Galileo Galilei Institute
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Strange Quark Mass and Kaon Decay Constant

On behalf of Cecilia Tarantino

Simulation Details

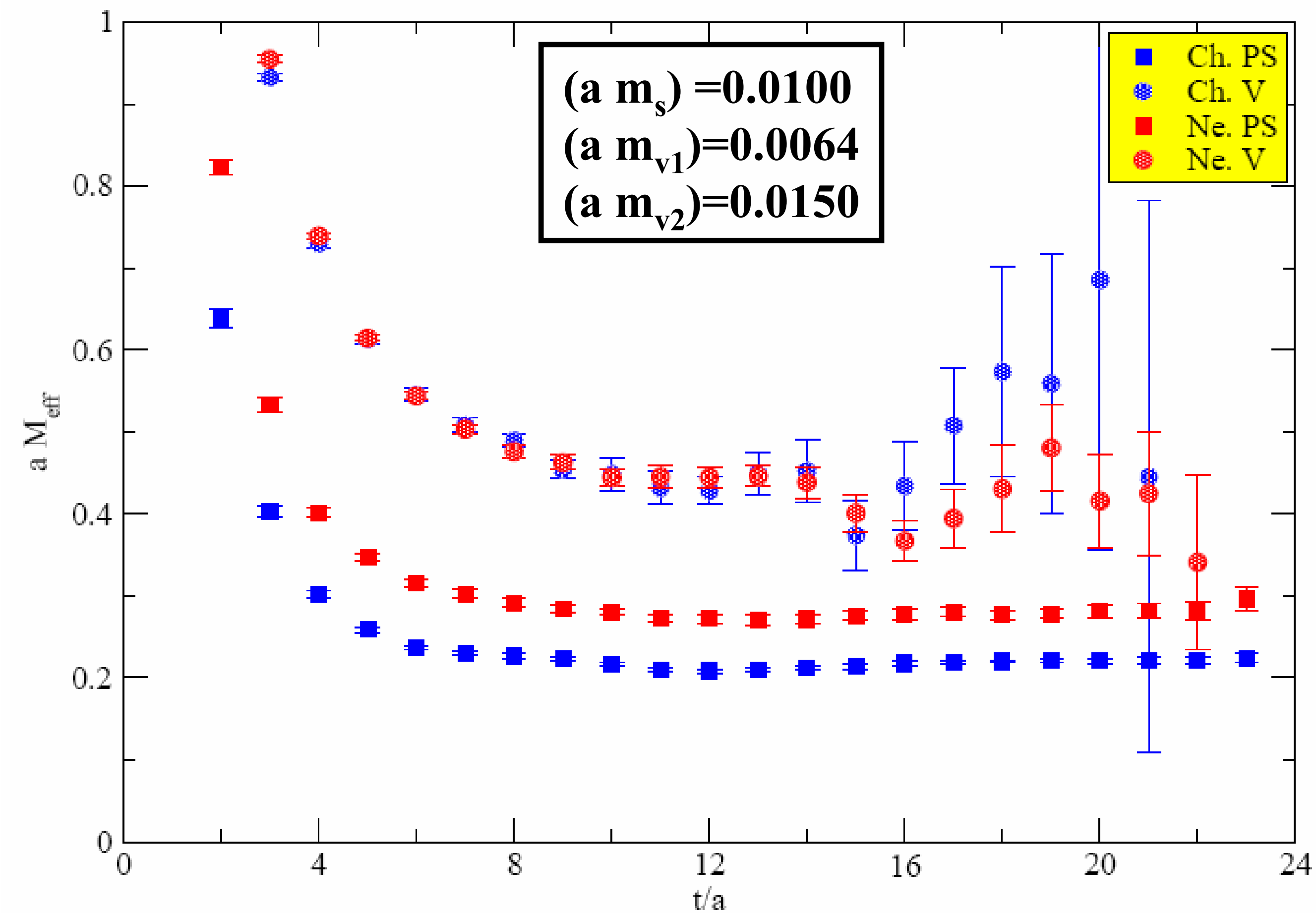
- TlSym gauge action + MtmQCD fermion action
- N_f=2 degenerate sea quarks
- β=3.9 , $V=24^3$ x 48 , k_c=0.0160856
- 240 confs. (24 jacknives and 1000 bootstrap events
  to combine data at different sea quark masses)
- 4 simulated quark mass values:(a m)=0.0040, 0.0064, 0.0100, 0.0150
- partially quenched: valence ≠ sea
- non-degeneracy in valence: $valence_1 \neq valence_2$

Future improvements

- Liverpool stochastic method for inversion
- Anti-periodic boundary conditions
- Inclusion of chiral logs (and finite volume) effects
- SU(3) breaking effects ($m_s >> m_{u,d}$)

Meson Masses

$M_P(m_s, m_{v1}, m_{v2})$
$M_V(m_s, m_{v1}, m_{v2})$



(*) Note: disconnected contributions are absent in this case

(**) It is crucial to improve the signal for the vector-vector correlation functions

Pseudoscalars:

- **Very good plateaux**
- **Neutral masses are systematically above charged masses (*)**

Vectors:

- **Similar plateaux from $T^{0k}T^{0k}$**
- **Neutral masses are more precise than charged masses (**)**

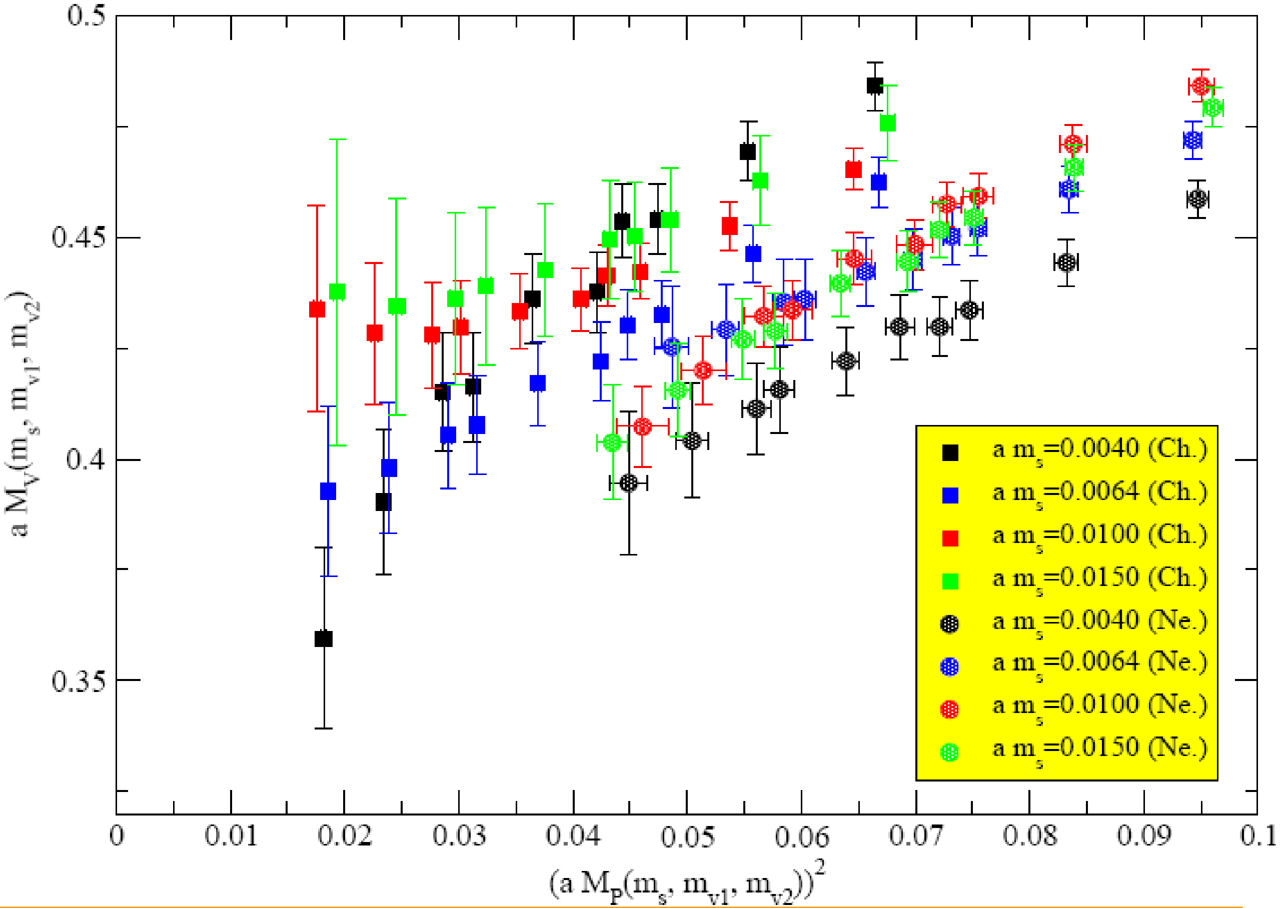
a^{-1} from the ``physical lattice plane´´ method

$$aM_V(m_s, m_{v1}, m_{v2}) = A + B\cdot(aM_P(m_s, m_{v1}, m_{v2}))^2 + C\cdot(aM_P(m_s, m_s, m_s))^2$$

$$\oplus$$

$$(M_K/M_{K^*}) = (M_K/M_{K^*})^{exp}$$

(negligible sea dependence)



Charged:
a^{-1}= 1.87(2)GeV
M_π= 130(3)MeV
M_ρ= 718(18)MeV
M_Φ= 1066(18)MeV

Neutral:
(more precise M_V)
a^{-1}= 2.11(3)GeV
M_π= 132(1)MeV
M_ρ= 758(7)MeV
M_Φ= 1039(7)MeV

Exp. values

$M_{\pi^\pm} = 140\text{MeV}, M_{\pi^0} = 135\text{MeV},$
$M_\rho = 776\text{MeV}, M_\Phi = 1019\text{MeV}$

$a^{-1}(\pi/\rho)$=2.02(5)GeV (Ch.), $a^{-1}(\pi/\rho)$=2.16(5)GeV (Ne.)

Light Quark Masses

$$(aM_P(m_s, m_{v1}, m_{v2}))^2 = A(m_s) + B\frac{(am_{v1} + am_{v2})}{2}$$

ChPT:

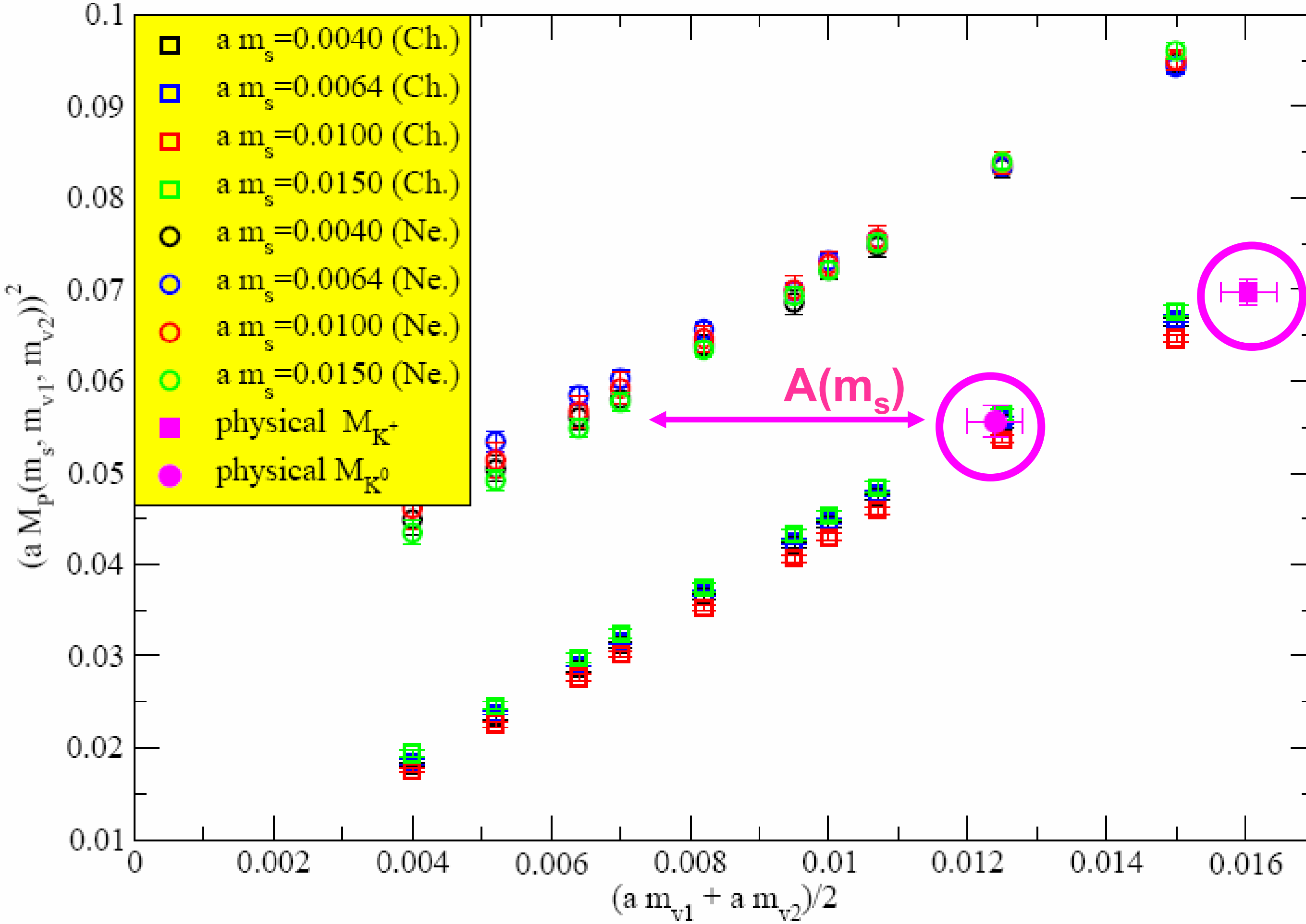
$$M_P(m_s, m_v)^2 = B \cdot m_v \cdot (1 + C_v\, m_v + C_s\, m_s + \ldots)$$

Discretization effects:

- Have a **slight sea dependence**
- are ~30 times **bigger** in **neutral** than in charged

$A(m_s)^{ch.} \simeq 8 \cdot 10^{-4}$

$A(m_s)^{ne.} \simeq 3 \cdot 10^{-3}$



Charged:

$(a\, m_{light}) = 0.00110(6)$

$(a\, m_{strange}) = 0.0309(7)$

Neutral:

$(a\, m_{light}) = 0.00087(4)$

$(a\, m_{strange}) = 0.0240(7)$

Non-perturbative Renormalization

[See previous talk]

$$\hat{m}_q = \frac{m_q}{Z_P}$$

$$Z_P^{RI-MOM}(\mu = 1/a) = 0.39(1)(2) < Z_P^{1loop\text{-}PT} = 0.52 - 0.62$$

In $\overline{MS}$ at μ=2GeV

Charged:
$\hat{m}_{light}$ = 4.1(2) MeV
$\hat{m}_{strange}$= 115(1) MeV

Neutral:
$\hat{m}_{light}$ = 3.8(1) MeV
$\hat{m}_{strange}$= 105(2) MeV

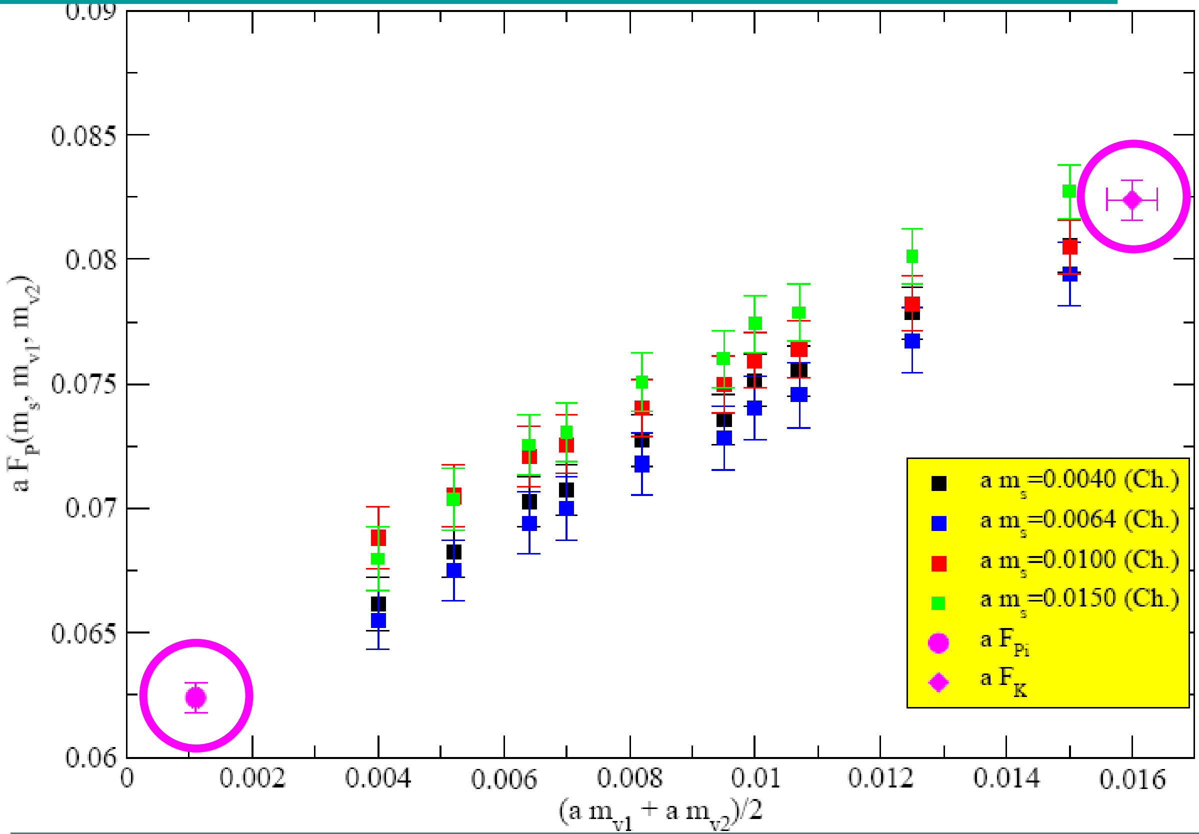
Statistical errors only

Importance of NP-renormalization

Pseudoscalar Decay Constants

$$aF_P(m_s,m_{v1},m_{v2}) = (am_{v1}+am_{v2})\frac{|\langle 0|P_5(0)|P\rangle|}{(aM_P(m_s,m_{v1},m_{v2}))^2}$$

Indirect Method
(for charged mesons only, due to mixing with the scalar operator in the neutral case)



- **Negligible sea dependence**
- **Compatible polynomial and logarithmic fits**

- F_π=117(2)MeV
- F_K/F_π=1.31(1)
- $a^{-1}(F_\pi)$=2.09(2)GeV

equal to a^{-1}(K/K*) ``neutral´´

F_π=132(2)MeV
from a^{-1}(K/K*) ``neutral´´

$$1)\quad aF_P(m_s,m_{v1},m_{v2}) = A + B\frac{(am_{v1}+am_{v2})}{2} + C\left(\frac{(am_{v1}+am_{v2})}{2}\right)^2$$

$$2)\quad aF_P(m_s,m_{v1},m_{v2}) = P_1 + P_2\frac{(am_{v1}+am_{v2})}{2}\mathrm{Log}\left[\frac{(am_{v1}+am_{v2})}{2}\right]$$

Exp. values

- F_π=131MeV
- F_K/F_π=1.22

Conclusions

- The strange quark mass is $am_s \simeq 0.03$: important for next calculations. $m_s \simeq 115$ MeV: importance of NP-renormalization
- It is important to improve the determination of m_V and the lattice spacing
- It is crucial to improve the systematic accuracy of f_K/f_π
  (MILC: $f_K/f_\pi = 1.210(4)(13) \rightarrow V_{us} = 0.2219(26)$)

Future plans

- Liverpool stochastic method for inversion
- Anti-periodic boundary conditions
- Inclusion of chiral logs (and finite volume) effects
- SU(3) breaking effects ($m_s >> m_{u,d}$)