Nucleon form factors

C. Alexandrou, T. Korzec, G. Koutsou
University of Cyprus

- definitions
- lattice techniques
- first results

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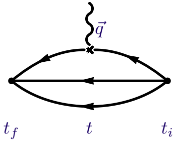
Matrix elements

We are interested in QCD matrix elements

$$\langle h'(p',s')|X|h(p,s)\rangle$$



where

- $h'(p',s')$, $h(p,s)$ are hadron states with initial (final) momentum $p(p')$ and spin $s(s')$
  - nucleon
  - Δ-baryon (later)
- X is a current or density
  - Electromagnetic current $V_\mu^{EM}(x) = \frac{2}{3}\bar{u}(x)\gamma_\mu u(x) - \frac{1}{3}\bar{d}(x)\gamma_\mu d(x)$
  - Axial current $A_\mu^a(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x)$
  - Pseudoscalar density $P^a(x) = \bar{\psi}(x)\gamma_5\frac{\tau^a}{2}\psi(x)$

Electromagnetic nucleon form factors

The electromagnetic matrix element of the nucleon can be expressed in terms of two form factors.

$$\begin{aligned}
\langle N(p',s')|V_\mu(0)|N(p,s)\rangle &= \sqrt{\frac{m_N^2}{E_{N(\vec{p}\,')}E_{N(\vec{p})}}}\,\bar{u}(p',s')\mathcal{O}_\mu u(p,s) \\
\mathcal{O}_\mu &= \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(q^2)
\end{aligned}$$

$q = p' - p$ is the momentum transfer
F_1, F_2 are the Dirac form factors.

$$\begin{aligned}
G_E(q^2) &= F_1(q^2) + \frac{q^2}{(2m_N)^2}F_2(q^2) \\
G_M(q^2) &= F_1(q^2) + F_2(q^2)
\end{aligned}$$

G_E, G_M are the electric and magnetic Sachs form factors.

Axial nucleon form factors

The axial current matrix element of the nucleon can be expressed in terms of the form factors G_A and G_p.

$$\begin{aligned}
\langle N(p',s')|A^a_\mu(0)|N(p,s)\rangle &= i\sqrt{\frac{m_N^2}{E_{N(\vec{p}')}E_{N(\vec{p})}}}\bar{u}(p',s')\mathcal{O}_\mu u(p,s) \\
\mathcal{O}_\mu &= \left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu}{2m_N}G_p(q^2)\right]\frac{\tau^a}{2}
\end{aligned}$$

Twisted mass QCD

Nucleon interpolating fields

Interpolating fields χ such that

$$\begin{aligned} \langle \chi_\alpha(0) | N(k,s) \rangle &= \sqrt{m_N/E_N}\, Z\, u_\alpha(k,s) \\ \langle N(k,s) | \bar{\chi}_\alpha(0) \rangle &= \sqrt{m_N/E_N}\, Z^*\, \bar{u}_\alpha(k,s) \end{aligned}$$

At maximal twist:

- $\chi^p = \sqrt{1/2}\,[\mathbb{1} + i\gamma_5]\,\chi^{p\prime}$ $\qquad \bar{\chi}^p = \bar{\chi}^{p\prime}\,\sqrt{1/2}\,[\mathbb{1} + i\gamma_5]$
- $\chi^n = \sqrt{1/2}\,[\mathbb{1} - i\gamma_5]\,\chi^{n\prime}$ $\qquad \bar{\chi}^n = \bar{\chi}^{n\prime}\,\sqrt{1/2}\,[\mathbb{1} - i\gamma_5]$

Operators

- $V_\mu^{0,3} = V'^{0,3}_\mu$, use Noether currents $\to Z_V = 1$
  $\Rightarrow$ Electromagnetic form factors
- $A_\mu^3 = A'^3_\mu$, use local current, need Z_A
  $\Rightarrow$ Axial form factors

Three-point functions

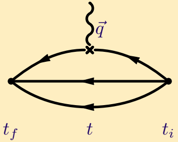
Measure three-point functions

$$G^{\mu}(\Gamma^{\nu},\vec{q},t)=\sum_{\vec{x},\vec{x}_f} e^{i\vec{x}\cdot\vec{q}}\,\Gamma^{\nu}_{\beta\alpha}\,\langle\chi_\alpha(t_f,\vec{x}_f)X^{\mu}(t,\vec{x})\bar{\chi}_\beta(0)\rangle$$

$\Gamma^k=\frac{i}{4}(\mathbb{1}+\gamma_4)\gamma_5\gamma_k$ and $\Gamma^4=\frac{1}{4}(\mathbb{1}+\gamma_4)$
$\chi,\bar{\chi}$: proton interpolating fields built from *smeared* quark-fields.

Kinematical setup

- Source at $t_i=0$, $\vec{x}=0$
- Sink at $t=t_f$, $\vec{p}_f=0$
- Operator X at t, $\vec{q}=-\vec{p}_i$



Reduced ratios

Leading time dependence and state normalizations cancel in ratios:

$$R^\mu = \frac{G^\mu(\Gamma, \vec{q}, t)}{G(\Gamma^4, \vec{0}, t_f)} \sqrt{\frac{G(\Gamma^4, \vec{p}_i, t_f - t) G(\Gamma^4, \vec{0}, t) G(\Gamma^4, \vec{0}, t_f)}{G(\Gamma^4, \vec{0}, t_f - t) G(\Gamma^4, \vec{p}_i, t) G(\Gamma^4, \vec{p}_i, t_f)}} \to \Pi^\mu(\Gamma, \vec{q})$$

If the separation between t_f and t_i sufficient: R^μ has a plateau

Optimal combinations for EM form factors

- Electric Sachs form factor from
  $\Pi^\mu(\Gamma^4, \vec{q}) = \frac{c}{2m}[(m+E)\delta_{4,\mu} + \sum_k iq_k \delta_{k,\mu}]\ G_E(q^2)$
- Magnetic Sachs form factor from
  $\Pi^i(\Gamma^1, \vec{q}) + \Pi^i(\Gamma^2, \vec{q}) + \Pi^i(\Gamma^3, \vec{q}) = -\frac{c}{2m} \sum_{jkl} \epsilon_{jkl} q_j \delta_{l,i}\ G_M(q^2)$
- Axial form factors
  $\Pi^{5i}(\Gamma^1, \vec{q}) + \Pi^{5i}(\Gamma^2, \vec{q}) + \Pi^{5i}(\Gamma^3, \vec{q}) = \frac{ic}{4m}[(E+m)G_A - (q_1+q_2+q_3)\frac{q_i}{2m} G_p]$

$c = \sqrt{\frac{2m^2}{E(E+m)}}$

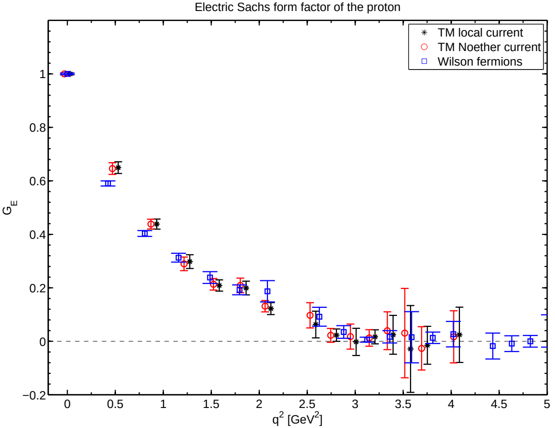
Sequential inversion

To calculate the connected diagrams, terms with $\bar{u}\gamma_\mu u$ and $\bar{d}\gamma_\mu d$ need separate treatment.

- construct "sequential sources" $\phi^a_\alpha(y) = \sum_{\vec{x}_f} \delta_{y_4,t_f} F^{ab}_{\alpha\beta}(\vec{y},\vec{x}_f) {\gamma_5}_{\beta\gamma} {\mathcal{M}^{bc}_{\gamma\delta}}^*$
  - F is the smearing function
  - c,δ fixed $\Rightarrow$ 12 sources
  - for current with d quarks:
    $\mathcal{M}^{cc'}_{\gamma\gamma'} = \epsilon^{abc}\epsilon^{a'b'c'}[C\gamma_5]_{\mu\gamma}[\overline{C\gamma_5}]_{\mu'\gamma'}\Gamma_{\beta\alpha}(\mathcal{U}^{aa'}_{\mu\mu'}\mathcal{U}^{bb'}_{\alpha\beta} + \mathcal{U}^{aa'}_{\mu\beta}\mathcal{U}^{bb'}_{\alpha\mu'})$
    where $\mathcal{U}$ is the smeared-smeared "up" propagator from x_i to x_f
- calculate "backward propagators" $\mathcal{B} = D^{-1}\phi$
  - "up" inversion for $\bar{d}\gamma_\mu d$ current
  - "down" inversion for $\bar{u}\gamma_\mu u$ current
- combine to three point function (here the $\bar{d}\gamma_\mu d$ case):
  $G = \sum_{\vec{x}} e^{i\vec{q}\cdot\vec{x}} {\gamma_5}_{\alpha\beta} {\mathcal{B}^{dc}_{\alpha\gamma}}^*(x)\gamma^\mu_{\beta\delta}\mathcal{D}^{dc}_{\delta\gamma}(x,x_i)$
  where $\mathcal{D}$ is the local-smeared "down" propagator

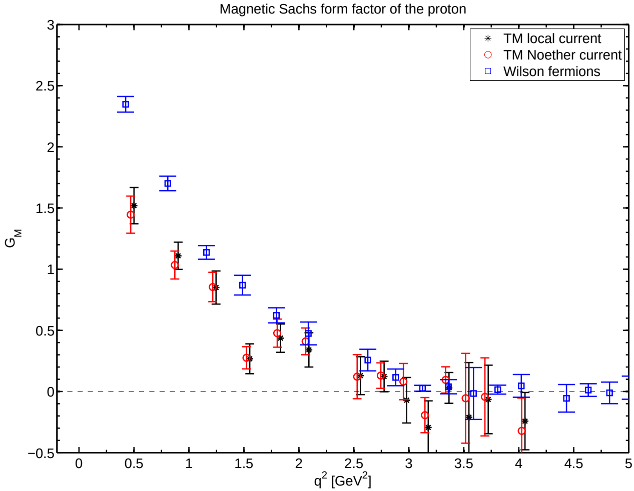
Electric Sachs form factor

- Connencted parts only
- 96 configurations
- $\beta = 3.9$
  $\mu = 0.0085$
  $L = 24$
  $m_\pi = 447 MeV$
- a from nucleon mass
- From local current:
  $Z_V = 0.6098(22)$
- Wilson: $m_\pi = 509 MeV$
  160 configurations

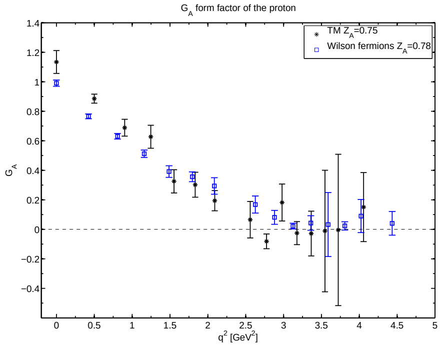


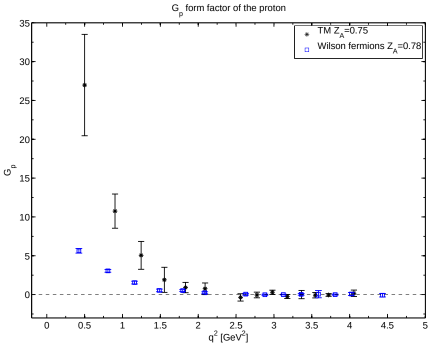
Wilson data: [C.A. et al Phys.Rev.D77:085012,2008]

Magnetic Sachs form factor



Axial form factors





- $Z_A = 0.75$ [P. Dimopoulos, internal notes]
- Wilson data: [C.A. et al Phys.Rev.D76:094511,2007]

Final remarks

- G_E, G_M, G_A, G_p calculable with 2 sources (4×12 inversions / conf)
- first results encouraging
- to do
  - decide on t_f, so far $t_f = t_i + 12$ (i.e. $> 1fm$)
  - perform a fully independent check (?)
  - alternative error analysis (so far: jackknife)
  - pseudo scalar form factor
  - flavor changing currents (require additional inversions)
  - better understanding of disconnected pieces
  - production runs