Lattice Computations for High Energy and Nuclear Physics

Karl Jansen

- **Progress in Lattice Field Theory computations**
- **The lattice and the Higgs boson**
- **Selected results from lattice QCD simulations**
  - Baryon spectrum
  - The anomalous magnetic moment of the muon
- **Challenges**
  - Nucleon structure, light-by-light scattering
    $\Rightarrow$ simulations at physical pion masses
- **New directions**
  - Nuclear physics
  - Search for the conformal window
- **Conclusion**

Schwinger model: 2-dimensional Quantum Electrodynamics

(Schwinger 1962)

Quantization via Feynman path integral

$$\mathcal{Z} = \int \mathcal{D}A_\mu \mathcal{D}\Psi \mathcal{D}\bar{\Psi} e^{-S_{\text{gauge}} - S_{\text{ferm}}}$$

Fermion action

$$S_{\text{ferm}} = \int d^2x \bar{\Psi}(x) \left[D_\mu + m \right] \Psi(x)$$

gauge covriant derivative

$$D_\mu \Psi(x) \equiv (\partial_\mu - i g_0 A_\mu(x)) \Psi(x)$$

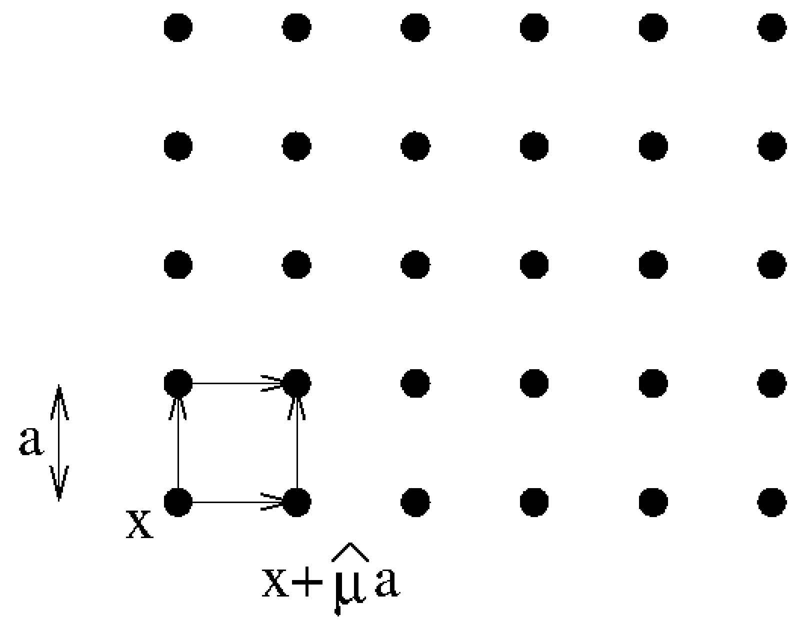
with A_μ gauge potential, g_0 bare coupling

$$S_{\text{gauge}} = \int d^2x F_{\mu\nu} F_{\mu\nu} \; , \; F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

equations of motion: obtain classical Maxwell equations

Lattice Schwinger model

introduce a 2-dimensional lattice with lattice spacing a



fields $\Psi(x)$, $\bar{\Psi}(x)$ on the lattice sites

$x = (t, \mathbf{x})$ integers

discretized fermion action

$$S \rightarrow a^2 \sum_x \bar{\Psi} \left[\gamma_\mu \partial_\mu - r \underbrace{\partial_\mu^2}_{\nabla_\mu^* \nabla_\mu} + m\right] \Psi(x)$$

$$\partial_\mu = \frac{1}{2}\left[\nabla_\mu^* + \nabla_\mu\right]$$

discrete derivatives

$$\nabla_\mu \Psi(x) = \frac{1}{a}\left[\Psi(x + a\hat{\mu}) - \Psi(x)\right] \ , \quad \nabla_\mu^* \Psi(x) = \frac{1}{a}\left[\Psi(x) - \Psi(x - a\hat{\mu})\right]$$

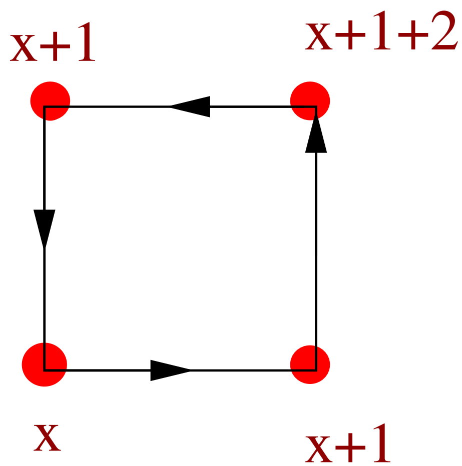
second order derivative $\rightarrow$ remove doubler $\leftarrow$ break chiral symmetry

Implementing gauge invariance

Wilson's fundamental observation: introduce Paralleltransporter connecting the points x and $y = x + a\hat{\mu}$:

$$U(x,\mu) = e^{iaA_\mu(x)} \in U(1)$$

$\Rightarrow$ lattice derivative: $\nabla_\mu \Psi(x) = \frac{1}{a}\left[U(x,\mu)\Psi(x+\mu) - \Psi(x)\right]$



$$U_p = U(x,\mu)U(x+\mu,\nu)U^\dagger(x+\nu,\mu)U^\dagger(x,\nu)$$

$$\rightarrow F_{\mu\nu}F^{\mu\nu}(x) \quad \text{for} \quad a \rightarrow 0$$

$$S = a^2 \sum_x \left\{\beta\left[1 - \mathrm{Re}(U_{(x,p)})\right] + \bar{\psi}\left[m_0 + \tfrac{1}{2}\{\gamma_\mu(\nabla_\mu + \nabla^\star_\mu) - a\nabla^\star_\mu\nabla_\mu\}\right]\psi\right\}$$

Partition functions (pathintegral) with Boltzmann weight (action) S

$$\mathcal{Z} = \int_{\text{fields}} e^{-S}$$

Physical Observables

expectation value of physical observables $\mathcal{O}$

$$\underbrace{\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int_{\text{fields}} \mathcal{O} e^{-S}}$$

$\downarrow$ lattice discretization

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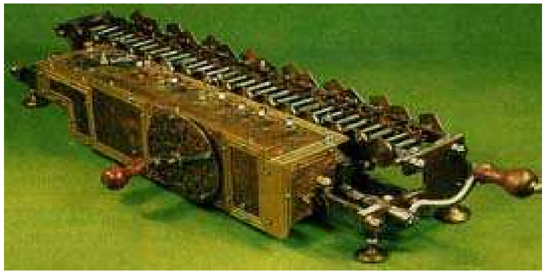
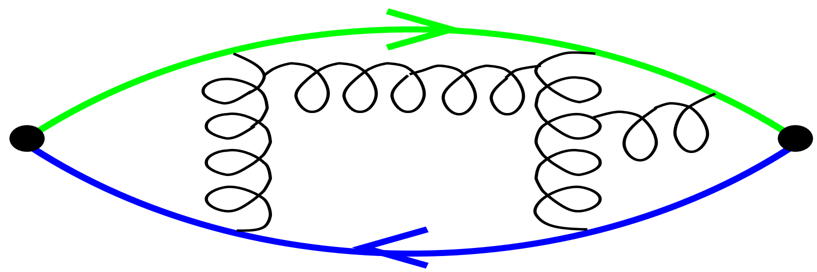
$\downarrow$



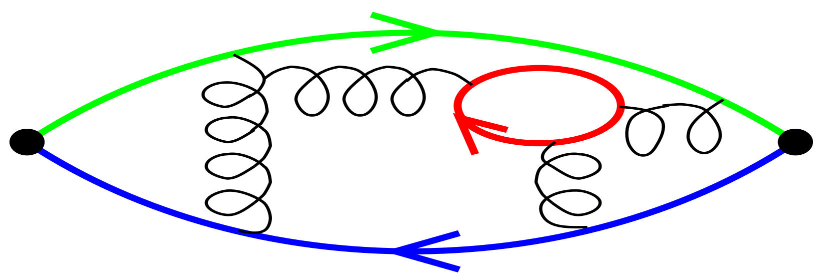
Lattice Field Theory in the 20th century: the Quenched Approximation

→ neglect internal quark-antiquark loops antiquarks in physical quantum processes

⇒ severe *truncation*



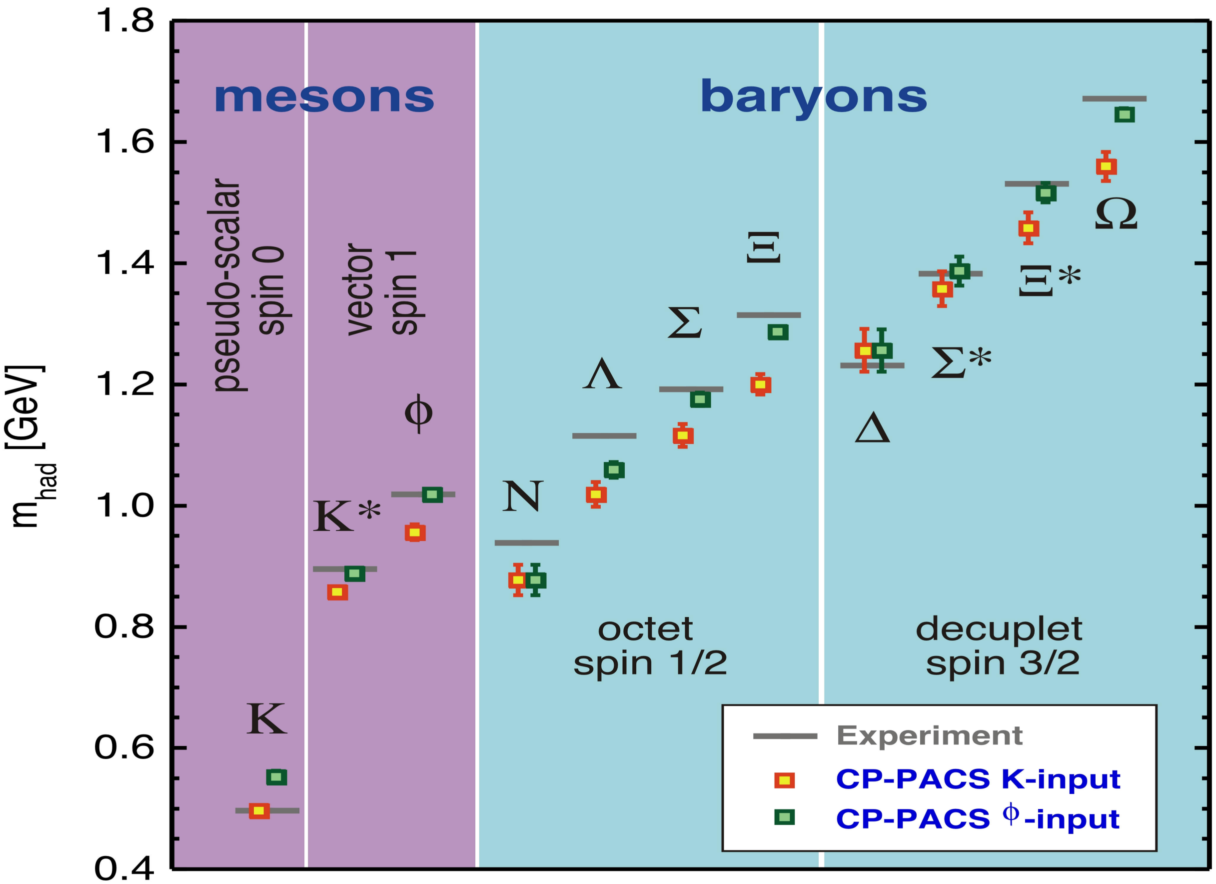
(A) Quenched QCD: no internal quark loops



(B) full QCD

Quenched approximation

Status in 2000: neglecting internal quark loops

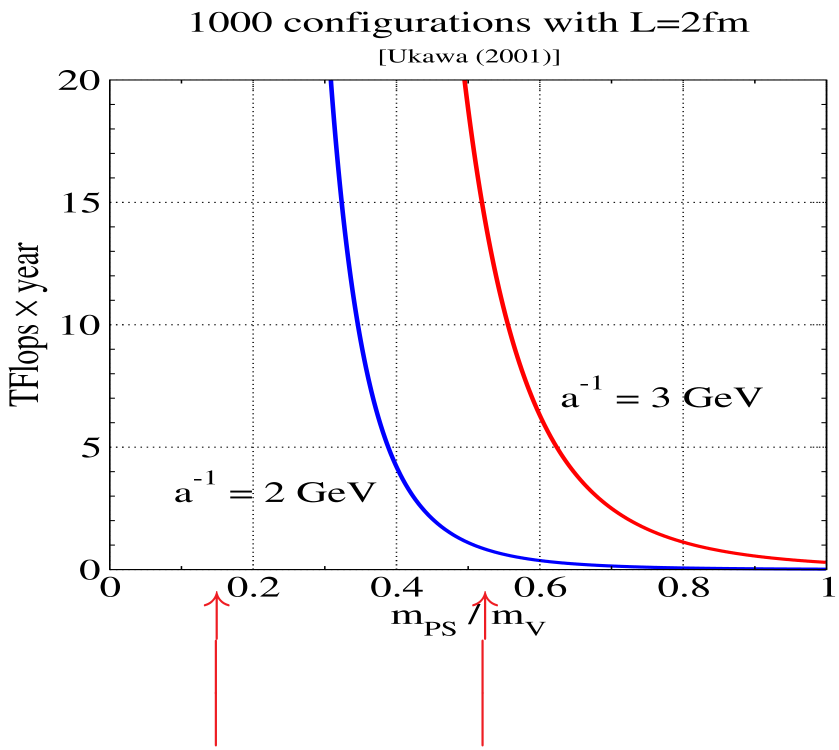


Japanese
CP-PACS collaboration, 2000

End of the "quenched area": need to take quark loops into account

Costs of dynamical fermions simulations, the “Berlin Wall”

see panel discussion in Lattice2001, Berlin, 2001



physical point

contact to χPT (?)

formula $C \propto \left(\frac{m_\pi}{m_\rho}\right)^{-z_\pi} (L)^{z_L} (a)^{-z_a}$

$$z_\pi = 6, \;\; z_L = 5, \;\; z_a = 7$$

“both a 10^8 increase in computing power AND spectacular algorithmic advances before a useful interaction with experiments starts taking place.”
(Wilson, 1989)

$\Rightarrow$ need of **Exaflops Computers**

Why are fermions so expensive?

- need to evaluate

$$\mathcal{Z} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{-\bar{\psi}\{D^{\rm Dirac}_{\rm lattice}\}\psi} \propto \det[D^{\rm Dirac}_{\rm lattice}]$$

- bosonic representation of determinant

$$\det[D^{\rm Dirac}_{\rm lattice}] \propto \int \mathcal{D}\Phi^\dagger\mathcal{D}\Phi e^{-\Phi^\dagger\{D^{-1}_{\rm lattice}\}\Phi}$$

- need vector $X = D^{-1}_{\rm lattice}\Phi$

- solve linear equation $D_{\rm lattice}X = \Phi$

  $D_{\rm lattice}$ matrix of dimension $100\text{million} \otimes 100\text{million} \approx 12 \cdot 48^3 \cdot 96$
  (however, matrix is sparse)

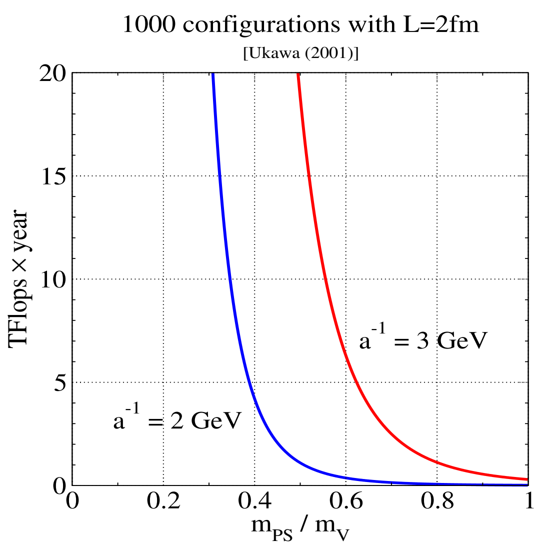
- number of such "inversions": $O(100 - 1000)$ for one field configuration

- want: $O(1000 - 10000)$ such field configurations

A generic improvement for Wilson type fermions

New variant of HMC algorithm (Urbach, Shindler, Wenger, K.J.)
(see also SAP (Lüscher) and RHMC (Clark and Kennedy) algorithms)

- even/odd preconditioning
- (twisted) mass-shift (Hasenbusch trick)
- multiple time steps



2001

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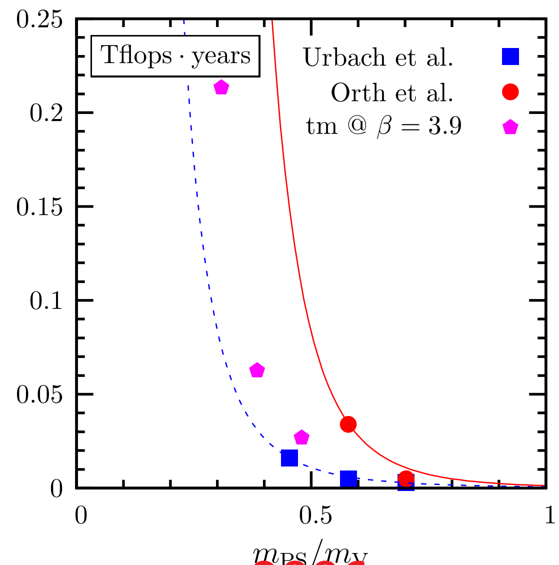
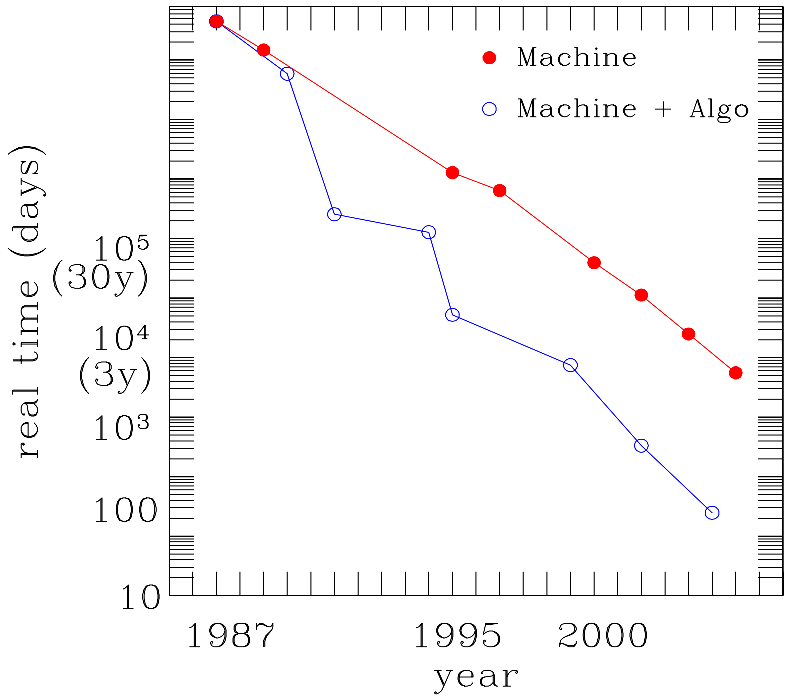


2006

– comparable to staggered
– reach small pseudo scalar masses $\approx 300\text{MeV}$

Computer and algorithm development over the years

time estimates for simulating $32^3 \cdot 64$ lattice, 5000 configurations



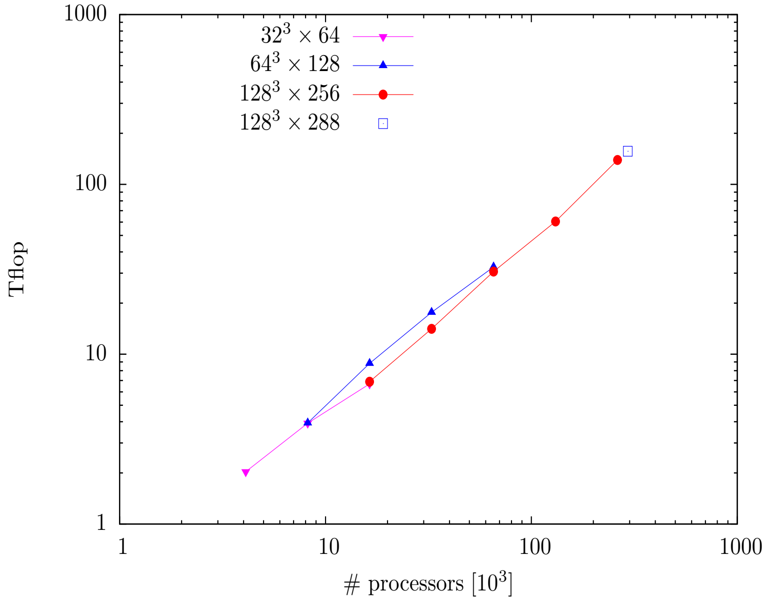
$\longrightarrow$ algorithm development very important

$\longrightarrow$ typical architectures: **BG/L,P,Q, GPUs**

Strong Scaling

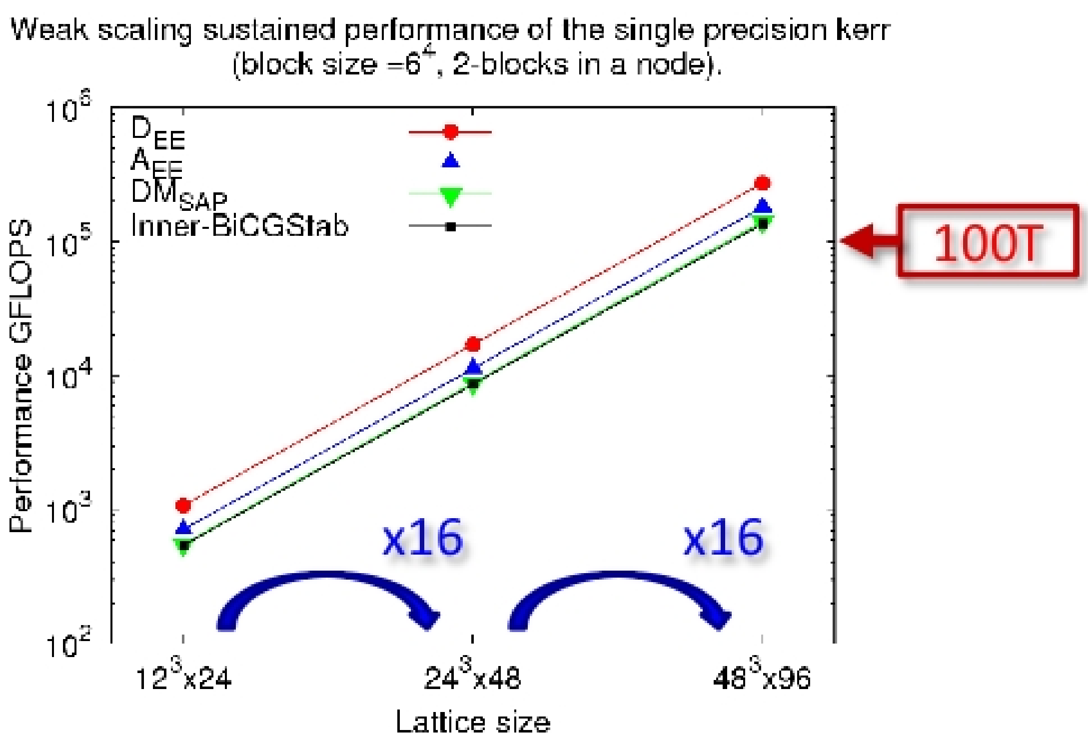
- Test on 72 racks BG/P installation at supercomputer center Jülich
  (Gerhold, Herdioza, Urbach, K.J.)

- using tmHMC code

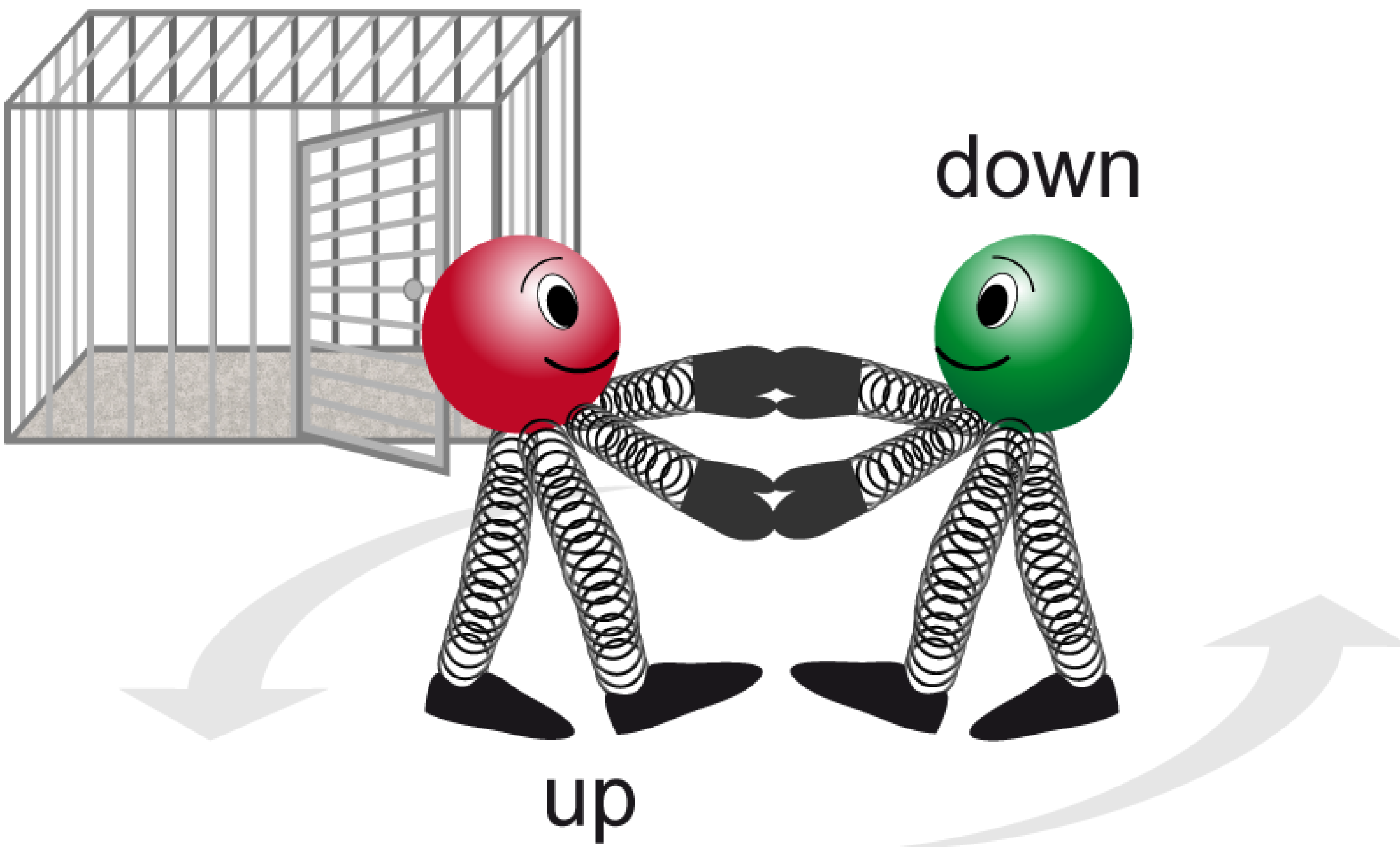


Lattice QCD code on the K-computer: weak scaling

(K. Ishikawa in collaboration with T.Boku, Y.Kuramasi, K.Minami, Y.Nakamura, F.Shoji, D.Takahashi, M.Terai, A.Ukawa, T.Yoshie (RIKEN-Tsukuba Joint Research))

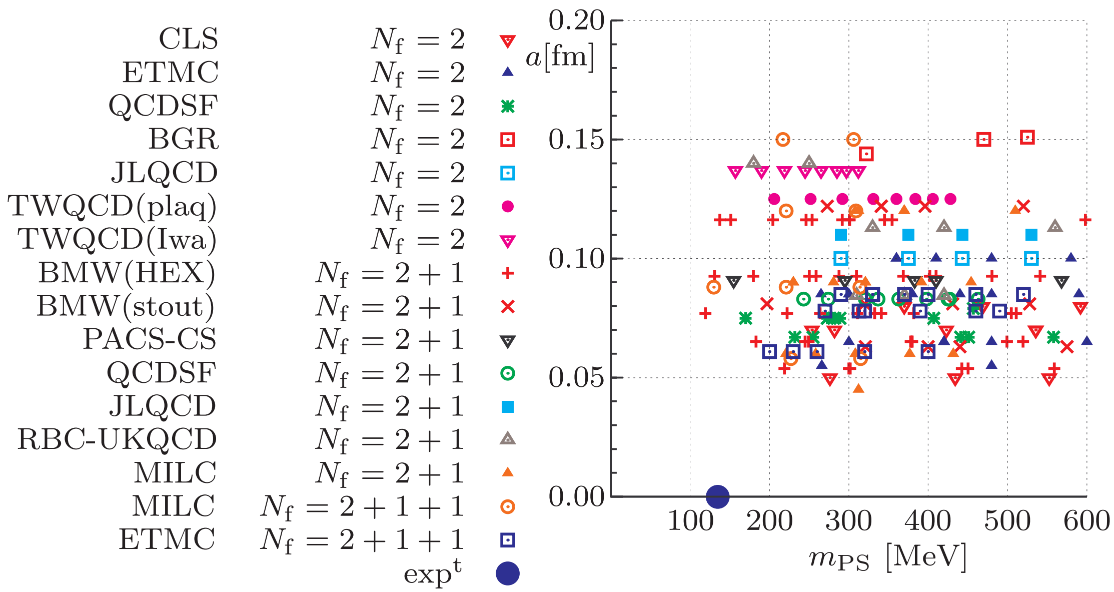


Releasing the quarks: 2 flavours



Simulation landscape

(thanks to G. Herdoiza)

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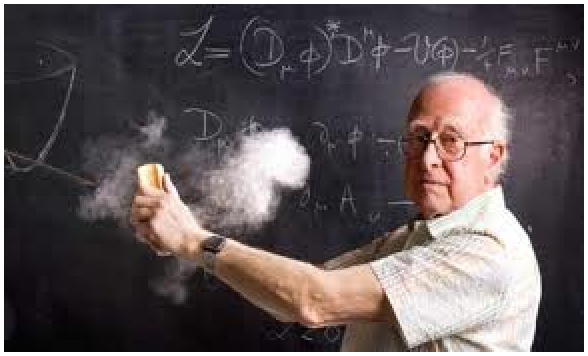
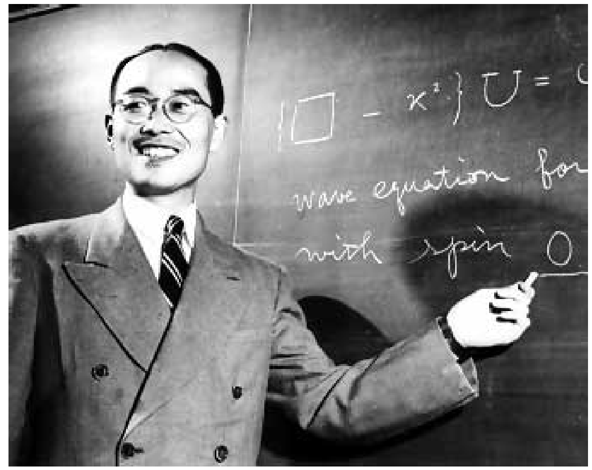
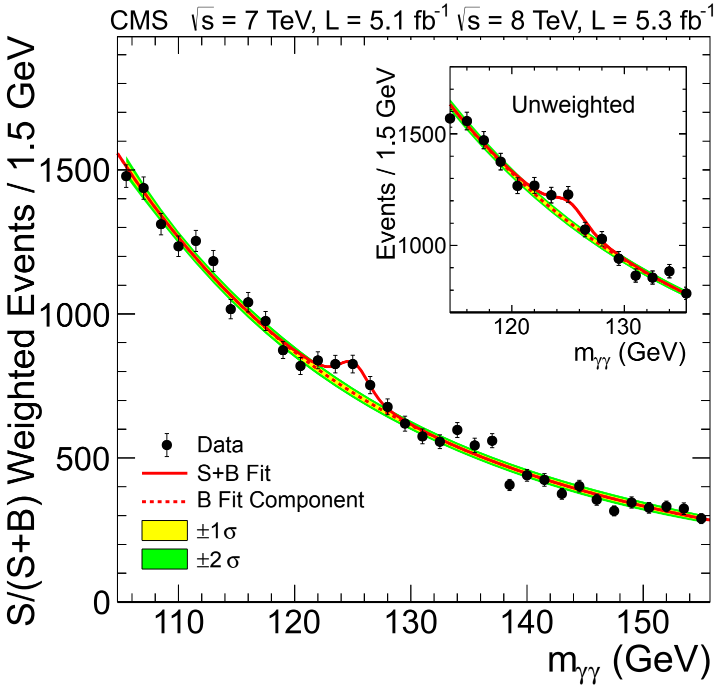
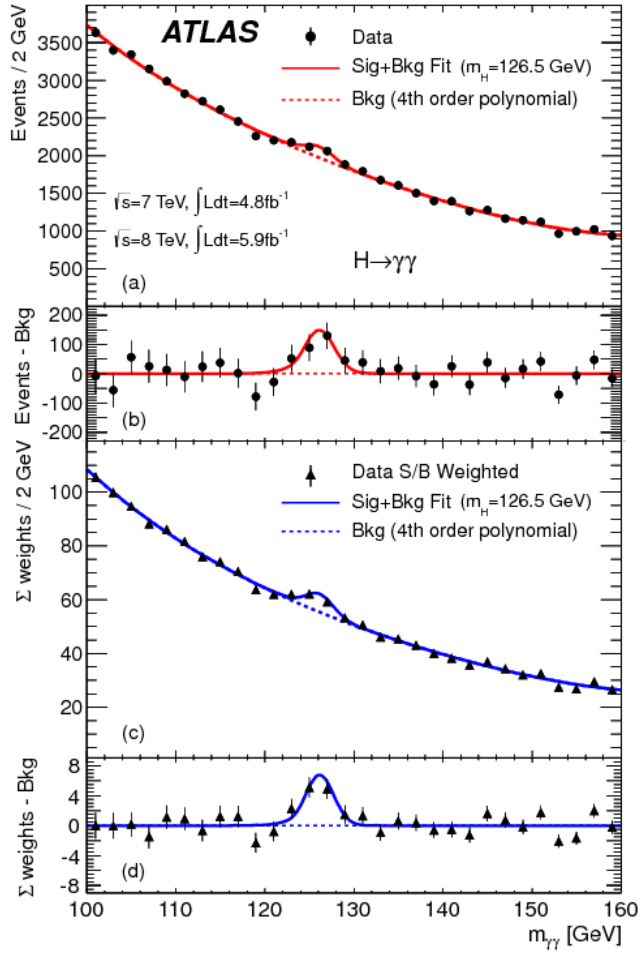
Higgs-Yukawa sector on the lattice





The evidence for a scalar particle at the LHC





CMS **ATLAS**

The Higgs-Yukawa sector of the standard model

- the scalar theory

$$L_\varphi[\varphi] = \frac{1}{2}\partial_\mu\varphi_x^\dagger\partial_\mu\varphi_x + \frac{1}{2}m_0^2\varphi_x^\dagger\varphi_x + \lambda\left(\varphi_x^\dagger\varphi_x\right)^2$$

- the fermionic and Yukawa parts

$$(L_F + L_Y)[\bar{\psi},\psi] = \bar{\psi}i\gamma_\mu\partial_\mu\psi + y_b\left(\bar{t},\bar{b}\right)_L\varphi b_R + y_t\left(\bar{t},\bar{b}\right)_L\tilde{\varphi}t_R + c.c.$$

exact $\mathrm{SU}(2)_L$ chiral symmetry: $\gamma_5[L_F + L_Y)] + [L_F + L_Y)]\gamma_5 = 0$

$$\psi \to P_+\psi + \Omega_L P_-\psi, \bar{\psi} \to \bar{\psi}P_+\Omega_L^\dagger + \bar{\psi}P_-,$$

$$\phi \to \phi\Omega_L^\dagger, \phi^\dagger \to \Omega_L\phi^\dagger.$$

with $\Omega_L \in \mathrm{SU}(2)$,
projectors: $P_\pm = \frac{1\pm\gamma_5}{2}$

Chiral invariant Higgs-Yukawa lattice action (Lüscher)

- the lattice fermionic and Yukawa parts

$$(L_F + L_Y)[\bar{\psi}, \psi] = \bar{\psi} D_{\mathrm{ov}} \psi + y_b \left(\bar{t}, \bar{b}\right)_L \varphi b_R + y_t \left(\bar{t}, \bar{b}\right)_L \tilde{\varphi} t_R + c.c.$$

- change from continuum:
  - $i\gamma_\mu \partial_\mu \to D_{\mathrm{ov}}$
  - $P_\pm = \frac{1 \pm \gamma_5}{2} \to \hat{P}_\pm = \frac{1 \pm \hat{\gamma}_5}{2}, \hat{\gamma}_5 = \gamma_5 \left(1 - aD_{\mathrm{ov}}\right)$

- exact *lattice* $\mathrm{SU}(2)_L$ chiral symmetry: $\gamma_5 D_{\mathrm{ov}} + D_{\mathrm{ov}} \gamma_5 = a D_{\mathrm{ov}} \gamma_5 D_{\mathrm{ov}}$

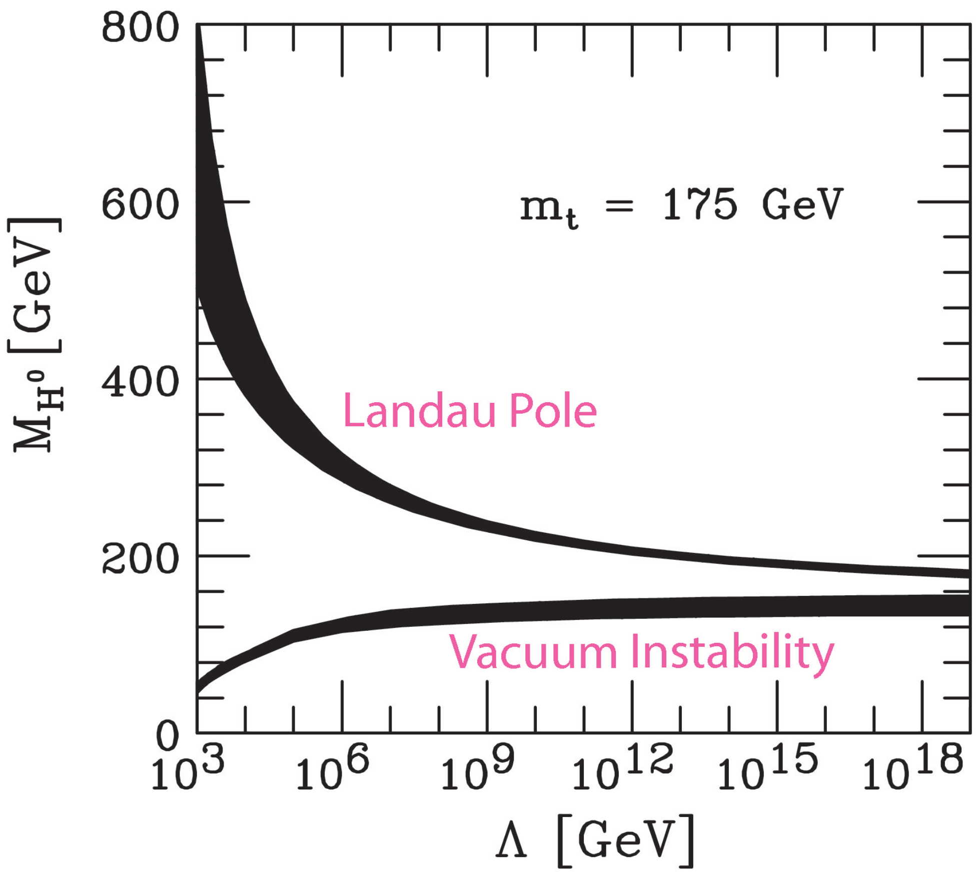
  Ginsparg-Wilson relation overlap operator D_{ov} Neuberger

$$\psi \to \hat{P}_+ \psi + \Omega_L \hat{P}_- \psi, \bar{\psi} \to \bar{\psi} P_+ \Omega_L^\dagger + \bar{\psi} P_-$$

$$\phi \to \phi \Omega_L^\dagger, \phi^\dagger \to \Omega_L \phi^\dagger.$$

  with $\Omega_L \in \mathrm{SU}(2)$,

Theoretical bounds on the Higgs boson mass



- upper bound from triviality
- lower bound from vacuum instability

unanswered questions

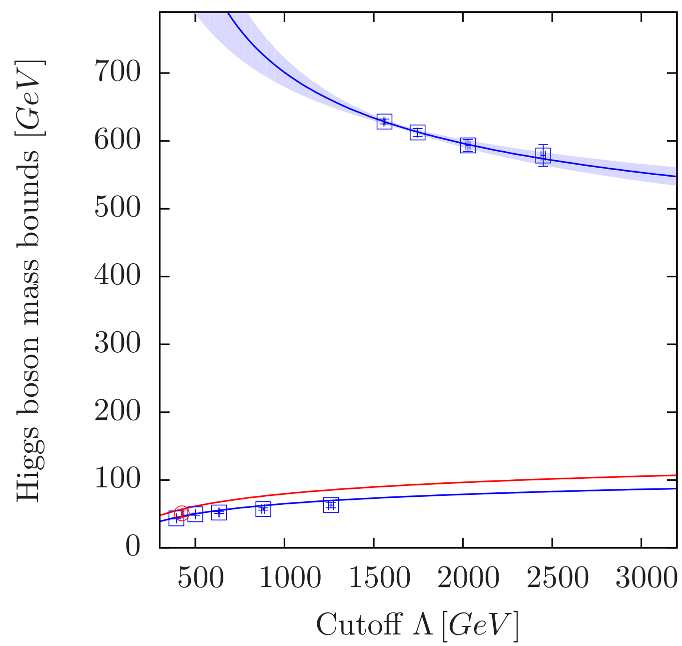
- upper bound:
  $\rightarrow$ coupling becomes strong, unclear whether perturbation theory is valid
- lower bound:
  $\rightarrow$ is vacuum instability an artefact of perturbation theory?
- effects of possible very heavy fermions

$\Rightarrow$ would like to have a first principles, non-perturbative calculation $\rightarrow$ lattice

Lower and upper Higgs boson mass bounds

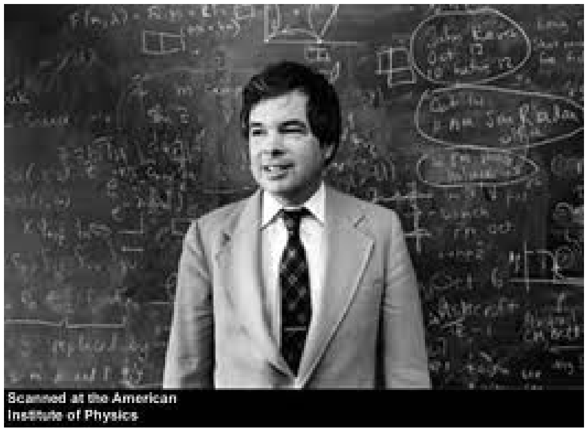
(J. Bulava, P. Gerhold, J. Kallarackal, A. Nagy, K.J.)

- cut-off depence of lower and upper bounds
- allowed range of Higgs boson mass:
  $50\text{GeV} < m_H < 650\text{GeV}$ at cut-off $\Lambda = 1.5\text{TeV}$



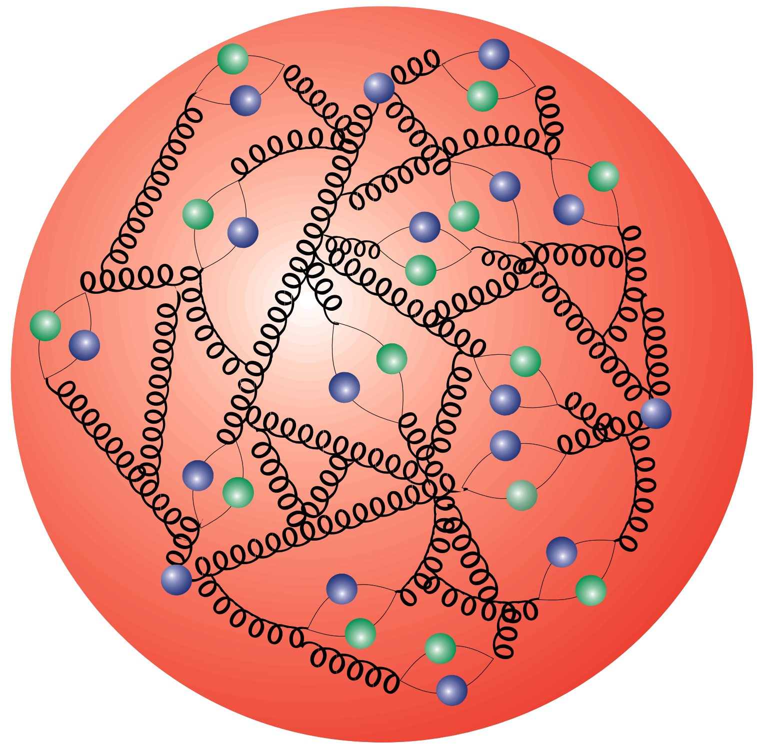
- When does experimental scalar boson mass cut the lower bound? (in progress)

Quantum Chromodynamics on the lattice



Why Perturbation Theory fails for the Strong Interaction

- situation becomes incredibly complicated



- value of the coupling (expansion parameter) $\alpha_{\mathrm{strong}}(1\mathrm{fm}) \approx 1$

⇒ need different ("exact") method

⇒ has to be non-perturbative

- Wilson's Proposal: Lattice Quantum Chromodynamics

Lattice Gauge Theory had to be invented

→ QuantumChromoDynamics

asymptotic freedom		confinement
distances $\ll 1\mathrm{fm}$		distances $\gtrsim 1\mathrm{fm}$
world of quarks and gluons		world of hadrons and glue balls
perturbative description		non-perturbative methods

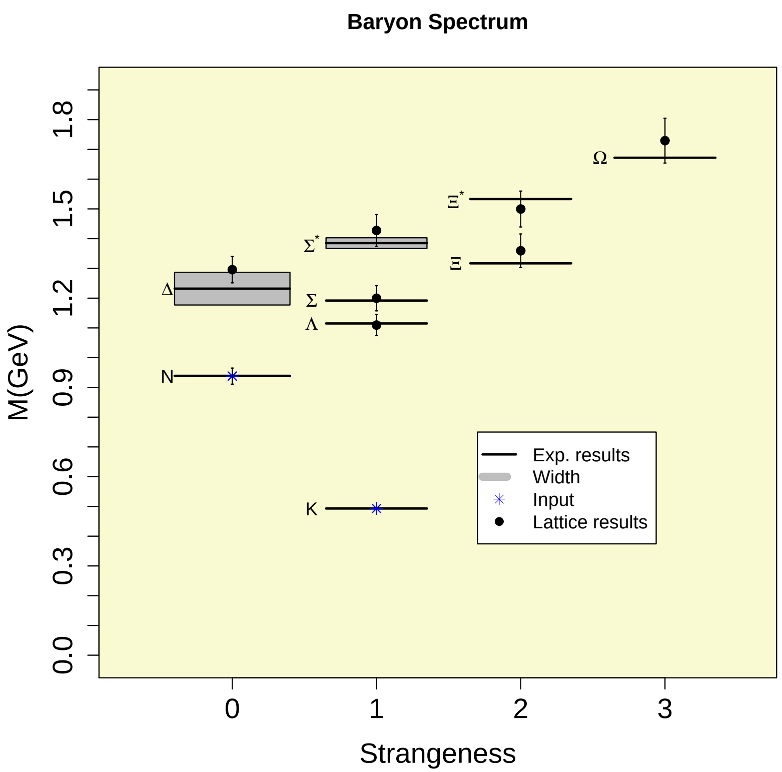
Unfortunately, it is not known yet whether the quarks in quantum chromodynamics actually form the required bound states. To establish whether these bound states exist one must solve a strong coupling problem and present methods for solving field theories don't work for strong coupling.

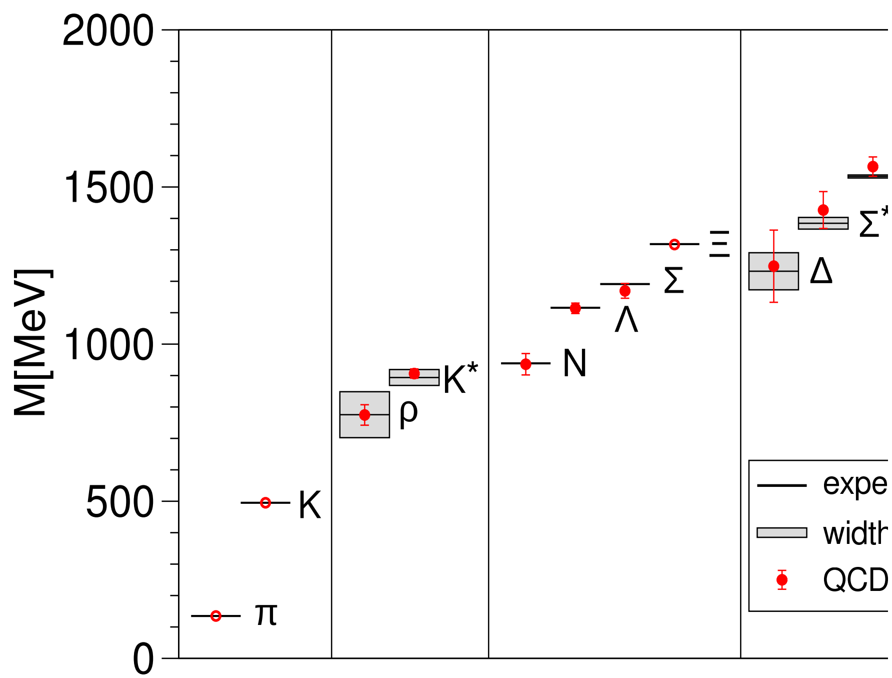
Wilson, Cargese Lecture notes 1976

The lattice QCD benchmark calculation: the spectrum

ETMC ($N_f = 2$), BMW ($N_f = 2 + 1$)

progress in lattice QCD: **Hadron Spectrum**





$N_f = 2$ $\qquad$ $N_f = 2 + 1$

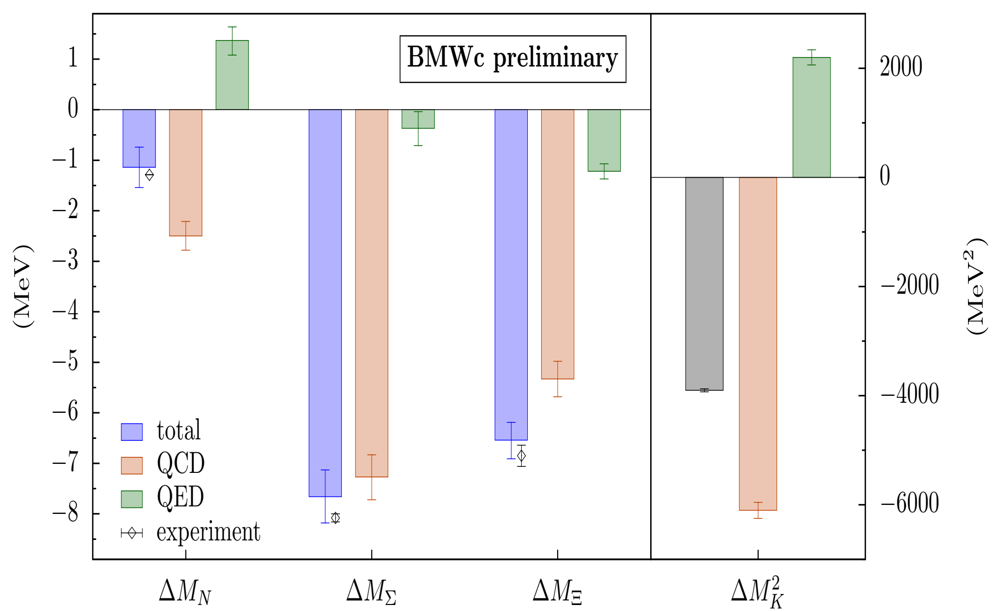
The mass splitting of baryons

(BMW collaboration)

progress in lattice QCD ... and even **Mass splitting**

inclusion of

- isospin-splitting
- (quenched) electromagnetism



→ proton-neutron mass difference from lattice computations

The spin of the elctron

- electron carries spin $\vec{S}$

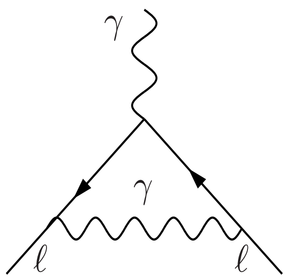
  $\Rightarrow$ magnetic moment $\vec{\mu}_m = -g_e \mu_0 \vec{S}$
  $\mu_0 = e/4m_e$, e electric charge, m_e electron mass

  g_e: gyro-magnetic ratio of electron, $g_e = 2$ (Dirac)

- quantum fluctuations
  $\Rightarrow$ *anomalous magnetic moment* $a_e = \frac{g_e - 2}{2}$

  $g_e = 2.00232$ theory (Schwinger, 1948)

  $g_e = 2.00238(10)$ experiment (Foley, 1948)



anomalous magnetic moment of muon a_μ

- nature has decided to have (at least) three families

  electron $m_e = 0.511\mathrm{MeV}$, **muon** $m_\mu = 105.7\mathrm{MeV}$, **tau** $m_\tau = 1777\mathrm{MeV}$

- muon anomalous magnetic moment

  $a_\mu^{\mathrm{exp}} = 1.16592080(63) \times 10^{-3}$

  $a_\mu^{\mathrm{theory}} = 1.16591790(65) \times 10^{-3}$

- tension between experiment and theory

  $a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}} = 2.90(91) \times 10^{-9}$

  $\Rightarrow$ larger than 3σ discrepancy

  - uncertainty in theoretical calculation?
  - signs of new physics?

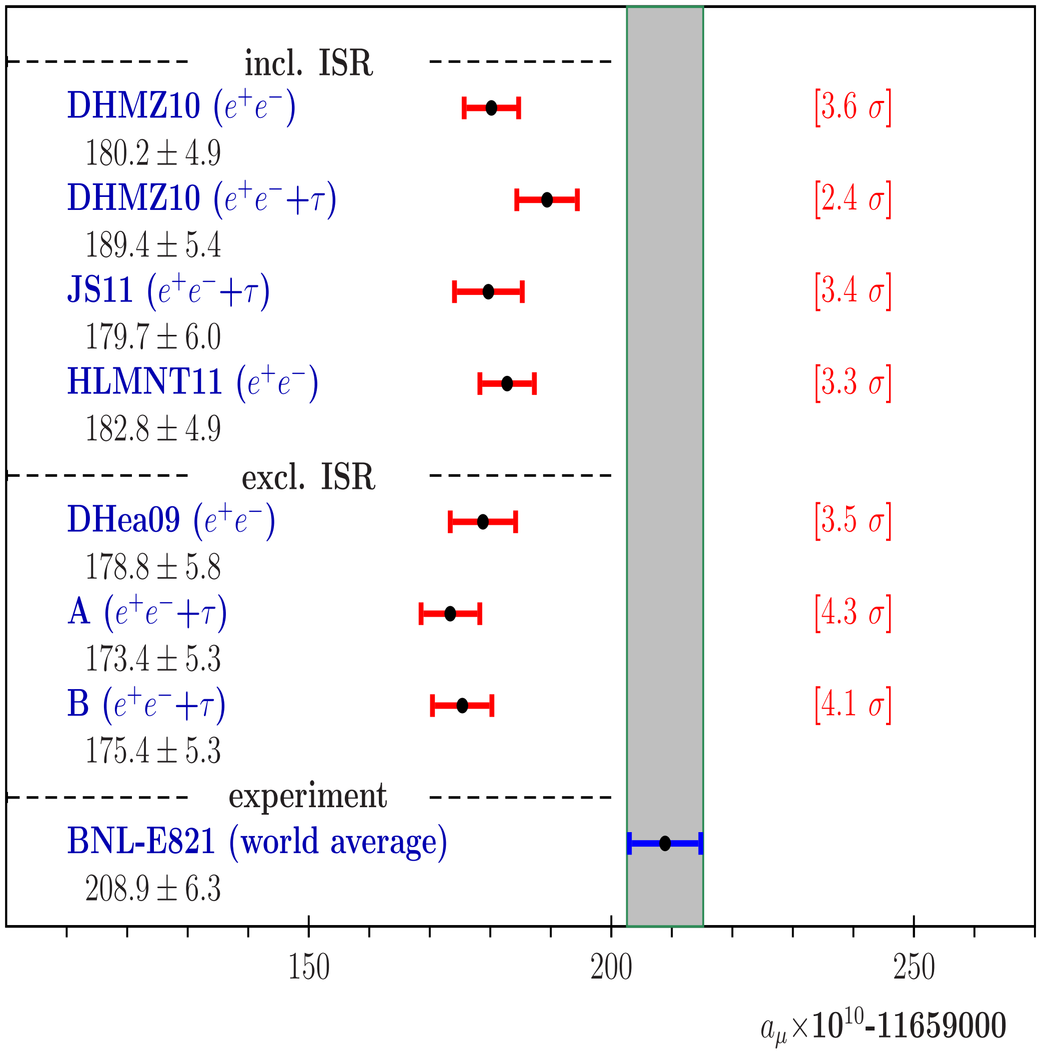
- $\delta(a_l^{\mathrm{newphysics}}) = m_{\mathrm{lepton}}^2 / M_{\mathrm{newphysics}}^2$

  since $m_\mu \approx 2 \cdot 10^4 m_e$: a_μ much more sensitive to new physics

- a_τ experimentally hard to measure

Muon magnetic moment: a tension between theory and experiment

$\rightarrow$ signs of new physics?



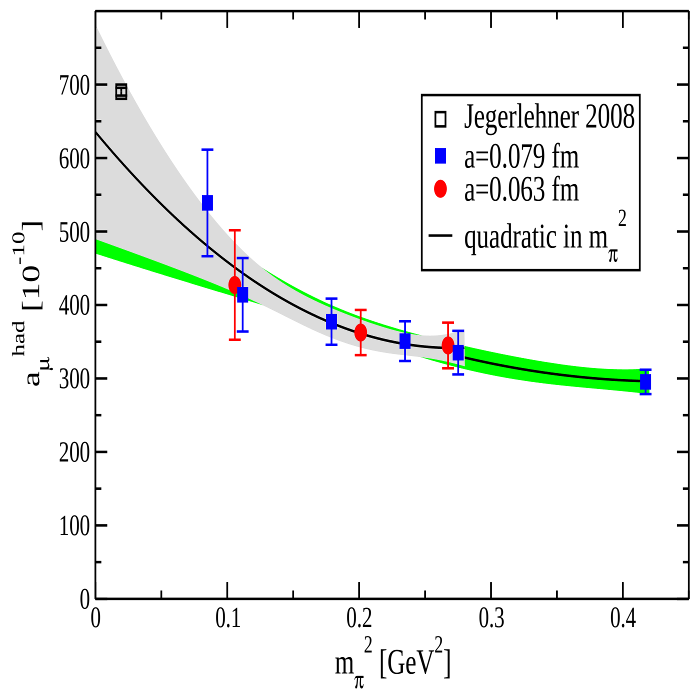
latest analysis

(Benayoun, David, DelBuono, Jegerlehner)

larger than $\underline{4\sigma}$ discrepancy

Do we control hadronic vacuum polarisation?

(Xu Feng, Dru Renner, Marcus Petschlies, K.J.; Lattice 2010)



- experiment: $a_{\mu,N_f=2}^{\rm hvp,exp} = 5.66(05)10^{-8}$
- lattice: $a_{\mu,N_f=2}^{\rm hvp,old} = 2.95(45)10^{-8}$

(numbers are scaled to $N_f = 4$ in plot)

$\longrightarrow$ misses the experimental value

$\longrightarrow$ order of magnitude larger error

- have used different volumes
- have used different values of lattice spacing

Different extrapolation to the physical point

lattice: simulations at unphysical quark masses, demand only

$$\lim_{m_{\mathrm{PS}}\to m_\pi} a_l^{\mathrm{hvp,latt}} = a_l^{\mathrm{hvp,phys}}$$

$\Rightarrow$ flexibility to define $a_l^{\mathrm{hvp,latt}}$

standard definitions in the continuum

$$a_l^{\mathrm{hvp}} = \alpha^2 \int_0^\infty dQ^2 \frac{1}{Q^2}\omega(r)\Pi_R(Q^2)$$

$$\Pi_R(Q^2) = \Pi(Q^2) - \Pi(0)$$

$$\omega(r) = \frac{64}{r^2\left(1+\sqrt{1+4/r}\right)^4\sqrt{1+4/r}}$$

with $r = Q^2/m_l^2$

Redefinition of $a_l^{\mathrm{hvp,latt}}$

redefinition of r for lattice computations

$$r_{\mathrm{latt}} = Q^2 \cdot \frac{H^{\mathrm{phys}}}{H}$$

choices

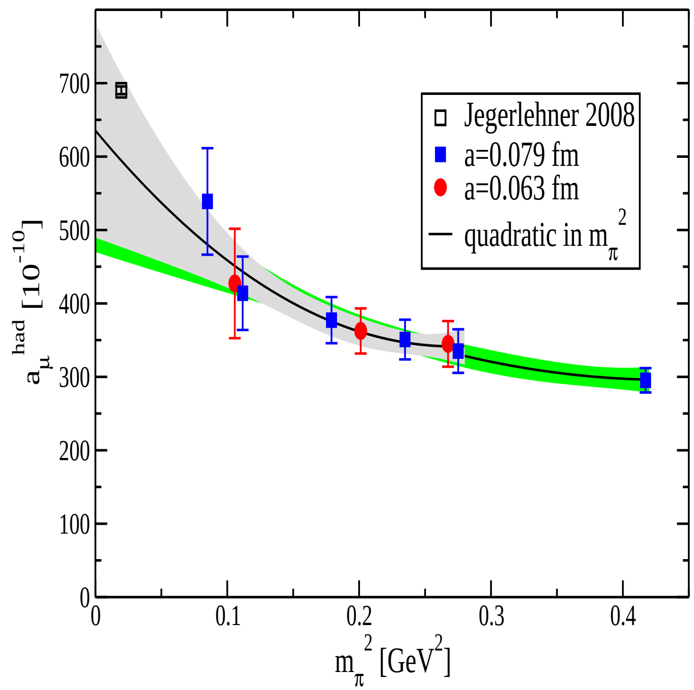
- r_1: $H = 1$; $H^{\mathrm{phys}} = 1/m_l^2$
- r_2: $H = m_V^2(m_{\mathrm{PS}})$; $H^{\mathrm{phys}} = m_\rho^2/m_l^2$
- r_3: $H = f_V^2(m_{\mathrm{PS}})$; $H^{\mathrm{phys}} = f_\rho^2/m_l^2$

each definition of r will show a different dependence on m_{PS} but agree *by construction* at the physical point

- for $m_\mu^2/m_\rho^2 << 1:\quad a_\mu^{\mathrm{hvp}} = \frac{4}{3}\alpha^2 g_\rho^2 m_\mu^2/m_\rho^2$

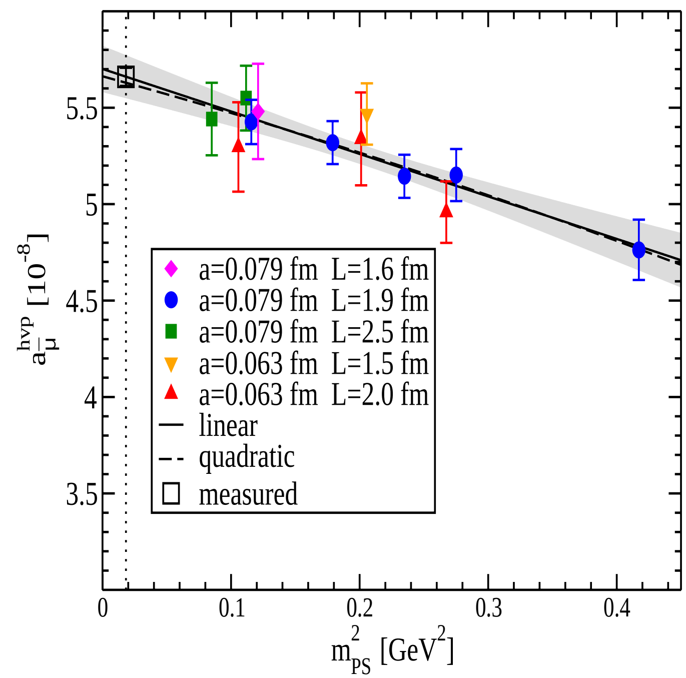
Progress: development of new, improved method

(X. Feng, M. Petschlies, D. Renner, K.J.)



standard method

→



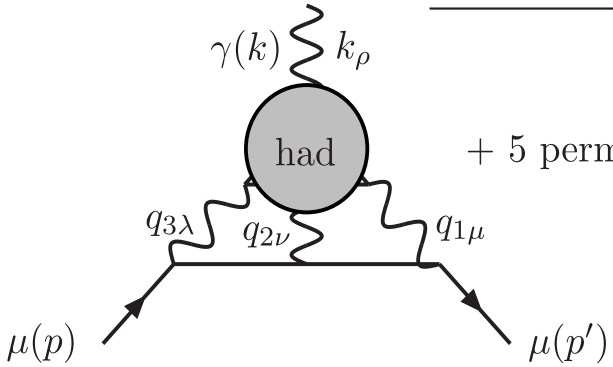
improved method

old: $a^{\rm hvp,old}_{\mu,N_f=2} = 2.95(45)10^{-8}$ new: $a^{\rm hvp,new}_{\mu,N_f=2} = 5.66(15)10^{-8}$

- experimental value: $a^{\rm hvp,exp}_{\mu,N_f=2} = 5.66(05)10^{-8}$
- different volumes and lattice spacings, included dis-connected contributions
- many groups: JLQCD, Mainz, Brookhaven
- ultimate goal: direct calculation at **physical point**


WARNING
CHALLENGES
AHEAD

Challenge I: next, α_s^3, contribution



$+$ 5 permutations of the q_i

light-by-light scattering

involves 4-point function

$$\Pi_{\mu\nu\alpha\beta}(q_1, q_2, q_3) = \int_{xyz} e^{iq_1\cdot x + iq_2\cdot y + iq_3\cdot z} \langle j_\mu(0) j_\nu(x) j_\alpha(y) j_\beta(z)\rangle$$

j_μ electromagnetic quark current

$$j_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c$$

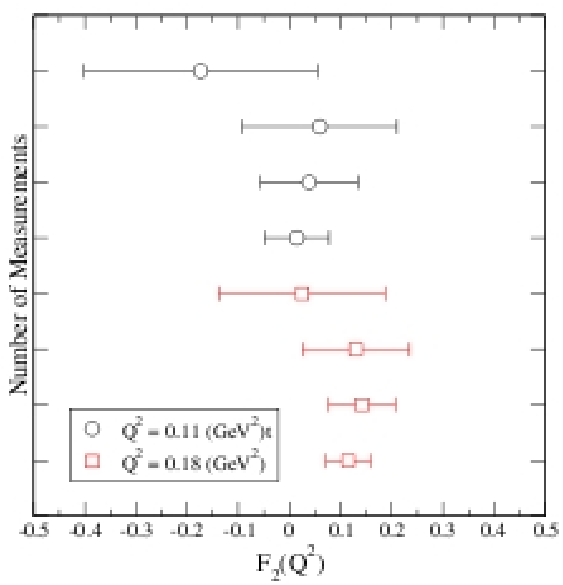
- need $O(V^3)$ momenta
- in turn: $O(V^3)$ "inversions"

$\Rightarrow$ direct calculations not possible

A first attempt

(T. Blum, T. Izubuchi and collaborators)

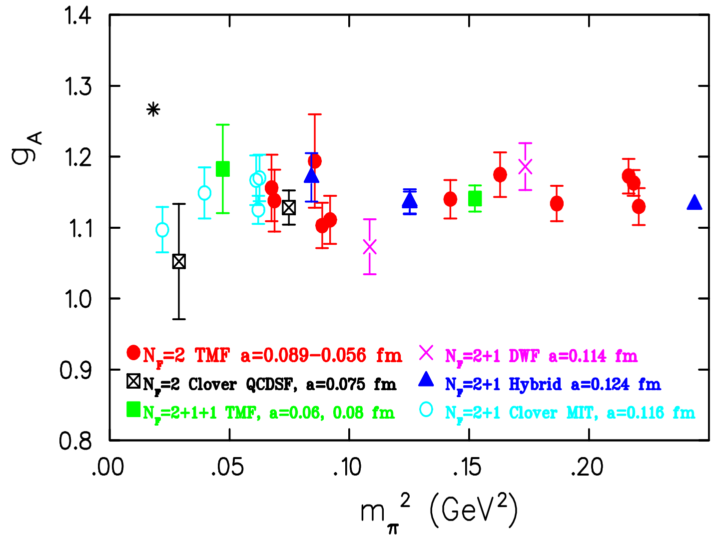
- include electromagnetism in simulations
- problem reduces to difference of 3-point functions



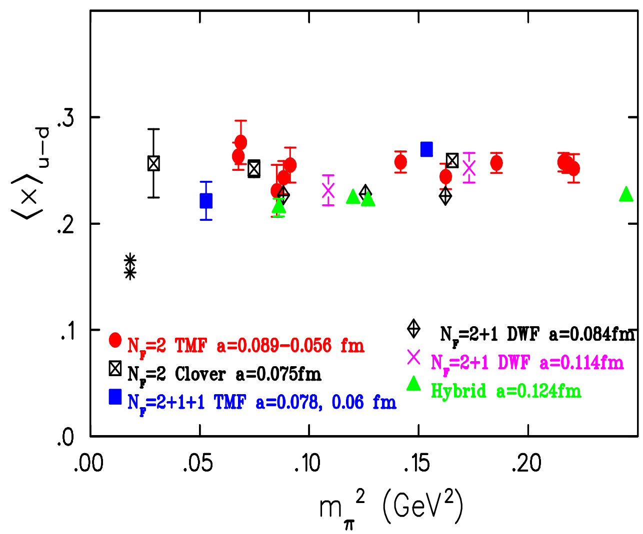
- presently unclear, whether method will be sufficiently precise

Challenge II: hadron structure

- the puzzle with hadron structure



the axial charge



average momentum quark in proton

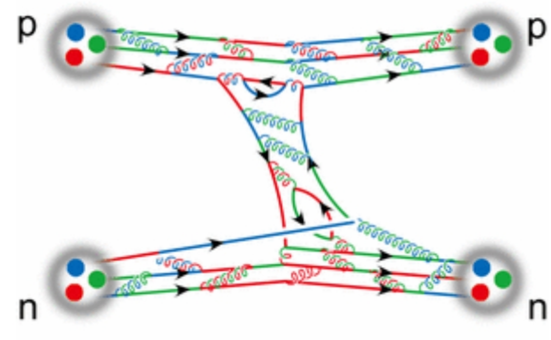
- misses experimental/phenomenological values

⇒ need simulations at physical point

Simulation at physical pion mass

- Lattice spacing: $a = 0.1\text{fm}$
- Pion mass: $m_\pi = 140\text{MeV}$
- Suppression of finite size effects: $L \cdot m_\pi > 5$
- Requirement
  - $L \approx 5\text{fm} \rightarrow 48^3 \cdot 96$ lattice ($\leftarrow$ present standard)
  - for $a = 0.05\text{fm} \rightarrow 96^3 \cdot 192$ lattice
  - grows of simulation cost $\propto a^{-6}$

Nuclear physics from Lattice QCD



start with Schrödinger equation

$$[H_0 + V(x)]\,\phi_k(x) = \epsilon_k \phi(x)_k$$

$$\epsilon_k = \frac{k^2}{2\mu}\ ;\quad \mu = m_N/2\ ;\quad H_0 = \frac{-\nabla^2}{2\mu}$$

Reversing this: knwoing wave function $\rightarrow$ compute potential

$$[\epsilon_k - H_0]\,\phi_k(x) = \int d^3y V(x,y)\phi_k(y)$$

Wavefunction from the lattice

$$\phi_k(r) = \langle 0|N(x+r)N(x)|NNW_k\rangle$$

$W_k = \sqrt{k^2 + m_N^2}$

partial wave in infinite volume:

$$\phi_k^l \rightarrow A_l \frac{\sin(kt - l\pi/2 + \delta_l(k))}{kr} \; for \; r \rightarrow \infty$$

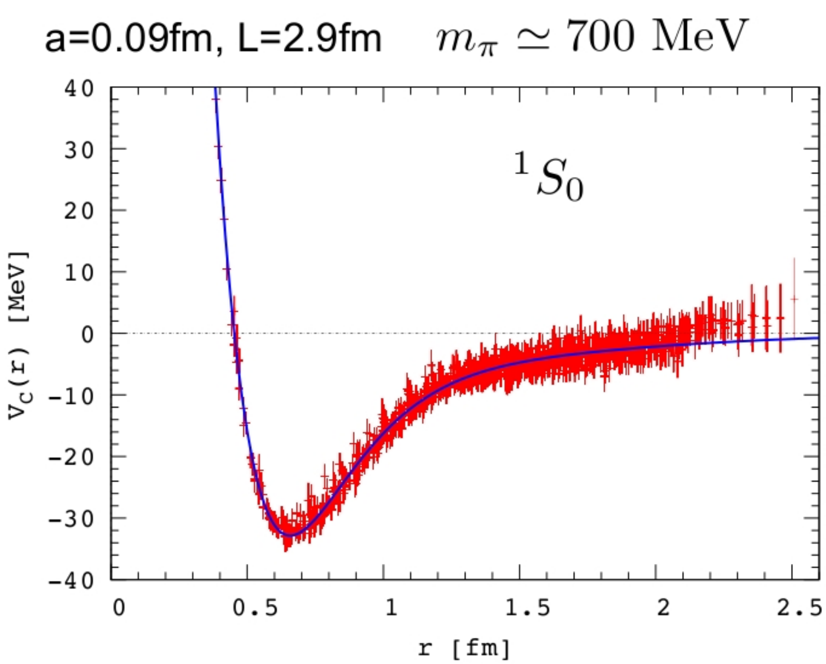
Strategy:

- compute wave function within lattice QCD in finite volume
- determine potential U(x)in finite volume
- use this potential to solve Schrödinger equation in infinite volume
- determine scattering phase or binding energy

⇒ study nuclear physics, neutron stars, supernovae, ...

Nuclear physics from Lattice QCD

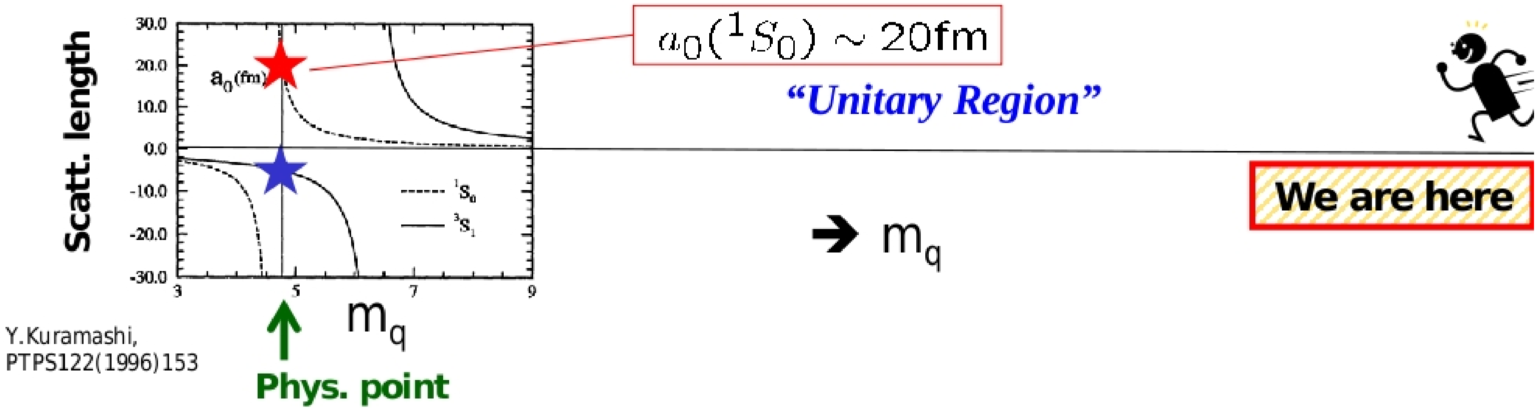
(Sinya Aoki (U. Tsukuba) Bruno Charron (U. Tokyo)
Takumi Doi (Riken) Tetsuo Hatsuda (Riken/U. Tokyo)
Yoichi Ikeda (TIT) Takashi Inoue (Nihon U.) Noriyoshi Ishii (U. Tsukuba) Keiko Murano (Riken)
Hidekatsu Nemura (U. Tsukuba) Kenji Sasaki (U. Tsukuba) Masanori Yamada (U. Tsukuba))



(Ishii-Aoki-Hatsuda, PRL90(2007)0022001)

→ paper has been selected as one of 21 papers in Nature Research Highlights 2007.

The challenge and the prospect

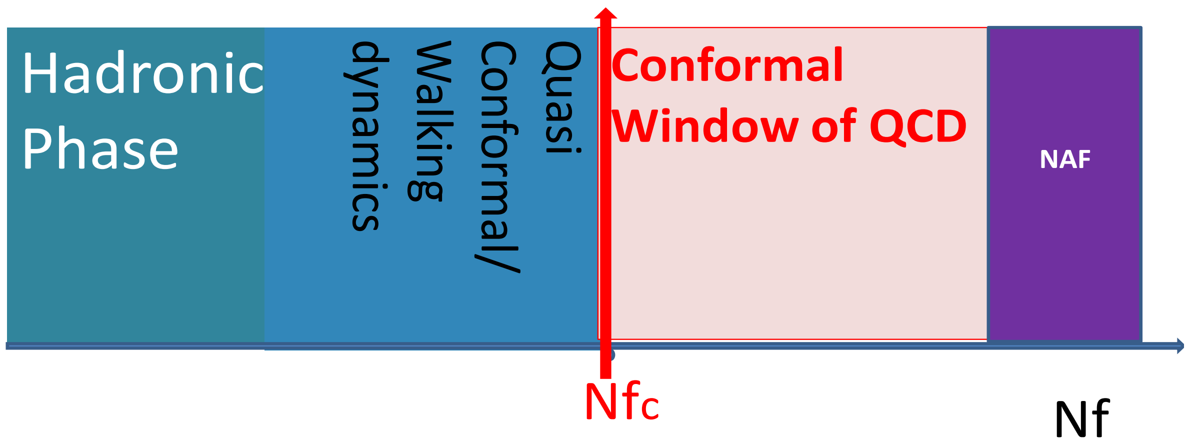


- in-elastic scattering
- resonances
- extension to weak interactions

⇒ need the **K-Computer**

A new game: looking for the conformal window

QCD with many flavors : Sketchy view of the phase diagram



conformal window: existence of **dilaton:** scalar particle

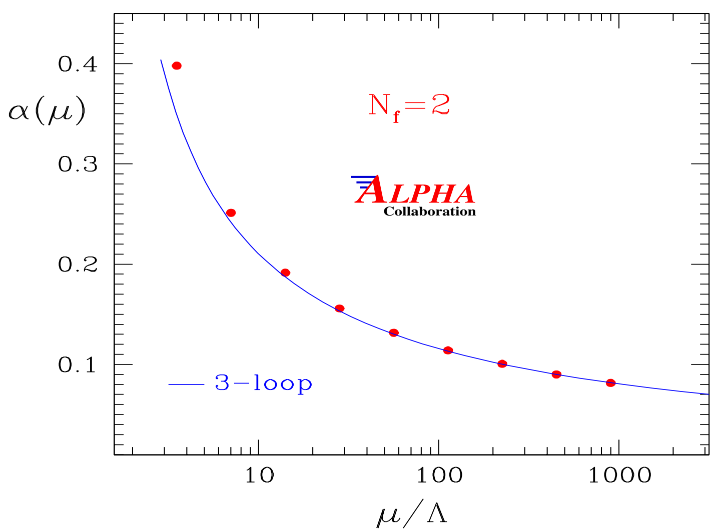
Higgs boson imposter?

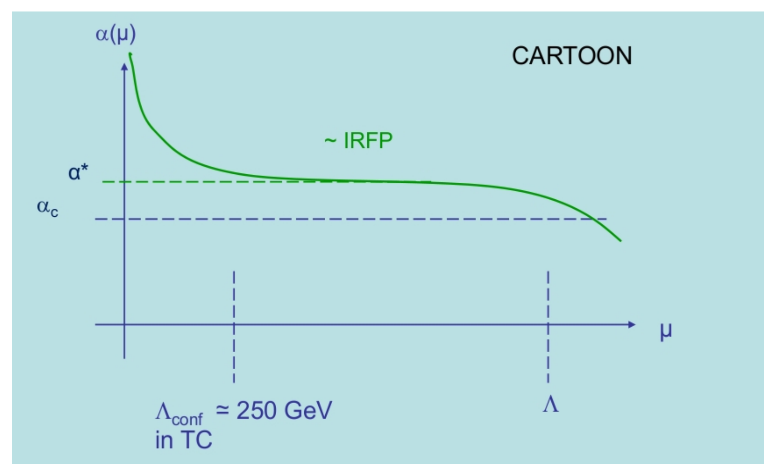
Scale depenedence of coupling

$$\mu\frac{\partial}{\partial\mu}\bar{g}^2(\mu) \equiv \beta(\bar{g}^2(\mu)) = \frac{2}{4\pi^2}\left(11 - \frac{2}{3}N_f\right) + \cdots$$

- for $N_f > 16$: β-function changes sign $\Rightarrow$ new fixed point

QCD: running coupling | **Conformal: walking coupling**

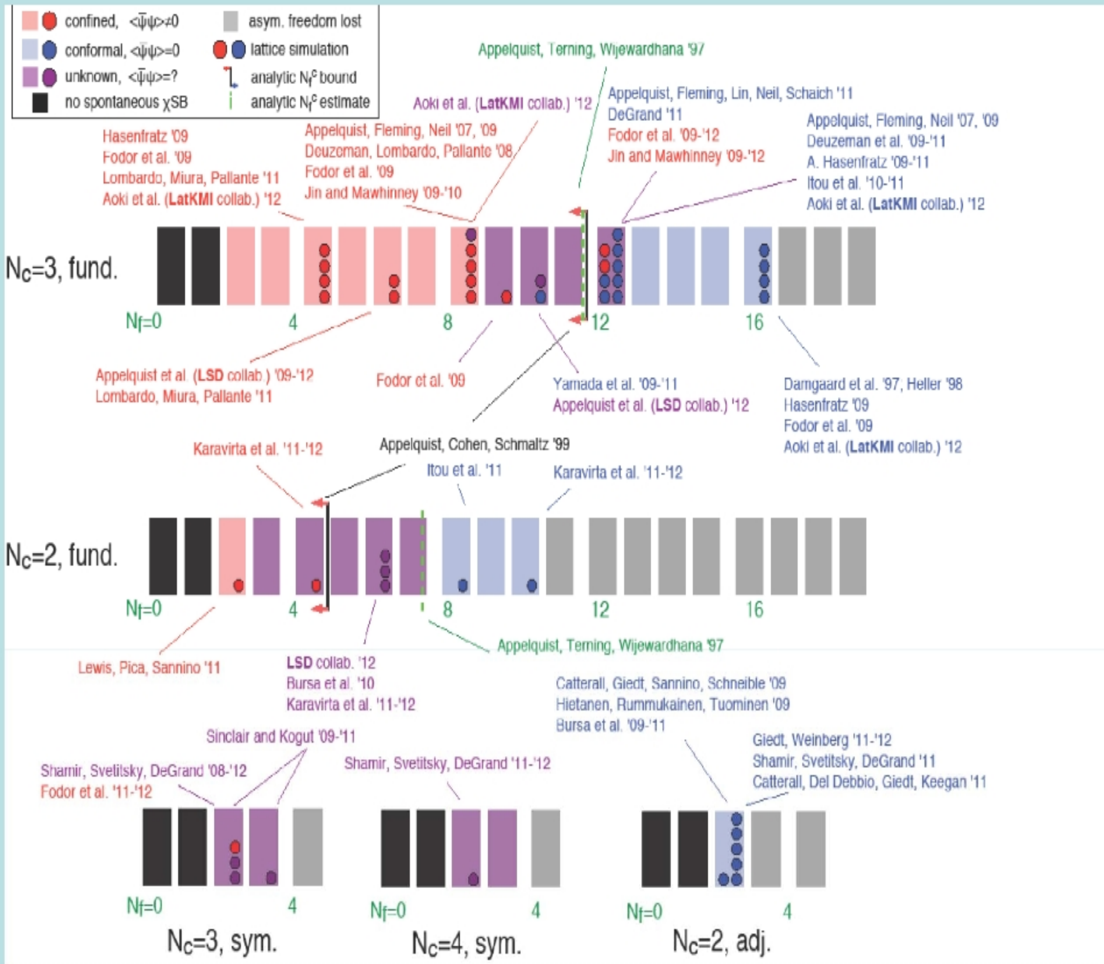




determine "running": difficult

$\rightarrow$ identifying "walking" much harder

Large activity in lattice community



The lattice offers much more ...

- Flavour physics, quark masses ($b - quark$), decay constants, $\alpha_{\rm strong}$
- CKM matrix elements, B_K
- non-zero temperature physics
- topology and chiral symmetry breaking
- charm physics

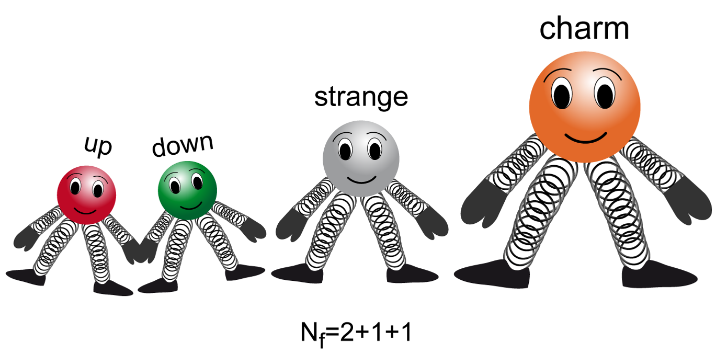
... and needs this at the physical point


DB
ICE

→

Summary

- Progress in solving QCD with lattice techniques
  - dramatic algorithm improvements
  - new supercomputer architectures
  - theoretical/conceptual developments
- offers to compute many physical quanties, showed
  - non-perturbative Higgs boson mass bounds
  - baryon spectrum
  - leading order hadronic contribution to muon anomalous magnetic moment
- challenges discussed (there are many more)
  - light-by-light contribution
  - nucleon structure puzzle
- simulations in physical situation:
  full first two generations at physical
  quark masses



- new directions
  - nuclear physics
  - conformal theories