Forces between static-light mesons

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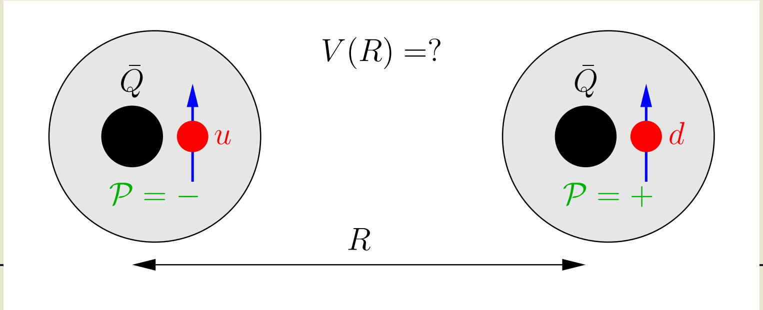
Introduction (1)

- Goal: compute the potential of (or equivalently the force between) two B mesons:
  - Treat the b quark in the static approximation.
  - Consider only pseudoscalar mesons ($j^{\mathcal{P}} = (1/2)^-$, denoted by S) and scalar mesons ($j^{\mathcal{P}} = (1/2)^+$, denoted by P_-), which are among the lightest static-light mesons.
  - Study the dependence of the mesonic potential $V(R)$ on
    * the light quark flavor u and/or d (isospin),
    * the light quark spin (the static quark spin is irrelevant),
    * the type of the meson S and/or P_-.

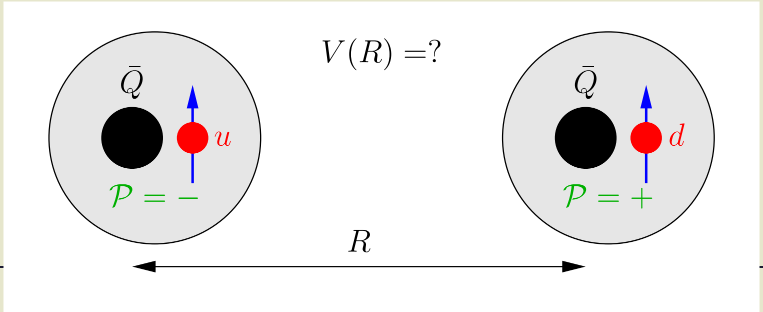


Introduction (2)

- Motivation:
  - First principles computation of a hadronic force.
  - Until now it has only been studied in the quenched approximation.

    [C. Michael and P. Pennanen [UKQCD Collaboration], Phys. Rev. D **60**, 054012 (1999)]

    [W. Detmold, K. Orginos and M. J. Savage, Phys. Rev. D **76**, 114503 (2007)]



(Pseudo)scalar B mesons

- Symmetries and quantum numbers of static-light mesons:
  - Isospin: $I = 1/2$, $I_z = \pm 1/2$, i.e. $B \equiv \bar{Q}u$ or $B \equiv \bar{Q}d$.
  - Parity: $\mathcal{P} = \pm$.
  - Rotations:
    * Light cloud angular momentum $j = 1/2$ (for S and P_-), $j_z = \pm 1/2$.
    * Static quark spin: irrelevant (static quarks can also be treated as spinless color charges).
- Static-light meson creation operators:
  - $\bar{Q}\gamma_5 q$ (pseudoscalar, i.e. S), $q \in \{u\,,\,d\}$,
  - $\bar{Q}q$ (scalar, i.e. P_-)

(j_z is not well-defined, when using these operators).

BB systems (1)

- Symmetries and quantum numbers of a pair of static-light mesons (separated along the z-axis):
  - Isospin: $I = 0, 1$, $I_z = -1, 0, +1$.
  - Rotations around the z-axis:
    * Angular momentum of the light degrees of freedom $j_z = -1, 0, +1$.
    * Static quark spin: irrelevant (static quarks can also be treated as spinless color charges).
  - Parity: $\mathcal{P} = \pm$.
  - If $j_z = 0$, reflection along the x-axis: $\mathcal{P}_x = \pm$.
  - Instead of using $j_z = \pm 1$ one can also label states by $|j_z| = 1$, $\mathcal{P}_x = \pm$.

$\rightarrow$ Label BB states by $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$.

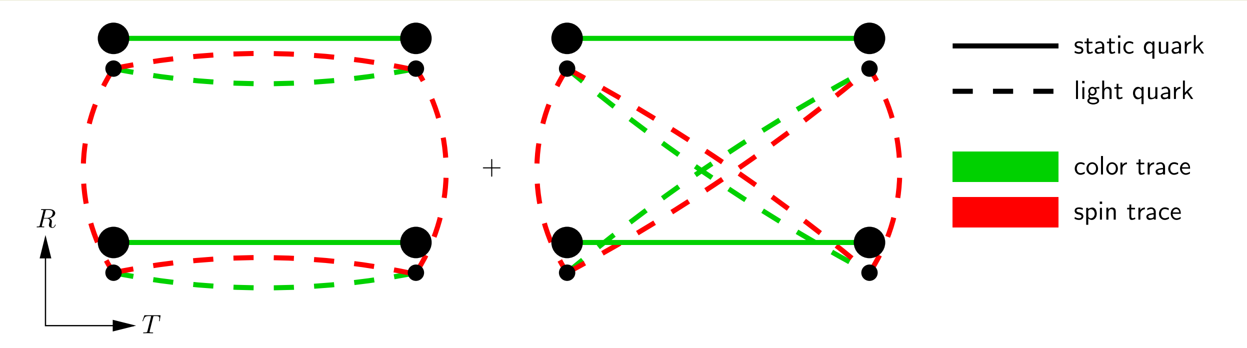
BB systems (2)

- To extract the potential(s) of a given sector (characterized by $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$), compute the temporal correlation function of the trial state

$$(\mathcal{C}\Gamma)_{AB}\Big(\bar{Q}_C(-R/2)q_A^{(1)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)q_B^{(2)}(+R/2)\Big)|\Omega\rangle,$$

  where

  - $\mathcal{C} = \gamma_0\gamma_2$ (charge conjugation matrix),
  - $q^{(1)}q^{(2)} \in \{ud - du \quad , \quad uu\, , \, dd\, , \, ud + du\}$ (isospin I, I_z) ,
  - Γ is an arbitrary combination of γ matrices (spin $|j_z|$, parity $\mathcal{P}$, $\mathcal{P}_x$).



BB systems (3)

- Wilson twisted mass action:

$$S_{\mathrm{F}}[\chi, \bar{\chi}, U] \;=\; a^4 \sum_x \bar{\chi}(x) \Big(D_{\mathrm{W}} + i\mu_{\mathrm{q}}\gamma_5\tau_3\Big) \chi(x) \quad , \quad \psi(x) \;=\; e^{i\gamma_5\tau_3\omega/2}\chi(x).$$

- Symmetries of Wilson twisted mass lattice QCD compared to QCD:
  - SU(2) isospin breaks down to U(1): I_z is still a good quantum number, I is not.
  - Parity $\mathcal{P}$ is replaced by $\mathcal{P}^{(\mathrm{tm})}$, which is parity combined with light flavor exchange.
  - Twisted mass BB sectors:
    * $I_z = \pm 1$: $(I_z \;,\; |j_z| \;,\; \underbrace{\mathcal{P}^{(\mathrm{tm})}\mathcal{P}_x^{(\mathrm{tm})}}_{=\mathcal{P}\mathcal{P}_x})$,
    * $I_z = 0$: $(I_z \;,\; |j_z| \;,\; \underbrace{\mathcal{P}^{(\mathrm{tm})}}_{=\mathcal{P}\times(2I-1)} \;,\; \underbrace{\mathcal{P}_x^{(\mathrm{tm})}}_{=\mathcal{P}_x\times(2I-1)})$.

  $\rightarrow$ QCD sectors $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$ are pairwise combined.

BB systems (4)

- BB creation operators for $I_z = +1$: 16 operators of type

$$(\mathcal{C}\Gamma)_{AB}\Big(\bar{Q}_C(-R/2)\chi_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)\chi_B^{(u)}(+R/2)\Big).$$

Γ twisted	$\|j_z\|$, $\mathcal{P}^{(\mathrm{tm,light})}\mathcal{P}_x^{(\mathrm{tm,light})}$	Γ pseudo physical	$\|j_z\|$, $\mathcal{P}^{(\mathrm{light})}$, $\mathcal{P}_x^{(\mathrm{light})}$
γ_5	$0, +$	$\mp i$	$0, -, -$
$\gamma_0\gamma_5$	$0, +$	$+\gamma_0\gamma_5$	$0, +, +$
1	$0, +$	$\mp i\gamma_5$	$0, +, +$
γ_0	$0, -$	$+\gamma_0$	$0, +, -$
γ_3	$0, +$	$+\gamma_3$	$0, -, -$
$\gamma_0\gamma_3$	$0, +$	$\mp i\gamma_0\gamma_3\gamma_5$	$0, +, +$
$\gamma_3\gamma_5$	$0, -$	$+\gamma_3\gamma_5$	$0, -, +$
$\gamma_0\gamma_3\gamma_5$	$0, +$	$\mp i\gamma_0\gamma_3$	$0, -, -$
γ_1	$1, -$	$+\gamma_1$	$1, -, +$
$\gamma_0\gamma_1$	$1, -$	$\mp i\gamma_0\gamma_1\gamma_5$	$1, +, -$
$\gamma_1\gamma_5$	$1, +$	$+\gamma_1\gamma_5$	$1, -, -$
$\gamma_0\gamma_1\gamma_5$	$1, -$	$\mp i\gamma_0\gamma_1$	$1, -, +$
$\ldots$	$\ldots$	$\ldots$	$\ldots$

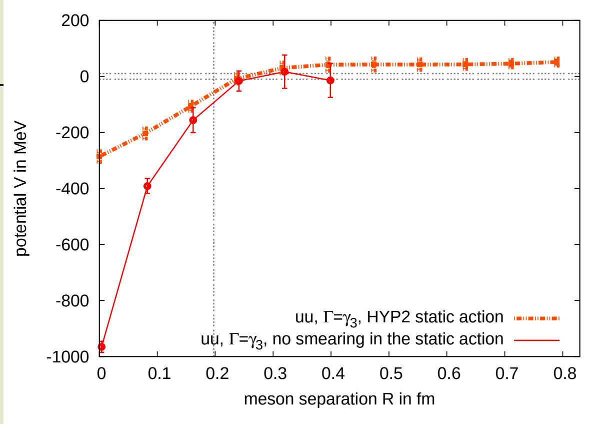
BB systems (5)

- BB creation operators for $I_z = 0$: 32 operators of type

$$(\mathcal{C}\Gamma)_{AB}\Big(\bar{Q}_C(-R/2)\chi_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)\chi_B^{(d)}(+R/2)\Big) \pm (u \leftrightarrow d).$$

Γ twisted, $\pm$	$\lvert j_z\rvert$, $\mathcal{P}^{(\mathrm{tm,light})}$, $\mathcal{P}_x^{(\mathrm{tm,light})}$	Γ pseudo physical, $\pm$	$\lvert j_z\rvert$, I, $\mathcal{P}^{(\mathrm{light})}$, $\mathcal{P}_x^{(\mathrm{light})}$
γ_5 , $+$	$0, +, +$	$+\gamma_5$, $+$	$0, 1, +, +$
$\gamma_0\gamma_5$, $+$	$0, +, +$	$+i\gamma_0$, $-$	$0, 0, -, -$
1 , $-$	$0, -, +$	$+1$, $-$	$0, 0, +, -$
γ_0 , $-$	$0, +, +$	$+i\gamma_0\gamma_5$, $+$	$0, 1, +, +$
γ_5 , $-$	$0, +, -$	$+\gamma_5$, $-$	$0, 0, -, +$
$\gamma_0\gamma_5$, $-$	$0, +, -$	$+i\gamma_0$, $+$	$0, 1, +, -$
1 , $+$	$0, -, -$	$+1$, $+$	$0, 1, -, -$
γ_0 , $+$	$0, +, -$	$+i\gamma_0\gamma_5$, $-$	$0, 0, -, +$
γ_3 , $+$	$1, -, -$	$+i\gamma_3\gamma_5$, $-$	$1, 0, +, +$
$\gamma_0\gamma_3$, $+$	$1, -, -$	$+\gamma_0\gamma_3$, $+$	$1, 1, -, -$
$\gamma_3\gamma_5$, $-$	$1, -, -$	$+i\gamma_3$, $+$	$1, 1, -, -$
$\gamma_0\gamma_3\gamma_5$, $-$	$1, +, -$	$+\gamma_0\gamma_3\gamma_5$, $-$	$1, 0, -, +$
$\ldots$	$\ldots$	$\ldots$	$\ldots$

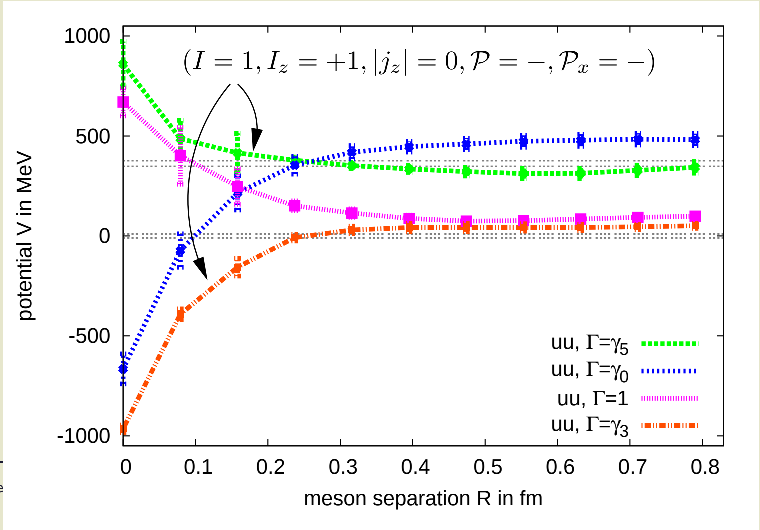
Simulation setup



- $\beta = 3.90$, $L^3 \times T = 24^3 \times 48$, $\mu = 0.0040$
  $\longrightarrow$ lattice spacing $a \approx 0.079\,\mathrm{fm}$
  $\longrightarrow$ lattice extension $L \approx 1.90\,\mathrm{fm}$
  $\longrightarrow$ pion mass $m_{\mathrm{PS}} \approx 340\,\mathrm{MeV}$.
- Inversions/contractions on
  - 210 gauge configurations for light u/d quarks.
  - 18 gauge configurations for partially quenched s quarks.
- 12 u and 12 d inversions per gauge configuration (stochastic timeslice sources located on the same timeslice).
- APE smearing of spatial links and Gaussian smearing of light quark fields to "optimize" the ground state overlap of trial states.
- Wilson lines of static quarks are discretized by path ordered products of ordinary links (small separations) and HYP2 smeared links (large separations).

Discussion of results (1)

- Four "types of potentials":
  - Two attractive, two repulsive.
  - Two have asymptotic values, which are larger by $\approx 400\,\mathrm{MeV}$.
- There are cases, where two potentials with identical quantum numbers are completely different (i.e. of different type)
  $\rightarrow$ at least one of the corresponding trial states must have very small ground state overlap
  $\rightarrow$ physical understanding, i.e. interpretation of trial states needed.



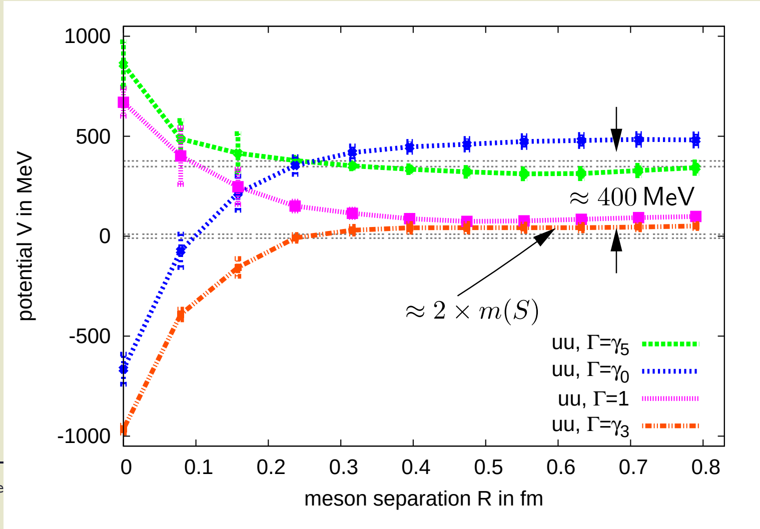
Discussion of results (2)

- Expectation at large meson separation R: $V(R) \approx 2 \times$ meson mass.
  - Potentials with smaller asymptotic value at $\approx 2 \times m(S)$.
  - $m(P_-) - m(S) \approx 400$ MeV: approximately the observed difference between different types of potentials.

$\rightarrow$ Two types correspond to S–S potentials.

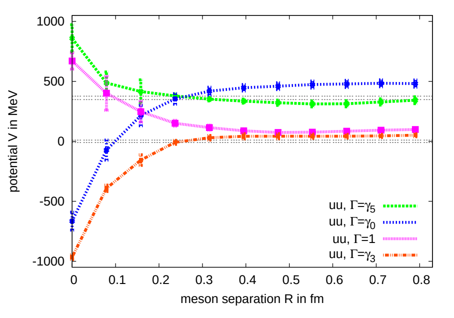
$\rightarrow$ Two types correspond to S–P_- potentials.

- Can this be understood in detail on the level of the used BB creation operators?

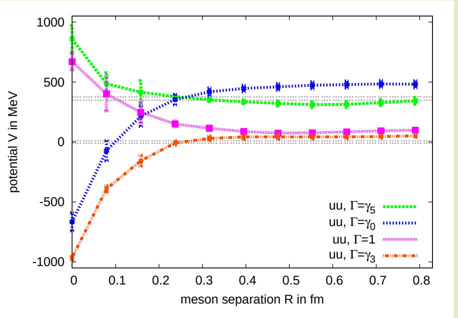


Discussion of results (3)

- Rotate the BB creation operators to the pseudo physical basis and express them in terms of static-light meson creation operators (use suitable spin and parity projectors for the light quarks).
  - Examples:
    - uu, $\Gamma = \gamma_5 \quad \rightarrow \quad \Gamma^{(\mathrm{ppb})} = -i \quad \rightarrow \quad \mathcal{P} = -$, $\mathcal{P}_x = -$:
    $$(\mathcal{C}\gamma_5)_{AB}\Big(\bar{Q}_C(-R/2)\chi_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)\chi_B^{(u)}(+R/2)\Big) =$$
    $$= +i\Big(S_\uparrow P_\downarrow - S_\downarrow P_\uparrow + P_\uparrow S_\downarrow - P_\downarrow S_\uparrow\Big).$$
    - uu, $\Gamma = \gamma_3 \quad \rightarrow \quad , \Gamma^{(\mathrm{ppb})} = \gamma_3 \quad \rightarrow \quad \mathcal{P} = -$, $\mathcal{P}_x = -$:
    $$(\mathcal{C}\gamma_3)_{AB}\Big(\bar{Q}_C(-R/2)\chi_A^{(u)}(-R/2)\Big)\Big(\bar{Q}_C(+R/2)\chi_B^{(u)}(+R/2)\Big) =$$
    $$= -i\Big(S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow - P_\downarrow P_\uparrow\Big).$$
  - SS/SP_- content and asymptotic values in agreement for all $12 + 24$ independent potentials $\rightarrow$ asymptotic differences understood.

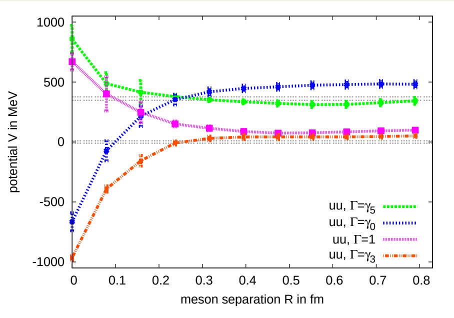


Discussion of results (4)



- Is there a general rule, about when a potential is repulsive and when attractive?
  - S–S potentials:
    - $(I = 0\ ,\ s = 0)$ or $(I = 1\ ,\ s = 1)$, i.e. $I = s$ $\quad\rightarrow\quad$ attractive
      $(I = 0\ ,\ s = 1)$ or $(I = 1\ ,\ s = 0)$, i.e. $I \neq s$ $\quad\rightarrow\quad$ repulsive
      (s: combined angular momentum of the two mesons).
    - Example: uu, $\Gamma = \gamma_3 \quad\rightarrow\quad \Gamma^{(\mathrm{ppb})} = \gamma_3 \quad\rightarrow\quad \mathcal{P} = -$, $\mathcal{P}_x = -$:
      $-i\Big(S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow - P_\downarrow P_\uparrow\Big)$,
      i.e. $I = 1$, $s = 1$; the numerically obtained potential is attractive, i.e. in agreement with the above stated rule.
    - All 6 + 12 independent S–S potentials fulfill the rule.
    - Agreement with similar quenched lattice studies.
      [C. Michael and P. Pennanen [UKQCD Collaboration], Phys. Rev. D **60**, 054012 (1999)]
      [W. Detmold, K. Orginos and M. J. Savage, Phys. Rev. D **76**, 114503 (2007)]

Discussion of results (5)



– S–P_- potentials:

- Do not obey the above stated rule.
- It can, however, easily be generalized by including parity, i.e. symmetry or antisymmetry under exchange of S and P_-:
  trial state symmetric under meson exchange $\quad \longrightarrow \quad$ attractive
  trial state antisymmetric under meson exchange $\quad \longrightarrow \quad$ repulsive
  (meson exchange $\equiv$ exchange of flavor, spin and parity).
- Example: uu, $\Gamma = \gamma_0 \quad \rightarrow \quad \Gamma^{(\mathrm{ppb})} = \gamma_0 \quad \rightarrow \quad \mathcal{P} = +$, $\mathcal{P}_x = -$:
  $-\Big(S_\uparrow P_\downarrow - S_\downarrow P_\uparrow - P_\uparrow S_\downarrow + P_\downarrow S_\uparrow\Big)$,
  i.e. $I = 1$ (symmetric), $s = 0$ (antisymmetric), antisymmetric with respect to $S \leftrightarrow P_-$; the numerically obtained potential is attractive, i.e. in agreement with the above stated general rule.
- All 6 + 12 independent S–P_- potentials (and all 6 + 12 independent S–S potentials) fulfill the general rule.

Summary, conclusions, future plans (1)

- Computation of BB potentials (arbitrary flavor, spin and parity) with light dynamical quarks ($m_{\mathrm{PS}} \approx 340\,\mathrm{MeV}$) in progress.
- Preliminary results promising:
  - Qualitative agreement with existing quenched results for S–S potentials.
  - Computation of S–P_- potentials seems feasible (for some channels correlation matrices will be needed).
  - Clear statements about whether a potential of a given channel is attractive or repulsive.

  Statistical accuracy needs to be improved:
  - Exponentially decaying signal is quickly lost in noise
    $\rightarrow$ BB potentials are extracted at rather small temporal separations
    $\rightarrow$ contamination from excited states cannot be excluded at the moment.
  - More inversions/contractions?
  - Better methods?

Summary, conclusions, future plans (2)

- Further plans:
  - Other β, $L^3 \times T$, μ values.
  - Partially quenched computations, to obtain $B_s B_s$ and/or $B_s B$ potentials.
  - Improve lattice meson potentials at small separations (where the suppression of UV fluctuations due to the lattice cutoff yields wrong results) with corresponding perturbative potentials.

Simulation setup (A)

- Fermionic action: Wilson twisted mass, $N_f = 2$ degenerate flavors,

$$S_\mathrm{F}[\chi, \bar{\chi}, U] = a^4 \sum_x \bar{\chi}(x) \Big(D_\mathrm{W} + i\mu_\mathrm{q}\gamma_5\tau_3\Big)\chi(x)$$

$$D_\mathrm{W} = \frac{1}{2}\Big(\gamma_\mu(\nabla_\mu + \nabla_\mu^*) - a\nabla_\mu^*\nabla_\mu\Big) + m_0$$

(m_0: untwisted mass; μ_q: twisted mass; τ_3: third Pauli matrix acting in flavor space).

- Relation between the physical basis ψ and the twisted basis χ (in the continuum):

$$\psi = \frac{1}{\sqrt{2}}\Big(\cos(\omega/2) + i\sin(\omega/2)\gamma_5\tau_3\Big)\chi$$

$$\bar{\psi} = \frac{1}{\sqrt{2}}\bar{\chi}\Big(\cos(\omega/2) + i\sin(\omega/2)\gamma_5\tau_3\Big)$$

(ω: twist angle; $\omega = \pi/2$: maximal twist).