Plots

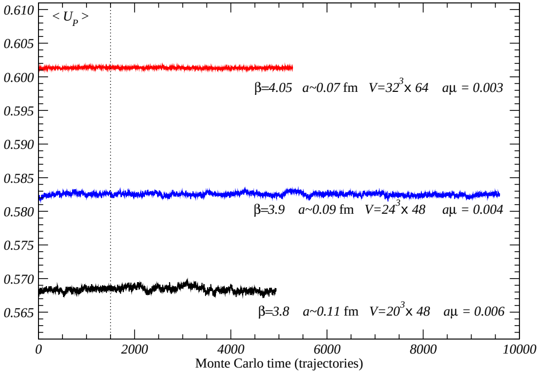
Rome II group
ETM Collaboration

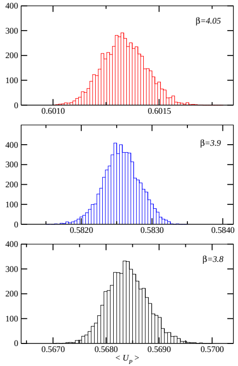
Florence – Feb. 2007

Simulations: setup

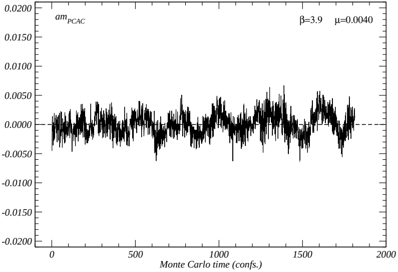
β	target a (fm)	$L^3 \cdot T$	$\kappa_{\rm crit}$	$a\mu$	$N_{\rm traj}$	target $m_{\rm PS}$ (MeV)
4.05	~ 0.075	$32^3 \cdot 64$	0.15701	0.0030	5000	~ 270
				0.0060	5000	~ 380
				0.0080	3600	~ 430
				0.0120	2400	~ 530
3.9	~ 0.09	$24^3 \cdot 48$	0.160856	0.0040	10000	~ 300
				0.0064	5000	~ 380
				0.0085	5000	~ 440
				0.0100	5000	~ 480
				0.0150	5000	~ 580
		$32^3 \cdot 64$		0.0040	5000	~ 300
3.8	~ 0.11	$20^3 \cdot 48$	0.164099	0.0060	3900	~ 290
				0.0090	3600	~ 390
				0.0120	5000	~ 450
				0.0150	3300	~ 510

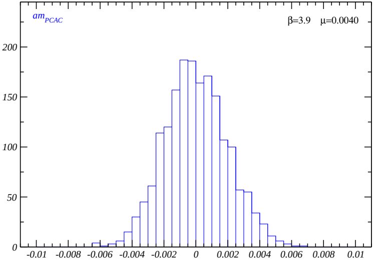
Monte Carlo histories of plaquette





Monte Carlo history of m_{PCAC} $\beta = 3.9$

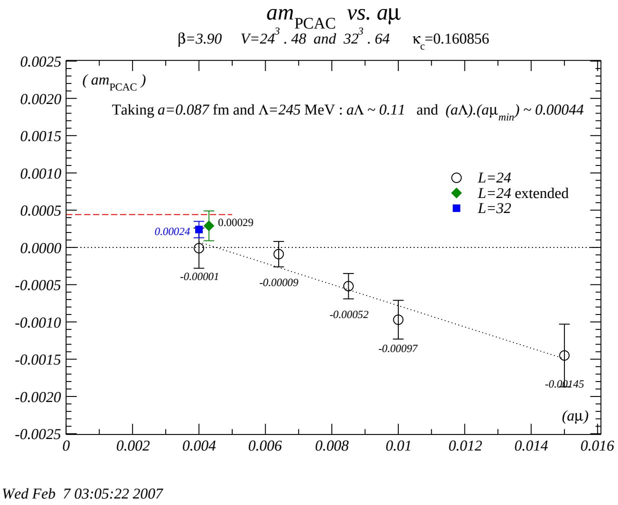




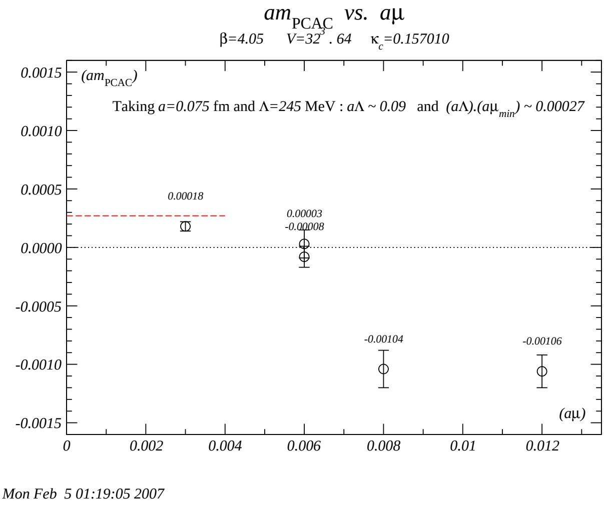
- $V = 24^3 \cdot 48, \;\; \beta = 3.9, \;\; a\mu = a\mu_{\mathrm{min}} = 0.004$

$$am_{\mathrm{PCAC}}(a\mu = a\mu_{\mathrm{min}}) = 0 \pm \mathcal{O}((a\mu_{\mathrm{min}})(a\Lambda_{\mathrm{QCD}}))$$

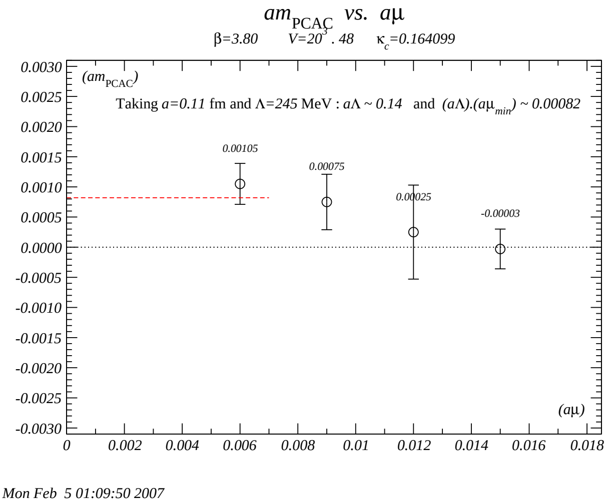
am_{PCAC} vs. $a\mu$ $\beta = 3.9$



am_{PCAC} vs. $a\mu$ $\beta = 4.05$



$am_{\rm PCAC}$ vs. $a\mu$ $\beta = 3.8$



Pion: decay constant and χPT fits

Pseudo-scalar decay constant:

$$f_{\rm PS} = \frac{2\mu}{m_{\rm PS}^2} |\langle 0|P^1(0)|\pi\rangle|$$

- fit to continuum $N_f = 2$ χPT at NLO

(Gasser, Leutwyler, 1987; Colangelo *et al.*, 2005)

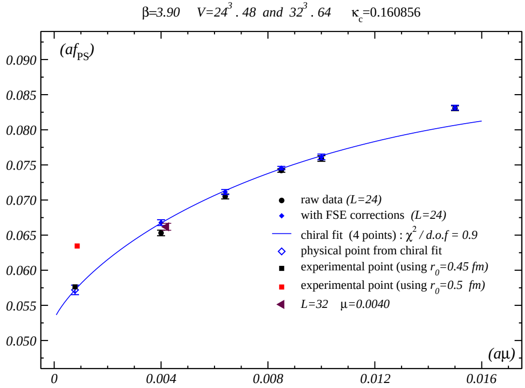
$$m_{\rm PS}^2(L) = 2B_0\mu \left[1 + \frac{1}{2}\xi \tilde{g}_1(\lambda)\right]^2 \left[1 + \xi \log(2B_0\mu/\Lambda_3^2)\right] ,$$

$$f_{\rm PS}(L) = F\,[1 - 2\xi \tilde{g}_1(\lambda)] \left[1 - 2\xi \log(2B_0\mu/\Lambda_4^2)\right]$$

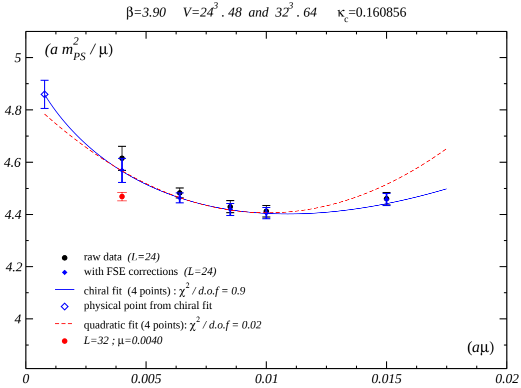
where $\xi = 2B_0\mu/(4\pi F)^2$, $\lambda = \sqrt{2B_0\mu L^2}$, $\tilde{g}_1(\lambda)$ is a known function

- fit parameters: B_0, F, Λ_3 and Λ_4
- extract low-energy constants: $\bar{l}_{3,4} \equiv \log(\Lambda_{3,4}^2/m_\pi^2)$

Pion sector: af_{PS} vs. $a\mu$ $\beta = 3.9$



Pion sector: m^2_{PS}/μ vs. μ $\beta = 3.9$



Pion: results from χPT fits $\beta = 3.9$

$$m^2_{\rm PS}(L) = 2B_0\mu \left[1 + \frac{1}{2}\xi\tilde{g}_1(\lambda)\right]^2 \left[1 + \xi\log(2B_0\mu/\Lambda_3^2)\right]$$

$$f_{\rm PS}(L) = F\,[1 - 2\xi\tilde{g}_1(\lambda)]\,\left[1 - 2\xi\log(2B_0\mu/\Lambda_4^2)\right]$$

where $\xi = 2B_0\mu/(4\pi F)^2$, $\lambda = \sqrt{2B_0\mu L^2}$

$L = 24$

$2aB_0 = 4.99(6)$

$aF = 0.0534(6)$

$\log(a^2\Lambda_3^2) = -1.93(10)$

$\log(a^2\Lambda_4^2) = -1.06(4)$

$\chi^2/dof = 3.5/4 \sim 0.9$

$L = 32$ (for $a\mu = 0.0040$)

$2aB_0 = 4.66(10)$

$aF = 0.0758(17)$

$\log(a^2\Lambda_3^2) = -2.09(12)$

$\chi^2/dof \sim 1.2$

- The "physical point" $a\mu_\pi$ is determined by requiring $m_{\rm PS}/f_{\rm PS} = 139.6/130.7 = 1.068$ $\rightsquigarrow$ we get: $a\mu_\pi = 0.00078(2)$
- Taking $f_\pi = 130.7$ MeV, we obtain $a = 0.087(1)$ fm
- Using $r_0/a = 5.22(2)$ we get: $r_0 = 0.454(7)$ fm
- We determine : $\bar{l}_{3,4} \equiv \log(\Lambda^2_{3,4}/m^2_\pi)$

Pion: low-energy constants $\beta = 3.9$

Determinations of $\bar{l}_{3,4} \equiv \log(\Lambda^2_{3,4}/m^2_\pi)$

$$\bar{l}_3 = 3.65 \pm 0.12 \quad (\text{with } L = 24)$$

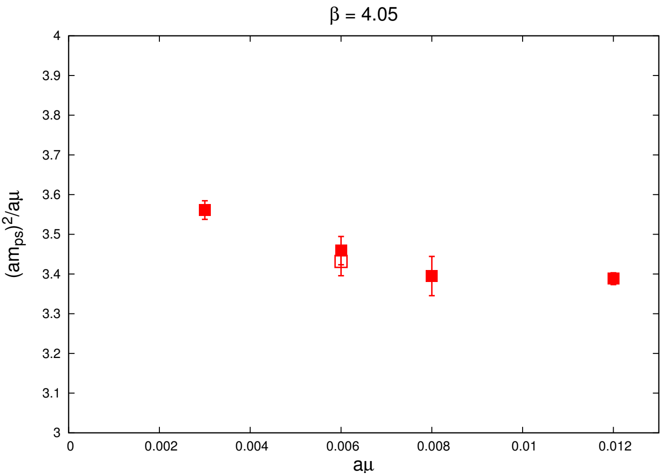
$$\bar{l}_3 = 3.49 \pm 0.12 \quad (\text{with } L = 32)$$

$$\bar{l}_4 = 4.52 \pm 0.06$$

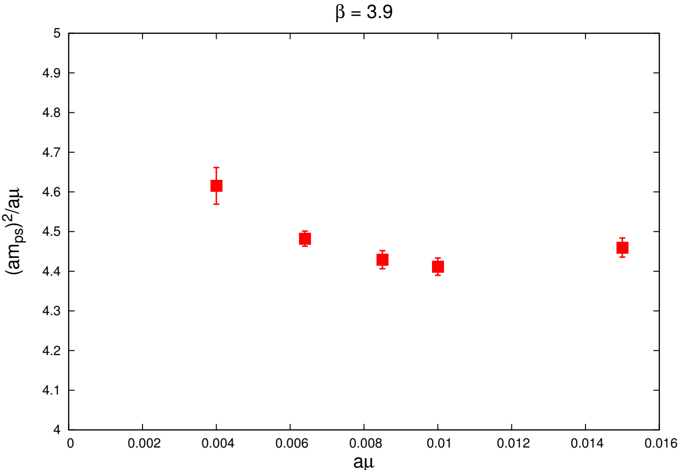
Other estimates (Leutwyler, hep-ph/0612112)

- $\bar{l}_3$:
  - $\bar{l}_3 = 2.9 \pm 2.4$ from the mass spectrum of the pseudoscalar octet
  - $\bar{l}_3 = 0.8 \pm 2.3$ from MILC
  - $\bar{l}_3 = 3.0 \pm 0.6$ from lattice CERN group
- $\bar{l}_4$:
  - $\bar{l}_4 = 4.3 \pm 0.9$ from f_K/f_π
  - $\bar{l}_4 = 4.4 \pm 0.2$ from the radius of the scalar pion form factor
  - $\bar{l}_4 = 4.0 \pm 0.6$ from MILC

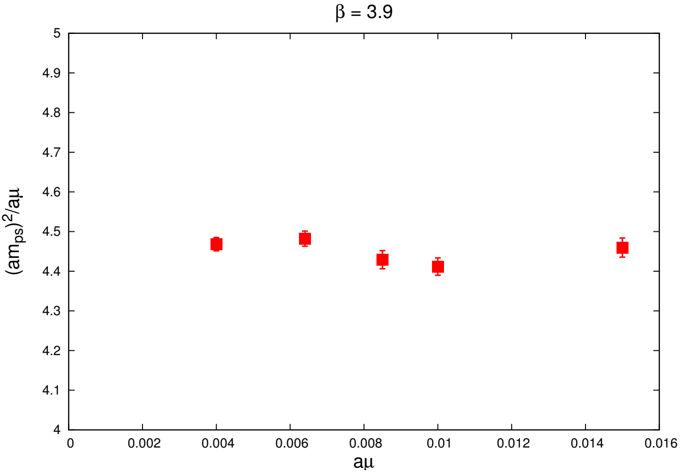
Pion sector: m_{PS}^2/μ vs. μ — $\beta = 4.05$



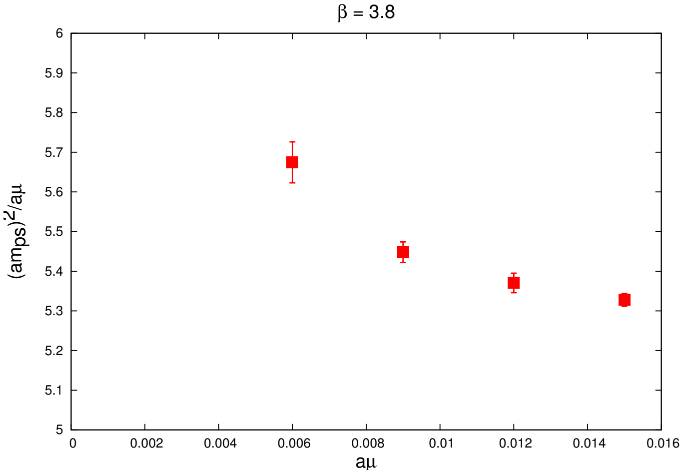
Pion sector: m_{PS}^2/μ vs. μ $\beta = 3.9$



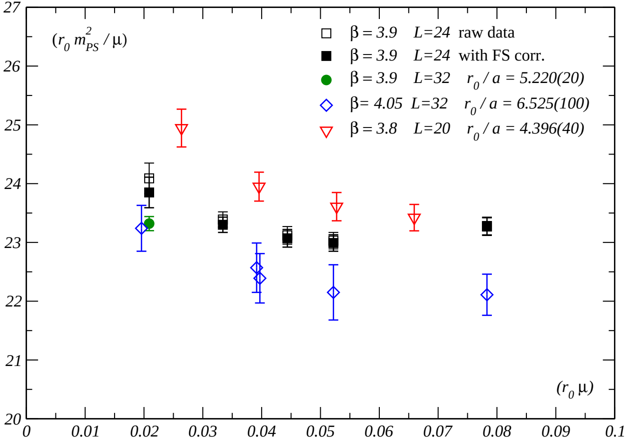
Pion sector: m_{PS}^2/μ vs. μ with $L = 32$ $\beta = 3.9$



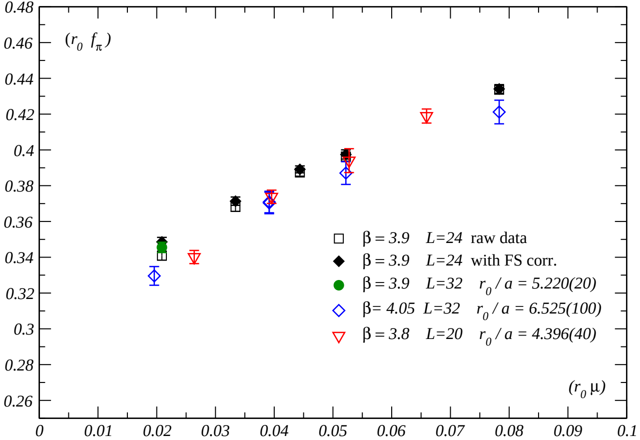
Pion sector: m_{PS}^2/μ vs. μ $\beta = 3.8$



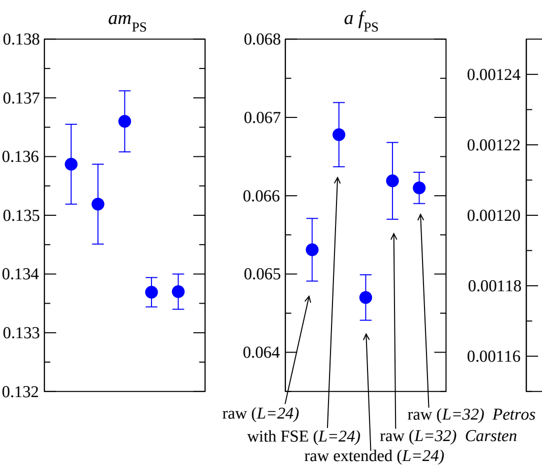
Pion sector: $r_0 m_{PS}^2/\mu$ vs. $r_0\mu$



Pion sector: $r_0 f_{\mathrm{PS}}$ vs. $r_0 \mu$

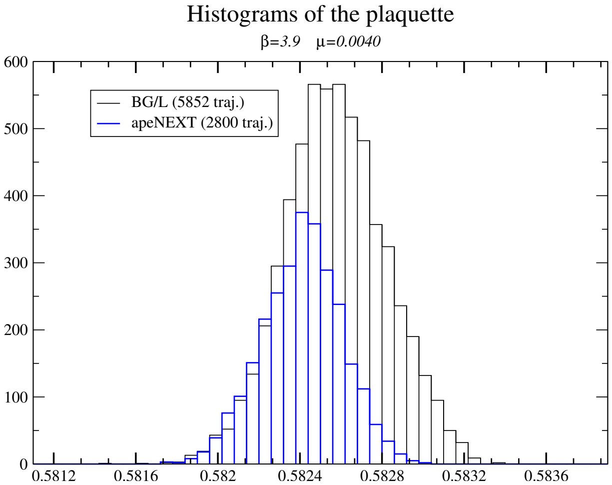


Comparison $\beta = 3.9$ $a\mu = 0.0040$



Monte Carlo histories of plaquette

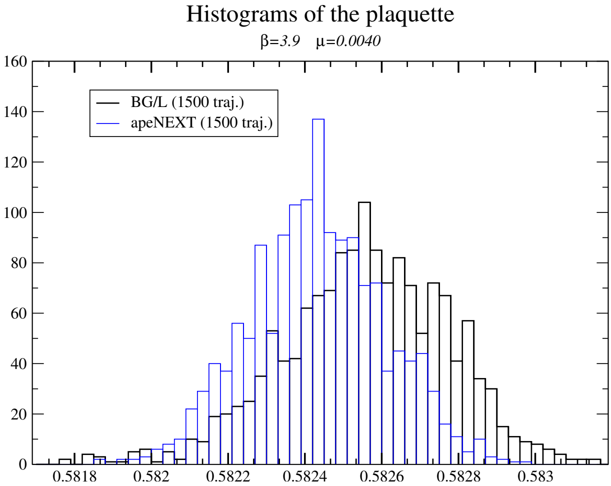
$\beta = 3.9$ $a\mu = 0.0040$ BG/L and apeNEXT



$\langle P_B \rangle = 0.5825724(33)(351)$ $N_{\rm traj} = 5852$
$\langle P_A \rangle = 0.5824103(37)(400)$ $N_{\rm traj} = 2800$

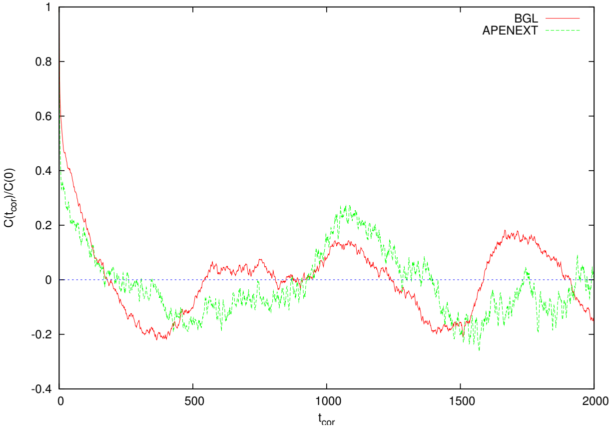
Monte Carlo histories of plaquette

$\beta = 3.9 \quad a\mu = 0.0040$ BG/L and apeNEXT



Monte Carlo histories of plaquette

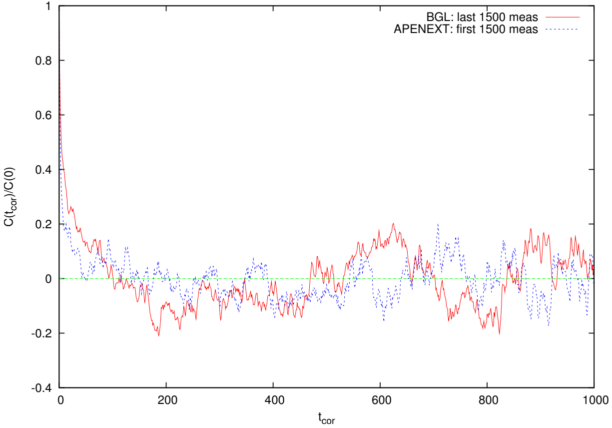
$\beta = 3.9$ $a\mu = 0.0040$ BG/L and apeNEXT



Autocorrelation function: all data
$\tau_{\rm int}^{B} = 45(16)$
$\tau_{\rm int}^{A} = 31(15)$

Monte Carlo histories of plaquette

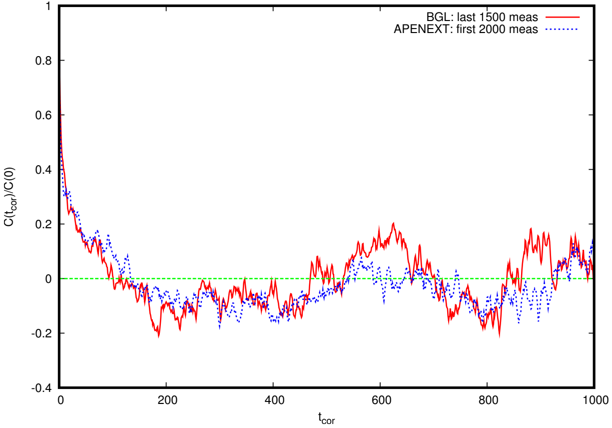
$\beta = 3.9$ $a\mu = 0.0040$ BG/L and apeNEXT



Autocorrelation function: 1500 traj.
$\tau_{\mathrm{int}}^{B} = 18(10)$
$\tau_{\mathrm{int}}^{A} = 8(3)$

Monte Carlo histories of plaquette

$\beta = 3.9 \quad a\mu = 0.0040 \quad$ BG/L and apeNEXT



Autocorrelation function: 1500 traj. for BG/L and 2000 for apeNEXT

$\tau_{\mathrm{int}}^{B} = 18(10)$

$\tau_{\mathrm{int}}^{A} = 22(12)$