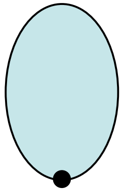
* the calculation of disconnected diagrams requires the evaluation of fermion bubbles



$$= B_\Gamma(t) = \sum_{\vec{x}} \Gamma_{\alpha\beta}\, S^{aa}_{\beta\alpha}(x,x)$$

Γ= Dirac matrix

* all-to-all propagator: $$\sum_z D^{ab}_{\alpha\beta}(x,z)\, S^{bc}_{\beta\gamma}(z,y) = \delta_{\alpha\gamma}\delta_{ac}\delta_{x,y}$$

* stochastic method: $$S(x,y) = \lim_{N_s\to\infty} \frac{1}{N_s}\sum_{r=1}^{N_s} \Phi^r(x)\left[\eta^r(y)\right]^\dagger$$

(drop color and spin labels)

η^r = stochastic source (Z_2-noise)

$$\begin{cases} \left[\eta^r(x)\right]^\dagger \eta^r(x) = 1 \\ \lim_{N_s\to\infty} \frac{1}{N_s}\sum_{r=1}^{N_s}\left[\eta^r(x)\right]^\dagger \eta^r(y) = \delta_{x,y} \end{cases}$$

$$\sum_z D(x,z)\,\Phi^r(z) = \eta^r(x)$$

$$B_\Gamma(t) = \lim_{N_s\to\infty} \frac{1}{N_s}\sum_{r=1}^{N_s}\sum_{\vec{x}}\left[\eta^r(x)\right]^\dagger \Gamma\, \Phi^r(x)$$

* finite number of stochastic sources: $\frac{1}{N_s}\sum_{r=1}^{N_s}\sum_{\vec{x}}\left[\eta^r(x)\right]^\dagger \Gamma\, \Phi^r(x) = \frac{1}{N_s}\sum_{r=1}^{N_s}\sum_{\vec{x}}\left[\eta^r(x)\right]^\dagger \Gamma\, S(x,y)\,\eta^r(y)$

$$\frac{1}{N_s}\sum_{r=1}^{N_s}\left[\eta^r(x)\right]^\dagger \eta^r(y) \neq \delta_{x,y}$$

$B_\Gamma(t)$ + gauge-variant noise (non-local bubble)

* recently the TrinLat collaboration has proposed a new technique: the dilution method (hep-lat/0505023)

- in few words: exact treatment of the all-to-all problem in a subset of variables only

* **time dilution**: L_t stochastic sources defined for each time slice

$$\eta^r(x) = \eta^r(\vec{x})\,\delta_{t,t_r} \qquad r = 1,2,\ldots L_t$$

* **color-spin dilution**: 12 stochastic sources defined for each color-spin combination

$$\left[\eta_\alpha^a(x)\right]^r = \eta^r(x)\,\delta_{\alpha,\alpha_r}\delta_{a,a_r} \qquad \alpha_r = 1,2,3,4,\ a_r = 1,2,3$$

* we want to compare **three methods:** time dilution, color-spin dilution and no dilution (direct method)

* quenched gauge configurations at V T = 16^3 32 and $\beta = 6.0$
  twisted-mass Wilson fermions at $\kappa_c = 0.157409$ (χLF collaboration)

* disconnected diagrams for π^0 and η' correlators:

$$\pi^0:\ \text{connected} + \left\langle \sum_t \left\{ \mathrm{B}^{\mathrm{u}}_{\gamma_5}(t)\mathrm{B}^{\mathrm{u}}_{\gamma_5}(t+\delta t) + \mathrm{B}^{\mathrm{d}}_{\gamma_5}(t)\mathrm{B}^{\mathrm{d}}_{\gamma_5}(t+\delta t) - \mathrm{B}^{\mathrm{u}}_{\gamma_5}(t)\mathrm{B}^{\mathrm{d}}_{\gamma_5}(t+\delta t) - \mathrm{B}^{\mathrm{d}}_{\gamma_5}(t)\mathrm{B}^{\mathrm{u}}_{\gamma_5}(t+\delta t) \right\} \right\rangle$$

$$\eta':\ \text{connected} + \left\langle \sum_t \left\{ \mathrm{B}^{\mathrm{u}}_{\gamma_5}(t)\mathrm{B}^{\mathrm{u}}_{\gamma_5}(t+\delta t) + \mathrm{B}^{\mathrm{d}}_{\gamma_5}(t)\mathrm{B}^{\mathrm{d}}_{\gamma_5}(t+\delta t) + \mathrm{B}^{\mathrm{u}}_{\gamma_5}(t)\mathrm{B}^{\mathrm{d}}_{\gamma_5}(t+\delta t) + \mathrm{B}^{\mathrm{d}}_{\gamma_5}(t)\mathrm{B}^{\mathrm{u}}_{\gamma_5}(t+\delta t) \right\} \right\rangle$$

* in tmLQCD, π^0 mixes with identity:

$$B^{\tau_3}_{\gamma_5}(t) = \hat{B}^{\tau_3}_{\gamma_5}(t) + (\pm)^{\tau_3} \frac{1}{a^3} \rho_{\gamma_5}(\mu)$$

subtraction of $\sum_t \left\langle B^{\tau_3}_{\gamma_5}(t) \right\rangle \left\langle B^{\tau_3'}_{\gamma_5}(t+\delta t) \right\rangle$

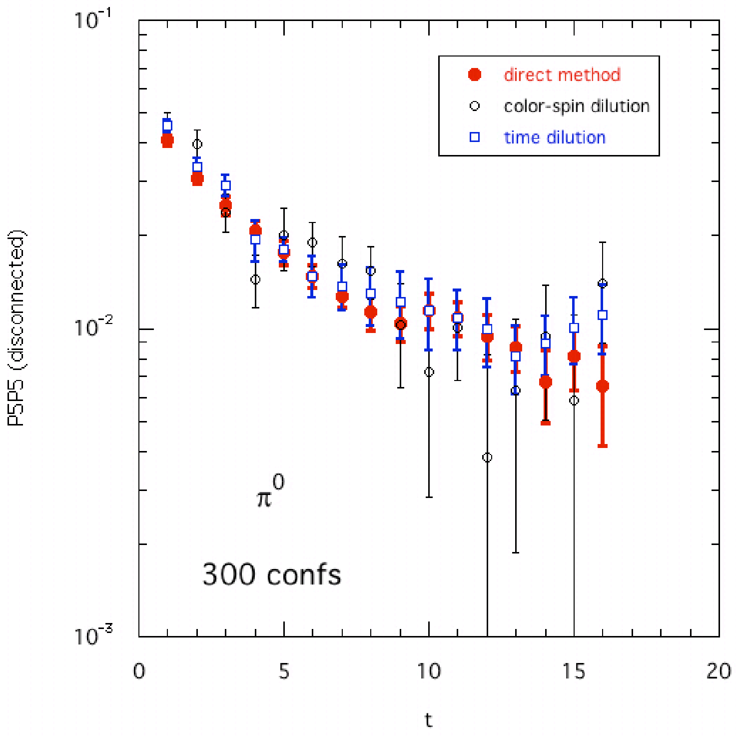
* η' is free from such a mixing

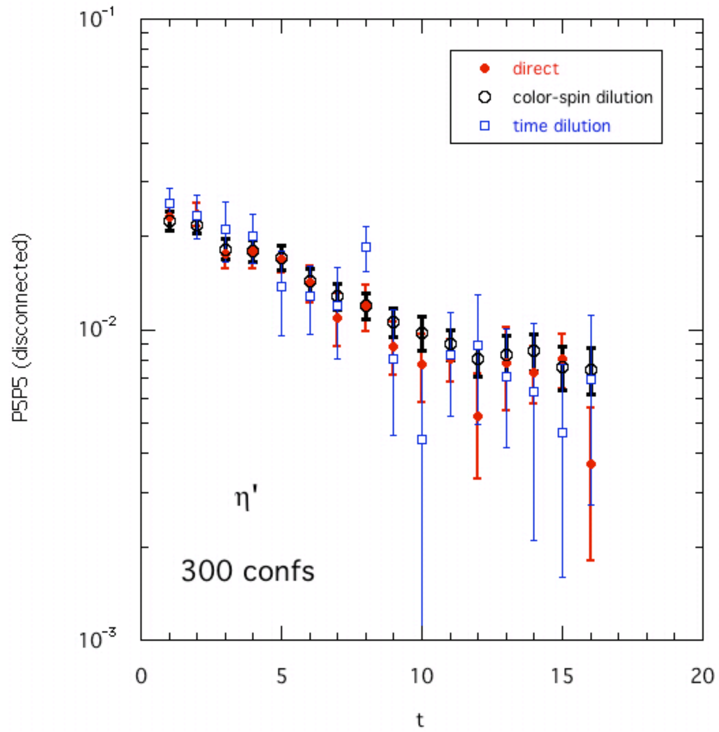
computational costs (per gauge conf.)

2*2*32 = 128 inversions (time dilution)

2*2*12 = 48 inversions (direct method)

2*12 = 24 inversions (color-spin dilution) a $\mu = 0.0038$ [a $M_{\pi+} = 0.131$ (2)]





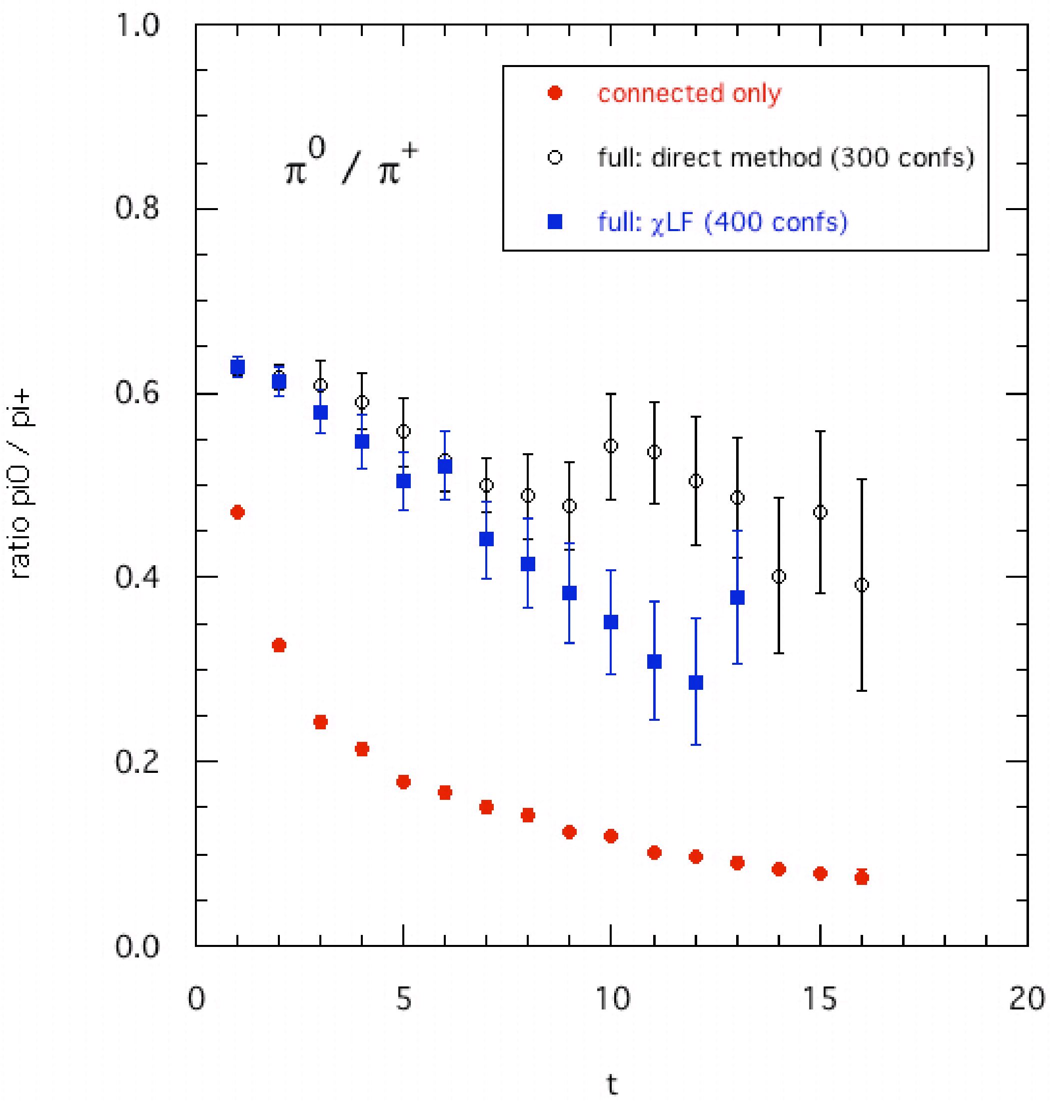
best: direct method

worst: color-spin dilution

best: color-spin dilution

worst: time dilution

<u>next method</u>: a hybrid one composed by spin dilution + direct method



χLF ('05): direct method +
noise variance reduction trick

$a * m(\pi^0) = 0.18 \pm 0.03$ (direct)

$a * m(\pi^0) = 0.19 \pm 0.02$ (χLF)

$a * \Delta m(\pi^0 - \pi^+) = 0.05 \pm 0.03$ (direct)

$a * \Delta m(\pi^0 - \pi^+) = 0.07 \pm 0.02$ (χLF)