

ETMColl.: Discussion on ChPT

Open issues on ChPT fits (left from hep-lat/0701012):

- Lattice Artifacts
- Finite Size Effects
- NNLO
- in particular:
effects on Chiral fit related to Flavor symm. breaking.

Other issues:

- error analysis
- $a=a(\beta)$
- ...

Finite Size Effects

$[\beta=3.9, \mu=0.004]$

L	$M_{\pi}(L)$	$M_{\pi}(1)$	$M_{\pi}(2)$	$M_{\pi}(3)$
24	0.1359(7)	^{-0.9%} 0.1347	0.1353	0.1350
32	^{-1.6%} 0.1337(2)	0.1335	0.1336	0.1335

L	$F_{\pi}(L)$	$F_{\pi}(1)$	$F_{\pi}(2)$	$F_{\pi}(3)$
24	0.0653(4)	0.0669	0.0664	0.0669
32	0.0662(5)	0.0665	0.0664	0.0665

- (1) CDH, expansion parameter for Finite Size effects: $1/F$
- (2) GL, $1/F$
- (3) GL, $1/F_0$

Update of the results

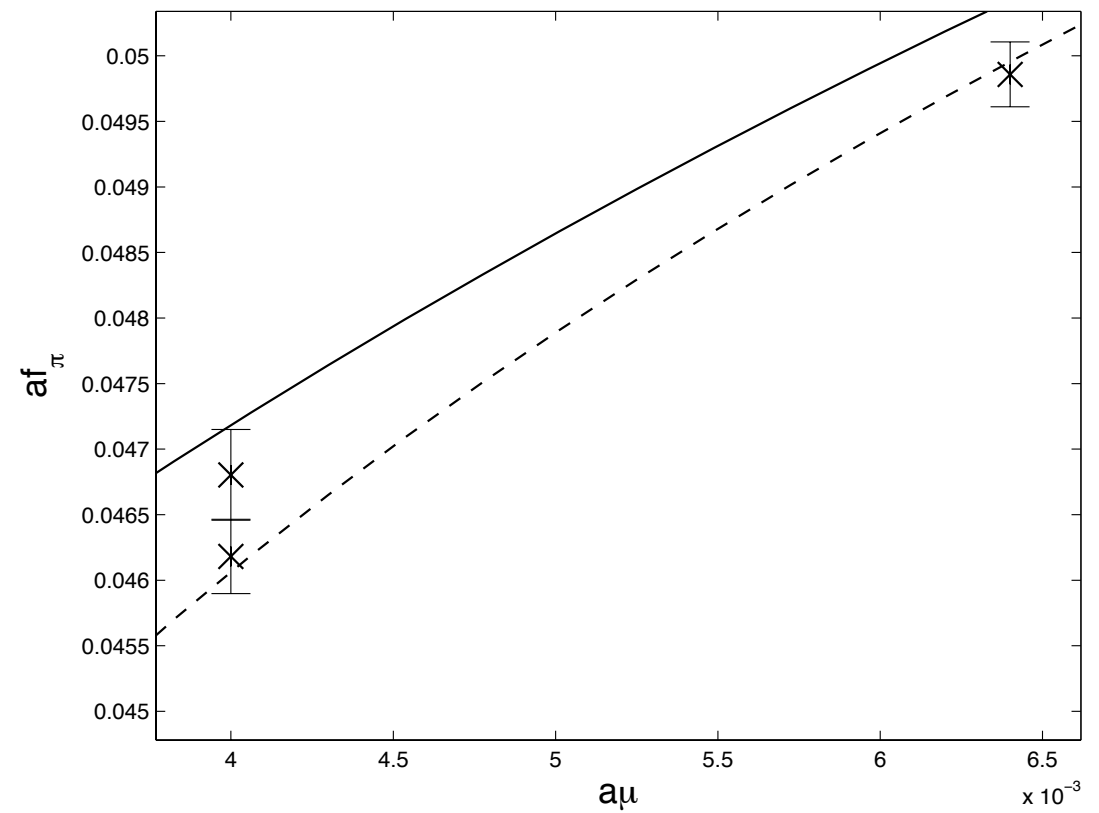
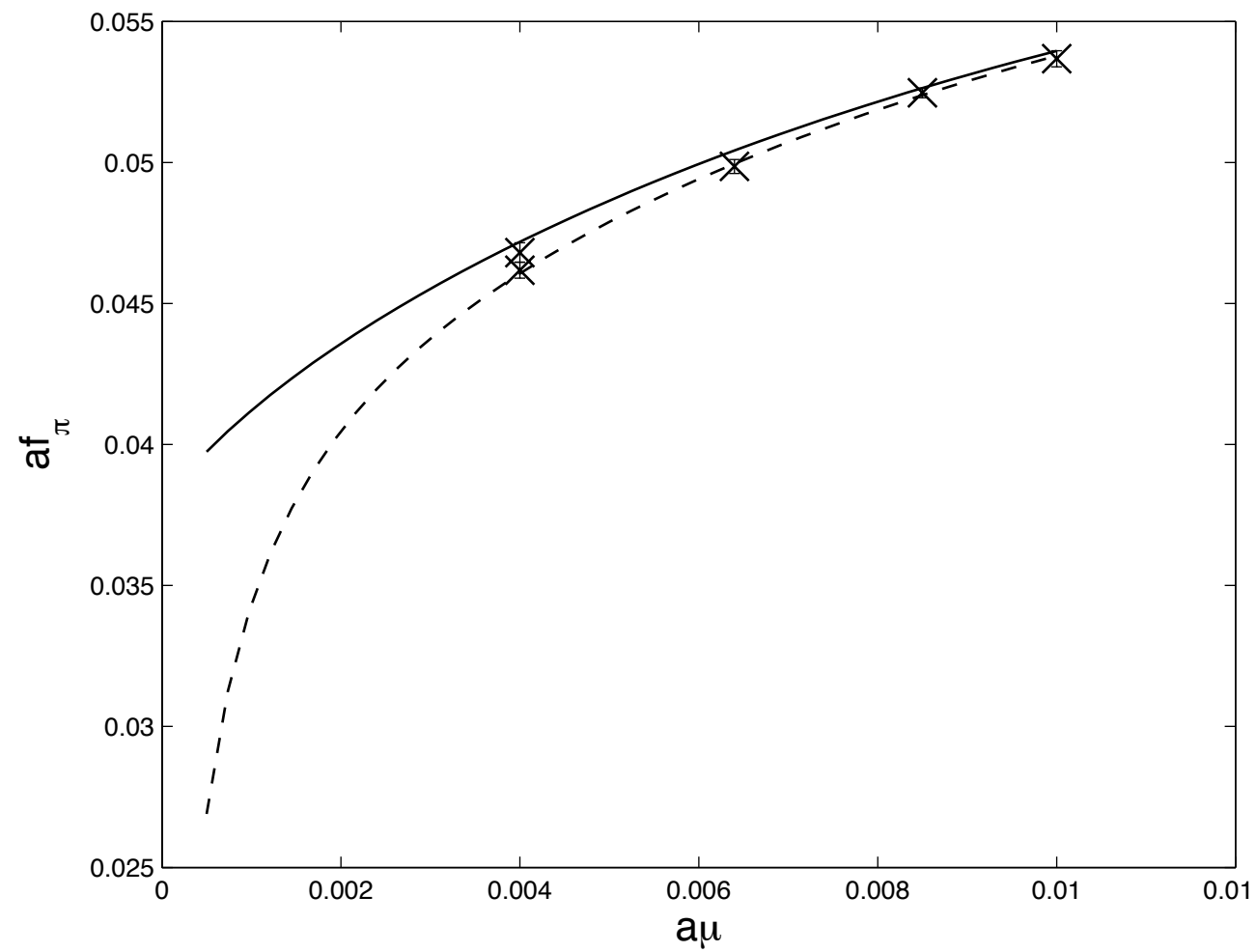
$\beta=3.9$ ONLY
old: only L=24
new: include L=32

	(3) old	(2) old	(1) old	(3) new	(2) new	(1) new
$2aB_0$	4.99(7)	5.04(7)	4.98(7)	4.84(3)	4.86(3)	4.83(3)
aF_0	0.0534(7)	0.0523(8)	0.0532(8)	0.0535(6)	0.0528(6)	0.0533(6)
$2 \log(a\Lambda_3)$	-1.92(12)	-1.90(11)	-1.96(12)	-2.22(8)	-2.23(8)	-2.23(8)
$2 \log(a\Lambda_4)$	-1.06(5)	-1.03(5)	-1.05(5)	-1.03(4)	-1.01(4)	-1.03(4)
χ^2 / n	0.15	0.14	0.15	1.65	2.05	1.35

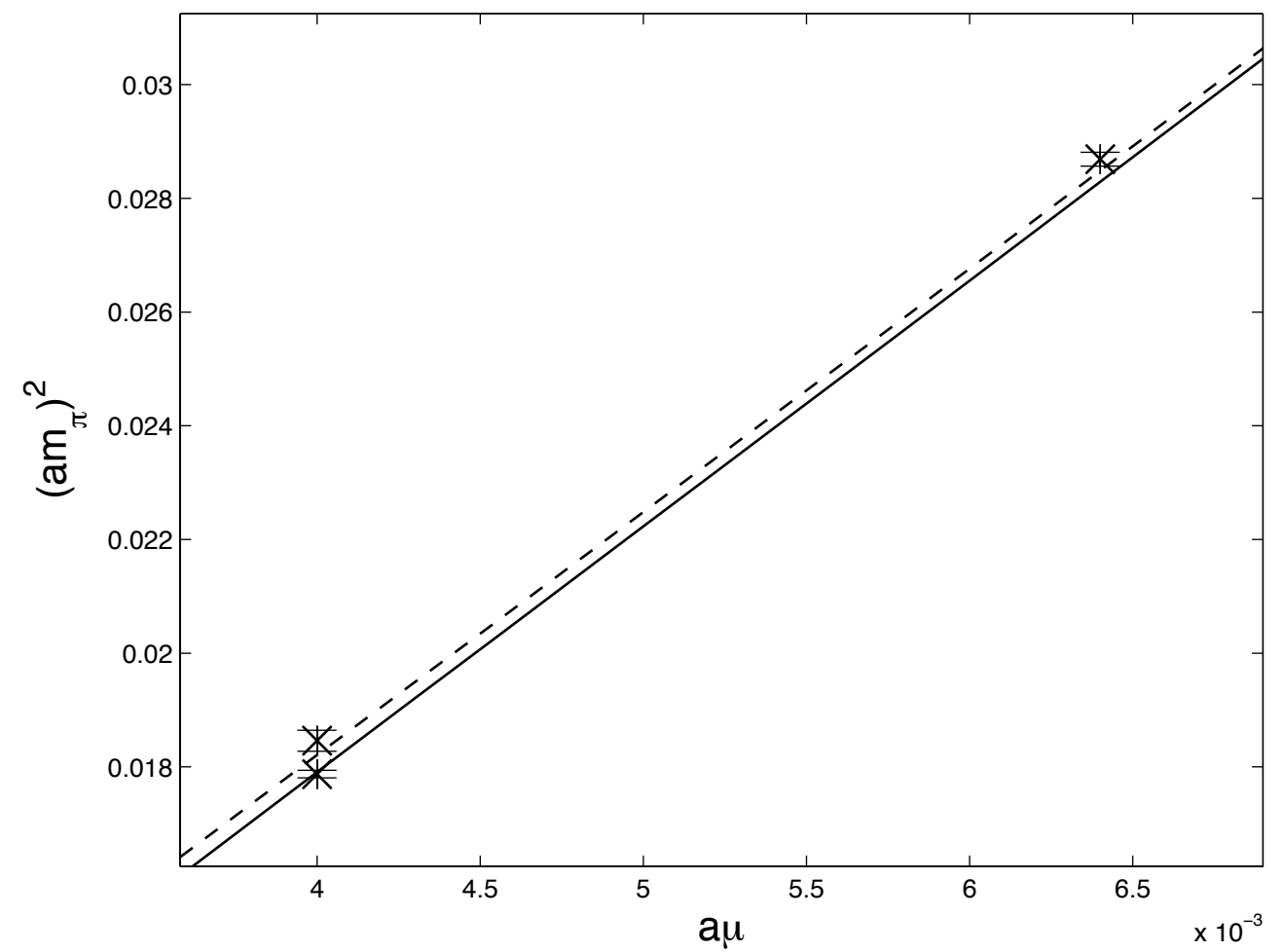
Important effect on Λ_3 (and B_0), while Λ_4 and F_0 are stable

The **errorbars** take into account non-linear implicit fitting procedure but:

- neglect cross correlation of basic parameters (easy to include)
- neglect autocorrelation (more work to adapt the Γ -method).



- X: data
- dashed lines: fit of Finite Volume formulae
- cont. lines: Infinite Volume extrapolation.



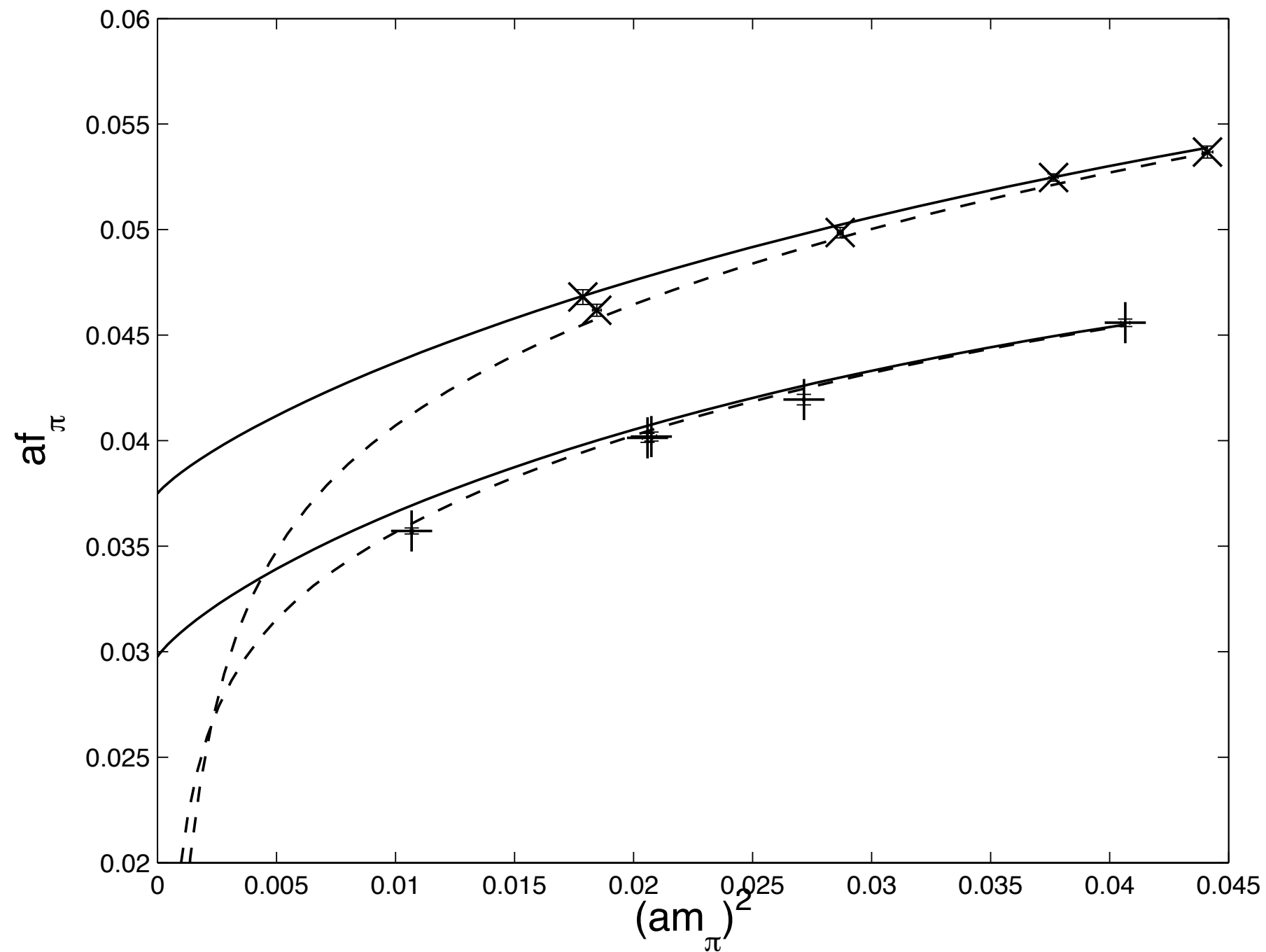
Does ChPT have an explanation?

- FSE Leading Order from Gasser Leutwyler NO
- FSE NLO from Colangelo Dürr Haefli: tried NO
- ChPT with non degenerate u, d quarks: very different
- compute Matrix element which enter CDH formulae with Wilson ChPT: maybe ?!?
- other reason: T not large enough ?!?

Lattice Artifacts

Plan:

- check the scaling.
- If fails, compelling to check WChPT.
- If the scaling is good, still might be helpful to check lattice artifacts with WChPT.



Scaling seems reasonable,
but only marginally within errorbars.

NNLO

A simple test: (inspired to the substitution $1/F_0 \rightarrow 1/F_\pi$ as expansion parameter)

$$M_\pi^2 = 2B_0\mu(1 - \frac{x}{2}l_3)$$

$$F_\pi = F_0(1 + xl_4)$$

$$x = \frac{2B_0\mu}{(4\pi F_0)^2} \longrightarrow x' = x(1 \pm 2xl_4)$$

$$l_{3,4} = \log(\Lambda_{3,4}^2/(2B_0\mu))$$

	usual x	x'(+)	x'(-)
2aB ₀	4.99(7)	5.20(8)	
aF ₀	0.0534(7)	0.0516(9)	
2 log(aΛ ₃)	-1.92(12)	-2.19(7)	
2 log(aΛ ₄)	-1.06(5)	-1.76(1)	
χ ² /n	0.15	1.5	

No convergence

Definition of fixed *lattice spacing* at different masses

- Pion mass splitting is mass independent up to NNLO: $m_{\pi\pm}^2 - m_{\pi 3}^2 = c a^2 \sin(\omega)^2$
- This offers a probably impractical but theoretically clear definition of *lattice spacing* a , which is compatible with ChPT:
If I use such definition of a , I am sure that I will get the continuum limit LEC's from ChPT fits.
- In the physical point all definition of a' are ok, even if:
(even $a = a F_\pi / (92 \text{ MeV})$)
$$a' = a(1 + \lambda m / \Lambda_{\text{QCD}})$$
- However, to compare with ChPT, only those are good such that at least. Otherwise LEC's would be wrong in the continuum limit
$$a' = a(1 + \lambda a m)$$
- No problems for ratios of quantities which have ChPT predictions (for example: F_π / M_π)
- But it is a problem when fitting for example: $a F_\pi$

Is it possible to prove that the usual definition $a = a(\beta)$ has a good relation with the natural one in ChPT ?

- The relation between the two definitions $a = a(\beta)$ and $a = a(r_0)$ has been discussed in [Aoki at Lattice2000, Sommer at Lattice 2003].