Parity mixing in the TM spectrum

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Hadronic Spectrum - mixing

Let me illustrate this: in a simple mass mixing model of π_0 and σ
In the continuum we have two states π and σ which do not mix.
To order a, the L_5 term (odd in parity) will induce a mixing.
This will shift the masses of the two states as well as inducing off-diagonal correlators.
This can be encapsulated by using a mass mixing matrix.
The mass mixing matrix of bare (or continuum) states (π, σ) is

$$\begin{pmatrix} m_1 & \epsilon \\ \epsilon & m_2 \end{pmatrix} \quad (1)$$

where ϵ is given by $< \pi|L_5|\sigma >$ (up to some normalisation factor).
The mass shift is $-\epsilon^2/(m_2 - m_1)$ for the pion (π_0 rel π_+ where π_+ mixing is negligible)

Hadronic Spectrum - mixing II

The mixed (lattice) states are approx $p \equiv \pi + \eta\sigma$ and $s \equiv \sigma - \eta\pi$
with $\eta = \epsilon/(m_2 - m_1)$, $\Delta = m_2 - m_1$
The two state lattice fit with states p and s has amplitudes:

$$\begin{aligned} c_5 &=< 0|P|p >=< 0|P|\pi > & c_1 &=< 0|S|p >= \eta < 0|S|\sigma > \\ d_5 &=< 0|P|s >= -\eta < 0|P|\pi > & d_1 &=< 0|S|s >=< 0|S|\sigma > \end{aligned} \quad (2)$$

So one can get η from $\eta^2 = -(c_1/c_5) * (d_5/d_1)$
Here S and P refer to "physical basis" for clarity - (lattice results have "lattice basis" with operators S, P interchanged, Here I assume that we are at maximal twist - so that S and P are exchanged completely.) The lattice correlators are vacuum subtracted ($< AB > - < A >< B >$) but include fermionic disconnected loops as required.
(You can also use fuzzed operators to check this - namely $\eta^2 = -(c_2/c_6)(d_6/d_2)$ getting a consistent result)
Also there is a cross-check that (in this two state mixing approach)
$c1 * c5 = -d1 * d5$

Hadronic Spectrum - mixing III

From the fit (eg. μ=0.004 β=3.9), one can estmate η^2 with bootstrap method getting 0.029(19)

Then with $\Delta = m(s) - m(p) = 0.131(25)$, one has expected mass shift of π_0 (downwards) rel π_+ due to L_5 induced mixing of $ma = \eta^2 \Delta = 0.0038(25)$ compared to observed shift of $\pi_+ - \pi_0$=0.027.

($\mu = 0.003$, $\beta = 4.05$ $\eta^2 = 0.046(17)$ so rel mass shift not smaller - but big errors)

Therefore L_6 must be important compared to $L_5.L_5$ (or my simple 2 state fit and 2 state model is over-naive).

Mixing heavy-light mesons

$J^P = 0^-$ and 0^+ static-light mesons

$\mu = 0.004$, $\beta = 3.9$, levels Ea 0.171 apart

Fit agrees with model and gives mixing parameter $\eta^2 = 0.082$.

Then $\Delta = 0.171$ so expected mass shift $\eta^2 \Delta = 0.014$, so 8% effect.

More tricky here since "physical states" need Z's to evaluate them since mixing from lattice states is at 45^0. Qualitatively situation is as above.

To be checked at $\beta = 4.05$.

Mixing parity

Similar effects occur all over (in TMQCD)

$\rho \; a_1$

$D(0^-) \; D(0^+)$

$n(\frac{1}{2}^+) \; n(\frac{1}{2}^-)$

Essential to measure full correlator (including "parity-violating" part) to get at mixing and estimate possible effect on spectrum.

Also important to control effects of ground state contamination in study of excited (opposite parity) state.