Remarks on $N_f = 2 + 1 + 1$ LQCD at maximal twist

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Action for 2+1+1 MtmLQCD

$S^{\rm latt} = S_{YM} + a^4 \sum_x [L^\ell_{Mtm} + L^h_{\rm Mtm}]$

with S_{YM} a suitable (chosen: Iwasaki) gluonic action and

a) $N_f = 2$ light degenerate flavours ($\psi_\ell = (u,d)^t$) with $\gamma_5\tau^3$–twist

$$L^l_{Mtm} = \bar\psi_\ell \, [\gamma \cdot \widetilde\nabla \; - \; i\gamma_5\tau^3(W + m_0) \; + \; \mu_l \,] \, \psi_\ell \, , \qquad W \equiv -r(a/2)\nabla^* \cdot \nabla$$

b) $N_f = 1 + 1$ flavours ($\psi_h = (c,s)^t$) with $\gamma_5\tau^1$–twist

$$L^h_{\rm Mtm} = \bar\psi_h[\gamma \cdot \widetilde\nabla - i\gamma_5\tau^1(W + m_0) + \mu_h + \epsilon_h\tau^3]\psi_h$$

Mass parameter renormalization:

$m_0 = M_{\rm cr}$ for max. twist – $O(a)$ part of m_0 tunable to reduce $O(a^2)$ artifacts

$\hat m_{ud} = Z_P^{-1}\mu_\ell$

$\hat m_s = Z_P^{-1}(\mu_h - Z_P Z_S^{-1}\epsilon_h) \, , \qquad \hat m_c = Z_P^{-1}(\mu_h + Z_P Z_S^{-1}\epsilon_h)$

Gauge links in $L^{\ell,h}_{\rm Mtm}$ can be replaced with stout ones without affecting symmetries and renormalization pattern...

Symanzik LEL and optimal M_{cr} – I

Symanzix expansion of lattice operator vev's in terms of continuum ones

$$\langle O\rangle^{\mathrm{MtmL}} = \langle O\rangle^{\mathrm{cont}} - a\int d^4y\langle O\mathcal{L}_5^{Mtm}(y)\rangle^{\mathrm{cont}} + a\langle\Delta_1 O\rangle^{\mathrm{cont}} + a^2\langle\Delta_1 O\rangle^{\mathrm{cont}} +$$
$$+\frac{a^2}{2}\int d^4y d^4z\langle O\mathcal{L}_5^{Mtm}(y)\mathcal{L}_5^{Mtm}(z)\rangle^{\mathrm{cont}} - a^2\int d^4y\langle O\mathcal{L}_6^{Mtm}(y)\rangle^{\mathrm{cont}} + \dots$$

with matched renorm. cond. on the two sides above and

$$\mathcal{L}_5^{Mtm} = \mathcal{L}_5^{\ell} + \mathcal{L}_5^{h} \quad ,$$
$$\mathcal{L}_5^{\ell} = \eta_{1\mathrm{SW}}\bar\psi_\ell\sigma\cdot F\gamma_5\tau^3\psi_\ell + (\eta^\ell_{1a}\mu_\ell^2 + \eta^\ell_{1b}\mu_h^2 + \eta^\ell_{1c}\epsilon_h^2)\bar\psi_\ell i\gamma_5\tau^3\psi_\ell + \Lambda^2_{\mathrm{QCD}}\delta_1\bar\psi_\ell i\gamma_5\tau^3\psi_\ell,$$
$$\mathcal{L}_5^{h} = \eta_{1\mathrm{SW}}\bar\psi_h\sigma\cdot F\gamma_5\tau^1\psi_h + (\eta^h_{1a}\mu_\ell^2 + \eta^h_{1b}\mu_h^2 + \eta^h_{1c}\epsilon_h^2)\bar\psi_h i\gamma_5\tau^1\psi_h + \Lambda^2_{\mathrm{QCD}}\delta_1\bar\psi_h i\gamma_5\tau^1\psi_h$$

The structure of $\mathcal{L}_5^{Mtm}$ follows from (spurionic) symmetries (see JHEP 0410 (2004) 070)

$H(4)$, charge conjugation, $P\times\mathcal{D}_d\times(\mu_\ell\to-\mu_\ell)\times(\mu_h\to-\mu_h)\times(\epsilon_h\to-\epsilon_h)$
Vector$^{3\ell}$, Axial$^{2\ell}_{\pi/2}\times(\mu_\ell\to-\mu_\ell)$, Axial$^{1\ell}_{\pi/2}\times(\mu_\ell\to-\mu_\ell)$
Vector$^{1h}_{\pi/2}\times(\epsilon_h\to-\epsilon_h)$, Axial$^{2h}_{\pi/2}\times(\mu_h\to-\mu_h)$, Axial$^{3h}_{\pi/2}\times(\mu_h\to-\mu_h)\times(\epsilon_h\to-\epsilon_h)$

Terms containing Λ^2_{QCD} are needed to describe N.P. O(a) artifacts in the determination of M_{cr} (its value is part of the specification of $\langle\ \ \rangle^{\mathrm{MtmL}}$)

Symanzik LEL and optimal $M_{\rm cr}$ – II

Generic estimate of $M_{\rm cr}$: the Symanzik expansion of $\langle O \rangle^{\rm MtmL}$ contains

powers of $[\xi_\pi m_\pi^{-2} \Lambda_{\rm QCD}^{-1}]_{\mu_\ell,(\mu_h,\epsilon_h)}$ with

$$\xi_\pi|_{\mu_\ell,(\mu_h,\epsilon_h)} \equiv \langle \Omega | a\mathcal{L}_5^{Mtm} | \pi \rangle|^{\rm cont}_{\mu_\ell,(\mu_h,\epsilon_h)} = \langle \Omega | a\mathcal{L}_5^{\ell} | \pi \rangle|^{\rm cont}_{\mu_\ell,(\mu_h,\epsilon_h)}$$

- last equality above follows from vector$^{2\ell,1\ell}$-invariance of continuum theory;
- $\mathcal{L}_5^\ell = \eta_{\rm 1sw} \bar\psi_\ell \sigma \cdot F \gamma_5 \tau^3 \psi_\ell + (\eta^\ell_{1a} \mu_\ell^2 + \eta^\ell_{1b} \mu_h^2 + \eta^\ell_{1c} \epsilon_h^2) \bar\psi_\ell i \gamma_5 \tau^3 \psi_\ell + \Lambda^2_{\rm QCD} \delta_1 \bar\psi_\ell i \gamma_5 \tau^3 \psi_\ell$

As in the $N_f = 2$ theory (see Frezzotti-Martinelli-Papinutto-Rossi'05), also here

$$C^{\ell\ell}_{VP}(x_0) \equiv a^3 \sum_{\vec x} \langle V_0^{2\ell}(x) P^{1\ell}(0) \rangle^{\rm MtmL} = 0 \quad \Rightarrow \quad \xi_\pi|_{\mu_\ell,(\mu_h,\epsilon_h)} = {\rm O}(a\mu_\ell)\Lambda^3_{\rm QCD} + \ldots$$

Define the optimal critical mass $m_0 = M^{\rm opt}_{\rm cr}[\mu_\ell, (\mu_h, \epsilon_h)]$ by

$|m_{\rm PCAC}| = |\partial_{x_0} C^{\ell\ell}_{VP}(x_0) / C^{\ell\ell}_{PP}(x_0)|_{m_0,\mu_\ell,(\mu_h,\epsilon_h)} = 0$ (in practice $\ll \mu_\ell$)

$M^{\rm opt}_{\rm cr}$ (in the sense of FMPR'05 – and also of χPT: Aoki-Bär'04 and Sharpe-Wu'04) depends significantly on (μ_h, ϵ_h) – however the mild μ_ℓ–dependence is treated

m_0-value for evaluation of Z's

For practical reasons: mass-independent renormalization schemes (RIMOM, SF)

$\Rightarrow$ must work at small $\to$ zero mass parameters, e.g. with
$[\mu_\ell, (\mu_h, \epsilon_h)] = [\mu, (\mu, 0)] \to [0, (0, 0)]$ and $m_0 \to M_{\rm cr}$ (generic).

How to choose m_0 if the Z's "feel" χSSB-enhanced (e.g. $\sim a/m_\pi^2$) cutoff effects?

A) work at $[\mu, (\mu, 0)]$, μ small and $m_0 = M_{\rm cr}^{\rm opt}[\mu_\ell, (\mu_h, \epsilon_h)]$ $\Rightarrow$

$$\xi_\pi|^{m_0}_{\mu,(\mu,0)} = {\rm O}(a\mu_h^2, a\epsilon_h^2)\Lambda^2_{\rm QCD} + {\rm O}(a\mu_\ell)\Lambda^3_{\rm QCD} \quad \gg \quad {\rm O}(a\mu)\Lambda^3_{\rm QCD}$$

m_0 already known, $m_{\rm PCAC}|^{m_0}_{\mu,(\mu,0)} \neq 0$: any χSSB-enhanced cutoff effects in Z's?

B) work at $[\mu, (\mu, 0)]$, μ small and $m_0' = M_{\rm cr}^{\rm opt}[\mu, (\mu, 0)]$ $\Rightarrow$

$$\xi_\pi|^{m_0}_{\mu,(\mu,0)} = {\rm O}(a\mu)\Lambda^3_{\rm QCD} \quad \text{by construction,}$$

extra tuning to find m_0': $m_{\rm PCAC}|^{m_0'}_{\mu,(\mu,0)} \simeq 0$! Can show: $m_0' - m_0 \sim a$

Exploit suppression of χSSB-enhanced artifacts at small L (SF) or large q (RIMOM)

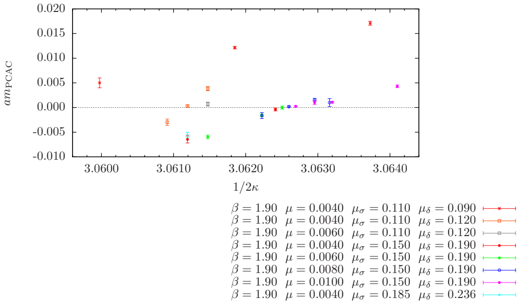
Choice A): m_{PCAC} at $[\mu_\ell, (\mu_h, \epsilon_h)]$ and $[\mu, (\mu, 0)]$

Theory & data at $\beta = 1.9 \quad \rightarrow \quad \Delta(aM_{\mathrm{cr}}^{\mathrm{opt}}) \sim 0.40\Delta(a^2\epsilon_h^2) - 0.71\Delta(a^2\mu_h^2)$

With choice $m_0 = M_{\mathrm{cr}}^{\mathrm{opt}}[\mu_\ell, (\mu_h, \epsilon_h)]$: expect $\Delta(aM_{\mathrm{cr}}^{\mathrm{opt}}) \simeq -0.0015$

i.e. $am_{\mathrm{PCAC}}|^{m_0}_{\sim 0,(\sim 0,0)} \simeq 0.0075$ or (bare) $m_{\mathrm{PCAC}}|^{m_0}_{\sim 0,(\sim 0,0)} \simeq 16$ MeV

To get an idea of χSSB vs FS effects at $L = 1$ fm: $(L^2 2Bm_{\mathrm{PCAC}})^{1/2} \simeq \sqrt{2}$



det($D^h_{\rm Mtm}$) and the case of $\mu_h^2 < \epsilon_h^2$

$\mu_h^2 > \epsilon_h^2 \quad \Rightarrow \quad$ det($D^h_{\rm Mtm}$) > 0 on each gauge configuration, for any V.

$\mu_h^2 < \epsilon_h^2$: det($D^h_{\rm Mtm}$) has fixed sign on the relevant U-ensembles, as we find

- median of the spectral gap of $Q_h \equiv \gamma_5 \tau^3 D^h_{\rm Mtm} = Q_h^\dagger$ (though EO-precond.) well > 0 and nearly proportional to $\mu_h - Z_P Z_S^{-1} \epsilon_h = Z_P \hat{m}_s$ (Gregorio's plot)
- width of the spectral gap of Q_h seems to decrease with $1/\sqrt{V}$ (G.'s plots)

This makes plausible the hypothesis that in infinite volume Q_h^2 has zero (positive) eigenvalue density for $\alpha < \bar{\alpha}$ ($\alpha = \bar{\alpha}$) – at least for sufficiently small a

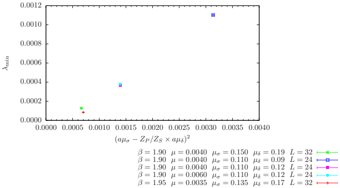
Under this hypothesis and in infinite volume: $\bar{\alpha} = (\mu_h - Z_P Z_S^{-1} \epsilon_h)^2 + {\rm O}(a^2)$

renormalization of eigenvalue density (and gap) understood and related to that of quark mass(es), up to O(a^2) $\Leftrightarrow$ consistent with G.'s plot

Proof parallels that in hep-lat/0512021 (Del Debbio *et al.*) (γ_5-hermiticity of D_m and individual valence quarks there $\rightarrow$ $\gamma_5 \tau^3$-hermiticity of $D^h_{\rm Mtm}$ and pairs of valence quarks here)

The case of $\mu_h^2 < \epsilon_h^2$ on finite lattices

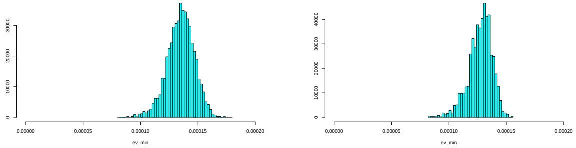
Spectral gap of Q_h^2 vs $(\mu_h - Z_P Z_S^{-1} \epsilon_h)^2$ (but EO-preconditioned)



Hypothesis on the spectral density: true in the continuum; can and should be checked (for the unpreconditioned Q_h) on our lattices (large V, small a)

The case of $\mu_h^2 < \epsilon_h^2$ on finite lattices – cont'd

$\beta = 1.9$, $(a\mu_h, a\epsilon_h) = (0.15, 0.19)$: distribution of Q_h^2 (but EO-preconditioned)



$L/a = 24$, $a\mu_\ell = 0.006$ and $L/a = 32$, $a\mu_\ell = 0.004$

Width of the Q_h^2 spectral gap distribution: check whether it scales with $a/\sqrt{V}$

Philosophy: if in our small a, large V simulations, by numerical checks of this type, we find good evidence that $\det(D^h_{\rm Mtm})$ has constant sign on the relevant (for PHMC importance sampling) U-ensembles, then we also understand why: because we work close enough to the continuum and $V = \infty$ limits. $\Rightarrow$ self-consistent picture supporting the safety of our MC simulations at $\epsilon_h^2 > \mu_h^2$.

Unitary vs. mixed action setup

Possibility of using OS fermions for the valence strange and charm quarks: avoid s-c mixings, retain unitarity in the continuum limit.

Easy matching of s and c bare masses (requires knowledge of Z_P/Z_S):

$m_s = (\mu_h - Z_P/Z_S\epsilon_h)$ and $m_c = (mu_h + Z_P/Z_S\epsilon_h)$

No anomalously large cutoff effects seen in $N_f = 2$ observables (other than m_π^{OS}): see e.g. B_K. Indeed no such problems expected for masses and matrix elements involving only one-particle states.