Pseudoscalar Decay Constants from $N_f = 2 + 1 + 1$ Flavour LQCD

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Motivation

- ETMC $N_f = 2$ flavour simulations very successful
- but systematic effects stemming from missing strange and charm cannot be controled

$\Rightarrow$ include strange and charm in simulations maintaining $\mathcal{O}(a)$ improvement

- do we see any effects of strange and charm on observables?
- some (like η, η', η_c) can be computed only in this set-up
- prime quantities to look at: f_K, f_D and f_{D_s}

Outline

Action

- gauge action: Iwasaki
- Twisted Mass Dirac operator in the light sector:

$$D_\ell = D_W + m_0 + i\mu_\ell\gamma_5\tau^3$$

[Frezzotti, Rossi, Sint, Weisz (1999)]

- μ_ℓ bare light twisted mass, τ^3 Pauli matrix in flavour space
- m_0 set to $m_0 = m_{\mathrm{crit}}(\mu_\ell, \beta)$ by tuning m_{PCAC} to zero.

$\Rightarrow$ physical observables $\mathcal{O}(a)$ improved

[Frezzotti, Rossi (2003)]

- talk of S. Reker

Action

- Twisted Mass charm/strange doublet:

  [Frezzotti, Rossi (2004)]

$$D_h = D_W + m_{\text{crit}} + i\mu_\sigma\tau^1 + \mu_\delta\tau^3$$

- quark masses from bare parameters

$$m_s = \mu_\sigma \ - \ (Z_{\text{P}}/Z_{\text{S}})\ \mu_\delta, \quad m_c = \mu_\sigma \ + \ (Z_{\text{P}}/Z_{\text{S}})\ \mu_\delta$$

- requires knowledge of Z-ratio
- flavour and parity symmetry broken at $\mathcal{O}(a^2)$

$N_f = 2+1+1$ Ensembles

- ensembles as produced by ETMC
  [ETMC (2010)]
- here: m_{PS} from 270 MeV to 510 MeV
  otherwise soon 230 MeV to 510 MeV
- $m_{\mathrm{PS}} \cdot L \geq 4$, $L \lesssim 3$ fm
- three values of the lattice spacing from 0.1 fm to 0.06 fm
  β-values: 1.90, 1.95 and 2.10
- for single β-value: fixed values of μ_σ and μ_δ

Mixing in the Heavy-Light Sector

- twist rotations (χ twisted, ψ physical basis):

$$\psi_\ell = e^{i\omega\gamma_5\tau^3/2}\chi_\ell, \qquad \psi_h = e^{i\omega\gamma_5\tau^1/2}\chi_h$$

- scalar and pseudoscalar, charm and strange sectors mix:

$$\mathcal{V} = \begin{pmatrix} \bar{\chi}_d\gamma_5\chi_s \\ \bar{\chi}_d\gamma_5\chi_c \\ \bar{\chi}_d\chi_s \\ \bar{\chi}_d\chi_c \end{pmatrix}, \qquad \mathcal{C} = \mathcal{V}\otimes\bar{\mathcal{V}}$$

[Chiarappa, et al. (2007), ETMC (2010)]

- Determine m_K and m_D from 4×4 correlation matrix $\mathcal{C}$
  various methods available [ETMC (2010)]
- Poster by E. Pallante

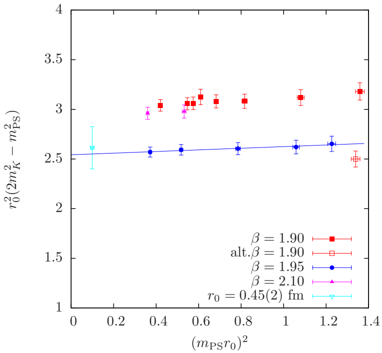
Mixing in the Heavy-Light Sector

- mixing looks complicated
- but for known twist angle ω and $Z_{\mathrm{P,S}}$
  we know exactly the matrix $\mathcal{M}$ which diagonalises $\mathcal{C}$

$$\hat{\mathcal{C}} = \mathcal{M}(\omega, Z_{\mathrm{P}}/Z_{\mathrm{S}})\, \mathcal{C}\, \mathcal{M}^{-1}(\omega, Z_{\mathrm{P}}/Z_{\mathrm{S}})$$

- for maximal twist this opens a possibility to estimate $(Z_{\mathrm{P}}/Z_{\mathrm{S}})$
- however, mixing can be avoided using a different action
  in the valence sector $\quad\rightarrow\quad$ mixed action
- for instance an Osterwalder-Seiler type action
  without any flavour mixing

Tuning of Unitary Kaon Mass



[ETMC (2010)]

- for $\beta = 1.90$ and 2.10 the Kaon is slightly too heavy
- $\beta = 1.95$ better tuned
- mild dependence on m_π^2
- retuning at $\beta = 1.90$ gives much better value for m_K
- also D-meson slightly too heavy

Use mixed action to match to physical K- and D-Meson masses

Unitary f_K

- the unitary f_K can be computed from

$$f_K = (m_\ell + m_s)\frac{\langle 0|P_K|K\rangle}{m_K^2}$$

with $m_s = \mu_\sigma - (Z_\mathrm{P}/Z_\mathrm{S})\mu_\delta$

- similar formula for f_D
- P_K is the physical Kaon projecting operator determined e.g. from diagonalising $\mathcal{C}$
- unitary f_K value depends strongly on estimate of $Z_\mathrm{P}/Z_\mathrm{S}$ via m_s

Outline

Mixed Action Set-up

- introduce Wilson twisted mass doublets in the valence sector

$$D_{tm}(\mu_{val}) = D + m_{\mathrm{crit}} + i\ \mu_{val}\gamma_5\tau^3$$

[Pena et al. (2004); Frezzotti, Rossi (2004)]

- m_{crit} from unitary set-up
- $4-6$ values for μ_{val} in the strange μ_s and the charm μ_c region inversions with multi-mass solver
- matching to unitary set-up using m_K and m_D
  $\Rightarrow$ obtain simulated μ_s and μ_c

Mixed Action Set-up

- at matching point we can determine $Z_{\rm P}/Z_{\rm S}$ from equating

$$\mu_s \;=\; \mu_\sigma - (Z_{\rm P}/Z_{\rm S})\,\mu_\delta$$

- valid up to lattice artifacts

- the mixed action (MA) pseudoscalar decay constants from

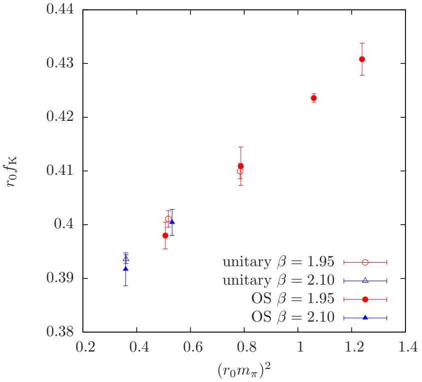
$$f_{\rm PS} = \left(\mu_{val}^{(1)} + \mu_{val}^{(2)}\right) \frac{|\langle 0|P|PS\rangle|}{m_{\rm PS}\,\sinh m_{\rm PS}}\,,$$

- $m_{\rm PS}$ and $f_{\rm PS}$ both determined from combined fit
  using smeared-smeared and local-smeared correlators

- the sinh in lattice $f_{\rm PS}$ definition
  helps reducing discretisation errors

Outline

f_K: Unitary versus Mixed Action



- matched values for unitary and MA m_K
- compare unitary with MA f_K in units of r_0
- very good agreement between MA and unitary f_K
- lattice artifacts seem to be small

$SU(2)$ χPT Fit Formulae

- $SU(2)$ χPT Fit Formulae for f_K and f_π:

$$f_{\mathrm{PS}}(\mu_\ell,\mu_\ell,\mu_\ell) = f_0 \cdot (1 - 2\,\xi_{\ell\ell}\ln\xi_{\ell\ell} + b\,\xi_{\ell\ell})\ ,$$

$$f_{\mathrm{PS}}(\mu_\ell,\mu_\ell,\mu_s) = (f_0^{(K)} + f_m^{(K)}\,\xi_{ss}) \cdot \left[1 - \frac{3}{4}\xi_{\ell\ell}\ln\xi_{\ell\ell} + (b_0^{(K)} + b_m^{(K)}\,\xi_{ss})\,\xi_{\ell\ell}\right]\ .$$

where

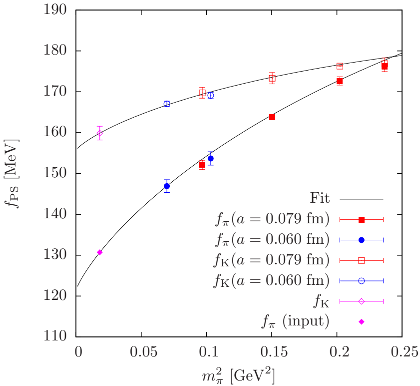
$$\xi_{XY} = \frac{m_{\mathrm{PS}}^2(\mu_\ell,\mu_X,\mu_Y)}{(4\pi f_0)^2}$$

[Gasser, Leutwyler (1984); Allton et al (2008); ETMC, Blossier et al. (2010)]

- correct for finite size effects using NLO χPT

[Gasser, Leutwyler (1987); Becirevic, Villadoro (2004)]

f_K and f_π versus m_π^2



- fit $\beta = 1.95$ and $\beta = 2.10$ simultaneously
- from setting

$$m_{\rm PS}^2(\mu_\ell, \mu_s, \mu_s) = 2m_K^2 - m_\pi^2$$

- using f_π and m_π as light input
- nice agreement between $\beta = 1.95$ and $\beta = 2.10$
- curvature visible
- no usage of r_0/a

Fitresults Kaon-Sector

- physical input :

$$m_\pi = 135 \text{ MeV}, \quad f_\pi = 130.7 \text{ MeV}, \quad m_K = 497.7 \text{ MeV}$$

- prelinimary fit results:

$$f_K/f_\pi = 1.224(13), \qquad f_K = 160(2) \text{ MeV}, \qquad \bar{\ell}_4 = 4.78(2)$$

- $\chi^2/\mathrm{dof} = 50/30$
- from f_K/f_π and additional input

$$|V_{us}| = 0.220(2)$$

[Marciano (2008)]

- errors statistical only

D-Meson Sector

- preliminary analysis of f_D and f_{D_s} in MA set-up
- $SU(2)$ heavy meson χPT fit to our data for $f_{D_s}\sqrt{m_{D_s}}$ and $f_{D_s}\sqrt{m_{D_s}}/(f_D\sqrt{m_D})$
  [ETMC, Blossier et al. (2009)]
- including terms proportional to $a^2 m_{D_s}^2$ and $1/m_{D_s}$
- first results very encouraging

$$f_{D_s} = 250(3)\ \mathrm{MeV}, \quad f_D = 204(3)\ \mathrm{MeV}, \quad f_{D_s}/f_D = 1.230(6)$$

- very preliminary!

Conclusion

- results for f_K, f_{D_s} and f_D from mixed action investigation on ETMC $2+1+1$ flavour confiturations
- unitary and MA f_K at matched am_K agree within errors
- results for f_K/f_π and f_{D_s} and f_D look encouraging
- comparing to other groups shows agreement
- no difference to $N_f = 2$ within errors

Outlook

- investigate different fit formulae
- use lighter pion masses
- control finite size effects
- third value of the lattice spacing
- investigate dependence on sea strange and charm quark mass

Thanks to all ETMC $2+1+1$ collaborators!

Kaon Projecting Operator

- unitary kaon decay constant

$$f_K = \frac{\mu_\ell + \mu_\sigma - (Z_P/Z_S)\mu_\delta}{2m_K^2} \cdot \langle 0|(P_K - P_D) + i(Z_S/Z_P)(S_K + S_D)|K\rangle$$

- Kaon is lowest state, so flavour mixings should play no role
- mixing of scalar and pseudoscalar