Pre-Print Classification

NOTES PHONE CALL: Chirally rotated SF boundary conditions. Non-perturbative tuning of κ and z_f

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Abstract

Key words: .... lattice QCD

1 Introduction

1.1 Lattice propagator with chirally rotated SF boundary conditions. Orbifolding version II

With chirally rotated SF b.c. and an orbifold construction of the lattice Dirac operator with $O(a)$-offset (in case of version II), the lattice fermion action takes the form

$$S_f = a^4 \sum_{x_0=0}^{T} \sum_{\vec{x}} \bar{\psi}(x)\mathcal{D}\psi(x) \tag{1.1}$$

with the lattice Dirac operator defined by

$$a\mathcal{D}\psi(x) = \begin{cases} -P_-\psi(x+a\hat{0}) + \left[aK + i\gamma_5\tau^3 P_-\right]\psi(x) & x_0 = 0 \\ aD_W\psi(x) & 0 < x_0 < T \\ \left[aK + i\gamma_5\tau^3 P_+\right]\psi(x) - P_+\psi(x-a\hat{0}) & x_0 = T. \end{cases}$$

First we add to the action a boundary (relevant) counterterm of dimension three:

$$\delta S = (z_f - 1)a^3 \sum_{\vec{x}} \left(\bar{\psi}\psi|_{x_0=0} + \bar{\psi}\psi|_{x_0=T}\right) \tag{1.2}$$

so the Dirac operator reads now as

$$a\mathcal{D}\psi(x) = \begin{cases} -P_-\psi(x+a\hat{0}) + \left[aK + i\gamma_5\tau^3 P_- + (z_f-1)\right]\psi(x) & x_0 = 0 \\ aD_W\psi(x) & 0 < x_0 < T \\ \left[aK + i\gamma_5\tau^3 P_+ + (z_f-1)\right]\psi(x) - P_+\psi(x-a\hat{0}) & x_0 = T. \end{cases}$$

We have denoted the Wilson-Dirac operator with a standard mass term m_0 as D_W, whose action on the quark fields is given by

$$aD_W\psi(x) = -P_-\psi(x+a\hat{0}) + aK\psi(x) - P_+\psi(x-a\hat{0}). \tag{1.3}$$

The spatial part of the Wilson-Dirac operator is given by

$$aK\psi(x) = \left[1 + am_0 + \frac{1}{2}\sum_{k=1}^{3}\left\{a\gamma_k\left(\nabla_k + \nabla_k^*\right) - a^2\nabla_k^*\nabla_k\right\}\right]\psi(x). \tag{1.4}$$

$P_\pm$ are the projectors

$$P_\pm = \frac{1}{2}\left(\mathbb{1} \pm \gamma_0\right). \tag{1.5}$$

Writing this in matrix form in space-time, we have

$$S_f = a^8 \sum_{x_0,y_0=0}^{T} \sum_{\vec{x},\vec{y}} \bar{\psi}(x)\mathcal{D}(x,y)\psi(y), \tag{1.6}$$

with

$$aD_W(x,y) = -P_- \delta_{y_0,x_0+a}\delta_{\vec{y},\vec{x}} + aK(x,y) - P_+\delta_{y_0,x_0-a}\delta_{\vec{y},\vec{x}} \tag{1.7}$$

and

$$aK(x,y) = \left[\frac{1}{2\kappa}\delta_{\vec{y},\vec{x}} - \frac{1}{2}\sum_{k=1}^{3}\left\{(1-\gamma_k)\delta_{\vec{y},\vec{x}+a\hat{k}} + (1+\gamma_k)\delta_{\vec{y},\vec{x}-a\hat{k}}\right\}\right]\delta_{y_0,x_0}, \tag{1.8}$$

where the substitution $\kappa = \frac{1}{8+2am_0}$ has been performed.

In order to find the expression of the Dirac operator in the time-momentum representation, we perform a Fourier transform of the quark fields only in the space directions and act with $D(x,y)$ on the Fourier transformed field. In this case, the form of the Dirac operator is

$$a\mathcal{D}(x_0,y_0;\vec{p}) = \begin{cases} -P_-\delta_{y_0,a} + aK(x_0,y_0;\vec{p}) + \left[i\gamma_5\tau^3 P_- + (z_f-1)\right]\delta_{y_0,0} & x_0 = 0 \\ aD_W(x_0,y_0;\vec{p}) & 0 < x_0 < T \\ aK(x_0,y_0;\vec{p}) + \left[i\gamma_5\tau^3 P_+ + (z_f-1)\right]\delta_{y_0,T} - P_+\delta_{y_0,T-a} & x_0 = T \end{cases}$$

with

$$aD_W(x_0,y_0;\vec{p}) = -P_-\delta_{y_0,x_0+a} + aK(x_0,y_0;\vec{p}) - P_+\delta_{y_0,x_0-a} \tag{1.9}$$

and

$$aK(x_0,y_0;\vec{p}) = \left[\frac{1}{2\kappa} - \frac{1}{2}\sum_{k=1}^{3}\left\{(1-\gamma_k)\,e^{iap_k} + (1+\gamma_k)\,e^{-iap_k}\right\}\right]\delta_{y_0,x_0}. \tag{1.10}$$

The spatial part can also be expressed in the form

$$aK(x_0,y_0;\vec{p}) = \left[\frac{1}{2\kappa} - 3 + \frac{1}{2}\sum_{k=1}^{3}\left\{\gamma_k 2ia\mathring{p}_k + a^2\hat{p}_k^2\right\}\right]\delta_{y_0,x_0}. \tag{1.11}$$

Next, we add the dimension four fermionic (irrelevant) conunterterm at the boundaries:

$$\delta S = a(d_s-1)a^3\sum_{\vec{x}}\left(\bar{\psi}\gamma_k D_k\psi|_{x_0=0} + \bar{\psi}\gamma_k D_k\psi|_{x_0=T}\right) \tag{1.12}$$

and the Dirac operator takes the final form

$$a\mathcal{D} = \begin{cases} -P_-\delta_{y_0,a} + aK(x_0,y_0;\vec{p}) + \left[i\gamma_5\tau^3 P_- + (z_f-1) + a\,(d_s-1)\,i\gamma_k \mathring{p}_k\right]\delta_{y_0,0} & x_0 = 0 \\ aD_W(x_0,y_0;\vec{p}) & 0 < x_0 < T \\ aK(x_0,y_0;\vec{p}) + \left[i\gamma_5\tau^3 P_+ + (z_f-1) + a\,(d_s-1)\,i\gamma_k \mathring{p}_k\right]\delta_{y_0,T} - P_+\delta_{y_0,T-a} & x_0 = T \end{cases}$$

In our case, the tree-level value of d_s is $d_s^0 = \frac{1}{2}$. (Stefan and Bjoern (offset in opposite direction): $d_s^0 = \frac{3}{2}$).

2 Tuning

2.1 Scheme of how to perform the tuning

For each value of the renormalised coupling $\bar{g}^2$:

- Determine z_f^* at fix number of lattice points $L/a = fix$:
  - · Choose a value of z_f:
    - Choose four values of κ
    - Calculate $g_{A_-}(T/2a)$ and $m_{\mathrm{PCAC}}(T/2a)$ for each κ
    - Plot $g_{A_-}(T/2a)$ vs. $m_{\mathrm{PCAC}}(T/2a)$
    - Interpolating the data, determine $g_{A_-}(T/2a)$ such that $m_{\mathrm{PCAC}}(T/2a) = 0$ at this z_f
  - · Repeat the previous steps for other three values of z_f
  - · Plot the previous interpolated values of $g_{A_-}(T/2a)$ vs. z_f
  - · Interpolating the data, determine $z_f^* = z_f : g_{A_-} = 0$
- Obtain κ_c at the previously determined value of z_f^*
  - · For each of the previously chosen values of z_f, determine $\kappa_c = \kappa : m_{\mathrm{PCAC}}(T/2a) = 0$
  - · Plot κ_c vs. z_f and interpolate to $z_f = z_f^*$
  - · This will give the value of κ_c at z_f^*
- Repeat all the previous steps at different values of $L/a = fix$

Then repeat the procedure for the other values of $\bar{g}^2$.

Fixed parameters			
d_s	θ_1	θ_2	θ_3
0.5	0.0	0.0	0.0

Table 1
Fixed parameters during the tuning.

Tuning points				
$\bar{g}^2$	L/a	β	source	κ_c(source)
0.9944	8	10.3000	hep-lat/0402022	0.135457(5)
	12	10.6086		0.135160(4)
	16	10.8910		0.134849(6)
2.4484	8	7.0197	hep-lat/0402022	0.134639(7)
	12	7.3551		0.134141(5)
	16	7.6101		0.133729(4)
$L = 2L_{max} = 1.436r_0$	8	6.0219	hep-lat/9810063	0.153597(14)
	10	6.1628		0.152277(10)
	12	6.2885		0.151024(7)
	16	6.4956		0.149012(12)

Table 2
Tuning points.

2.2 Parameters and tuning points

We have sonsidered the set of fixed parameters presented in Tab. 1. The sources have been set at $x_0 = a$.

The tuning is being performed for the cases summarised in Tab. 2

Tuning results at $\bar{g}^2$ such that $L = 2L_{max}$				
L/a	β	κ_c	z_f^*	$z_f^*(Dublin)$
8	6.0219	0.153531(40)	1.8090(32)	0.7885(21)
10	6.1628	0.152133(34)	1.7920(30)	———
12	6.2885	running	running	0.8272(23)
16	6.4956	running	running	0.8385(14)

Table 3
Tuning parameters and results at $L = 1.436r_0$.

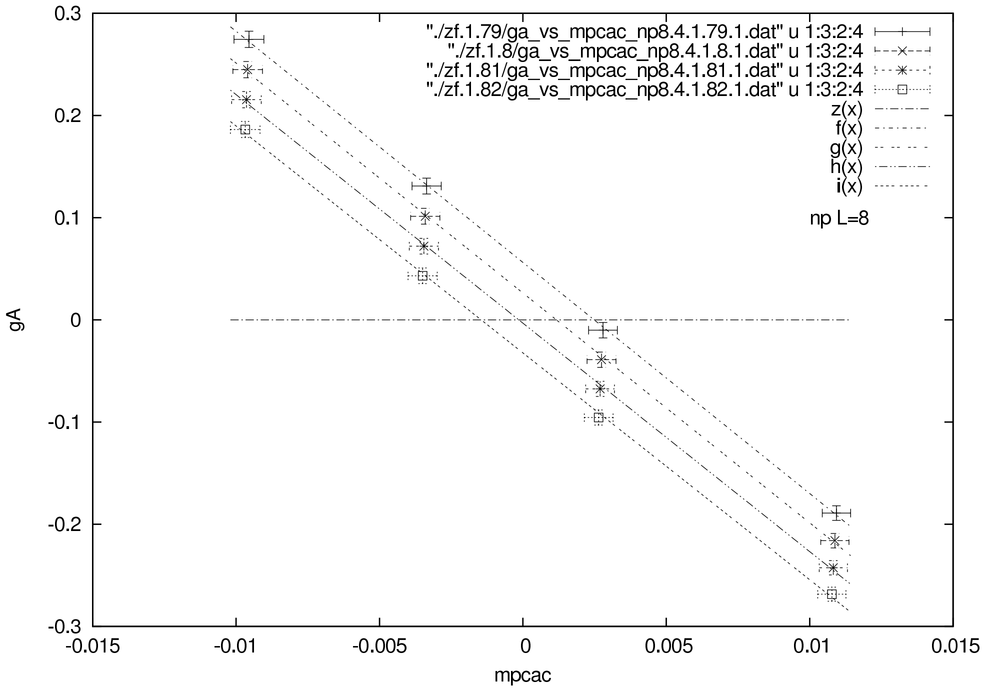

Fig. 1. g_{A-} vs. m_{PCAC} at fixed $L = 2L_{max}$ and $L/a = 8$ for $z_f = 1.79, 1.8, 1.81, 1.82$.

2.3 Tuning of κ and z_f at fixed $\bar{g}^2$: $L = 2L_{max}$

The results obatined for the tuning of κ and z_f to the critical values at the most non-perturbative point are summarised in Tab. 3.

The values of g_{A-} as a function of the m_{PCAC} for each value of z_f are presented in Fig. 1 and Fig. 2 for 8^4 and 10^4 lattices respectively.

The values of $g_{A-}(m_{PCAC} = 0)$ as a function of z_f are presented in Fig. 3 and Fig. 4 for 8^4 and 10^4 lattices respectively.

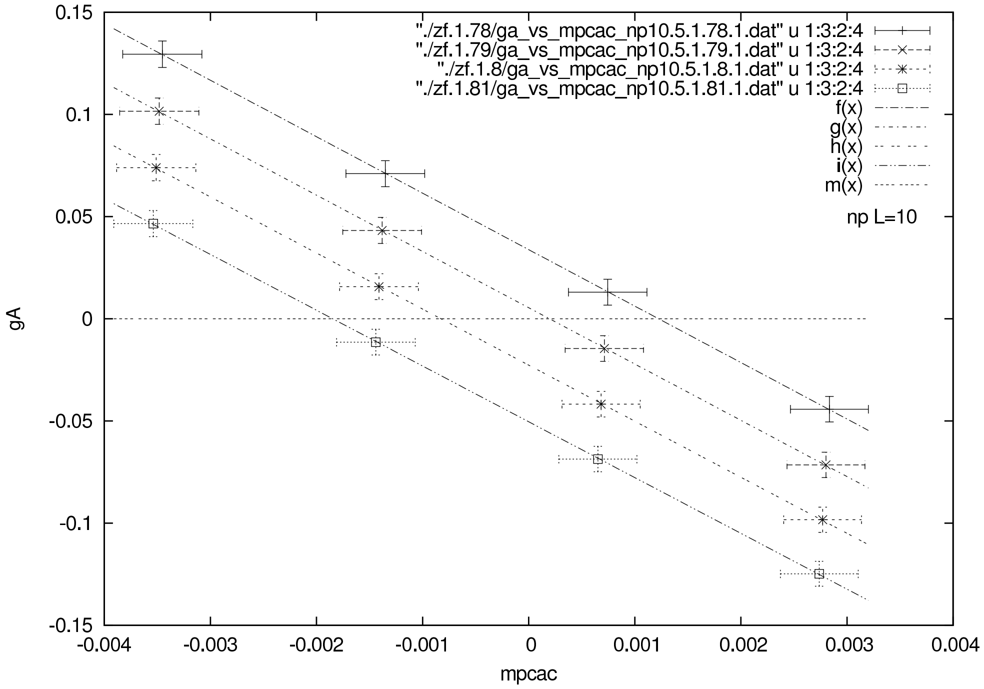

Fig. 2. g_{A-} vs. m_{PCAC} at fixed $L = 2L_{max}$ and $L/a = 10$ for $z_f = 1.78, 1.79, 1.8, 1.81$.

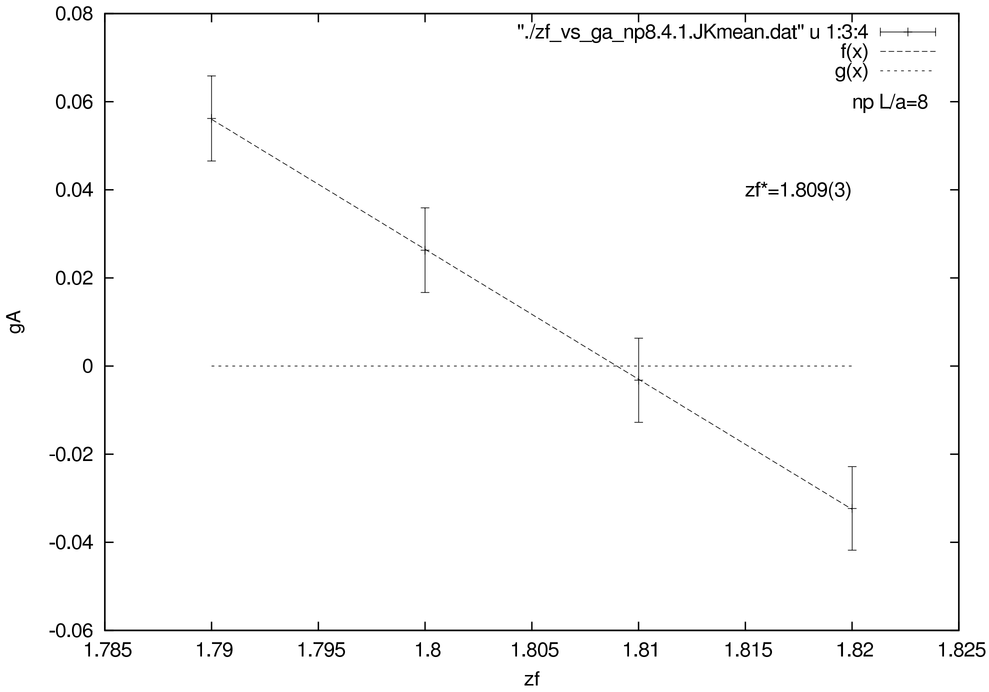

Fig. 3. $g_{A-}(m_{PCAC} = 0)$ vs. z_f at fixed $L = 2L_{max}$ and $L/a = 8$.

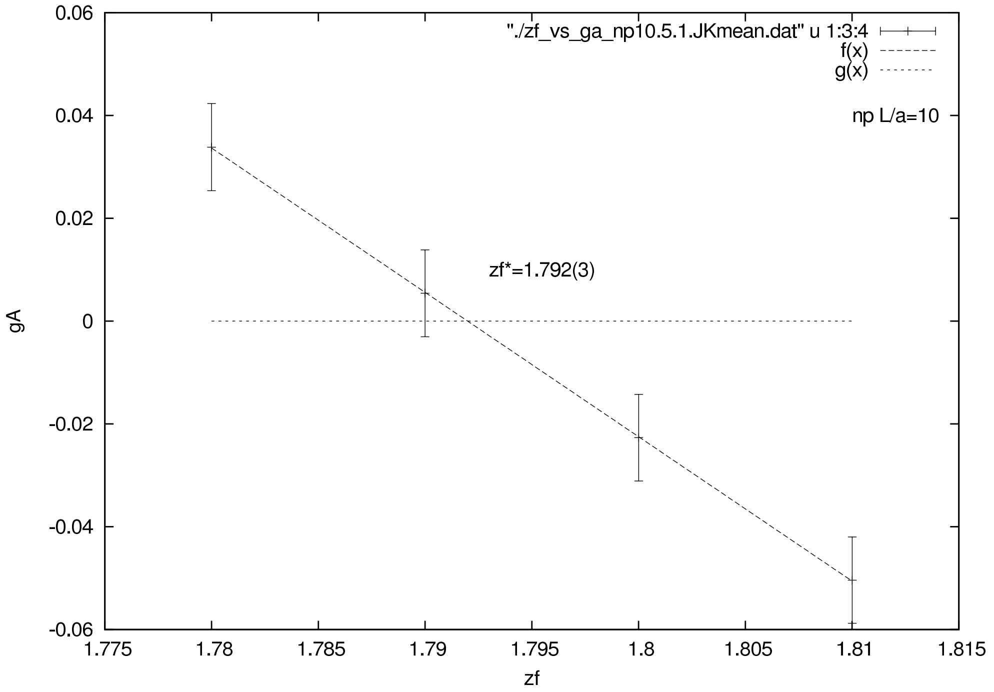

Fig. 4. $g_{A-}(m_{PCAC} = 0)$ vs. z_f at fixed $L = 2L_{max}$ and $L/a = 10$.

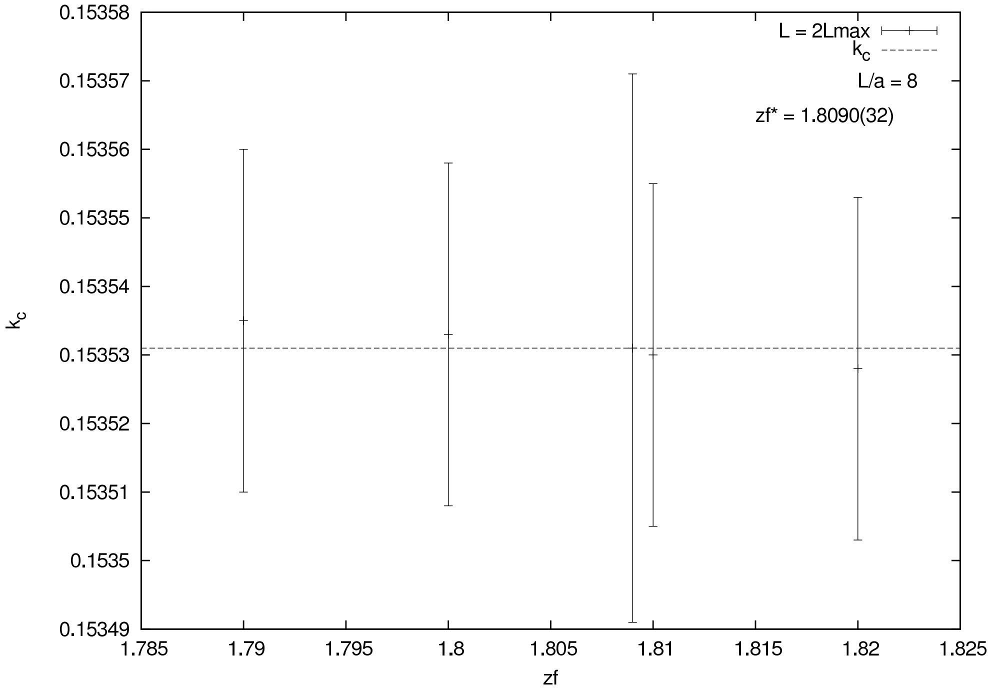

Fig. 5. κ_c vs. z_f at fixed $L = 2L_{max}$ and $L/a = 8$.

The values of κ_c as a function of z_f are presented in Fig. 5 and Fig. 6 for 8^4 and 10^4 lattices respectively.

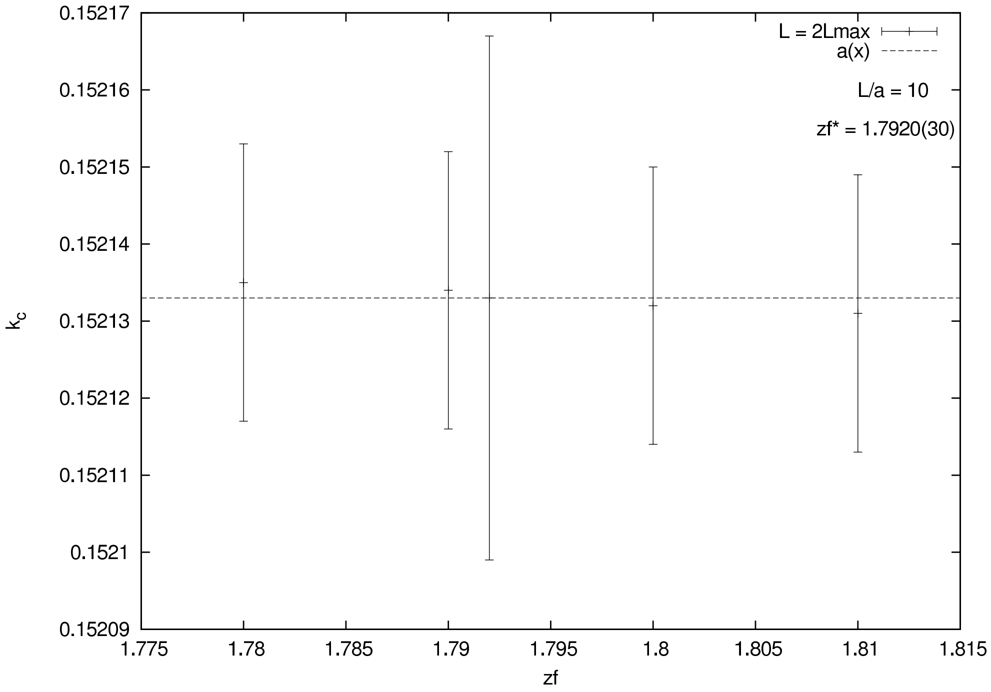

Fig. 6. κ_c vs. z_f at fixed $L = 2L_{max}$ and $L/a = 10$.

Tuning results at $\bar{g}^2 = 0.9944$				
L/a	β	κ_c	z_f^*	$z_f^*(Dublin)$
8	10.3000	running	running	——
12	10.6086	running	running	——
16	10.8910	running	running	——

Table 4
Tuning parameters and results at $\bar{g}^2 = 0.9944$

Tuning results at $\bar{g}^2 = 2.4484$				
L/a	β	κ_c	z_f^*	$z_f^*(Dublin)$
8	7.0197	running	running	0.8571(14)
12	7.3551	running	running	0.8767(13)
16	7.6101	running	running	0.87978(79)

Table 5
Tuning parameters and results at $\bar{g}^2 = 2.4484$

2.4 Tuning of κ and z_f at fixed $\bar{g}^2 = 0.9944$

The results obatined for the tuning of κ and z_f to the critical values at the most perturbative point are summarised in Tab. 4.

2.5 Tuning of κ and z_f at fixed $\bar{g}^2 = 2.4484$

The results obatined for the tuning of κ and z_f to the critical values at the intermediate point are summarised in Tab. 5.