B_K from N_f=2

(Work in progress)

A project of **… INFN Frascati – ROME 1 -2- 3 …**

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Set-up for B_K
(Frezzotti-Rossi, hep-lat/ 0407002)

Consider the ***Mixed action*:**

$$S^{L} = S_{YM} + S^{Mtm}_{q,sea} + S^{OS}_{q_f,val} + S^{OS}_{gh,\ val}$$

with: $$S^{OS}_{q_f,val} = \sum_{f=1}^{N_{val}} \overline{q}_f \left[\gamma\cdot\tilde{\nabla} - i\gamma_5 W_{cr}(r_f) + \mu_f\right] q_f$$ *(q_f a <u>single</u> flavour f)*

and: $$W_{cr}(r_f) = -r_f \frac{a}{2}\nabla^{*}\nabla + M_{cr}(r_f; r^2_{sea})$$

Work in the ***Partial-Quenched*** set-up:

$$(\,q_1 = d,\quad q_2 = d',\quad q_3 = s,\quad q_4 = s'\,)$$

$$M_0^{sea} = M_0^{f} = M_{cr}(1;1^2) \qquad |\,r^{u,d}_{sea}\,| = |\,r_f\,| = 1$$

$$\mu^{u}_{sea} = \mu^{d}_{sea} = \mu_d = \mu_{d'} \qquad r^{u}_{sea} = -r^{d}_{sea} = r_d = -r_{d'} = 1$$

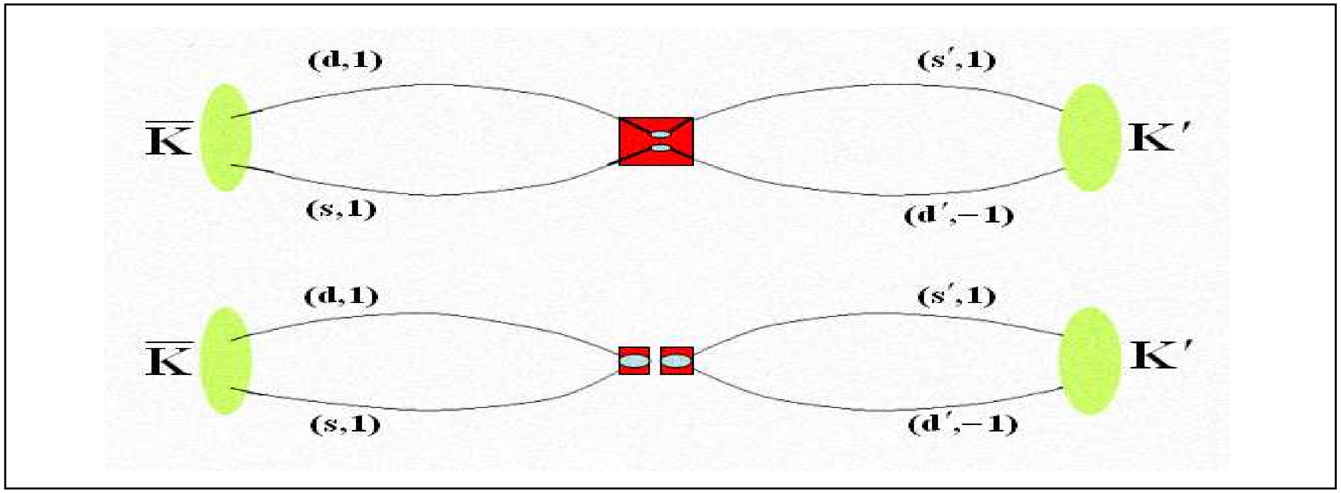
$$\mu_s = \mu_{s'} \qquad r_s = r_{s'} = 1$$

Calculate the three-point correlator:

$$C_{K'QK}(z_0 - x_0, z_0 - y_0) = \sum_{\vec{x},\vec{y},\vec{z}} \left\langle (\bar{d}'\gamma_5 s')(x)\ Q^{\Delta S=2}_{VV+AA}(z)\ (\bar{d}\gamma_5 s)(y) \right\rangle$$

with the 4-fermion operator:

$$Q^{\Delta S=2}_{VV+AA} = 2\{ (\bar{s}\gamma_\mu d)(\bar{s}'\gamma_\mu d') + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}'\gamma_\mu\gamma_5 d') + (\bar{s}\gamma_\mu d')(\bar{s}'\gamma_\mu d) + (\bar{s}\gamma_\mu\gamma_5 d')(\bar{s}'\gamma_\mu\gamma_5 d) \}$$



$\phi_{K'} = \bar{d}'\gamma_5 s'$ $\quad -r_{d'} = r_{s'} = 1$ $\quad$ (**tm-like**)

$\phi_K = \bar{d}\gamma_5 s$ $\quad r_d = r_s = 1$ $\quad$ (**OS-like**)

GAIN: no mixing in the renormalization of the 4-fermion operator + O(a) improvement

Produce the OS-like Kaon contribution in ***two*** possible ways:

1. $\mathbf{M_{cr}^{OS} = M_{cr}^{opt}}$

- **It guarantees the O(a)-improvement**
- **It leads to different O(a^2) effects on the masses of tm and OS-like Kaons (Unphysical O(a^2) energy transfer in $K' \leftrightarrow \overline{K}$ in B_K calculation)**

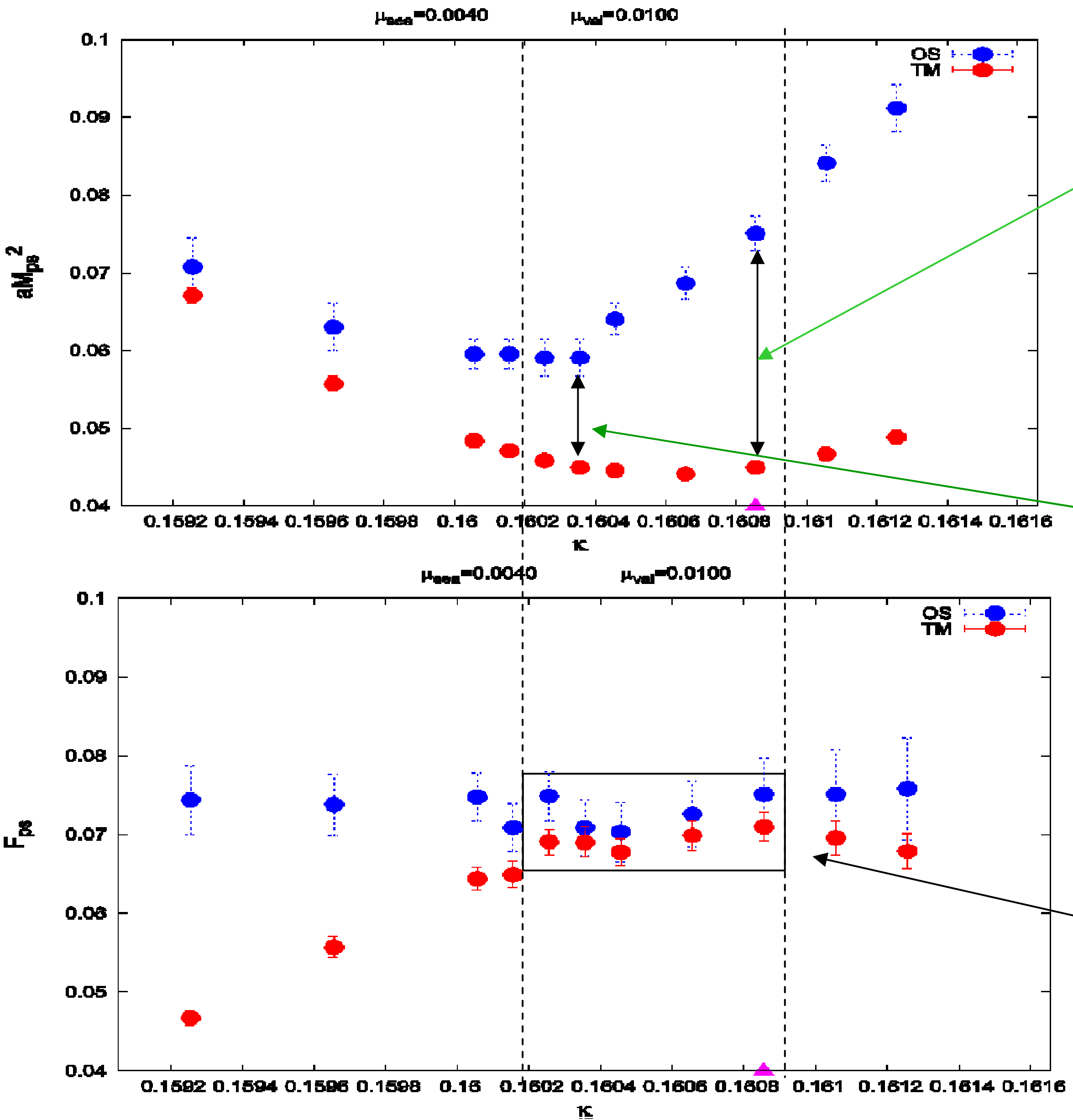
2. $\mathbf{M_{cr}^{OS} \neq M_{cr}^{opt}}$

- **Different (hopefully *smaller*) O(a^2) mass-splitting between** $\mathrm{m}_K^{2\ \mathrm{OS}}$ **and** $\mathrm{m}_K^{2\ \mathrm{tm}}$
- **It, still, guarantees the O(a)-improvement**
- **Tune *κ_{cr}(OS)* in order to *minimize* the mass splitting:** $(\mathrm{m}_K^{2\ \mathrm{OS}} - \mathrm{m}_K^{2\ \mathrm{tm}})$

(*See the ToV presentation at the Cyprus ETMC meeting …*)

Numerical search of M_{cr}^{OS} $(\neq M_{cr}^{opt})$

β=3.90



O(a²) effect in the pseudoscalar masses

O(a²) effect is reduced

"κ_{cr} region"
[F_{ps} (tm) - F_{ps} (OS)] ~ small and constant

similar results for β=4.05 …

We conclude that determining a useful $M_{cr}^{OS} \left(\neq M_{cr}^{opt} \right)$ is feasible for the N_f=2 case.

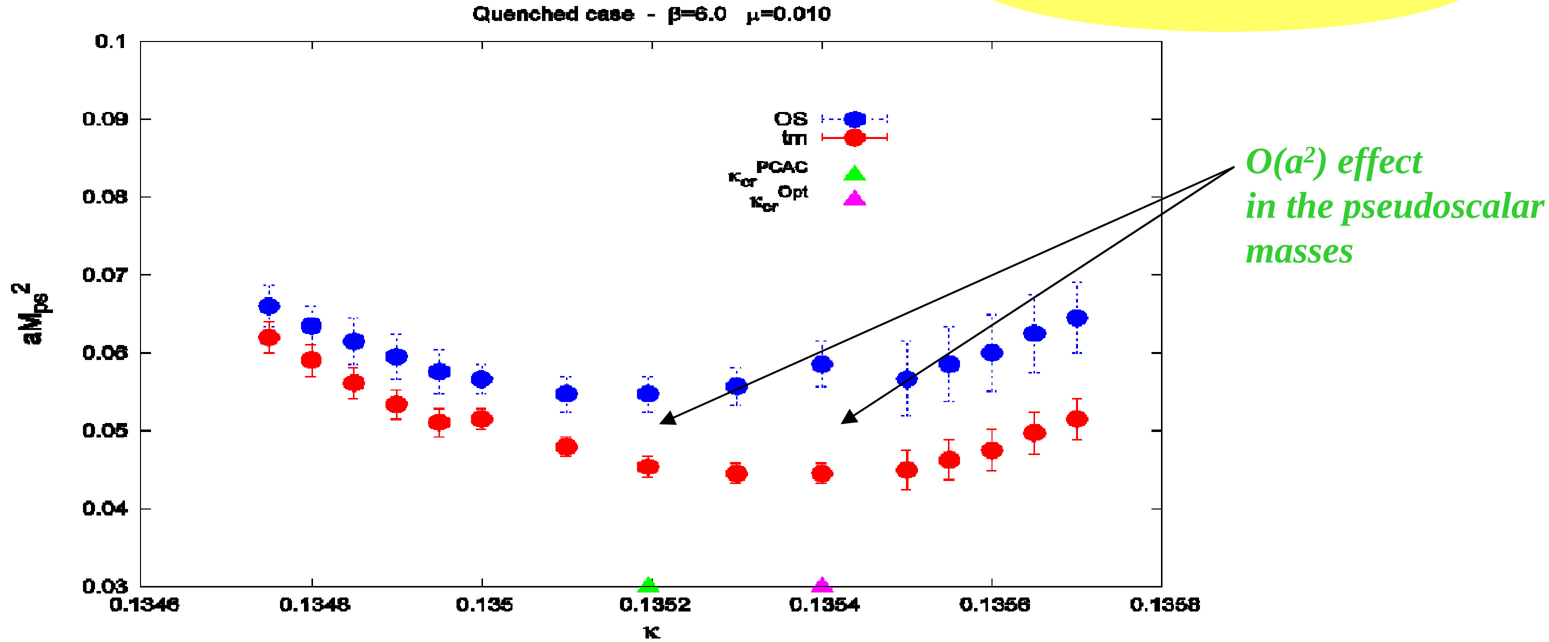
But, this is a *numerical fact* and surely not guaranteed in general.

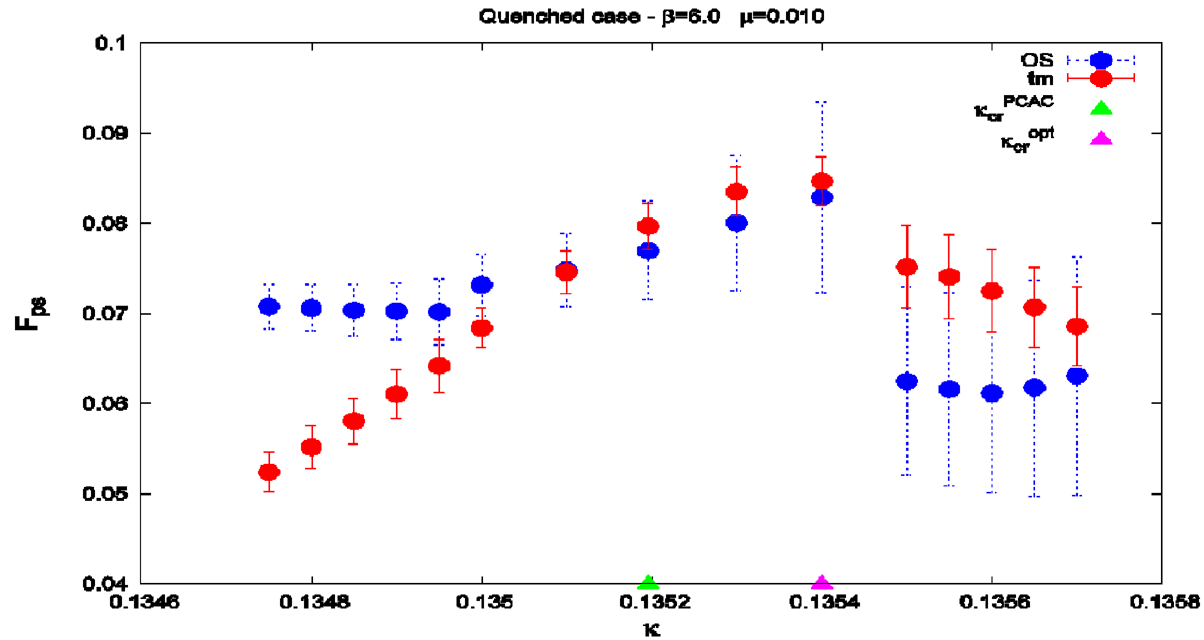
As an exercise, let's have a look at the quenched case…

Numerical search of M_{cr}^{OS} $(\neq M_{cr}^{opt})$

β=6.0 – quenched case

More difficult to judge …





Until now, we only have runs with

$$\mathbf{M_{cr}^{OS} = M_{cr}^{opt}}$$

B_K calculation

- **Local-local calculation**

  - Choose and fix a couple of time slices x_0 such as to isolate the Kaon (first Kaon source)
  - Locate the 4-fermion operator at $z_0=0$

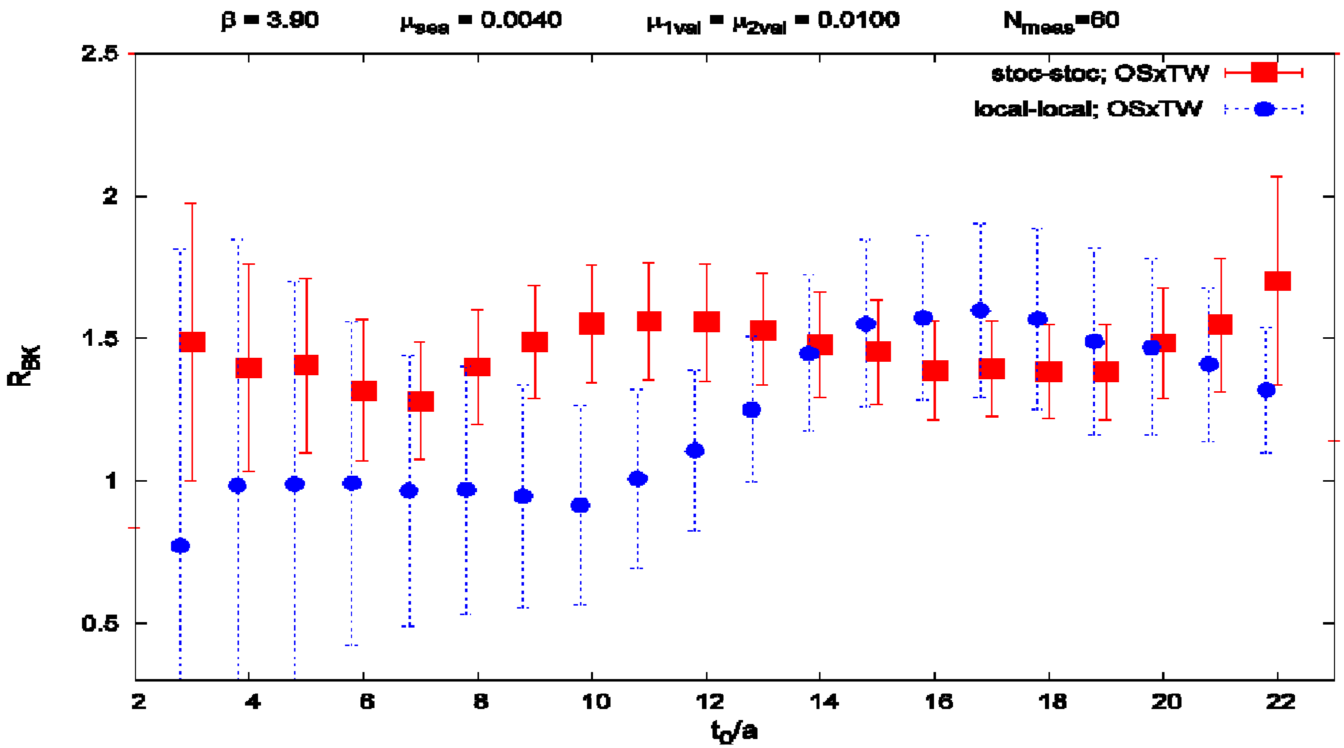
$$R_{B_K} = \frac{C^{(3)}_{K'QK}(z_0 - x_0^{fixed}, z_0 - y_0)}{\frac{8}{3} C^{(2)}_{K'}(z_0 - x_0^{fixed}) C^{(2)}_{K}(z_0 - y_0)} \Big|_{z_0=0} \xrightarrow{T/2 \ll y_0 \ll T} B_K$$

- **Stochastic-stochastic calculation**

  - Locate the two stochastic sources at fixed time slices, x_0 and y_0
  - Free moving in time, z_0, the 4-fermion operator

$$R^{stoc}_{B_K} = \frac{C^{(3)}_{K'QK}(z_0 - x_0^{fixed}, z_0 - y_0^{fixed})}{\frac{8}{3} C^{(2)}_{K'}(z_0 - x_0^{fixed}) C^{(2)}_{K}(z_0 - y_0^{fixed})} \xrightarrow{x_0 \ll z_0 \ll y_0} B_K$$

stoc-stoc vs. local-local set-up



$$\sigma_{\text{local-local}} \approx 2\ \sigma_{\text{stoc-stoc}}$$

In the final statistics, ~ 400 confs and keeping into account autocorrelations, we expect $\sigma_{BK(bare)} \approx$ 3-5%

- ***Results for the bare quantity (β=3.90) will be available in a couple of weeks' run …***

- ***Of course, for the physical estimate, the calculation of Z_Q is requested …***