# Heavy-light decay constants with twisted-mass fermions

Benoît Blossier

- Statistics and simulation parameters
- Extraction of $f_{D_{(s)}}$ by a scaling analysis
- Scaling analysis in the light and strange sectors
- Scaling analysis in the heavy sector
- Extraction of physical quantities in the continuum

# Statistics and simulation parameters

| | $\beta$ | $\kappa$ | $L^3 \times T$ | $a\mu_{\rm sea}$ | # measurements |
|---|---|---|---|---|---|
| $A_2$ | 3.8 | 0.164111 | $24^3 \times 48$ | 0.008 | 240 |
| $A_3$ | | | | 0.011 | 240 |
| $A_4$ | | | | 0.0165 | 240 |
| $B_1$ | 3.9 | 0.160856 | $24^3 \times 48$ | 0.004 | 480 |
| $B_2$ | | | | 0.0064 | 240 |
| $B_3$ | | | | 0.0085 | 240 |
| $B_4$ | | | | 0.010 | 240 |
| $B_5$ | | | | 0.015 | 240 |
| $B_6$ | 3.9 | 0.160856 | $32^3 \times 64$ | 0.004 | 240 |
| $C_1$ | 4.05 | 0.157010 | $32^3 \times 64$ | 0.003 | 152 |
| $C_2$ | | | | 0.006 | 131 |
| $C_3$ | | | | 0.008 | 135 |
| $C_4$ | | | | 0.012 | 133 |

$A_i$: 1 measurement every 20 trajectory of length 1
$B_i$: 1 measurement every 20 trajectory of length 0.5
$C_i$: 1 measurement every 40 trajectory of length 0.5

The totality of the MC history has been analysed for all the ensembles.

We did not have taken into account the ensemble $A_1$: a bit far from maximal twist.

| | $A_2 - A_4$ | $B_1 - B_5$ | $B_6$ | $C_1 - C_4$ |
|---|---|---|---|---|
| $a\mu_l$ | 0.008 0.011 0.0165 | 0.004 0.0064 0.0085 0.01 0.015 | 0.004 | 0.003 0.006 0.008 0.012 |
| $a\mu_s$ | 0.02 0.025 0.03 0.036 | 0.022 0.027 0.032 | 0.022 0.027 | 0.015 0.018 0.022 0.026 |
| $a\mu_h$ | 0.27 0.31 0.355 0.435 0.52 | 0.25 0.32 0.39 0.46 | 0.25 0.32 | 0.20 0.23 0.26 0.315 |

$a\mu_l \in [m_s/6, 2m_s/3]$ to perform the chiral extrapolations;

$a\mu_s$ and $a\mu_h$ around $am_s$ and $am_c$ to perform the appropriate interpolations.

$$C_{PP}(t) = \frac{Z_{PS}}{\sinh(am_{PS})} e^{-\frac{m_{PS}T}{2}} \cosh[m_{PS}(T/2 - t)]$$

Ward identity at maximal twist: $af_{PS}(\mu_1, \mu_2) = \frac{(\mu_1+\mu_2)\sqrt{Z_{PS}}}{\sinh(am_{PS})m_{PS}}$

# Extraction of $f_{D_{(s)}}$ based on a scaling analysis

We follow the strategy presented in [P. Dimopoulos *et al*, PoS (LATTICE 2007)102] and extend it to the strange and heavy sectors.

1) At each lattice spacing we perform an interpolation of $r_0 f_{PS}$ and $r_0^{3/2} f_{PS}\sqrt{m_{PS}}$ to reference points $\mu_l^{\rm ref}$, $\mu_s^{\rm ref}$ and $\mu_h^{\rm ref}$ which are defined by $r_0 m_{PS}(\mu_{l,s,h}^{\rm ref})$.

2) Then we extrapolate to the continuum limit the quantities defined at the reference points.

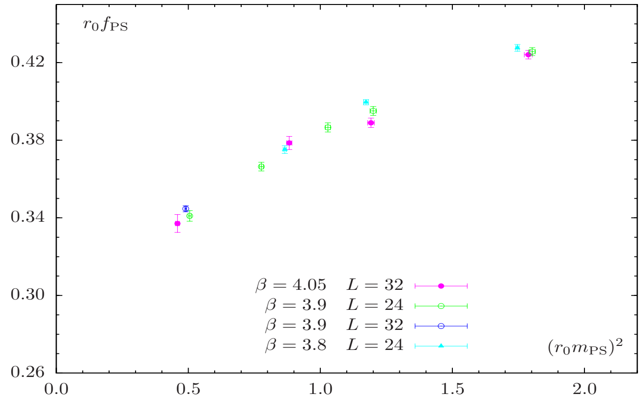
3) Finally we perform the chiral extrapolation and the interpolation to the physical point ($m_\pi$, $m_K$, $m_D$).

$$r_0 m_{PS}(\mu_l^{\rm ref}, \mu_l^{\rm ref}) = \underbrace{0.7 \;\; 0.8 \;\; 0.9}_{< m_s/3} \;\; 1.0 \;\; 1.1$$

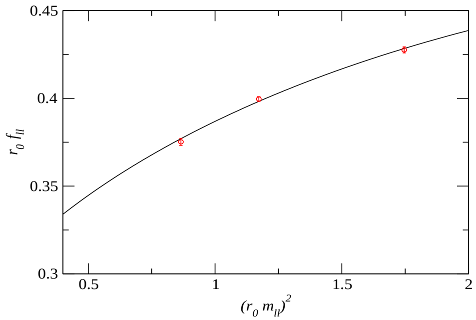
$$r_0 m_{PS}(\mu_l^{\rm ref}, \mu_s^{\rm ref}) = 1.24 \;\; 1.30 \;\; 1.36 \;\; 1.46$$

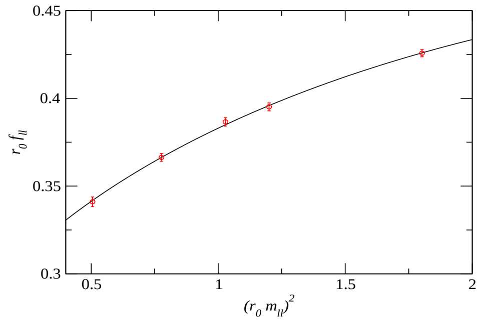
$$r_0 m_{PS}(\mu_l^{\rm ref}, \mu_h^{\rm ref}) = 4.0 \;\; 4.5 \;\; 5.0$$

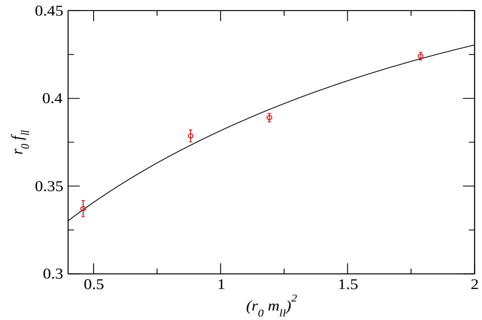
# Scaling analysis in the light and strange sectors

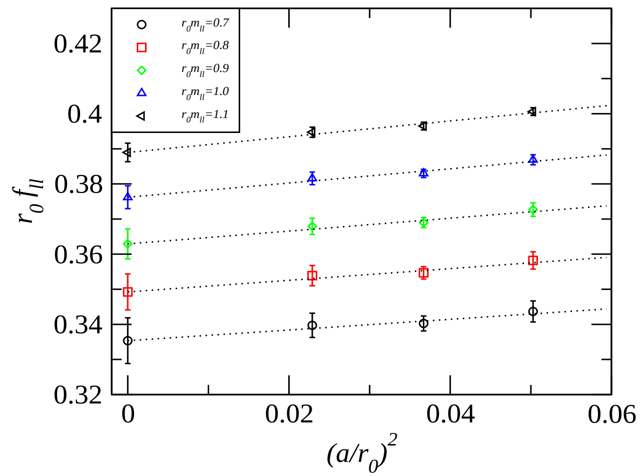



$$f_{ll} = F_0 \left[ 1 - \frac{m_{ll}^2}{8\pi^2 F_0^2} \ln\left( \frac{m_{ll}^2}{\Lambda_4^2} \right) \right]$$

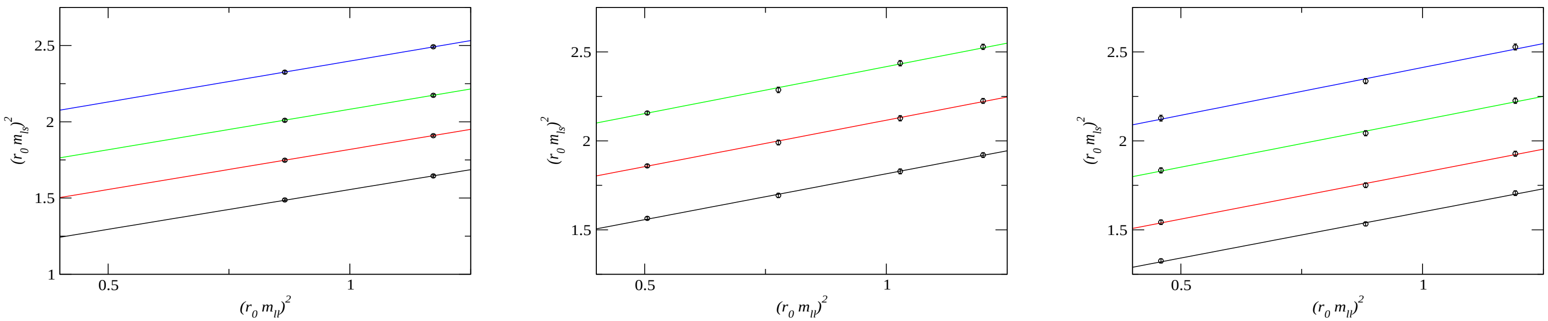


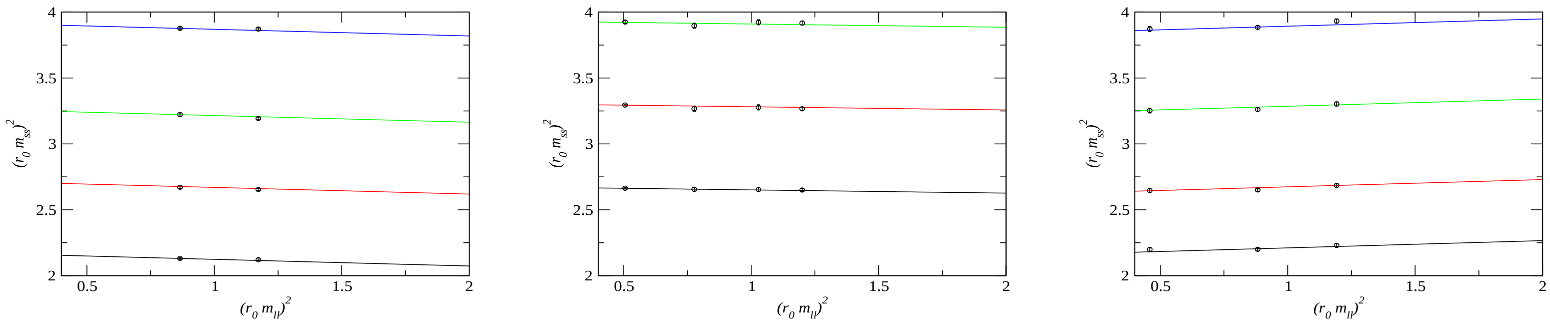



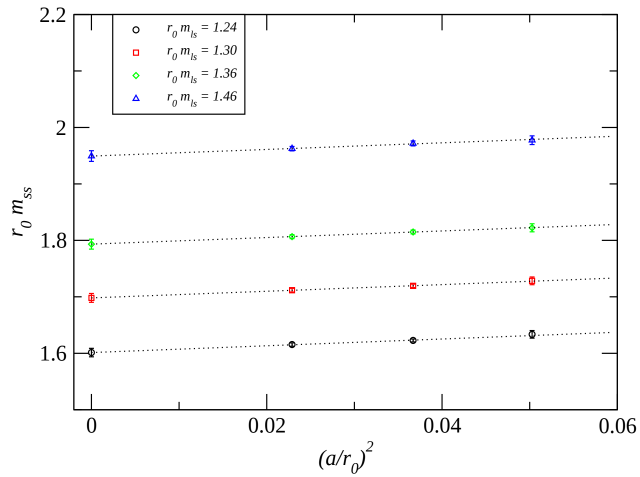



$$r_0^2 m_{ls}^2 = a_0 r_0 \mu_s + r_0^2 m_{ll}^2 (a_1 + a_2 r_0 \mu_s)$$

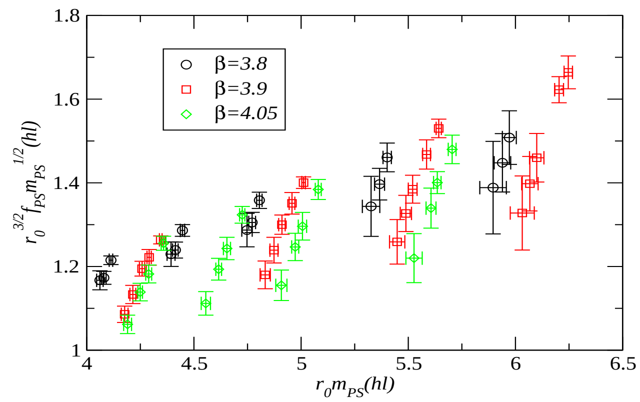



$$r_0^2 m_{ss}^2 = b_0 \mu_s + b_1/\mu_s + b_2 m_{ll}^2$$

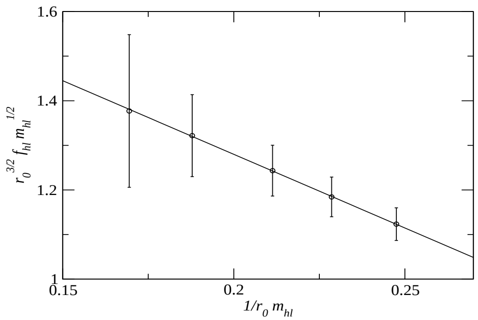


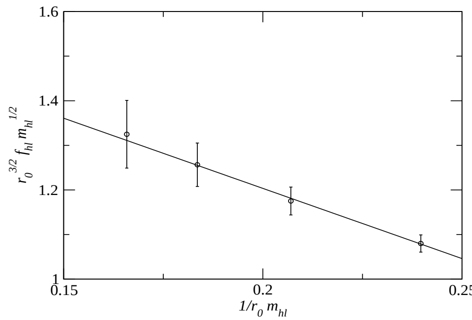



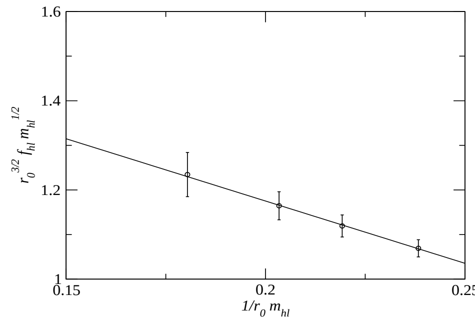
# Scaling analysis in the heavy sector (I)

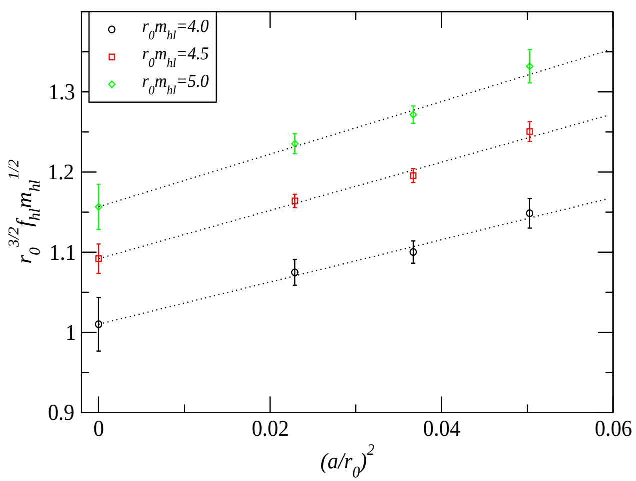



$$f_{hl}\sqrt{m_{hl}} = \hat{f}\left[1-\frac{3}{4}(1+3\hat{g}^2)\frac{m_{ll}^2}{16\pi^2F_0^2}\ln(m_{ll}^2)+b\,m_{ll}^2\right] \qquad r_0^{3/2} f_{hl}\sqrt{m_{hl}} = c_0+\frac{c_1}{m_{hl}}$$

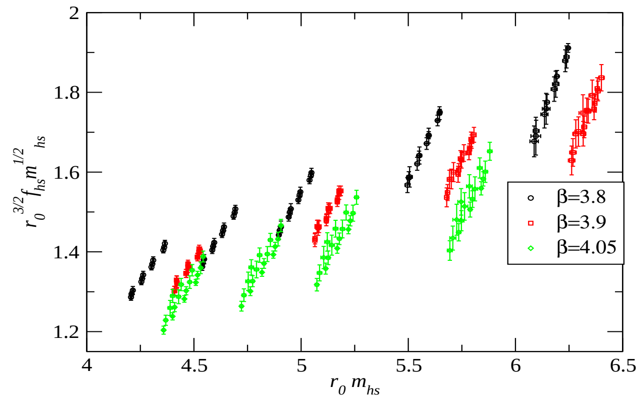


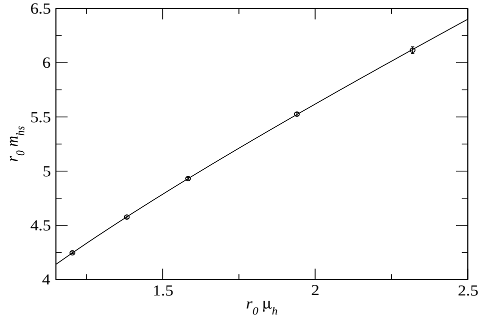



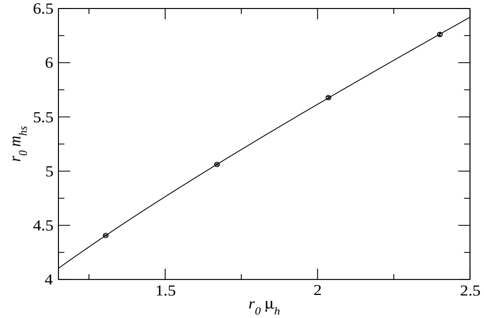



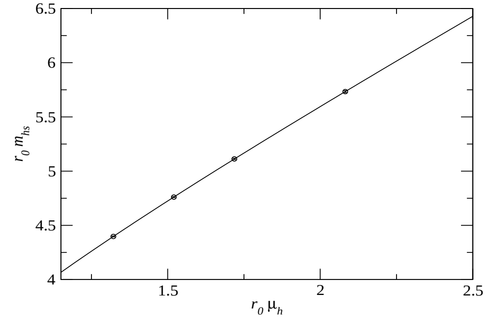
# Scaling analysis in the heavy sector (II)

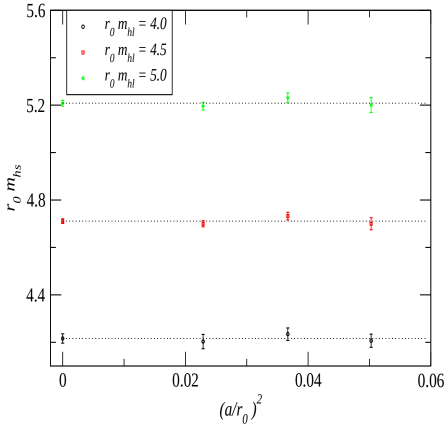


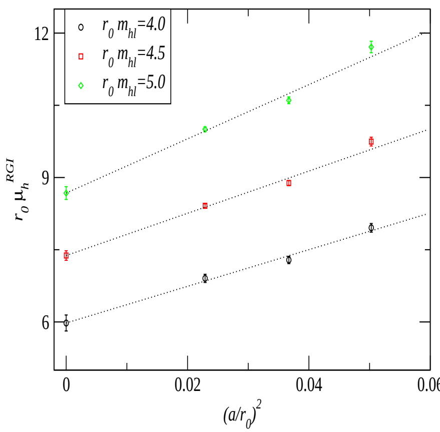

$$r_0^{3/2} f_{hs}\sqrt{m_{hs}} = d_0 + \frac{d_1}{m_{hs}} \qquad m_{PS}(\mu_h,\mu_2) = m_0(\mu_2)a\mu_h + m_1(\mu_2) + m_2(\mu_2)/\mu_h$$

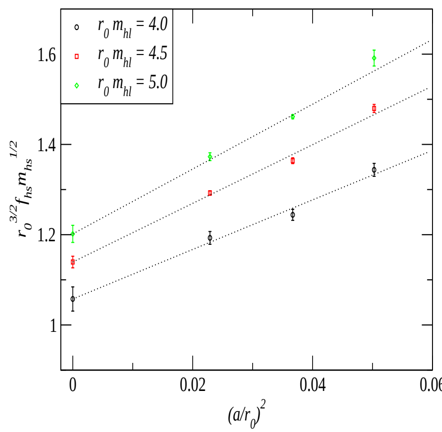


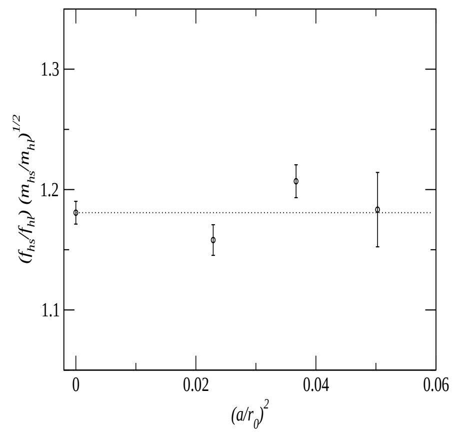



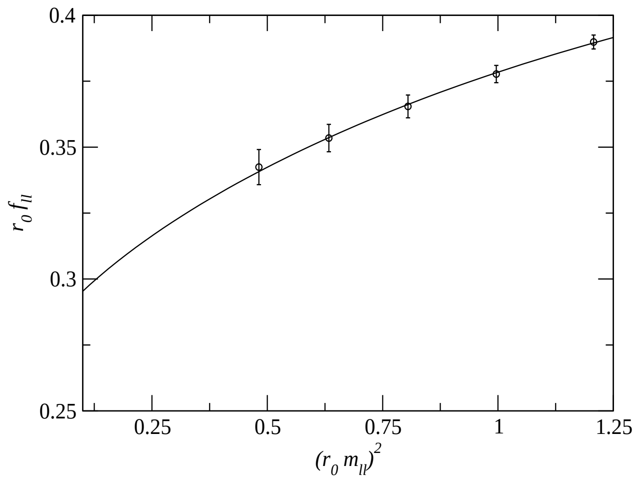



# Extraction of physical quantities in the continuum

All the quantities have been normalised so far with $r_0 = ??$. We will normalise $f_D$ and $f_{D_s}$ by $f_\pi$ and $m_{D_s}$ respectively.
The finite volume data have been extrapolated to the infinite volume limit according to numerical tables given in [G. Colangelo, S. Dürr and C. Haefeli, Nucl. Phys. B **721**, 136 (2005)].
First we have to fix $r_0$:

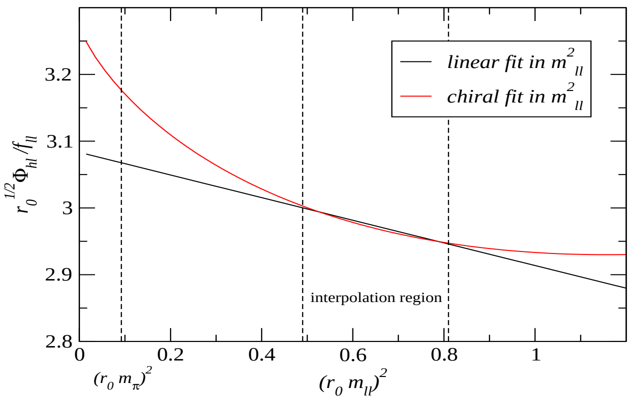
$$r_0 f_\pi = A_0 \left[ 1 - \frac{r_0^2 m_\pi^2}{8\pi^2 A_0^2} \left( \ln(r_0^2 m_\pi^2) - B_0 \right) \right] ,$$

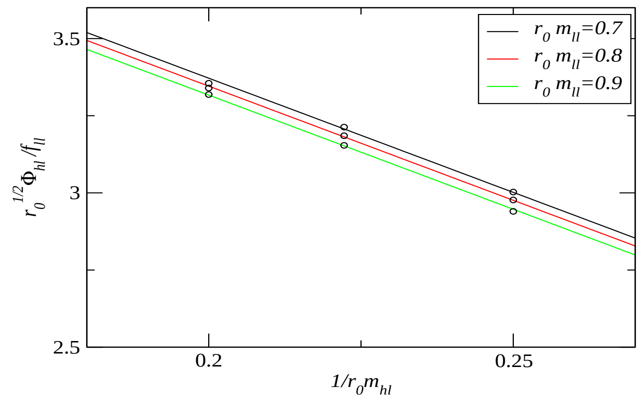
$$A_0 = 0.274(8) \quad B_0 = 2.25(11) \quad r_0 = 0.443(12)\text{fm}$$



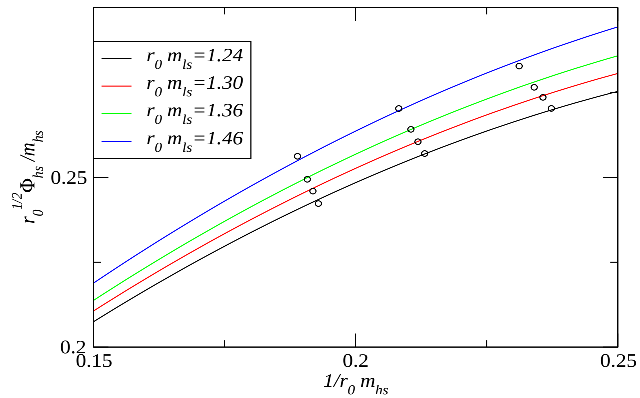

$$m_{ss}^2 = A m_{ls}^2 + B m_{ll}^2 \quad A = 2.0110(69) \quad B = -1.007(11)$$

Extraction of $f_D/f_\pi$ by a linear/chiral fit of $\frac{f_{hl}\sqrt{r_0 m_{hl}}}{f_{ll}} \left(\frac{\alpha_s(m_{hl})}{\alpha_s(m_B)}\right)^{2/\beta_0}$.



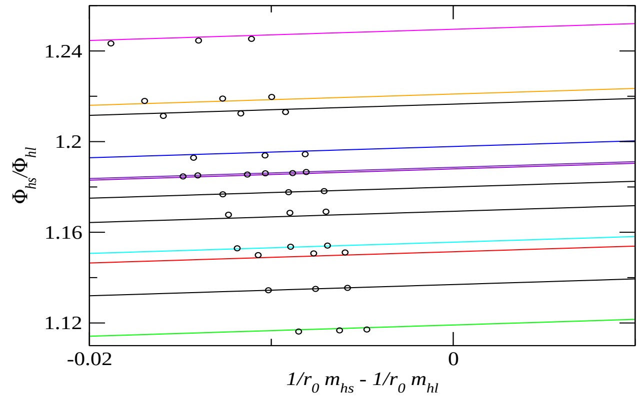




Extraction of $f_{D_s}/m_{D_s}$ by a linear/chiral fit of $\frac{f_{hs}\sqrt{r_0 m_{hs}}}{m_{hs}} \left(\frac{\alpha_s(m_{hs})}{\alpha_s(m_B)}\right)^{2/\beta_0}$.




An alternative is the linear/chiral fit of $\frac{f_{hs}\sqrt{r_0 m_{hs}}}{f_{ll}}$ to extract $f_{D_s}/f_\pi$.

Extraction of $f_{D_s}/f_D$ by the linear/chiral fit of $\frac{f_{hs}\sqrt{m_{hs}}}{f_{hl}\sqrt{m_h}}\left(\frac{\alpha_s(m_{hs})}{\alpha_s(m_{hl})}\right)^{2/\beta_0}$.




Improvement of the extrapolation to the continuum limit by considering $\mathcal{O}(a^4)$ effects?
Waiting for the very fine lattice at $\beta = 4.2$?