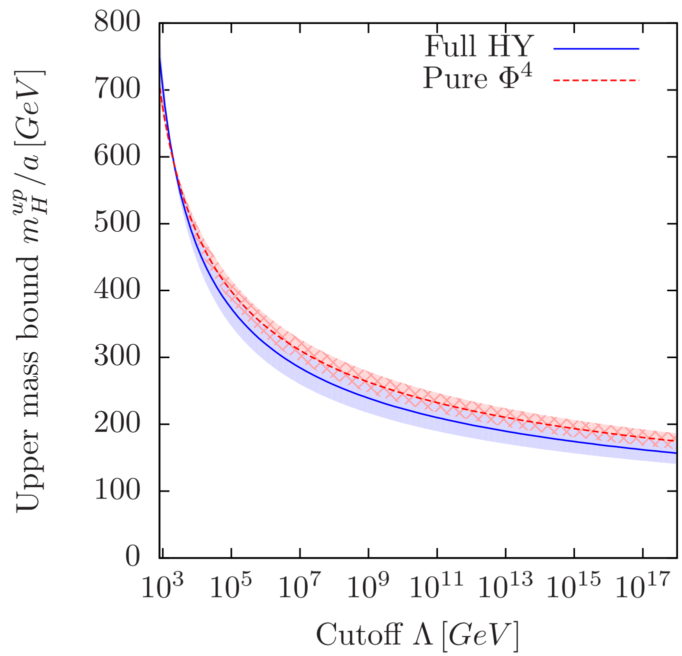
Upper Higgs boson mass bounds

fit data to expected theoretical dependence on cut-off Λ

$$\frac{m_{Hp}}{a} = A_m \cdot \left[\log(\Lambda^2/\mu^2) + B_m\right]^{-1/2}$$

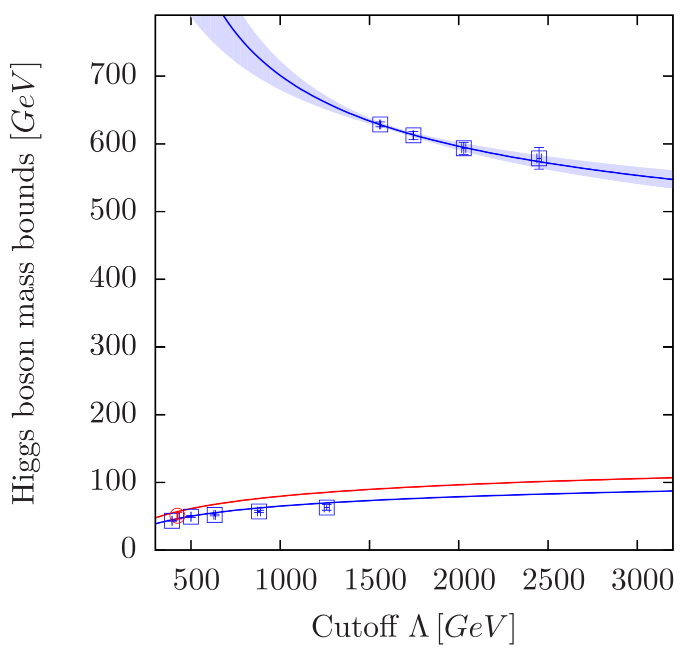
- *infinite volume* data are well described by theoretical expectation
  $\longrightarrow$ consistent with triviality of Higgs-Yukawa model
- compare pure Φ^4 theory and Higgs-Yukawa:
  $\longrightarrow$ see no significant effect





Lower and upper Higgs boson mass bounds

- cut-off depence of lower and upper bounds
- allowed range of Higgs boson mass:
  $50\text{GeV} < m_H < 650\text{GeV}$ at cut-off $\Lambda = 1.5\text{TeV}$



- When does experimental scalar boson mass cut the lower bound? (in progress)

Resonance parameters of Higgs boson from the lattice

Finite volume energy levels:

- measure two-particle Goldstone energy in center of mass frame

$$W = 2\sqrt{m^2 + k^2}$$

$\Rightarrow$ value of k

$\Rightarrow$ infinite volume scattering phase δ_0 (Lüscher)

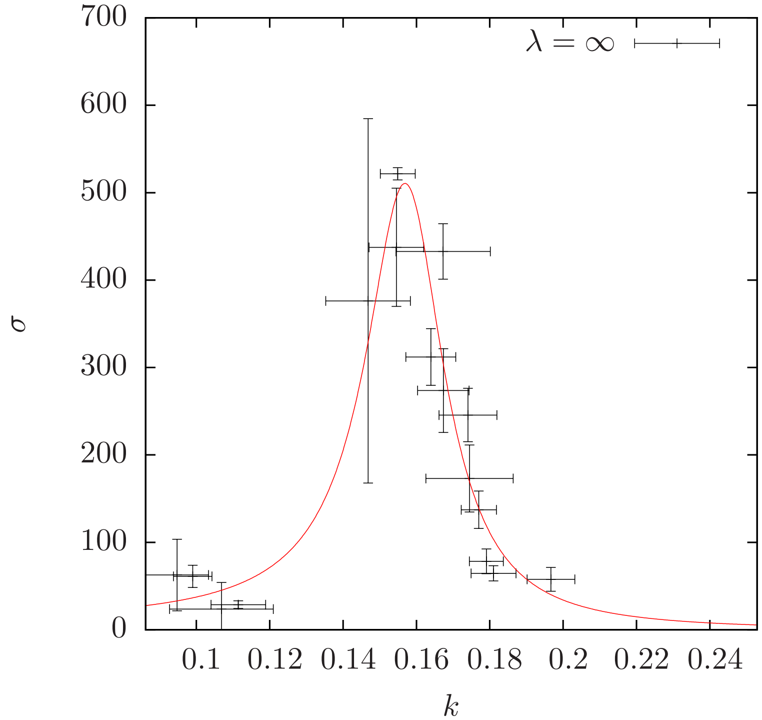
$$\tan\delta_0(k) = \frac{\pi^{\frac{3}{2}} q}{\mathcal{Z}_{00}(q^2)}, \quad q = \frac{kL}{2\pi}$$

$$\mathcal{Z}_{00}(q^2) = \sum_{\vec{n}\in\mathbb{Z}^3} \frac{1}{\sqrt{4\pi}} \frac{1}{n^2 - q^2}.$$

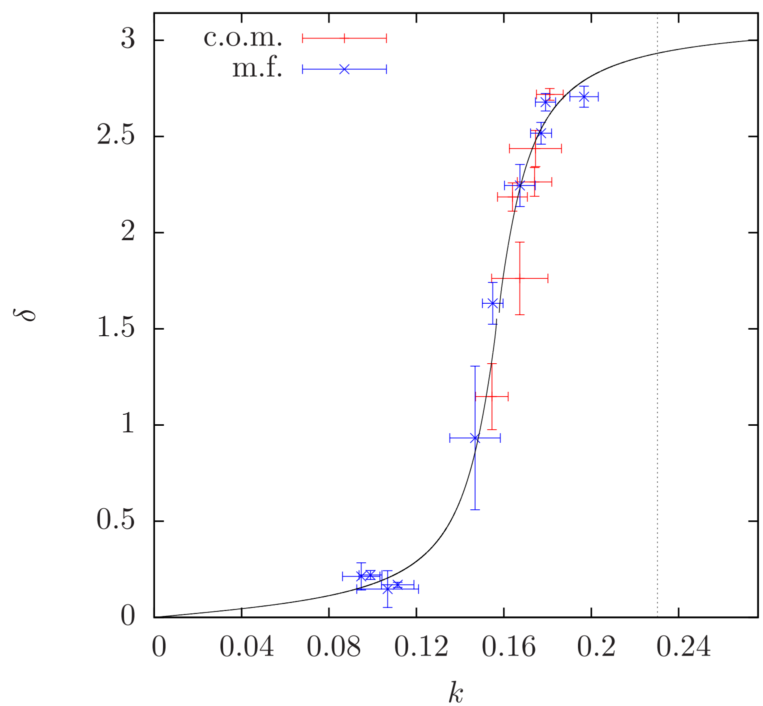
Generalization to moving frames (Gottlieb, Rummukainen; Feng, Renner, K.J.)

$\rightarrow$ many more finite volume enegy levels

Scattering phase and cross section

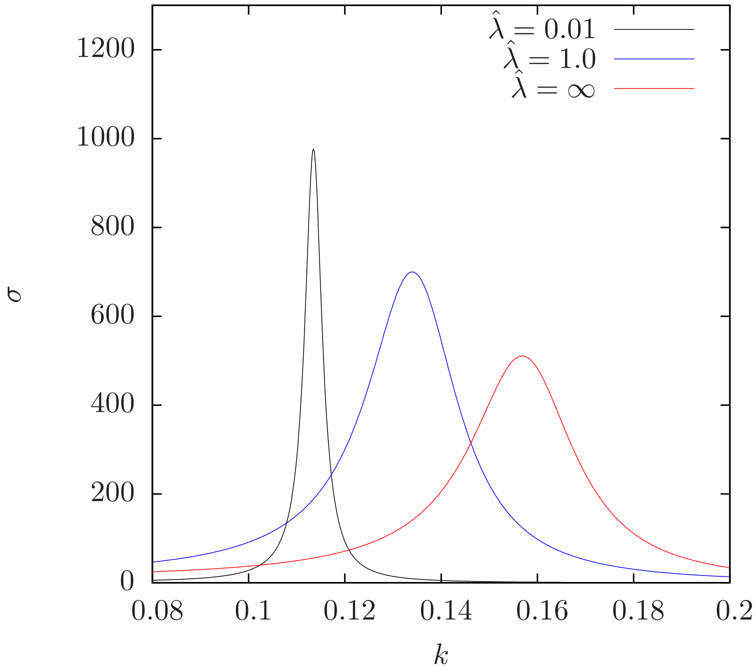


cross section



scattering phase

Coupling dependence of Higgs boson width



Breit-Wigner fit

$$f(k) = 16\pi \frac{M_H^2 \Gamma_H^2}{(M_H^2 - 4m_G^2)((W_k^2 - M_H^2)^2 + M_H^2 \Gamma_H^2)}$$

λ	aM_H	$a\Gamma_H$	$a\Gamma_H^{\mathrm{pert}}$
0.01	0.2811(6)	0.007(1)	0.0054(1)
1.0	0.374(4)	0.033(4)	0.036(8)
∞	0.411(3)	0.040(4)	0.052(2)

Extension to a fourth fermion generation

(Hou; Holdom, Hou, Hurth, Mangano, Sultansoy, Ünel)

Motivation:

- offers potential to generate sufficient amount of CP violation (Hou)

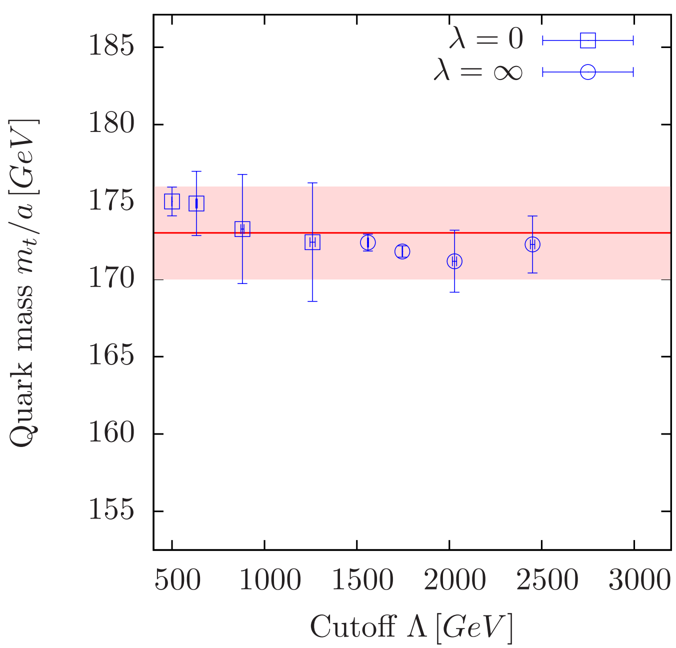
- heavy fermion mass
  $\rightarrow$ large Yukawa couplings
  $\rightarrow$ need of non-perturbative study

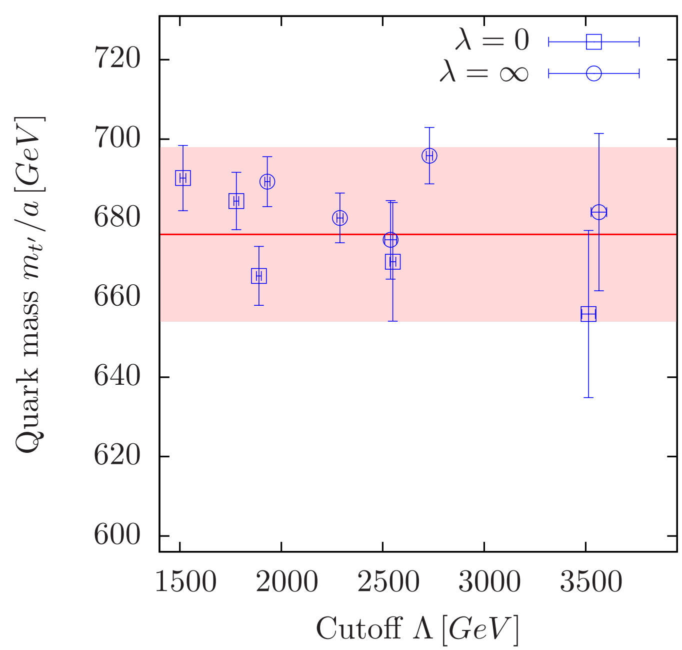
- here: effect of 4th fermion generation on Higgs boson mass bounds

- strong dynamics due to large Yukawa coupling?

Fourth generation Higgs boson mass bounds

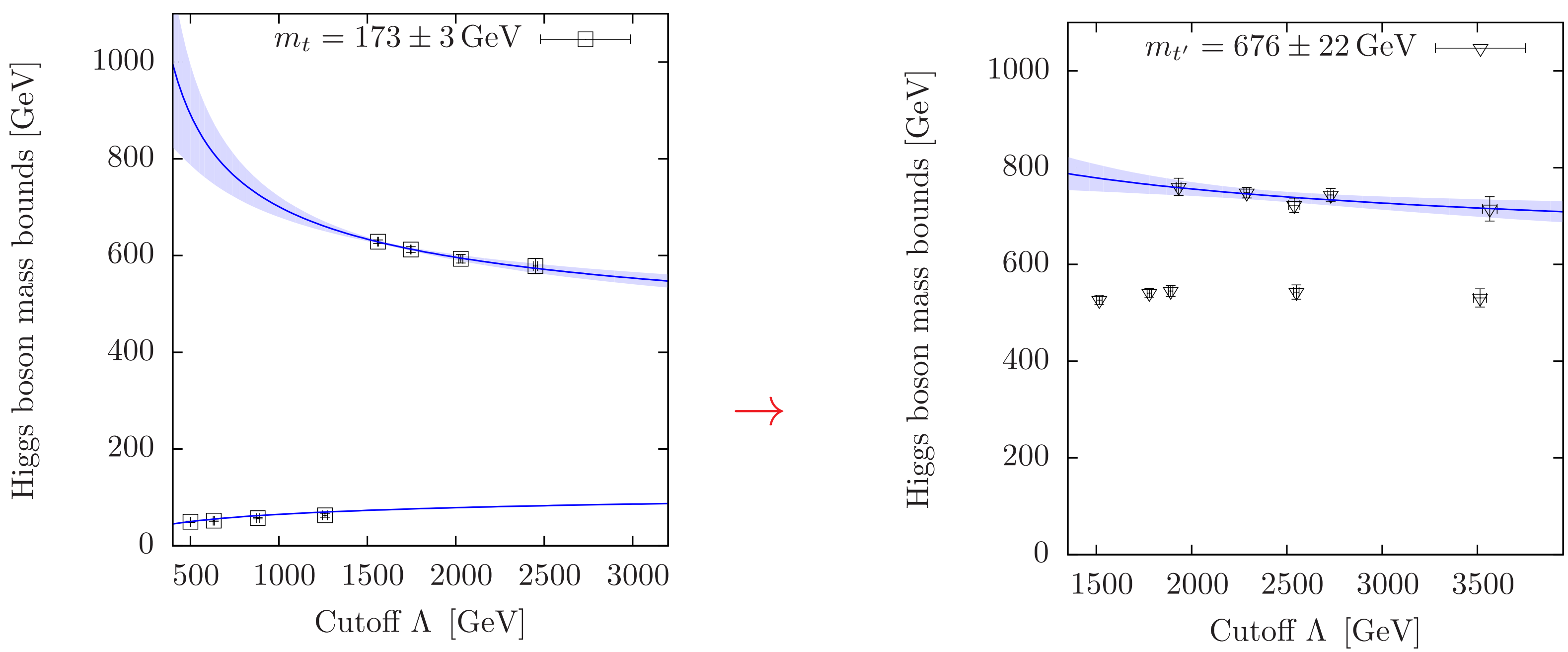
- fixing the top quark masses
- managed to fix m_t and $m_{t'}$ to a few % precision



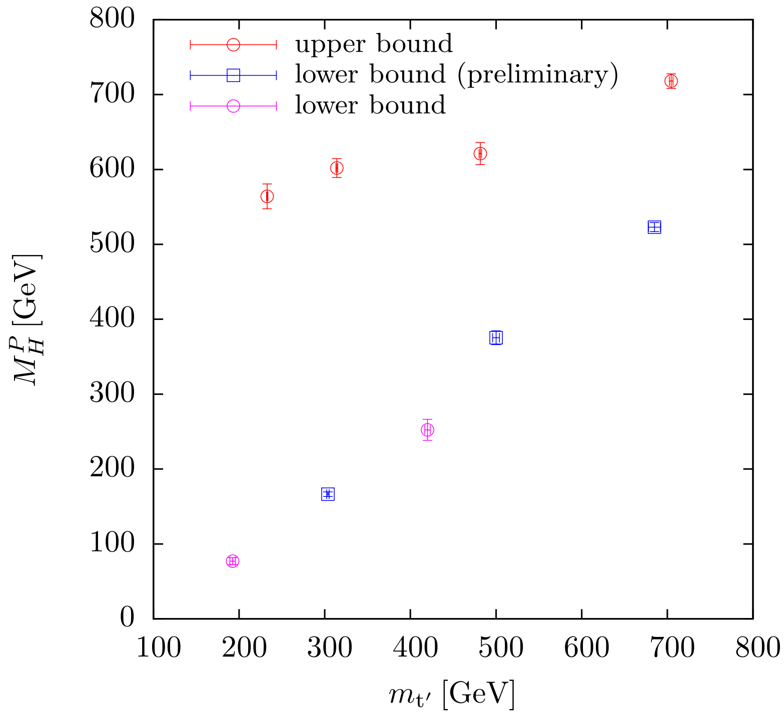


Moving to 4th generation

- apply same technology to heavy fermions
- comparing $m_t = 175$GeV and $m_{t'} = 700$GeV:
  $\rightarrow$ signficant narrowing of allowed Higgs boson mass range
- slight shift of upper bound $\approx 20\%$
- large shift of lower bound: 50GeV $\rightarrow$ 500GeV

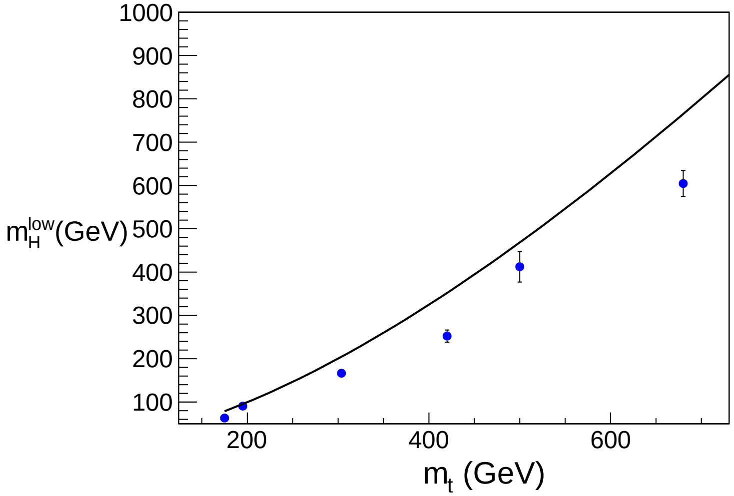


Fermion mass dependence of Higgs boson mass bounds



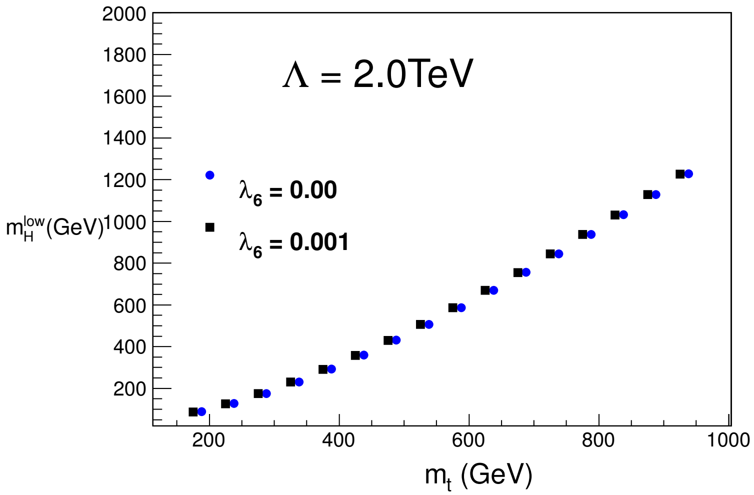
- strong dependence on fermion mass
- questions to be addressed
  - for a few data points: infinite volume limit missing
  - b' and t' are mass degenerate

Heavy Fermion mass $\rightarrow$ system becomes non-perturbative?



- 1-loop lattice effective potential for lower bound
  $\rightarrow$ good description of simulation results

Effect of higher dimensional operators



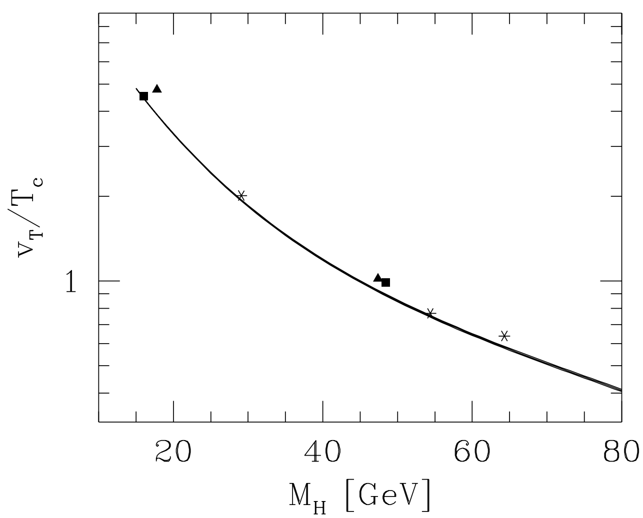
- analysis from lattice perturbative effective potential
- fix $m_{\rm top} = 175{\rm GeV}$, $\lambda_6 = 0.001$ and cut-off $\Lambda = 2{\rm TeV}$
- change λ with constraint $d^2V_{\rm eff}/d\Phi^2 > 0$ for $v < \Phi < 0.5\Lambda$

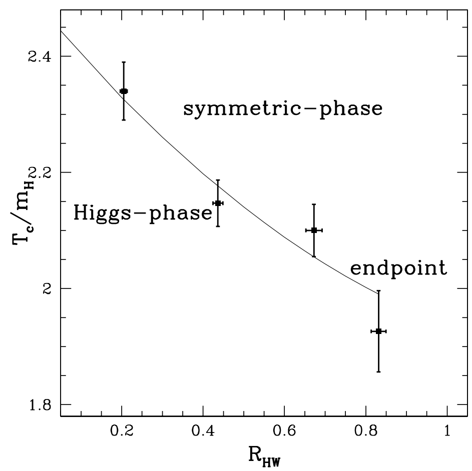
Non-zero temperature electroweak phase transition

Sakharov condition: sufficiently out of thermal equilibrium

$\Rightarrow$ first order phase transition with $v/T_c > 1$

Situation in the standard model:





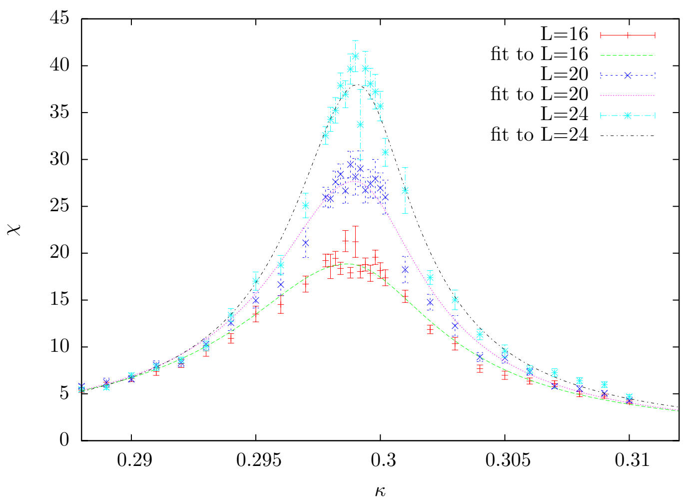
- at physical Higgs boson masses: cross-over
  (side remark QCD: at physical quark masses: cross-over)
  Why are the phase transitions avoided?

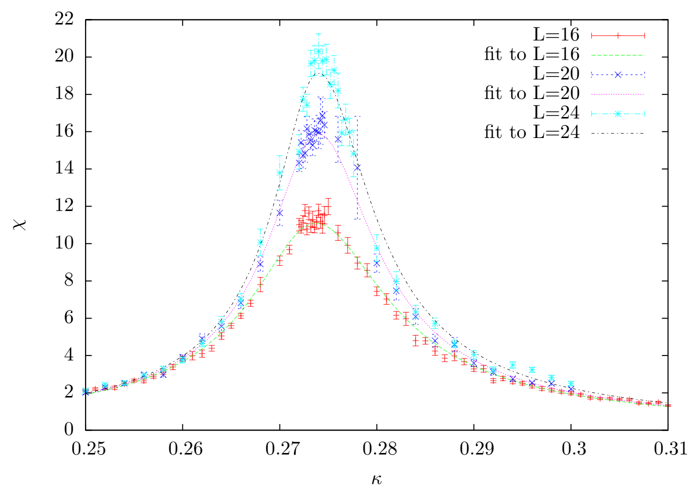
- fourth generation: could be stronger first oder phase transition

Higgs-Yukawa model at Non-zero temperature

large fermion mass $\rightarrow$ strong first order phase transition
from effectice potential in Higgs-Yukawa model
(Kikukawa, Kohda, Yasuda)

Check scenario in our lattice Higgs-Yukawa model





$m_{\mathrm{top}} = 175\mathrm{GeV}$ $\qquad$ $m_{\mathrm{top}'} = 400\mathrm{GeV}$

- T_c at $m_{\mathrm{top}} = 175\mathrm{GeV} \rightarrow T_c = 509(18)\mathrm{GeV}$
- T_c at $m_{\mathrm{top}'} = 400\mathrm{GeV}$ still to be determined
- check effective potential and order of phase transition

Summary

- Lattice Higgs-Yukawa model
  - *exact* chiral symmetry
  - analytical control from effective potential
  - Higgs bosn treated as true resonance
- established lower and upper bounds on the Higgs boson mass:

$$\text{at} \quad \Lambda = 1.5\text{TeV} \qquad 50\text{GeV} \lesssim M_H \lesssim 650\text{GeV}$$

- extended study to 4th generation with $190\text{GeV} \lesssim m_{t'} \lesssim 700\text{GeV}$
  - moderate (20%) shift of upper bound
  - large shift of lower bound: $\rightarrow$ $M_H^{\text{lower}} \approx 500\text{GeV}$ $m_{t'} = 700\text{GeV}$
  - no effect of higher dimensional operators
- calculations of Higgs boson resonance parameters
  - width remains small $O(10\%)$ even for $\lambda = \infty$
- consequences of a 125GeV Higgs boson mass
  - 4th generation ruled out
  - energy scale of breadown of standard model (in progress)