

# Ghost-gluon coupling, power corrections and $\Lambda_{\overline{MS}}$ from twisted-mass QCD at $N_f = 2$

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# Outline

## 1 QCD strong coupling

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- 2  $N_f = 2$  twisted-mass QCD
- 3 Conclusion

## Ghost-gluon vertex

$$\tilde{\Gamma}_{\nu}^{abc}(-q, k; q - k) = \text{Diagram} = ig_0 f^{abc} (q_{\nu} H_1(q, k) + (q - k)_{\nu} H_2(q, k))$$

## Strong coupling

$$g_R(\mu^2) = \lim_{\Lambda \rightarrow \infty} Z_g^{-1}(\mu^2, \Lambda^2) g_0(\Lambda^2) = \lim_{\Lambda \rightarrow \infty} \frac{Z_3^{1/2}(\mu^2, \Lambda^2) \tilde{Z}_3(\mu^2, \Lambda^2)}{\tilde{Z}_1(\mu^2, \Lambda^2)} g_0(\Lambda^2)$$

Taylor scheme: **zero incoming ghost momentum**<sup>1</sup>

$$\tilde{Z}_1(\mu^2, \Lambda^2) \equiv 1$$

$$\alpha_T(\mu^2) \equiv \frac{g_T^2(\mu^2)}{4\pi} = \lim_{\Lambda \rightarrow \infty} \frac{g_0^2(\Lambda^2)}{4\pi} Z_3(\mu^2, \Lambda^2) \tilde{Z}_3^2(\mu^2, \Lambda^2)$$

<sup>1</sup>J. C. Taylor, Nuclear Physics B33 (1971) 436

Lattice:  $\frac{1}{L^2} \ll p^2 \ll \frac{1}{a^2}$

Propagators in Landau gauge

$$\begin{aligned}\left(G^{(2)}\right)_{\mu\nu}^{ab}(p^2, \Lambda) &= \frac{G(p^2, \Lambda)}{p^2} \delta_{ab} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \\ \left(F^{(2)}\right)^{a,b}(p^2, \Lambda) &= -\delta_{ab} \frac{F(p^2, \Lambda)}{p^2}\end{aligned}$$

$$\begin{aligned}G_R(p^2, \mu^2) &= \lim_{\Lambda \rightarrow \infty} Z_3^{-1}(\mu^2, \Lambda) G(p^2, \Lambda) \\ F_R(p^2, \mu^2) &= \lim_{\Lambda \rightarrow \infty} \tilde{Z}_3^{-1}(\mu^2, \Lambda) F(p^2, \Lambda)\end{aligned}$$

Non-perturbative renormalization: **MOM scheme**

$$G_R(\mu^2, \mu^2) = F_R(\mu^2, \mu^2) = 1$$

Strategy

Matching lattice-perturbation theory

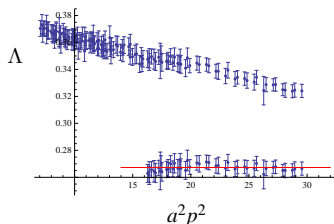
# 4-loops perturbation theory<sup>3</sup>: $p \gg \Lambda_{QCD}$

$$\alpha_T(\mu^2) = \frac{4\pi}{\beta_0 t} \left( 1 - \frac{\beta_1}{\beta_0^2} \frac{\log(t)}{t} + \frac{\beta_1^2}{\beta_0^4} \frac{1}{t^2} \left( \left( \log(t) - \frac{1}{2} \right)^2 + \frac{\tilde{\beta}_2 \beta_0}{\beta_1^2} - \frac{5}{4} \right) \right) \\ + \frac{1}{(\beta_0 t)^4} \left( \frac{\tilde{\beta}_3}{2\beta_0} + \frac{1}{2} \left( \frac{\beta_1}{\beta_0} \right)^3 \left( -2 \log^3(t) + 5 \log^2(t) + \left( 4 - 6 \frac{\tilde{\beta}_2 \beta_0}{\beta_1^2} \right) \log(t) - 1 \right) \right), t = \ln \frac{\mu^2}{\Lambda_T^2}$$

$$\beta_T(\alpha_T) = \frac{d\alpha_T}{d \ln \mu^2} = -4\pi \sum_{i=0} \tilde{\beta}_i \left( \frac{\alpha_T}{4\pi} \right)^{i+2}, \quad \frac{\Lambda_{\overline{\text{MS}}}}{\Lambda_T} = e^{-\frac{c_1}{2\beta_0}} = e^{-\frac{507 - 40N_f}{792 - 48N_f}} = 0.541449$$

Quenched QCD:  $N_f = 0$ <sup>2</sup>

**Matching lattice-perturbation theory**



<sup>2</sup> Ph. Boucaud *et al.*, Phys. Rev. D **79** (2009) 014508

<sup>3</sup> K. G. Chetyrkin, Nucl. Phys. B **710** (2005) 499

# OPE power corrections

$$\begin{aligned}
 (F^{(2)})^{ab}(q^2) &= (F_{\text{pert}}^{(2)})^{ab}(q^2) + w^{ab} \frac{\langle A^2 \rangle}{4(N_C^2 - 1)} + \dots, \quad w^{ab} = 2 \times \text{diagram} \\
 (G^{(2)})^{ab}_{\mu\nu}(q^2) &= (G_{\text{pert}}^{(2)})^{ab}_{\mu\nu}(q^2) + w^{ab}_{\mu\nu} \frac{\langle A^2 \rangle}{4(N_C^2 - 1)} + \dots, \quad w^{ab}_{\mu\nu} = \text{diagram} + 2 \times \text{diagram}
 \end{aligned}$$

$$F_R(q^2, \mu^2) = F_{R,\text{pert}}(q^2, \mu^2) \left( 1 + \frac{3}{q^2} \frac{g_R^2 \langle A^2 \rangle_{R, \mu^2}}{4(N_C^2 - 1)} \right), \quad G_R(q^2, \mu^2) = G_{R,\text{pert}}(q^2, \mu^2) \left( 1 + \frac{3}{q^2} \frac{g_R^2 \langle A^2 \rangle_{R, \mu^2}}{4(N_C^2 - 1)} \right)$$

$$\text{Leading logarithm}^4: \alpha_T(\mu^2) = \alpha_T^{\text{pert}}(\mu^2) \left( 1 + \frac{9}{\mu^2} \left( \frac{\alpha_T^{\text{pert}}(\mu^2)}{\alpha_T^{\text{pert}}(q_0^2)} \right)^{1 - \gamma_0^A / \beta_0} \frac{g_T^2(q_0^2) \langle A^2 \rangle_{R, q_0^2}}{4(N_C^2 - 1)} \right)$$

$$1 - \gamma_0^A / \beta_0 = 1 - \frac{105 - 8N_f}{132 - 8N_f} = \frac{9}{44 - \frac{8}{3}N_f}$$

Wilson coefficient is known at four-loop<sup>5</sup>

<sup>4</sup>R. Wilson, Phys. Rev. **179** (1969) 1499

<sup>5</sup>K. G. Chetyrkin and A. Maier, arXiv:0911.0594 [hep-ph]

# 4-loops perturbation theory<sup>7</sup>: $p \gg \Lambda_{QCD}$

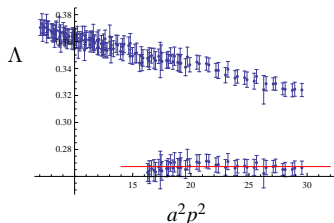
$$\alpha_T(\mu^2) = \frac{4\pi}{\beta_0 t} \left( 1 - \frac{\beta_1}{\beta_0^2} \frac{\log(t)}{t} + \frac{\beta_1^2}{\beta_0^4} \frac{1}{t^2} \left( \left( \log(t) - \frac{1}{2} \right)^2 + \frac{\tilde{\beta}_2 \beta_0}{\beta_1^2} - \frac{5}{4} \right) \right) \\ + \frac{1}{(\beta_0 t)^4} \left( \frac{\tilde{\beta}_3}{2\beta_0} + \frac{1}{2} \left( \frac{\beta_1}{\beta_0} \right)^3 \left( -2 \log^3(t) + 5 \log^2(t) + \left( 4 - 6 \frac{\tilde{\beta}_2 \beta_0}{\beta_1^2} \right) \log(t) - 1 \right) \right), t = \ln \frac{\mu^2}{\Lambda_T^2}$$

$$\beta_T(\alpha_T) = \frac{d\alpha_T}{d \ln \mu^2} = -4\pi \sum_{i=0} \tilde{\beta}_i \left( \frac{\alpha_T}{4\pi} \right)^{i+2}, \quad \frac{\Lambda_{\overline{MS}}}{\Lambda_T} = e^{-\frac{c_1}{2\beta_0}} = e^{-\frac{507 - 40N_f}{792 - 48N_f}} = 0.541449$$

Quenched QCD:  $N_f = 0$ <sup>6</sup>

## Matching lattice-perturbation theory

$$\Lambda_{\overline{MS}}^{N_f=0} = 224_{-5}^{+8} \text{ Mev}, g_T^2 \langle A^2 \rangle = 5.1_{-1.1}^{+0.7} \text{ GeV}^2$$



<sup>6</sup> Ph. Boucaud *et al.*, Phys. Rev. D **79** (2009) 014508

<sup>7</sup> K. G. Chetyrkin, Nucl. Phys. B **710** (2005) 499

# European Twisted Mass Collaboration

Fermions: twisted-mass action

$$S_{\text{tm}}^{\text{F}} = a^4 \sum_x \left\{ \bar{\chi}_x [D_{\text{W}} + m_0 + i\gamma_5 \tau_3 \mu_q] \chi_x \right\}$$



Gauge fields: tISym action

$$S_g = \frac{\beta}{3} \sum_x \left( b_0 \sum_{\substack{\mu, \nu=1 \\ 1 \leq \mu < \nu}}^4 \{1 - \text{ReTr}(U_{x, \mu, \nu}^{1 \times 1})\} + b_1 \sum_{\substack{\mu, \nu=1 \\ \mu \neq \nu}}^4 \{1 - \text{ReTr}(U_{x, \mu, \nu}^{1 \times 2})\} \right), \quad \beta \equiv 6/g_0^2$$

$$(b_0 = 1 - 8b_1, b_1 = -1/12) \quad + \quad \text{Maximal twist : } \underline{\mathcal{O}(a^2)}$$

$$V = 24^3 \times 48 \quad \beta = 3.9 \quad \mu = 0.004, 0.0064, 0.010$$

$$V = 32^3 \times 64 \quad \beta = 4.05 \quad \mu = 0.003, 0.006, 0.008, 0.012$$

$$\beta = 4.2 \quad \mu = 0.0065$$

$$\text{Artefacts: } \underline{\mathcal{O}(a^2 \Lambda_{\text{QCD}}^2)}, \quad \underline{\mathcal{O}(a^2 p^2)}, \quad \underline{\mathcal{O}(a^2 \mu^2)}$$

# Ghost and gluon on the lattice

Landau gauge

$$F_U[g] = \text{Re} \left[ \sum_x \sum_\mu \text{Tr} \left( 1 - \frac{1}{N} g(x) U_\mu(x) g^\dagger(x + \mu) \right) \right]$$

Gluon:

$$A_\mu(x + \hat{\mu}/2) = \frac{U_\mu(x) - U_\mu^\dagger(x)}{2ia g_0} - \frac{1}{3} \text{Tr} \left( \frac{U_\mu(x) - U_\mu^\dagger(x)}{2ia g_0} \right)$$

$$\underbrace{\phantom{0000000000}}_{\mu_1 \mu_2} \quad \left( G^{(2)} \right)_{\mu_1 \mu_2}^{a_1 a_2}(p) = \langle A_{\mu_1}^{a_1}(p) A_{\mu_2}^{a_2}(-p) \rangle$$

Ghost:

$$\text{-----} \rightarrow \text{-----} \quad \left( F^{(2)} \right)^{ab}(x-y) \equiv \langle \left( M^{-1} \right)_{xy}^{ab} \rangle, \quad M(U) = -\frac{1}{N} \nabla \cdot \tilde{D}(U)$$

$$\tilde{D}(U) \eta(x) = \frac{1}{2} \left( U_\mu(x) \eta(x + \mu) - \eta(x) U_\mu(x) + \eta(x + \mu) U_\mu^\dagger - U_\mu^\dagger(x) \eta(x) \right)$$

# $O(4)$ breaking: $H(4)$ discretization artefacts<sup>8</sup>

Orbit labeled by  $H(4)$ -invariants:  $p^{[2n]} = \sum_{\mu=1}^4 p_{\mu}^{2n}$ ,  $n = 1, 2, 3$

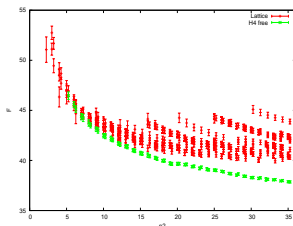
Momentum on the lattice:  $\tilde{p}_{\mu} = \frac{1}{a} \sin ap_{\mu}$ ,  $p_{\mu} = \frac{2\pi n}{Na}$   $n = 0, 1, \dots, N$

$$a^2 \tilde{p}^2 \equiv \sum_{\mu=1}^4 a^2 \tilde{p}_{\mu}^2 = a^2 p^2 + c_1 a^4 p^{[4]} + \dots = a^2 p^2 \left( 1 + c_1 a^2 \frac{p^{[4]}}{p^2} + \dots \right)$$

If  $\epsilon = a^2 p^{[4]} / p^2 \ll 1 \dots$

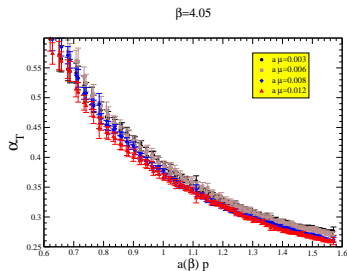
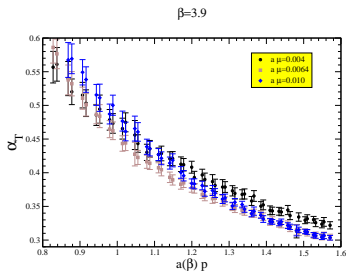
$$\begin{aligned} Q(a^2 \tilde{p}_{\mu}^2, a^2 \Lambda^2) &\equiv Q \left( a^2 p^2 \left( 1 + c_1 a^2 \frac{p^{[4]}}{p^2} + \dots \right), a^2 \Lambda^2 \right) \\ &= Q(a^2 p^2, a^2 \Lambda^2) + \left. \frac{dQ}{d\epsilon} \right|_{\epsilon=0} a^2 \frac{p^{[4]}}{p^2} + \dots \end{aligned}$$

$$R(a^2 p^2, a^2 \Lambda^2) = \left. \frac{dQ(a^2 p^2 (1 + c_1 \epsilon + \dots), a^2 \Lambda^2)}{d\epsilon} \right|_{\epsilon=0}, \quad R = R_0 + R_1 a^2 p^2$$



<sup>8</sup>F. de Soto and C. Roiesnel, JHEP **0709** (2007) 007

# Quark mass artefacts



Trying  $\mathcal{O}(a^2 \mu_q^2)$  dependence

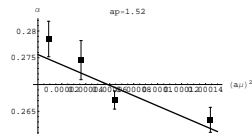
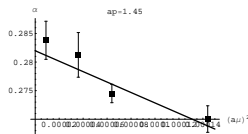
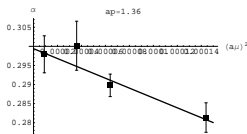
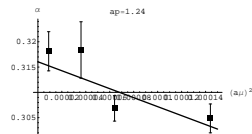
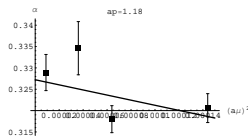
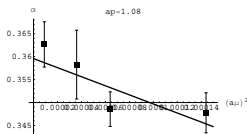
$$\begin{aligned}
 \hat{\alpha}_T(a^2 p^2, a^2 \mu_q^2) &= \frac{g_0^2(a^2)}{4\pi} \hat{G}(a^2 p^2, a^2 \mu_q^2) \hat{F}^2(a^2 p^2, a^2 \mu_q^2) \\
 &= \hat{\alpha}_T(a^2 p^2, 0) + \frac{\partial \hat{\alpha}_T}{\partial (a^2 \mu_q^2)} (a^2 p^2) a^2 \mu_q^2 + \dots
 \end{aligned}$$

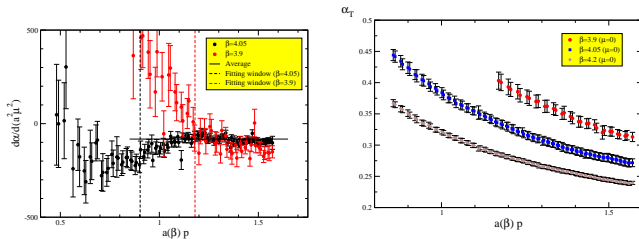
# Quark mass artefacts

$$\hat{\alpha}_T(a^2 p^2, a^2 \mu_q^2) = \alpha_T(p^2) + R_0(a^2 p^2) a^2 \mu_q^2$$

$$R_0(a^2 p^2) \equiv \frac{\partial \hat{\alpha}_T}{\partial (a^2 \mu_q^2)}$$

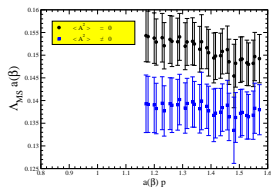
Example:  $\beta = 4.05$



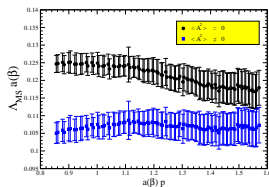


$$R_0 = -92(11), \quad p \geq p_{\min} \simeq 2.8 \text{ GeV} \quad (a(3.9)=0.0801(14) \text{ fm}^9)$$

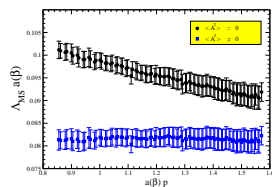
Requiring a plateau for  $\Lambda_{\overline{MS}}$  in the window  $ap \geq p_{\min}$



$$\beta = 3.9$$



$$\beta = 4.05$$

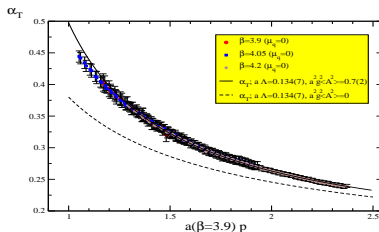


$$\beta = 4.2$$

# Global fit and calibration of lattice spacing

$$\chi^2 \left( a(\beta_0) \Lambda_{\overline{\text{MS}}}, c, \frac{a(\beta_1)}{a(\beta_0)}, \frac{a(\beta_2)}{a(\beta_0)} \right) = \sum_{j=0}^2 \sum_i \frac{\left( \Lambda_i(\beta_j) - \frac{a(\beta_j)}{a(\beta_0)} a(\beta_0) \Lambda_{\overline{\text{MS}}} \right)^2}{\delta^2(\Lambda_i)}$$

Variables:  $a(\beta_0) \Lambda_{\overline{\text{MS}}}, c, \frac{a(\beta_1)}{a(\beta_0)}, \frac{a(\beta_2)}{a(\beta_0)}$



	This paper	String tension <sup>10</sup>
$a(3.9)/a(4.05)$	1.224(23)	1.255(42)
$a(3.9)/a(4.2)$	1.510(32)	1.558(52)
$a(4.05)/a(4.2)$	1.233(25)	1.241(39)
$\Lambda_{\overline{\text{MS}}} a(3.9)$	0.134(7)	
$g^2 \langle A^2 \rangle a^2(3.9)$	0.70(23)	

# Systematics

Higher orders for Wilson coefficient...

	One loop	Two loops	Three loops	Four loops
$\Lambda_{\overline{\text{MS}}} a(3.9)$	0.134(7)	0.136(7)	0.137(7)	0.138(7)
$g^2 \langle A^2 \rangle a^2(3.9)$	0.70(23)	0.52(18)	0.44(14)	0.39(14)

Three-loop versus four-loop perturbative coupling constant...

	Four loops	Three loops
$a(3.9)/a(4.05)$	1.224(23)	1.229(23)
$a(3.9)/a(4.2)$	1.510(32)	1.510(29)
$a(4.05)/a(4.2)$	1.233(26)	1.234(25)
$\Lambda_{\overline{\text{MS}}} a(3.9)$	0.134(7)	0.125(6)
$g^2 \langle A^2 \rangle a^2(3.9)$	0.70(23)	0.80(20)

Higher orders in OPE...unstable!

$$\alpha_{T,P4}(\mu^2) = \alpha_T(\mu^2) + \frac{c_4}{\mu^4}$$

## Conclusions

$N_f = 2$ :

$$\begin{aligned}\Lambda_{\overline{\text{MS}}} &= (330 \pm 23 \pm 22_{-33}) \text{ MeV} \\ g^2(q_0^2) \langle A^2 \rangle_{q_0} &= \left( 4.2 \pm 1.5 \pm 0.7^{+?} \right) \text{ GeV}^2, \quad q_0 \sim 10 \text{ GeV}\end{aligned}$$

$N_f = 0$ :

$$\begin{aligned}\Lambda_{\overline{\text{MS}}} &= 224_{-5}^{+8} \text{ MeV} \\ g_T^2 \langle A^2 \rangle &= 5.1_{-1.1}^{+0.7} \text{ GeV}^2\end{aligned}$$

## Outlooks

ETMC new configurations:

$$\begin{aligned}N_f &= 4 \\ N_f &= 2 + 1 + 1\end{aligned}$$

THANK YOU!