The $1/2$ versus $3/2$ puzzle

ETM collaboration meeting, Glasgow

Marc Wagner

Humboldt University Berlin

mcwagner@physik.hu-berlin.de

http://people.physik.hu-berlin.de/~mcwagner/

September 29, 2008

Outline

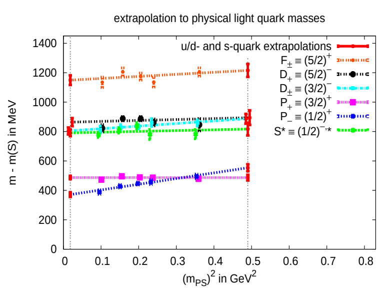
- Heavy-light mesons.
- $1/2$ versus $3/2$ puzzle:
  - Experimental side.
  - Theory side.
  - Possible explanations to resolve the puzzle.
- Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$.
- Conclusions.

Heavy-light mesons

- Heavy-light meson: a meson made from a heavy quark (b, c) and a light quark (u, d, s), e.g. $B = \{\bar{b}u\,,\ \bar{b}d\}$, $B_s = \bar{b}s$, $D = \{\bar{c}u\,,\ \bar{c}d\}$, $D_s = \bar{c}s$.
- Static limit, i.e. $m_b, m_c \rightarrow \infty$:
  - No interactions involving the static quark spin.
  - Classify states according to parity $\mathcal{P}$ and total angular momentum of the light cloud j.
- m_b, m_c finite, but heavy:
  - Classify states according to parity $\mathcal{P}$ and total angular momentum J.

$j^{\mathcal{P}}$	$J^{\mathcal{P}}$
$(1/2)^- \equiv S$	$0^- \equiv H$ $1^- \equiv H^*$
$(1/2)^+ \equiv P_-$	$0^+ \equiv H_0^* \equiv H_0^{1/2}$ $1^+ \equiv H_1^* \equiv H_1^{1/2}$
$(3/2)^+ \equiv P_+$	$1^+ \equiv H_1 \equiv H_1^{3/2}$ $2^+ \equiv H_2^* \equiv H_2^{3/2}$

static-light mass differences as functions of $(m_{\mathrm{PS}})^2$



$1/2$ versus $3/2$: experimental side

- Consider the semileptonic decay $B \to X_c \, l \, \nu$.
- Experiments, which have studied this decay: ALEPH, BaBar, BELLE, CDF, DELPHI, DØ.
- What is X_c?
  - $\approx 75\%$ D and D^*, i.e. S wave states (agreement with theory).
  - $\approx 10\%$ $D_1^{3/2}$ and $D_2^{3/2}$, i.e. $j = 3/2$ P wave states (agreement with theory).
  - For the remaining $\approx 15\%$ the situation is not clear:
    * A natural candidate would be $D_0^{1/2}$ and $D_1^{1/2}$, i.e. $j = 1/2$ P wave states.
    * This would imply $\Gamma(B \to D_{0,1}^{1/2} \, l \, \nu) > \Gamma(B \to D_{1,2}^{3/2} \, l \, \nu)$, which is in "conflict" with theory.
    * This "conflict" between experiment and theory is called the "$1/2$ versus $3/2$ puzzle".

$1/2$ versus $3/2$: theory side (1)

- Static limit ($m_b, m_c \rightarrow \infty$) with both b and c quark at rest:

$$\begin{aligned}
\langle D_0^{1/2} | \bar{c} \gamma_5 \gamma_j D_k b | B \rangle &= -i g_{jk} \Big(m(D_0^{1/2}) - m(B) \Big) \tau_{1/2} \\
\langle D_2^{3/2} | \bar{c} \gamma_5 \gamma_j D_k b | B \rangle &= +i \sqrt{3} \epsilon_{jk} \Big(m(D_2^{3/2}) - m(B) \Big) \tau_{3/2}
\end{aligned}$$

and

$$\frac{\Gamma(B \rightarrow D_{0,1}^{1/2} \, l \, \nu)}{\Gamma(B \rightarrow D_{1,2}^{2/2} \, l \, \nu)} = \frac{|\tau_{1/2}|^2}{|\tau_{3/2}|^2}.$$

($\tau_{1/2}$, $\tau_{3/2}$: Isgur-Wise form factors).

$1/2$ versus $3/2$: theory side (2)

- Phenomenological models:
  - $|\tau_{1/2}| < |\tau_{3/2}|$, which is in "conflict" with experiment.
- OPE:
  - Uraltsev sum rule:

    $$\sum_n |\tau_{3/2}^{(n)}|^2 - |\tau_{1/2}^{(n)}|^2 \quad = \quad \frac{1}{4}$$

    ($\tau_{1/2} \equiv \tau_{1/2}^{(0)}$ and $\tau_{3/2} \equiv \tau_{3/2}^{(0)}$).
  - From experience with sum rules one would expect approximate saturation from the ground states, i.e.

    $$|\tau_{3/2}^{(0)}|^2 - |\tau_{1/2}^{(0)}|^2 \quad \approx \quad \frac{1}{4},$$

    which also implies $|\tau_{1/2}| < |\tau_{3/2}|$, which is in "conflict" with experiment.

$1/2$ versus $3/2$: possible explanations

- **Experiment:**
  - The signal for the remaining 15% of X_c is rather vague; therefore, only a small part might be $D_{0,1}^{1/2}$.
- **Phenomenological models:**
  - Models might give a wrong answer.
- **OPE:**
  - Sum rules hold in the static limit and might change significantly for finite quark masses.
  - Sum rules might not be saturated by the ground states.
- A lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ could shed some light on this puzzle.

Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (1)

- Simulation setup:
  - $N_f = 2$, $24^3 \times 48$ lattice, $\beta = 3.9$, $\mu = 0.0040$ (i.e. $m_{\mathrm{PS}} \approx 300\,\mathrm{MeV}$).
  - Static-light meson trial states:
    * $|\tilde{S}\rangle = \bar{Q}\gamma_5\psi|\Omega\rangle$: trial state for $|B\rangle$.
    * $|\tilde{P}_-\rangle = \bar{Q}\psi|\Omega\rangle$: trial state for $|D_0^{1/2}\rangle$.
    * $|\tilde{P}_+\rangle = \bar{Q}(\gamma_x x - \gamma_y y)\psi|\Omega\rangle$: trial state for $|D_2^{3/2}\rangle$.
    * Gaussian smeared light quark operators ($N_{\mathrm{Gauss}} = 30$, $\kappa_{\mathrm{Gauss}} = 0.5$) with APE smeared spatial links ($N_{\mathrm{APE}} = 10$, $\alpha_{\mathrm{APE}} = 0.5$).
    * Separation between static antiquark and light quark operators: $r = 3$.
  - HYP2 static action.
  - Preliminary results with ≈ 100 gauge configurations.

Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (2)

- "Effective form factors",

$$\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2) \quad = \quad = \left|\frac{N(\tilde{P}_-) \;\; N(\tilde{S}) \;\; \langle \tilde{P}_-(T_0)|(\bar{Q}\gamma_5\gamma_3 D_3 Q)(T_1)|\tilde{S}(T_2)\rangle}{(m(\tilde{P}_-) - m(\tilde{S})) \;\; \langle \tilde{P}_-(T_0)|\tilde{P}_-(T_1)\rangle \;\; \langle \tilde{S}(T_1)|\tilde{S}(T_2)\rangle}\right|$$

$$\tau_{3/2,\text{effective}}(T_0 - T_1, T_1 - T_2) \quad = \quad = \sqrt{\frac{1}{6}} \left|\frac{N(\tilde{P}_+) \;\; N(\tilde{S}) \;\; \langle \tilde{P}_+(T_0)|(\bar{Q}\gamma_5(\gamma_1 D_1 - \gamma_2 D_2)Q)(T_1)|\tilde{S}(T_2)\rangle}{(m(\tilde{P}_+) - m(\tilde{S})) \;\; \langle \tilde{P}_+(T_0)|\tilde{P}_+(T_1)\rangle \;\; \langle \tilde{S}(T_1)|\tilde{S}(T_2)\rangle}\right| :$$

  – $N(X)$: norm of state $|X\rangle$.

  – $m(X)$: mass of state $|X\rangle$.

  – Three-point functions (T_0, T_1 and T_2).

  – Two-point functions (T_0 and T_1 or T_1 and T_2).

- $\tau_{1/2} \quad = \quad \lim_{T_0 - T_1 \, , T_1 - T_2 \rightarrow \infty} \tau_{1/2,\text{effective}} \quad , \quad \tau_{3/2} \quad = \quad \lim_{T_0 - T_1 \, , T_1 - T_2 \rightarrow \infty} \tau_{3/2,\text{effective}}.$

Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (3)

- Static-light meson trial states in the physical basis:

$$|\tilde{S}\rangle \;=\; \bar{Q}\gamma_5\chi|\Omega\rangle \quad , \quad |\tilde{P}_-\rangle \;=\; \bar{Q}\chi|\Omega\rangle .$$

- Twist rotation:

$$|\tilde{S}\rangle \;=\; \frac{1}{\sqrt{2}}\Big(Z(\gamma_5)|S\rangle + iZ(1)|P_-\rangle\Big)$$

$$|\tilde{P}_-\rangle \;=\; \frac{1}{\sqrt{2}}\Big(Z(1)|P_-\rangle + iZ(\gamma_5)|S\rangle\Big).$$

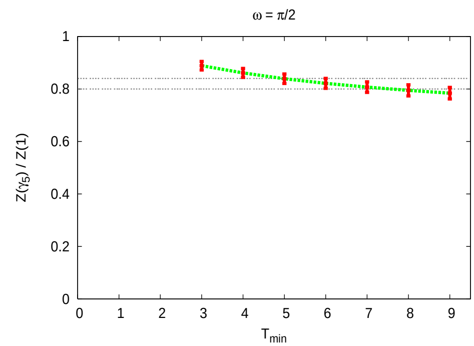
- Determine $Z(\gamma_5)/Z(1)$ by requiring

$$\langle\tilde{P}_-(T)|\tilde{S}(0)\rangle \;=\; 0$$

(cf. my "Trento talk" about extracting K and D meson masses).

- Analogously for $|P_+\rangle$ and $|D_-\rangle$.

$Z(\gamma_5)/Z(1)$ as a function of T_{min}



Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (4)

- $\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2) \quad =$

$$= \left| \frac{N(\tilde{P}_-) \ \ N(\tilde{S}) \ \ \langle \tilde{P}_-(T_0) | (\bar{Q}\gamma_5\gamma_3 D_3 Q)(T_1) | \tilde{S}(T_2) \rangle}{(m(\tilde{P}_-) - m(\tilde{S})) \ \ \langle \tilde{P}_-(T_0) | \tilde{P}_-(T_1) \rangle \ \ \langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle} \right| \quad , \quad \ldots$$

- Two-point functions:

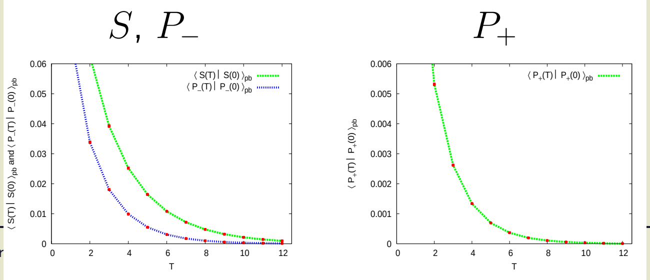
$$\langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle \quad = \quad \frac{1}{2}\Big(Z(\gamma_5)^2 \langle S(T_1) | S(T_2) \rangle + Z(1)^2 \langle P_-(T_1) | P_-(T_2) \rangle$$
$$+ i Z(\gamma_5) Z(1) \Big(\langle S(T_1) | P_-(T_2) \rangle - \langle P_-(T_1) | S(T_2) \rangle \Big) \Big).$$

- Determine the norm of $|\tilde{S}\rangle$, $N(\tilde{S})$, by performing a χ^2 minimizing fit with

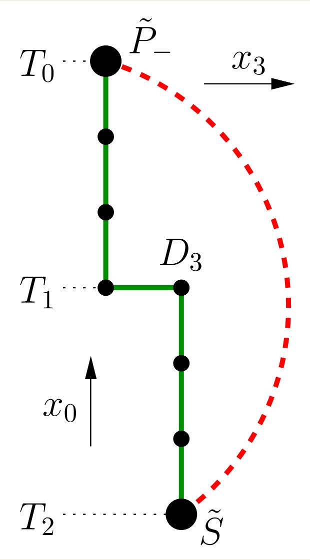
$$f(T) \quad = \quad N(\tilde{S})^2 e^{-m(S)T}$$

to $\langle \tilde{S}(T) | \tilde{S}(0) \rangle$.

- Analogously for the others.



Lattice computation of $\tau_{1/2}$ ...



- $\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2) \quad =$

$$= \left| \frac{N(\tilde{P}_-) \quad N(\tilde{S}) \quad \langle \tilde{P}_-(T_0) | (\bar{Q}\gamma_5\gamma_3 D_3 Q)(T_1) | \tilde{S}(T_2) \rangle}{(m(\tilde{P}_-) - m(\tilde{S})) \quad \langle \tilde{P}_-(T_0) | \tilde{P}_-(T_1) \rangle \quad \langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle} \right|$$

- Three-point functions:

$$\langle \tilde{P}_-(T_0) | (\bar{Q}\gamma_5\gamma_3 D_3 Q)(T_1) | \tilde{S}(T_2) \rangle \quad =$$
$$= \quad \frac{1}{2}\Big(Z(\gamma_5)Z(1)\Big(\langle P_-(T_0)| \ldots |S(T_2)\rangle + \langle S(T_0)| \ldots |P_-(T_2)\rangle\Big)$$
$$+i\Big(Z(1)^2\langle P_-(T_0)| \ldots |P_-(T_2)\rangle - Z(\gamma_5)^2\langle S(T_0)| \ldots |S(T_2)\rangle\Big)\Big).$$

- Analogously for the other three-point functions.

- Mass differences $m(P_-) - m(S)$ and $m(P_+) - m(S)$ from the "ETMC static-light spectrum paper".

Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (6)

- $\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2)$ and $\tau_{3/2,\text{effective}}(T_0 - T_1, T_1 - T_2)$ exhibit nice plateaus due to "optimized" trial states $|\tilde{S}\rangle$, $|\tilde{P}_-\rangle$ and $|\tilde{P}_+\rangle$.

- $T_0 - T_2 = 8$:
  - $\tau_{1/2} = 0.32$, $\tau_{3/2} = 0.47$.
  - $(\tau_{3/2})^2 - (\tau_{1/2})^2 = 0.11$.

- $T_0 - T_2 = 10$:
  - $\tau_{1/2} = 0.29$, $\tau_{3/2} = 0.54$.
  - $(\tau_{3/2})^2 - (\tau_{1/2})^2 = 0.20$.

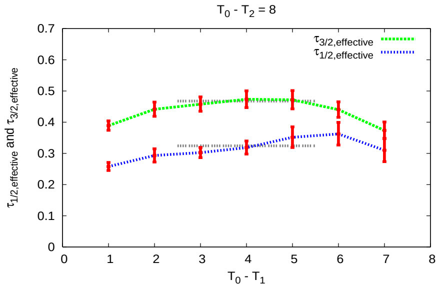
- $\tau_{3/2} > \tau_{1/2}$, i.e. theoretical expectation confirmed.

- "Consistent" with Uraltsev sum rule:

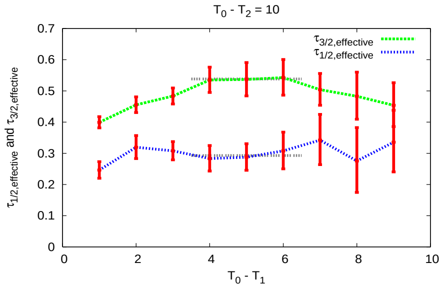
$$\sum_n |\tau_{3/2}^{(n)}|^2 - |\tau_{1/2}^{(n)}|^2 \quad = \quad \frac{1}{4}.$$

$\tau_{1/2}$ and $\tau_{3/2}$ as functions of $T_0 - T_1$

$T_0 - T_2 = 8$

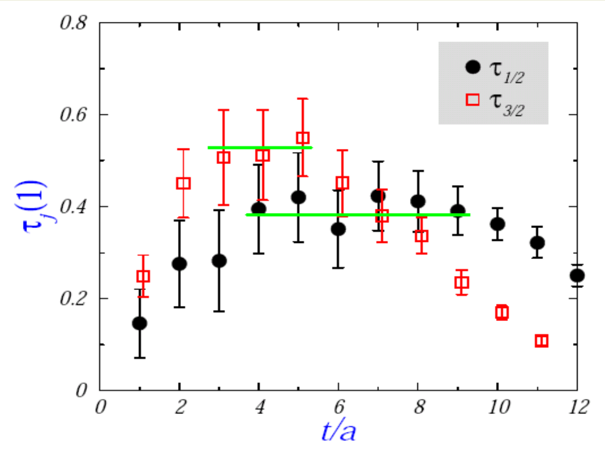


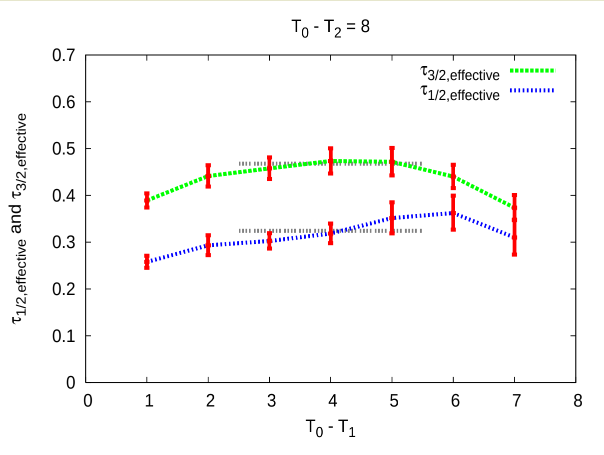
$T_0 - T_2 = 10$



Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (7)

- Comparison with the only existing lattice study (quenched, exploratory):
  - D. Becirevic *et al.*, "Lattice measurement of the Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$," Phys. Lett. B **609**, 298 (2005) [arXiv:hep-lat/0406031].
  - $16^3 \times 40$ lattice, $m_{\mathrm{sea}} = \infty$, $m_{\mathrm{PS}} = 800\,\mathrm{MeV}$.
  - $\tau_{1/2} = 0.38(4)$, $\tau_{3/2} = 0.53(8)$.





Conclusions

- $\tau_{1/2}$ and $\tau_{3/2}$ have been computed on dynamical ETMC gauge field configurations.
- Preliminary results indicate that in the static limit

  $$\Gamma(B \rightarrow D^{1/2}_{0,1} \, l \, \nu) \quad < \quad \Gamma(B \rightarrow D^{3/2}_{1,2} \, l \, \nu)$$

  (as expected from OPE and phenomenological models).
- "To do list":
  - Improve statistics.
  - Consider different light quark masses to extrapolate to u/d masses.
  - Perform the continuum limit.
  - Compute HQET $1/m_Q$ corrections.

I would like to thank Benoit Blossier, Vladimir Galkin and Olivier Pène for valuable discussions.