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SU(2) ChPT analysis of the scalar and vector form factors of the kaon semileptonic decay obtained from twisted-mass fermions with N_f=2

in collaboration with: V.Lubicz , F. Mescia, S. Simula, C. Tarantino and

on behalf of the

European Twisted Mass Collaboration

Outline

$K_{\ell 3}$ decays [arXiv:0906.4728]

-*Scaling study of $f_+(q^2)$ and $f_0(q^2)$: 4 values of a (0.054,0.068,0.086,0.102 fm)*

-*SU(2) ChPT extrapolation of f+(0) at the physical point*

-*new: SU(2) ChPT analysis of the q^2 dependence of $f_+(q^2)$ and $f_0(q^2)$*

- **ETMC** determination of the vector form factor: $f_+(0) = 0.9554\ (73)$ **preliminary**

- Cabibbo's angle: $|V_{us}| = 0.2264\ (5)_{exp}\ (18)_{f+(0)}$

- first-row CKM unitarity: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0003(12)$

main motivation:

extract V_{us} from $K_{\ell 3}$ decays

$$\langle \pi(p_\pi) | \hat{V}_\mu | K(p_K) \rangle = f_+(q^2)(p_K + p_\pi)_\mu + f_-(q^2)(p_K - p_\pi)_\mu$$

$$\underbrace{f_0(q^2)}_{\text{scalar f.f.}} = \underbrace{f_+(q^2)}_{\text{vector f.f.}} + f_-(q^2)\frac{q^2}{(M_K^2 - M_\pi^2)}$$

scalar f.f. vector f.f.: $f_+(0) = f_0(0)$

* from experiment: $\Gamma(K \to \pi l \bar{\nu}_l) \to |V_{us}| f_+(0) = 0.2163(5)$ [0.2%] [FLAVIANET '10 (1005.2323)]

* to match the experimental error one needs an accuracy of ~ 5% on [1 - $f_+(0)$]

* ChPT expansion of the vector form factor at zero momentum transfer, $f_+(0)$

$$f_+(0) = 1 + f_2 + f_4 + O(p^8) \xrightarrow[AG\ theorem]{} 1 + O\left[(m_s - m_l)^2\right]$$

* NLO f_2 is independent of LECs, calculable in terms of M_K, M_π and f_π : $f_2^{phys} = -0.023$

* NNLO f_4 depends on $O(p^6)$ LECs [Post and Schilcher ('01), Bijnens and Talavera ('03)]

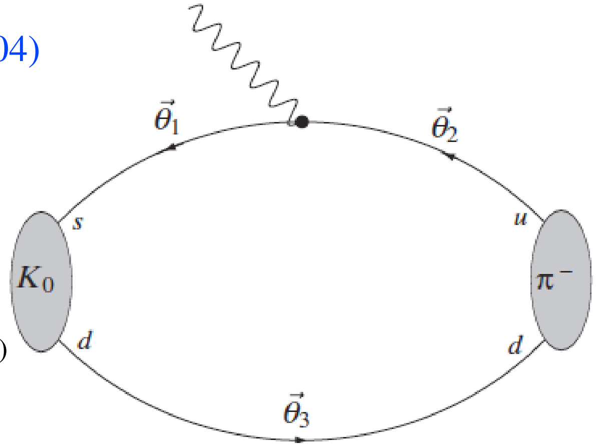
*** f_4 may be obtained from the slope and curvature of $f_0(q^2)$, but present data are not accurate enough *****

- from quark model [Leutwyler and Roos '84]: $f_4 = -0.016 \pm 0.008 \Rightarrow f_+(0) = 0.961 \pm 0.008$ [used by PDG]
- from NNLO ChPT + 1/N_c [Cirigliano et al. '06]: $f_4 = 0.007 \pm 0.012 \Rightarrow f_+(0) = 0.984 \pm 0.012$
- from NNLO ChPT + disp. rel. [Jamin et al. '04]: $f_4 = -0.003 \pm 0.011 \Rightarrow f_+(0) = 0.974 \pm 0.011$

semileptonic form factors on the lattice

* **tree-level Symanzik gauge action + tmQCD at maximal twist with N_f = 2** (ETMC '07, '08)

* **automatic O(a)-improvement in tmQCD at maximal twist** (Frezzotti and Rossi '04)



need of 2-point and 3-point correlators

2-point correlators: $C^{\pi(K)}(t;p) \equiv \sum_{x,z} \left\langle O_{\pi(K)}(\vec{x},t_x) O^{+}_{\pi(K)}(\vec{z},t_z) \right\rangle \times \delta_{t,t_x-t_z}\, e^{-ip\cdot(x-z)}$

$$\xrightarrow[t\to\infty]{} \frac{Z_{\pi(K)}}{2E_{\pi(K)}(p)} e^{-E_{\pi(K)}(p)t}$$

$$\vec{p}_K = \frac{2\pi}{L}(\vec{\theta}_3 - \vec{\theta}_1) \quad \vec{p}_\pi = \frac{2\pi}{L}(\vec{\theta}_3 - \vec{\theta}_2)$$

$$\vec{q} = \vec{p}_K - \vec{p}_\pi = \frac{2\pi}{L}(\vec{\theta}_2 - \vec{\theta}_1)$$

3-point correlators: $C^{K\pi}_{\mu}(t,t';p_K,p_\pi) \equiv \sum_{x,y,z} \left\langle O_\pi(\vec{y},t_y) \hat{V}_\mu(x,t_x) O^{+}_K(\vec{z},t_z) \right\rangle \times \delta_{t,t_x-t_z}\, \delta_{t',t_y-t_z} \times e^{-ip_K(x-z)} e^{ip_\pi(x-y)}$

$$\xrightarrow[\substack{t\to\infty \\ t'-t\to\infty}]{} \frac{\sqrt{Z_K Z_\pi}}{4E_K(p_K)E_\pi(p_\pi)} \left\langle \pi(p_\pi) \right| \hat{V}_\mu \left| K(p_K) \right\rangle e^{-E_K(p_K)t} e^{-E_\pi(p_\pi)(t'-t)}$$

* **suitable ratios of 3-point / 2-point correlators** $\longrightarrow$ $\left\langle \pi(p_\pi) \right| \hat{V}_\mu \left| K(p_K) \right\rangle$

* local interpolating PS fields and local vector current: $O_K = \bar{d}\gamma_5 s, \quad O_\pi = \bar{d}\gamma_5 u, \quad \hat{V}_\mu = Z_V\, \bar{s}\gamma_\mu u$

ETMC

β	a (fm)	$am_\ell = am_{sea}$	V • T	M_π (MeV)	$M_\pi L$	gauge confs
3.8	~ 0.103	0.0080	$24^3 \bullet 48$	~ 400	= 4.9	240
		0.0110	$24^3 \bullet 48$	~ 480	= 5.9	240
		0.0165	$24^3 \bullet 48$	~ 580	= 7.1	240
3.9	~ 0.088	0.0030	$32^3 \bullet 64$	~ 260	= 3.7	240
		0.0040	$32^3 \bullet 64$	~ 300	= 4.3	240
		0.0040	$24^3 \bullet 48$	~ 300	= 3.3	480
		0.0064	$24^3 \bullet 48$	~ 375	=4.0	240
		0.0085	$24^3 \bullet 48$	~ 435	= 4.7	240
		0.0100	$24^3 \bullet 48$	~ 470	= 5.0	240
		0.0150	$24^3 \bullet 48$	~ 575	= 6.2	240
4.05	~ 0.069	0.0030	$32^3 \bullet 64$	~ 300	= 3.3	240
		0.0060	$32^3 \bullet 64$	~ 410	= 4.5	240
		0.0080	$32^3 \bullet 64$	~ 470	= 5.3	240
4.2	~ 0.054	0.0065	$32^3 \bullet 64$	~ 470	=4.2	240

$\beta = 3.8$: a m_s = { 0.016, 0.020, 0.025, 0.030,0,036 } around a $m_s^{phys} \sim 0.020$
$\beta = 3.9$: a m_s = { 0.015, 0.022, 0.027, 0.032 } around a $m_s^{phys} \sim 0.018$
$\beta = 4.05$: a m_s = { 0.015, 0.018, 0.022, 0.026 } around a $m_s^{phys} \sim 0.015$
$\beta = 4.2$: a m_s = { 0.010, 0.012, 0.015, 0.019 } around a $m_s^{phys} \sim 0.012$

- use of all-to-all quark propagators evaluated with a stochastic method (the *one-end-trick* [UKQCD ’06])

- θ-boundary conditions to inject non-periodic momenta on the lattice [Bedaque ‘04, Petronzio et al. ‘04]

SU(2) analysis

SU(2) ChPT (expansion around the chiral point $m_\ell = 0$): [Flynn and Sachrajda '09]

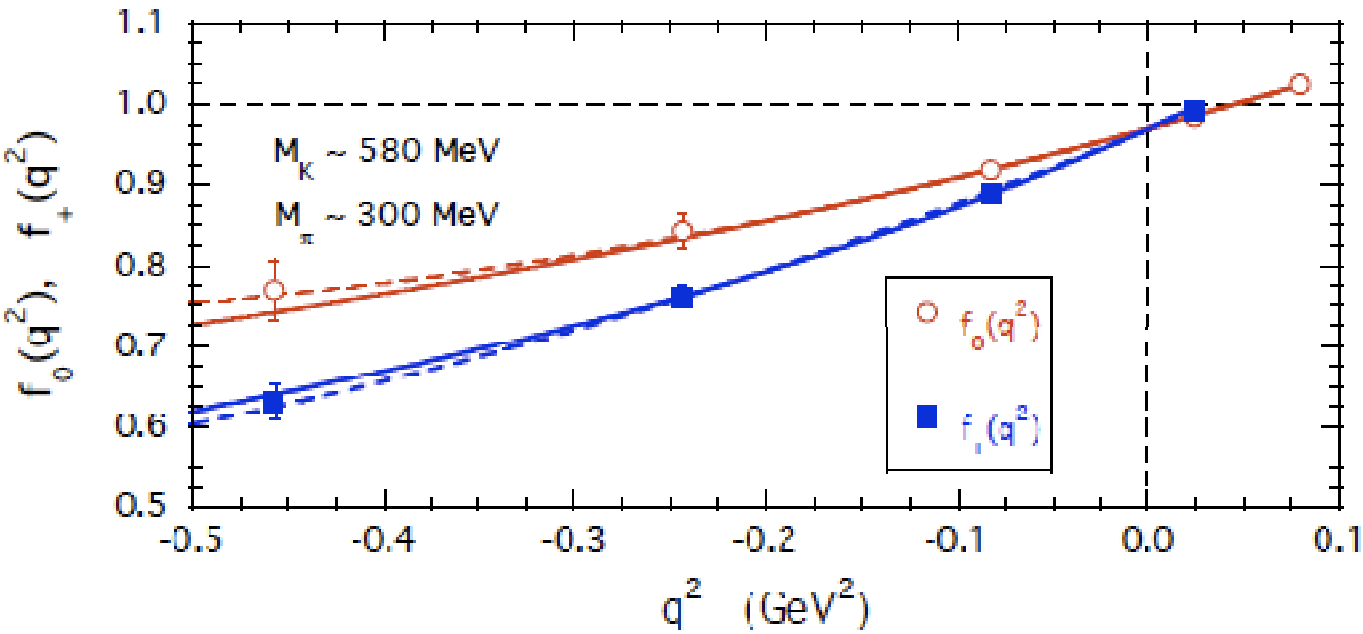
- the strange quark does not satisfy chiral symmetry and the dependence on m_s is reabsorbed into the LEC's of the effective theory
- applicability for $m_\ell << m_{sr}$

2 strategies:

- q^2 interpolation to get $f_{+,0}(0)$ then $\xrightarrow{\textit{Combined}}$ $\left\{ \begin{array}{l} \text{-Chiral analysis} \\ \text{-discretization analysis} \end{array} \right.$

- Starting from $f_{+,0}(q2)$ $\xrightarrow{\textit{Multi Combined}}$ $\left\{ \begin{array}{l} \text{- } q^2 \text{ analysis} \\ \text{- NLO Chiral analysis} \\ \text{-discretization analysis} \end{array} \right.$

extraction of $f_+(0; M_K, M_\pi)$



arXiv:0906.4728

Fig. 1: *Scalar $f_0(q^2)$ and vector $f_+(q^2)$ form factors obtained at $M_\pi \sim 300$ MeV and $M_K \sim 580$ MeV versus q^2 in physical units. The solid and dashed lines are the results of the fits based on Eqs. (5) and (6), respectively.*

———

$$f_{0,+}(q^2) = \frac{f_+(0)}{1 - s_{0,+} q^2}$$

- - - - -

$$f_{0,+}(q^2) = f_+(0)(1 + s_{0,+} q^2 + c_{0,+} q^4)$$

smooth interpolation at a reference kaon mass

$$2\left[M_K^{ref}\right]^2 - M_\pi^2 = 2\left[M_K^{phys}\right]^2 - \left[M_\pi^{phys}\right]^2 \propto m_s^{phys} \Rightarrow m_s^{ref} \approx m_s^{phys}$$

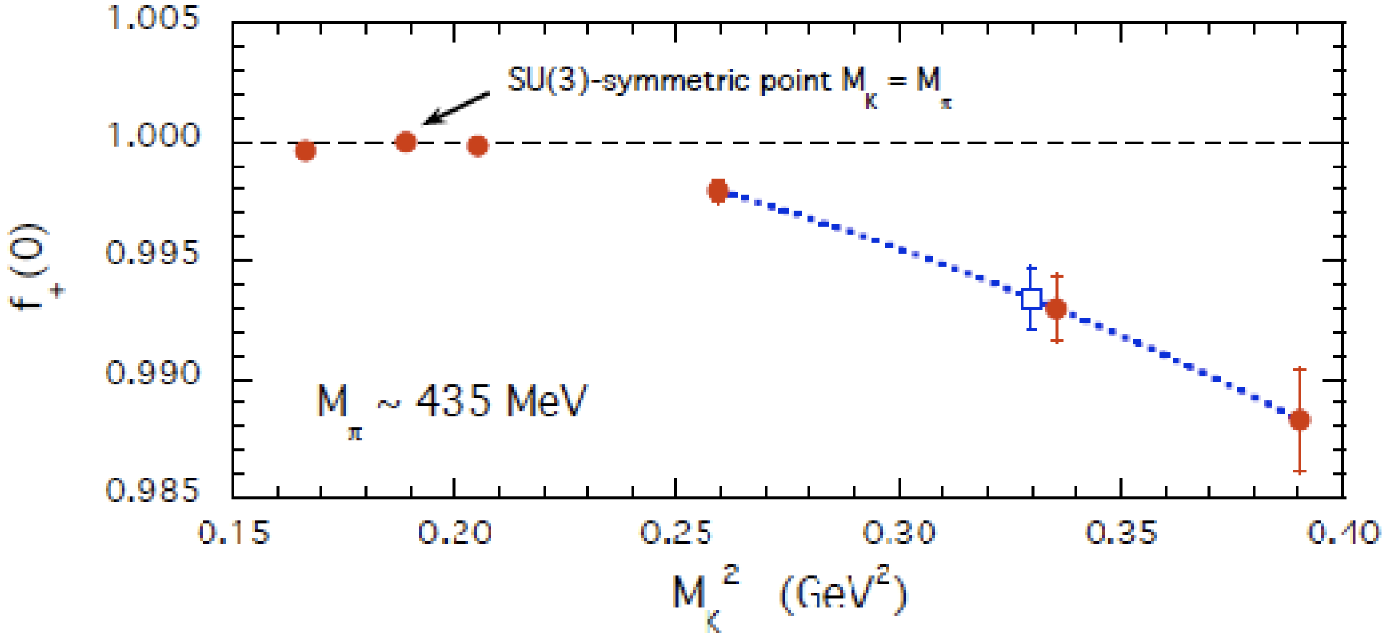


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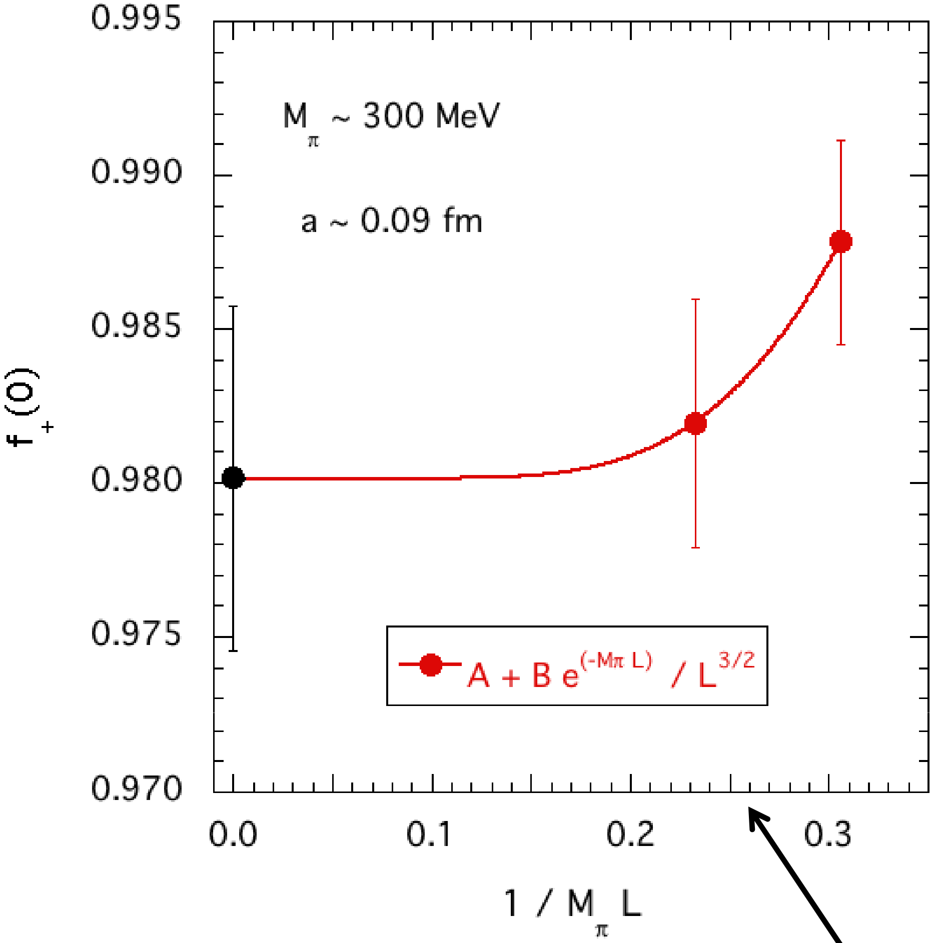
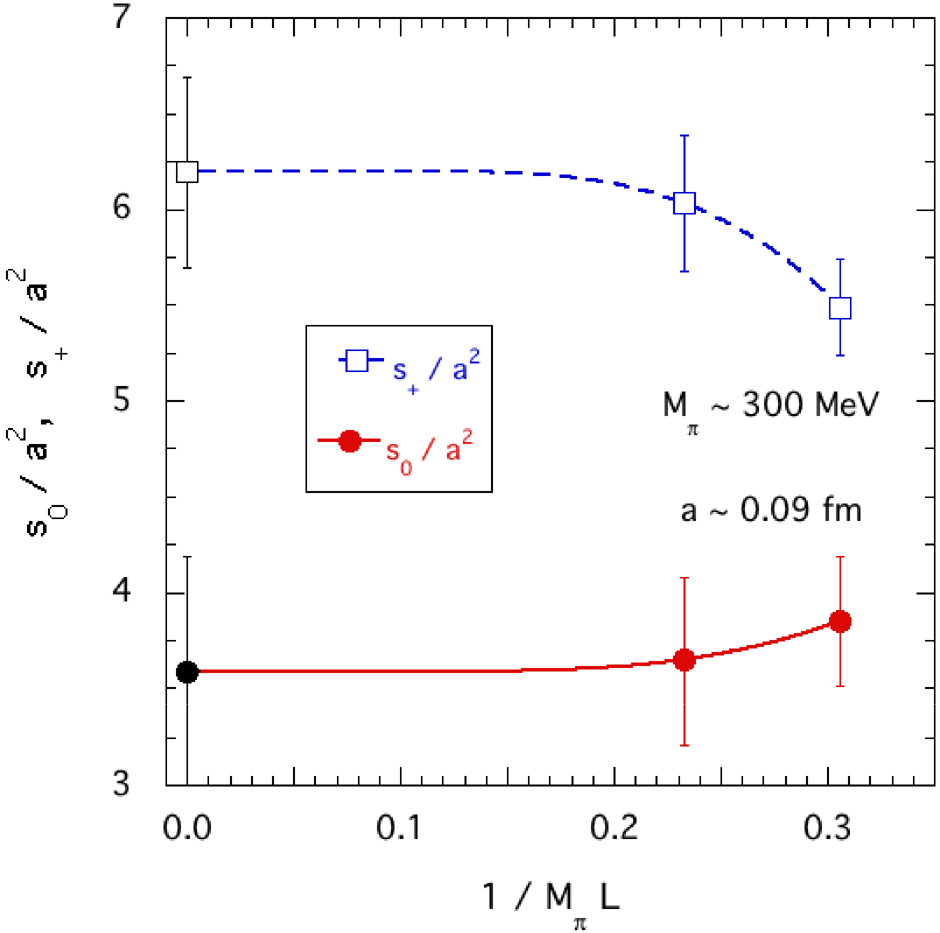
Fig. 2: *Results for $f_+(0)$ versus M_K^2 at $M_\pi \simeq 435$ MeV. The square corresponds to the value of $f_+(0)$ obtained by local interpolation via quadratic splines (dotted line) at the reference kaon mass $M_K^{ref} \simeq 575$ MeV from Eq. (7).*

Finite Size Effects

form factor



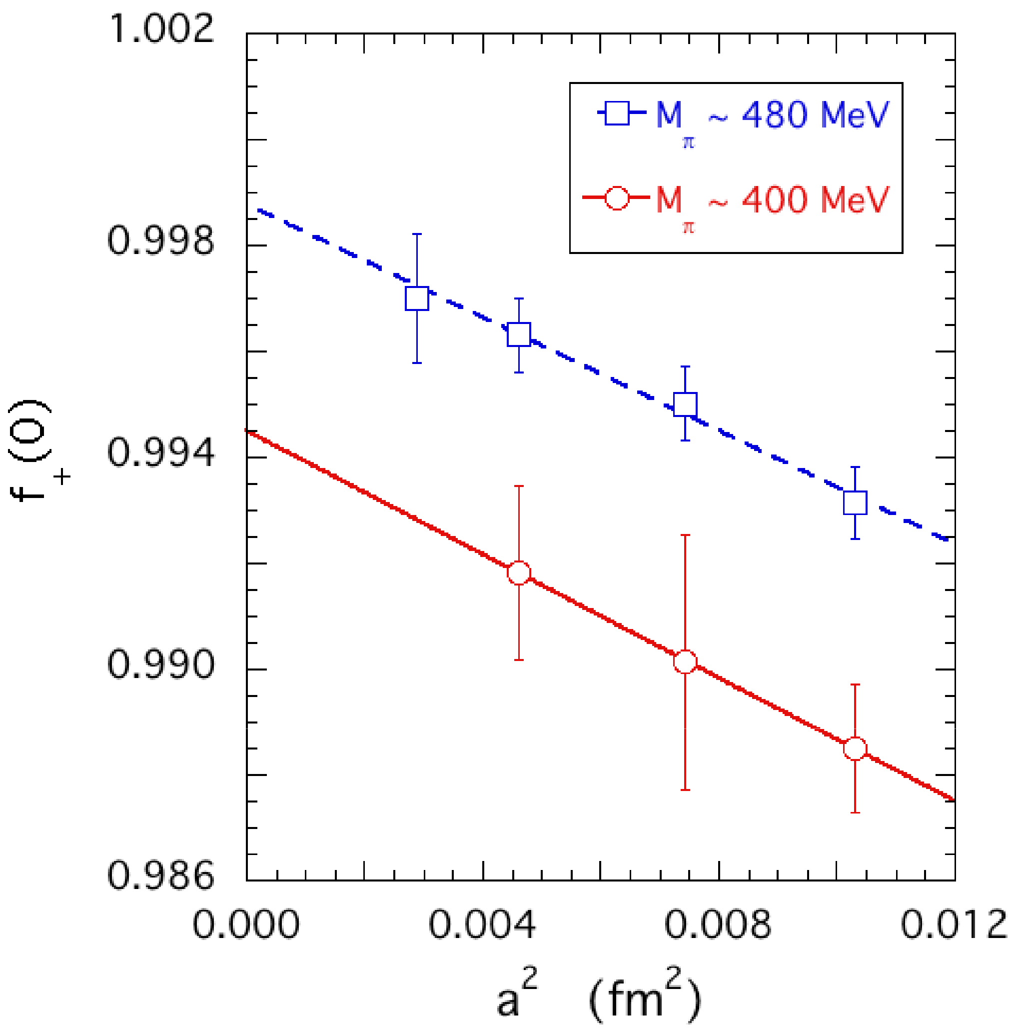
slopes: s_0 ,s_+



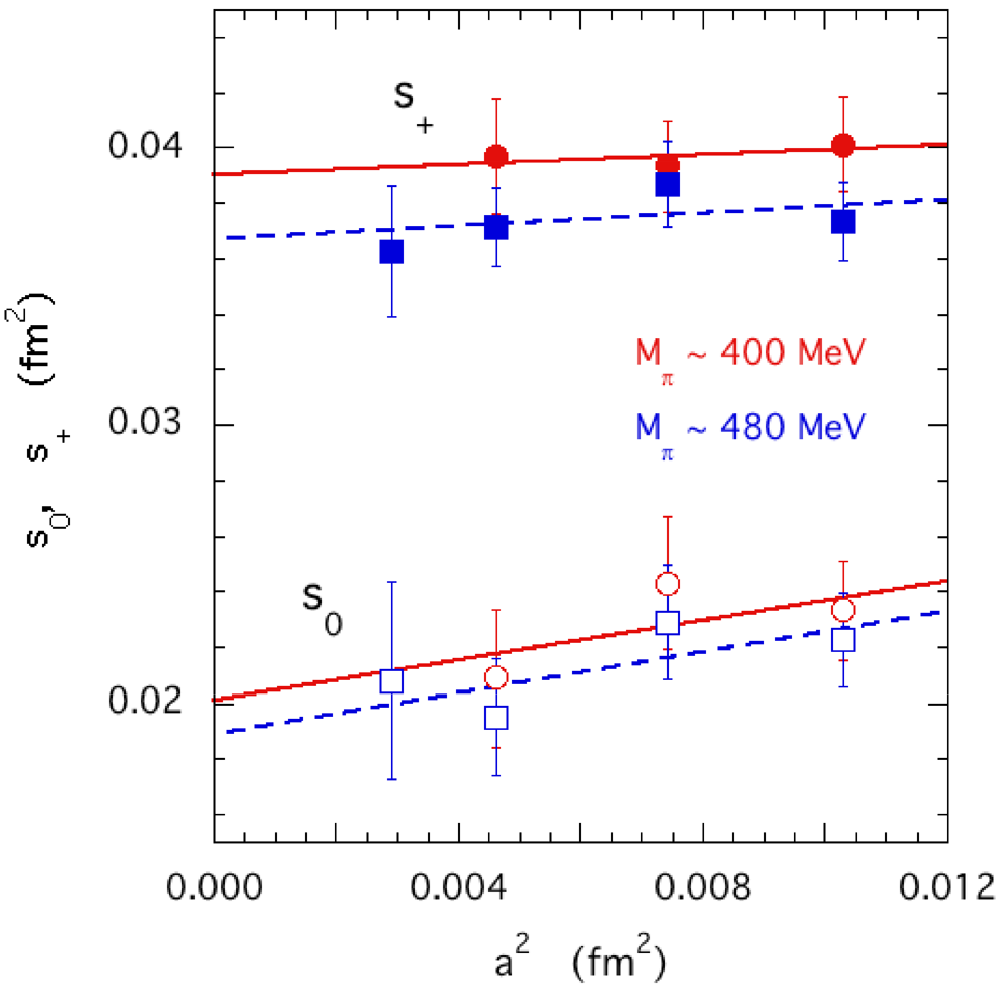
From $M_\pi L > 4$ negligible FSE

Discretization Effects

form factor

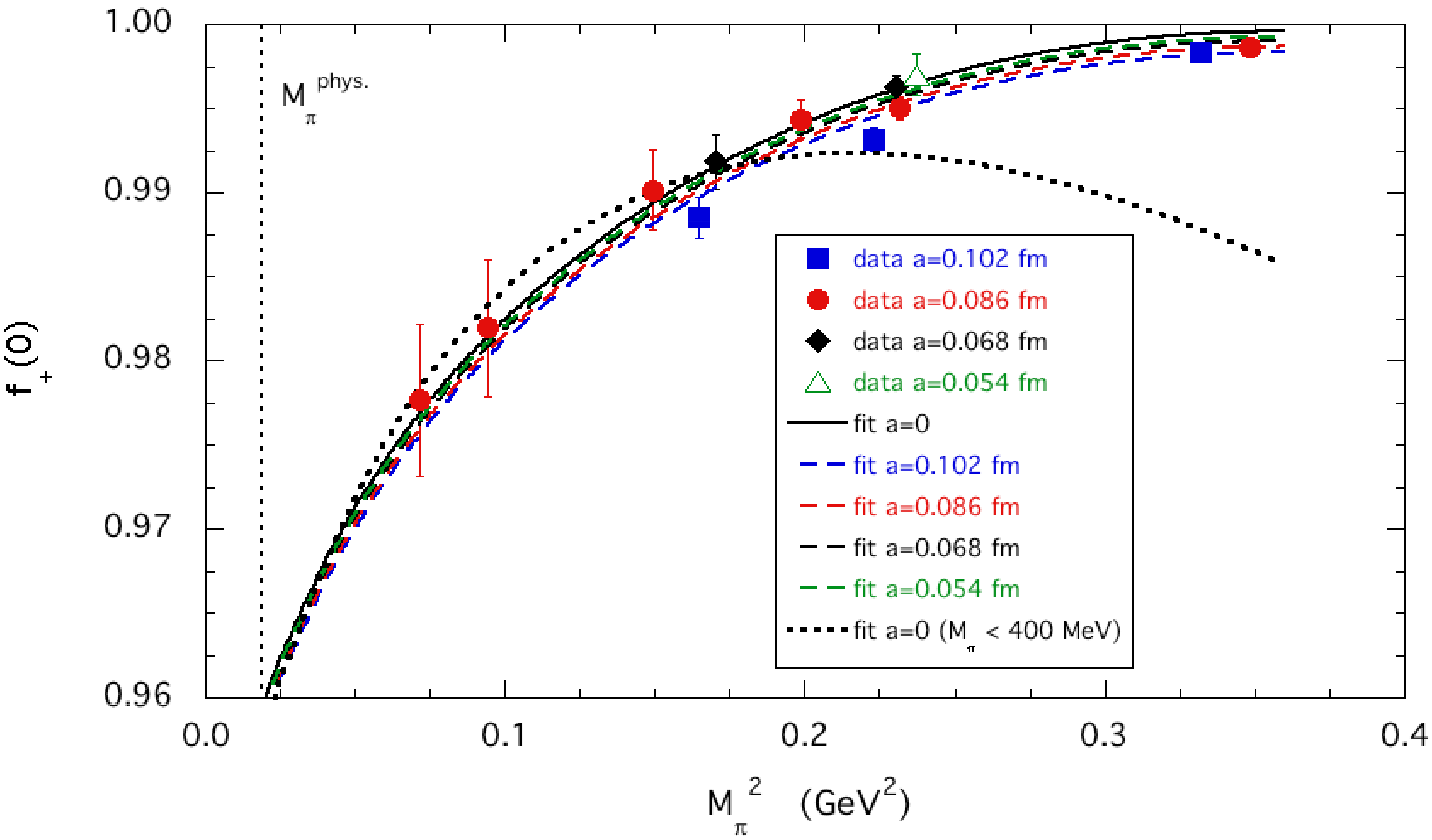


slopes: s_0 ,s_+



D.E. linear in a^2 in agreement with O(a) improvement of Mtm-LQCD

FIT Quality



$$f_+^{PQ}(0) = F_+\left(1 - \frac{3}{4}\frac{M_\pi^2}{(4\pi f_\pi)^2}\log(M_\pi^2) + c_+ M_\pi^2 + O(M_\pi^4) + da^2\right)$$ [Flynn and Sachrajda '09]

$$f_+^{PQ}(0) = 0.9612 \pm 0.0067$$

Quenching of the Strange Quark

arXiv:0906.4728

- estimate from SU(3)-ChPT:

$$f_2 - f_2^{PQ} = -0.0058 \qquad \text{(exactly known)} \qquad \Rightarrow \frac{f_2 - f_2^{PQ}}{f_2} \cong 26\%$$

* since the impact of chiral logs on Δf^{PQ} turns out to be quite small, we estimate

$$\frac{\Delta f - \Delta f^{PQ}}{\Delta f} \approx 50\% \quad \frac{f_2 - f_2^{PQ}}{f_2} \approx 13\% \Rightarrow \left|\Delta f - \Delta f^{PQ}\right| = 0.0028$$

- we shift the central value of $f_+(0)$ by $\delta_{quenching}$ = - 0.0058

- we add (quadratically) to the systematic error of $f_+(0)$ the value $\Delta_{quenching}$ = 0.0028

ETMC Results

* vector form factor at zero momentum transfer

$$f_+(0) = 0.9554 \pm 0.0067 \pm 0.0028 = 0.9554 \pm 0.0073$$ **(preliminary)**

in agreement with other determinations at $N_f = 0$, 2 and 2+1

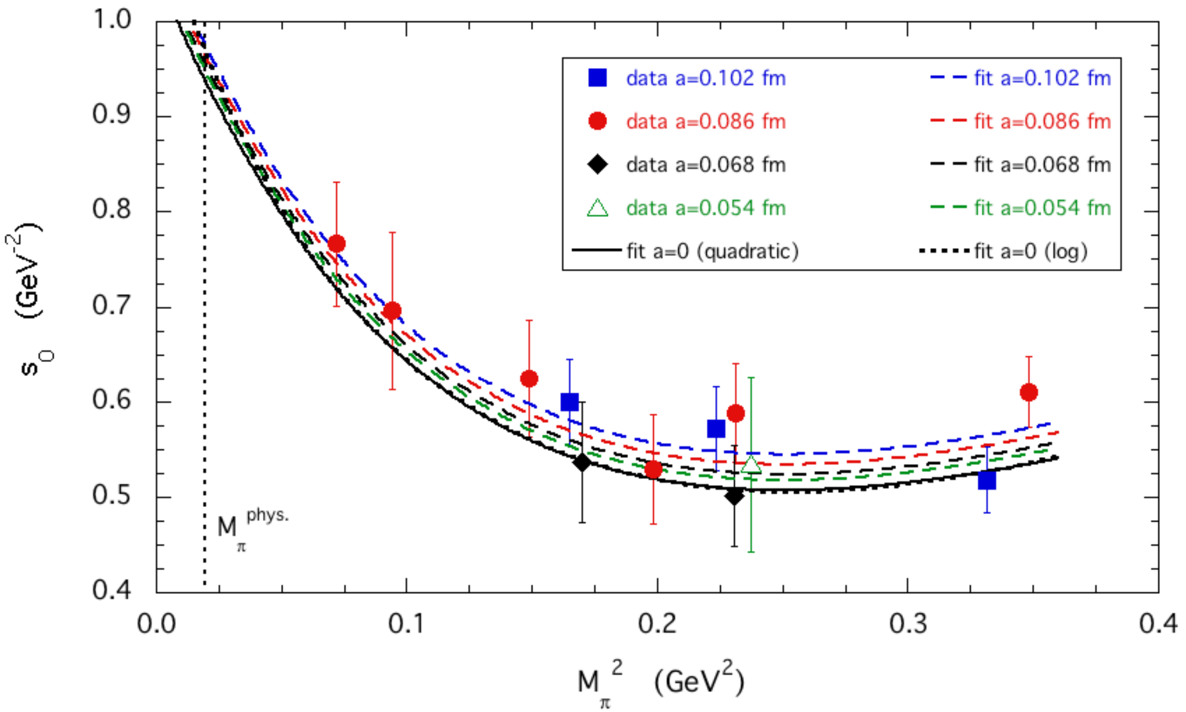
* using the latest experimental result $|V_{us}|\, f_+(0) = 0.2163 \pm 0.0005$ (FLAVIANET '10), one gets

$$V_{us} = 0.2264 \pm 0.0005 \pm 0.0018_{\, f+(0)}$$

* combining with $|V_{ud}| = 0.97425 \pm 0.00022$ and $|V_{ub}| = 0.00393 \pm 0.00036$ (PDG '08), the first-row CKM unitarity is

$$|V_{ud}| + |V_{us}| + |V_{ub}| = 1.0003 \pm 0.0012$$

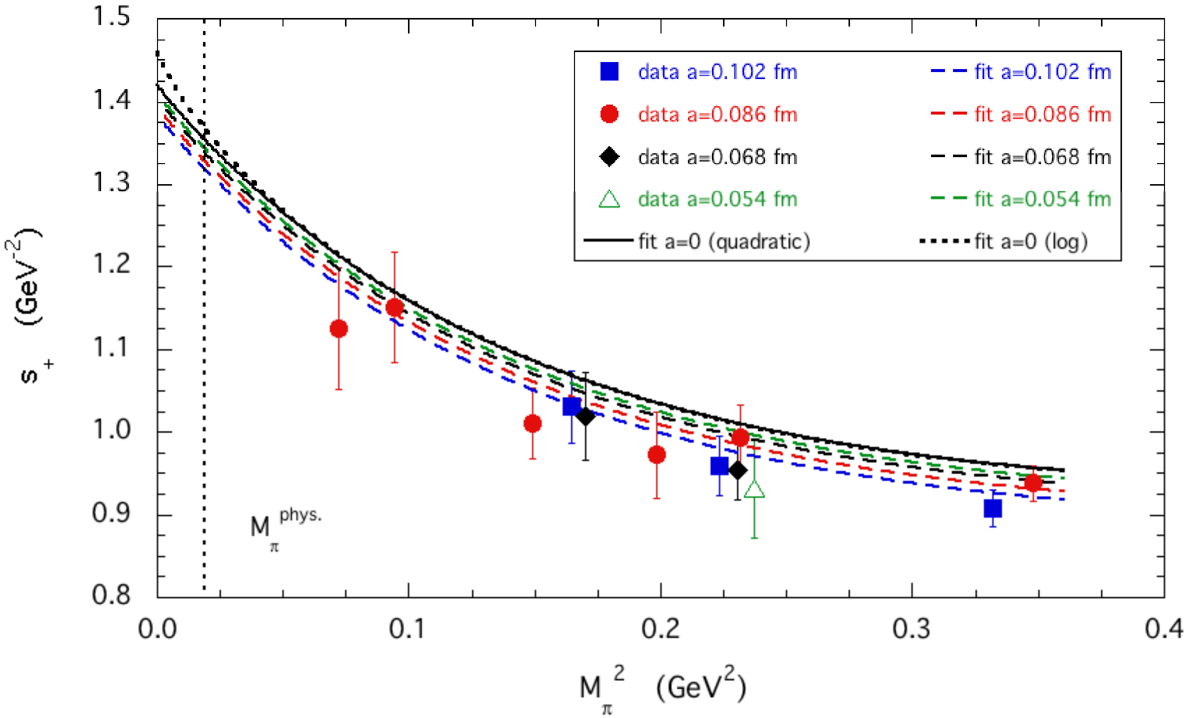
Scalar and Vector Slopes



$$s_j = a_j + b_j M_\pi^2 + c_j M_\pi^4 + d_j M_\pi^2 \log(M_\pi^2)$$

$\lambda_0 = M_\pi^2 \cdot s_0 = (15.7 \pm 0.8) \cdot 10^{-3}$

$\lambda_+ = M_\pi^2 \cdot s_+ = (24.1 \pm 1.1) \cdot 10^{-3}$



FlaviaNet '10 analysis of KLOE, KTeV, ISTRA+ and NA48 (no muons) experiments

$\lambda_0 = (15.90 \pm 0.79) \cdot 10^{-3}$

$\lambda_+ = (25.04 \pm 0.82) \cdot 10^{-3}$

2nd Analysis

$$\left\{ \begin{array}{l} x = \dfrac{M_\pi^2}{M_K^2}, \\ s = \dfrac{q^2}{M_K^2} \end{array} \right.$$

From the x expansion of the NLO SU(3) formulas of Gasser and Leutwyler [N.P. B250 '85]

* $$f_0(s) = F_0(s)\left\{1 + C_0(s)\times x + \frac{M_K^2}{(4\pi f)^2}\times\left\{-\frac{3}{4}x\ln x + T_1^0 - T_2^0\right\} + D\right\}$$ $D = da^2 + d'a^2 s,$

$$T_1^0 = x\times(9+7s^2)\times\left[\frac{\ln(1-s)+s(1+s/2)}{4s^2}\right], \qquad T_2^0 = (1-s)\times(3+5s)\times\left[\frac{(1-s)\times\ln(1-s)+s(1-s/2)}{4s^2}\right];$$

$F_0(s)$, $C_0(s)$: unknown LECs

* $$f_+(s) = F_+(s)\left\{1 + C_+(s)\times x + \frac{M_K^2}{(4\pi f)^2}\times\left\{-\frac{3}{4}x\ln x - T_1^+ - T_2^+\right\} + D\right\}$$

$$T_1^+ = 3x\times(1+s)\times\left[\frac{(1-s)\ln(1-s)+s(1-s/2)}{4s^2}\right], \qquad T_2^+ = (1-s)^2\times\left[\frac{(1-s)\times\ln(1-s)+s(1-s/2)}{4s^2}\right];$$

$F_+(s)$, $C_+(s)$: unknown LECs

Callan-Treiman relation

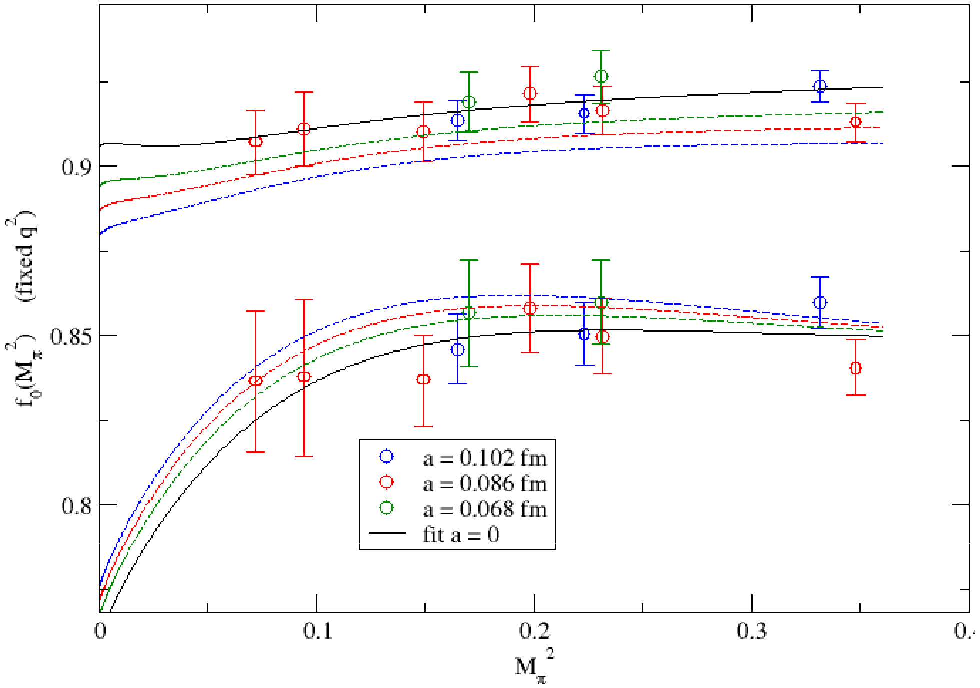
* $$f_0(M_K^2 - M_\pi^2) \xrightarrow[M_\pi\to 0]{} \left(\frac{F_K}{F_\pi}\right)_0$$

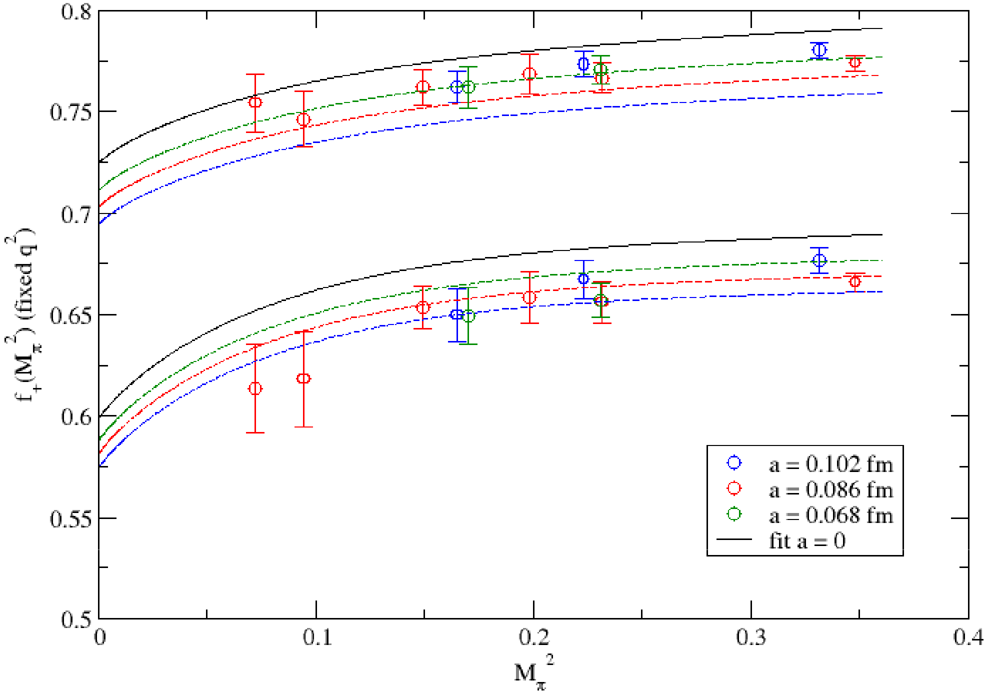
Right Chiral Log coeff
$$\left\{ \begin{array}{l} f_0(0) = f_+(0) \propto -\dfrac{3}{4}M_\pi^2 \log M_\pi^2 \\ f_0(q^2_{\max}) \propto -\dfrac{11}{4}M_\pi^2 \log M_\pi^2 \end{array} \right.$$

FIT Quality

Assuming: $F_0(s) = \frac{f_0}{1 - \lambda_0 s}$, $F_+(s) = \frac{f_0}{1 - \lambda_+ s}$, $C_0(s) = C_0 + C_1 s + C_2 s^2$

$C_+(s) = C_0 + C_1^+ s + C_2^+ s^2$





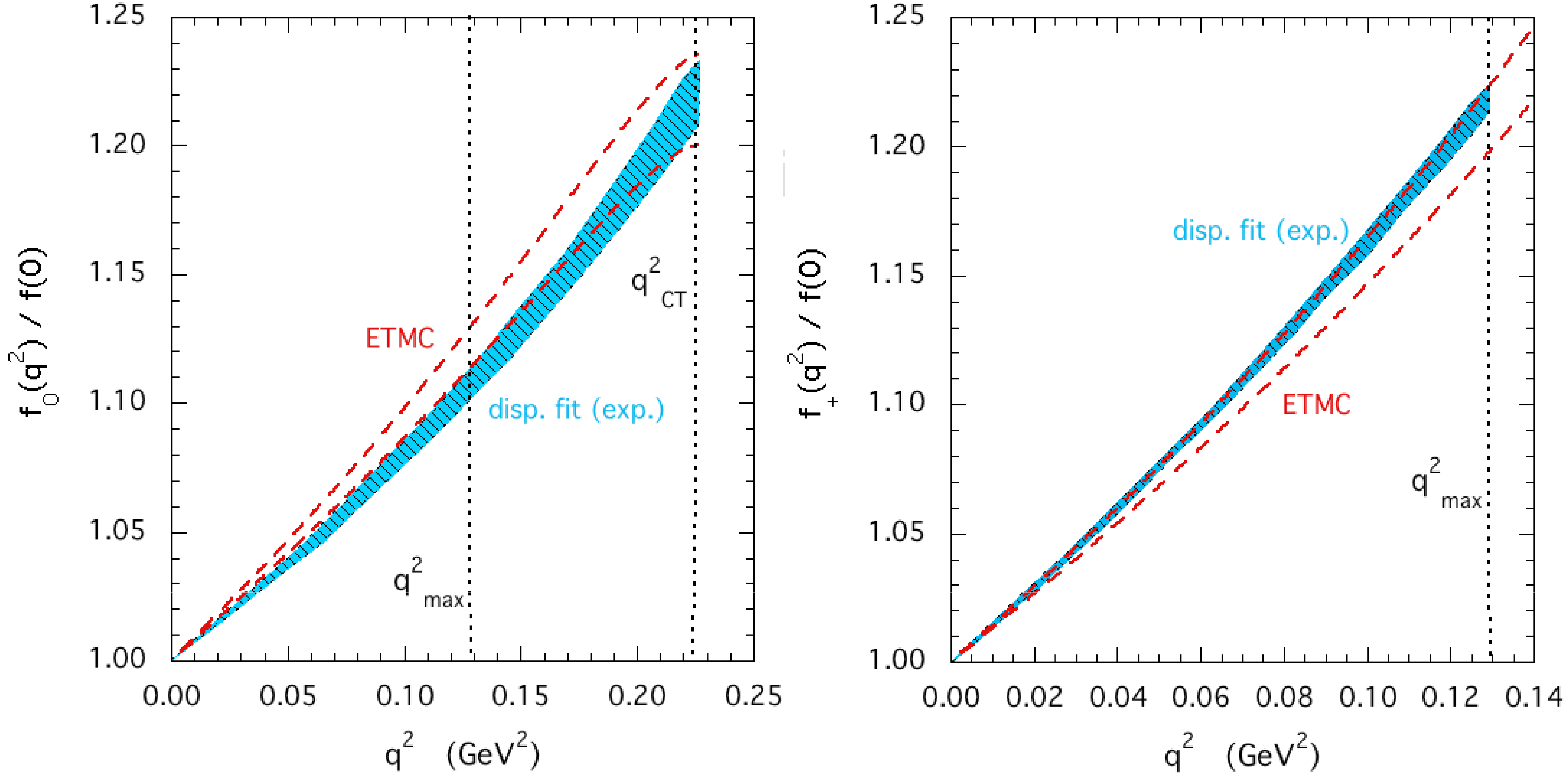
$f_+^{PQ}(0) = 0.9604 \pm 0.0073 \longrightarrow V_{us} = 0.2252 \pm 0.0017$

$(f_K / f_\pi)^{PQ} = 1.1899 \pm 0.0079 \longrightarrow V_{us} = 0.2258 \pm 0.0016$

$\lambda_0 = (16.01 \pm 0.8) \cdot 10^{-3}$

$\lambda_+ = (23.78 \pm 1.1) \cdot 10^{-3}$

Scalar and Vector Form Factors



dispersive fit: Bernard et al. '09

CONCLUSIONS

* K-meson semileptonic form factors has been calculated using unquenched ETMC gauge configurations with $N_f = 2$ dynamical quark flavors

* various volumes and lattice spacings with pion masses from ~ 260 to ~ 575 MeV

* **$K_{\ell 3}$ decays**: new determination of the key hadronic quantity

$$f_+(0) = 0.9554 \pm 0.0067 \pm 0.0028 = 0.9554 \pm 0.0073$$

where the error includes the uncertainties **calculated** for the chiral extrapolation, performed using SU(2) ChPT, finite size effects, discretization errors and the **estimate** of the effects due to the quenching of the strange quark

* Cabibbo's angle: $V_{us} = 0.2264 \pm 0.0018$

* first-row CKM unitarity: $|V_{ud}|+|V_{us}|+|V_{ub}| = 1.0003 \pm 0.0012$

• **$K_{\ell 3}$ decays (future)**:

a) Z-expansion for form factor momentum dependence

b) 2+1+1 dynamical quark flavours

back-up slides

ETMC strategy

Gauge action: tree-level Symanzik improved ($b_0 = 1 - 8b_1$, $b_1 = -1/12$)

$$S_g = \frac{\beta}{3} \sum_x \left(b_0 \sum_{\substack{\mu,\nu=1 \\ 1\leq\mu<\nu}}^{4} \{1 - \mathrm{Re\,Tr}(U^{1\times1}_{x,\mu,\nu})\} + b_1 \sum_{\substack{\mu,\nu=1 \\ \mu\neq\nu}}^{4} \{1 - \mathrm{Re\,Tr}(U^{1\times2}_{x,\mu,\nu})\} \right)$$

motivated by study of phase transitions

Fermionic action: twisted Wilson quarks [Frezzotti, Grassi, Sint, Weisz '99]

$$S_{tm} = a^4 \sum_x \bar{\chi}(x) D_{tm}[U] \chi(x)$$

$$D_{\mathrm{tm}} = \frac{1}{2} \sum_\mu \left[\gamma_\mu (\nabla_\mu + \nabla^*_\mu) - a \nabla^*_\mu \nabla_\mu \right] + m_0 + i\mu\gamma_5\tau_3$$

m_0 = untwisted (bare) quark mass

μ = twisted (bare) quark mass

Maximal twist: $m_0 = m_{cr}$ $\Rightarrow$ zero renormalized quark mass in the pure Wilson case

* automatic O(a)-improvement in tmQCD at maximal twist (Frezzotti & Rossi '04)