$\mathcal{O}(a^2)$ perturbative calculation of the bilinears

M. Constantinou, V. Giménez, V. Lubicz, D. Palao,
H. Panagopoulos, F. Stylianou

September 30, 2008

Table of contents

Motivation

Why to perform a 1-loop $\mathcal{O}(a^2)$ perturbative calculation of quark bilinears on the lattice?

1. RI-MOM analysis needs it to reduce errors
2. B_k analysis seems to need it too (!?)
3. 1 loop $\mathcal{O}(a^2)$ perturbative calculation never done

RI-MOM analysis

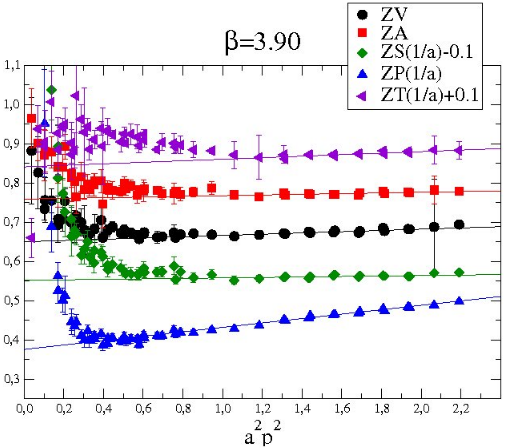


Figure: RCs with RI-MOM show linear residual $a^2 p^2$ dependency

B_k analysis

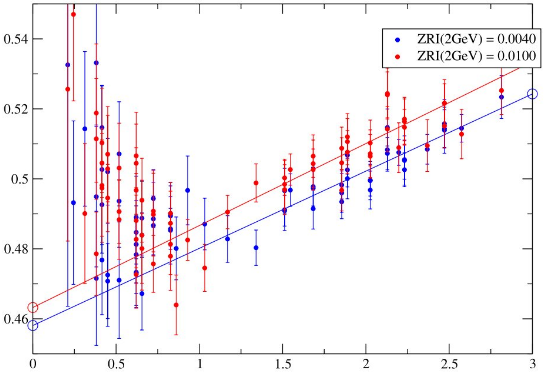
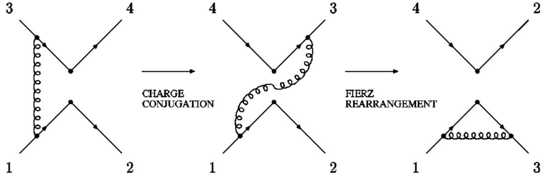


Figure: preliminary RI-MOM analysis seems to show linear residual a^2p^2 dependency in RCs

But, how?

how can we use 2 point functions to improve B_k?



We can relate the proper vertices of the four-point functions to the bilinears ones by doing:

1. charge conjugation
2. Fierz rearrangement

It is an interesting calculation, isn't it?

- 1-loop $\mathcal{O}(a^2)$ corrections to quark bilinears was never done
- Before this work:
  - Aoki et al; Capitani et al: 1-loop $\mathcal{O}(a)$ with $m \neq 0$
  - Panagopoulos+Skouroupathis: 2-loop $\mathcal{O}(a^0)$ with $m = 0$
- going to $\mathcal{O}(a^2)$ brings some technical difficulties:
  - new IR divergencies arise
  - non-Lorentz invariant terms appear:

$$\sum_\mu p_\mu^4/p^2\,,\ \sum_\mu \gamma_\mu p_\mu^3/p^2\,,\ldots$$

Framework

- we are interested in RCs at the chiral limit $\rightarrow$ we can set $m = 0$ and work with Wilson fermions
- improved actions for gluons (tlSym) $\rightarrow$ quite complicated expressions
- $r = 1$

Framework: diagrams

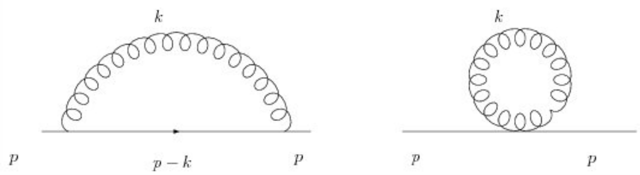


Figure: quark propagator diagrams

Framework: diagrams

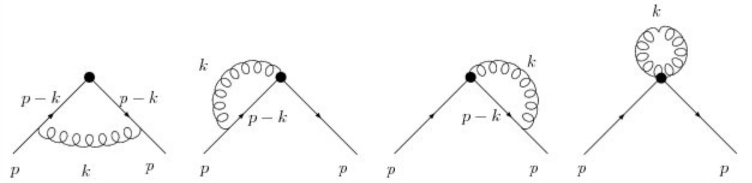


Figure: vertex diagrams

Procedure

1. Wick contractions
2. rescaling of integration variable:

$$\int_{-\pi/a}^{\pi/a} \frac{\mathrm{d}^d k}{(4\pi)^d}\ f(ak, ap) = \int_{-\pi}^{\pi} \frac{\mathrm{d}^d k}{(4\pi)^d}\ \frac{1}{a^d} f(k, ap)$$

3. Expanssion in *ap*
4. simplification of Dirac algebra (important: after expanssion, because some Dirac structure is hidden)
5. Isolation of divergences
6. Computation of finite parts

Dealing wiht UV divergences

We need to solve in the lattice the following integral, which is UV divergent:

$$I = \int \mathrm{d}k \, \mathcal{F}(k, p)$$

It could be difficult to handle as it is (due to external momentum).
But we can follow the Method introduced by Kawai et al.
What we do is to split it in two terms:

$$I = J + (I - J)$$

where J is the Taylor expansion in the external momentum of the original integral up to the needed order.
Then we integrate the two terms separately.

Dealing wiht UV divergences

What do we win?

- $J \rightarrow$ the integrals do not depend now on external momentum; they are much easier to calculate in the lattice
- $I - J \rightarrow$ contains external momentum, but because of the subtraction, it is UV finite. And (thanks to the theorem of Reisz) we can evaluate it doing a naive continuum limit.

These are good news, but there is one side effect, namely

- both J and $I - J$ are IR divergent (due to the somewhat unnatural splitting) $\Rightarrow$ we need to introduce an intermediate regularization.
- The new divergences must cancel between themselves, this is a useful check

Dealing with IR divergences

The general idea is to reduce the number of IR divergent integrals to a minimal set,
and, at the same time, express all primitive divergent integrals in terms of Wilson propagators

$$\frac{1}{\tilde{q}^2} = \frac{1}{\hat{q}^2} + \left\{ \frac{1}{\tilde{q}^2} - \frac{1}{\hat{q}^2} \right\}$$

with

$$\tilde{q}^2 = \sum_\mu \sin^2 q_\mu + \left(2r \sum_\mu \sin^2(q_\mu/2) \right)^2$$

and

$$\hat{q}^2 = 4 \sum_\mu \sin^2(q_\mu/2)$$

Computation of finite parts

1. obtain the value of integrals as a sum over the points of different lattices
2. extrapolate to infinite lattice size
3. extrapolation brings systematic errors; we need a method to have them under control

Status

- Cyprus (Panagopoulos et al): already preliminary results to $\mathcal{O}(a^2) \rightarrow$ see lattice talk by M. Constantinou
- we (Giménez, Lubicz, Palao):
  - $\mathcal{O}(a)$ already checked
  - still finishing $\mathcal{O}(a^2)$