The $1/2$ versus $3/2$ puzzle

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Most slides are borrowed from Marc wagner

$1/2$ versus $3/2$: experimental side

- Consider the semileptonic decay $B \rightarrow X_c \, l \, \nu$.
- Experiments, which have studied this decay: ALEPH, BaBar, BELLE, CDF, DELPHI, DØ.
- What is X_c?
  - $\approx 75\%$ D and D^*, i.e. S wave states (agreement with theory).
  - $\approx 10\%$ $D_1^{3/2}$ and $D_2^{3/2}$, i.e. $j = 3/2$ P wave states (agreement with theory).
  - For the remaining $\approx 15\%$ the situation is not clear:
    * A "natural candidate" would be $D_0^{1/2}$ and $D_1^{1/2}$, i.e. $j = 1/2$ P wave states.
    * This would imply $\Gamma(B \rightarrow D_{0,1}^{1/2} \, l \, \nu) > \Gamma(B \rightarrow D_{1,2}^{3/2} \, l \, \nu)$, which is in "conflict" with theory.
    * This "conflict" between experiment and theory is called the "$1/2$ versus $3/2$ puzzle".

An important issue to solve if we wish to measure accurately V_{cb}

Heavy-light mesons

- Heavy-light meson: a meson made from a heavy quark (b, c) and a light quark (u, d, s), e.g. $B = \{\bar{b}u\,,\,\bar{b}d\}$, $B_s = \bar{b}s$, $D = \{\bar{c}u\,,\,\bar{c}d\}$, $D_s = \bar{c}s$.
- Static limit, i.e. $m_b, m_c \to \infty$:
  - No interactions involving the static quark spin.
  - Classify states according to parity $\mathcal{P}$ and total angular momentum of the light cloud j.
- m_b, m_c finite, but heavy:
  - Classify states according to parity $\mathcal{P}$ and total angular momentum J.

$j^{\mathcal{P}}$	$J^{\mathcal{P}}$
$(1/2)^- \equiv S$	$0^- \equiv H$ $1^- \equiv H^*$
$(1/2)^+ \equiv P_-$ $(3/2)^+ \equiv P_+$	$0^+ \equiv H_0^* \equiv H_0^{1/2}$ $1^+ \equiv H_1^* \equiv H_1^{1/2}$ $1^+ \equiv H_1 \equiv H_1^{3/2}$ $2^+ \equiv H_2^* \equiv H_2^{3/2}$

$1/2$ versus $3/2$: theory side (1)

- Static limit ($m_b, m_c \rightarrow \infty$) with both b and c quark at rest:

$$\begin{aligned}\langle D_0^{1/2}|\bar{c}\gamma_5\gamma_j D_k b|B\rangle &= -ig_{jk}\Big(m(D_0^{1/2}) - m(D)\Big)\tau_{1/2} \\ \langle D_2^{3/2}|\bar{c}\gamma_5\gamma_j D_k b|B\rangle &= +i\sqrt{3}\epsilon_{jk}\Big(m(D_2^{3/2}) - m(D)\Big)\tau_{3/2}\end{aligned}$$

and

$$\frac{\Gamma(B \rightarrow D_{0,1}^{1/2}\, l\, \nu)}{\Gamma(B \rightarrow D_{1,2}^{3/2}\, l\, \nu)} \quad \text{``}=\text{''} \quad \frac{|\tau_{1/2}|^2}{|\tau_{3/2}|^2}.$$

($\tau_{1/2}$, $\tau_{3/2}$: Isgur-Wise form factors).

$1/2$ versus $3/2$: theory side (2)

- Phenomenological models:
  - $|\tau_{1/2}| < |\tau_{3/2}|$, which is in "conflict" with experiment.
- OPE:
  - Uraltsev sum rule:

    $$\sum_n |\tau_{3/2}^{(n)}|^2 - |\tau_{1/2}^{(n)}|^2 = \frac{1}{4}$$

    ($\tau_{1/2} \equiv \tau_{1/2}^{(0)}$ and $\tau_{3/2} \equiv \tau_{3/2}^{(0)}$).
  - From experience with sum rules one would expect approximate saturation from the ground states, i.e.

    $$|\tau_{3/2}^{(0)}|^2 - |\tau_{1/2}^{(0)}|^2 \approx \frac{1}{4},$$

    which also implies $|\tau_{1/2}| < |\tau_{3/2}|$, which is in "conflict" with experiment.

$1/2$ versus $3/2$: possible explanations

- **Experiment:**
  - The signal for the remaining 15% of X_c is rather vague; therefore, only a small part might be $D_{0,1}^{1/2}$.
- **Phenomenological models:**
  - Models might give a wrong answer.
- **OPE:**
  - Sum rules hold in the static limit and might change significantly for finite quark masses.
  - Sum rules might not be saturated by the ground states.
- A lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ could shed some light on this puzzle.

Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (1)

- Simulation setup:

μ	$m_{\rm PS}$ in MeV	number of gauges
0.0040	$314(2)$	1400
0.0064	$391(1)$	1450
0.0085	$448(1)$	1350

Interpolating fields

$\Gamma(\hat{\mathbf{n}})$	$\rm O_h$	j
γ_5	A_1	$1/2\ ,\ 7/2\ ,\ ...$
1		$1/2\ ,\ 7/2\ ,\ ...$
$\gamma_x\hat{n}_x - \gamma_y\hat{n}_y$ (and cyclic)	E	$3/2\ ,\ 5/2\ ,\ ...$
$\gamma_5(\gamma_x\hat{n}_x - \gamma_y\hat{n}_y)$ (and cyclic)		$3/2\ ,\ 5/2\ ,\ ...$

Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (2)

- "Effective form factors",

$$\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2) =$$
$$= \left|\frac{N(\tilde{P}_-) \ N(\tilde{S}) \ \langle \tilde{P}_-(T_0)|(\bar{Q}\gamma_5\gamma_3 D_3 Q)(T_1)|\tilde{S}(T_2)\rangle}{(m(\tilde{P}_-) - m(\tilde{S})) \ \langle \tilde{P}_-(T_0)|\tilde{P}_-(T_1)\rangle \ \langle \tilde{S}(T_1)|\tilde{S}(T_2)\rangle}\right|$$
$$\tau_{3/2,\text{effective}}(T_0 - T_1, T_1 - T_2) =$$
$$= \sqrt{\frac{1}{6}} \left|\frac{N(\tilde{P}_+) \ N(\tilde{S}) \ \langle \tilde{P}_+(T_0)|(\bar{Q}\gamma_5(\gamma_1 D_1 - \gamma_2 D_2) Q)(T_1)|\tilde{S}(T_2)\rangle}{(m(\tilde{P}_+) - m(\tilde{S})) \ \langle \tilde{P}_+(T_0)|\tilde{P}_+(T_1)\rangle \ \langle \tilde{S}(T_1)|\tilde{S}(T_2)\rangle}\right| :$$

  - $N(X)$: norm of state $|X\rangle$.
  - $m(X)$: mass of state $|X\rangle$.
  - Three-point functions (T_0, T_1 and T_2).
  - Two-point functions (T_0 and T_1 or T_1 and T_2).

- $\tau_{1/2} = \lim_{T_0 - T_1, T_1 - T_2 \to \infty} \tau_{1/2,\text{effective}} \quad , \quad \tau_{3/2} = \lim_{T_0 - T_1, T_1 - T_2 \to \infty} \tau_{3/2,\text{effective}}.$

Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (3)

- $\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2) \quad =$

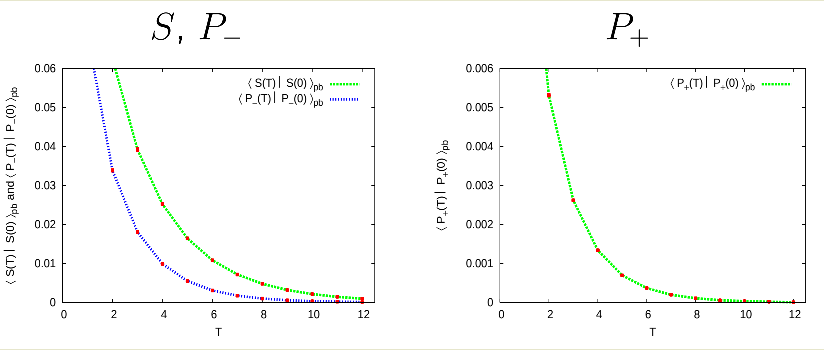
$$= \left| \frac{N(\tilde{P}_-) \;\; N(\tilde{S}) \;\; \langle \tilde{P}_-(T_0) | (\bar{Q}\gamma_5\gamma_3 D_3 Q)(T_1) | \tilde{S}(T_2) \rangle}{(m(\tilde{P}_-) - m(\tilde{S})) \;\; \langle \tilde{P}_-(T_0) | \tilde{P}_-(T_1) \rangle \;\; \langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle} \right| \quad , \quad \ldots$$

- Two-point function $\langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle$: a standard lattice computation.
- Determine the norm of $|\tilde{S}\rangle$, $N(\tilde{S})$, by performing a χ^2 minimizing fit with

$$f(T) \quad = \quad N(\tilde{S})^2 e^{-m(S)T}$$

to $\langle \tilde{S}(T) | \tilde{S}(0) \rangle$ at large T.

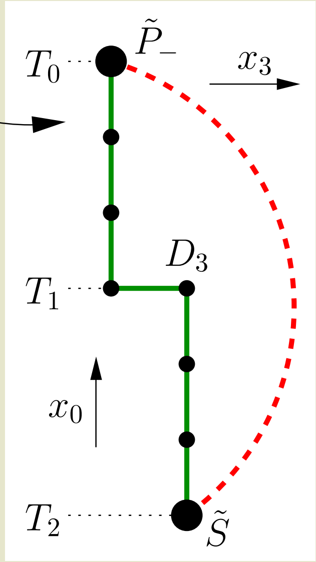
- Analogously for the others.



Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (4)

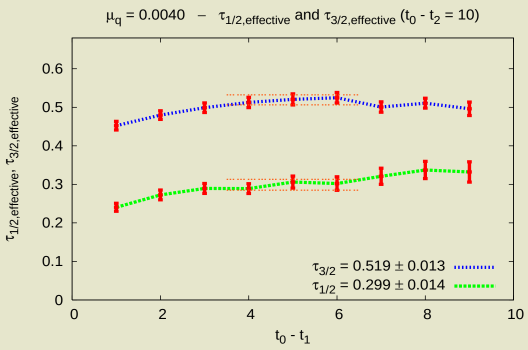
- $\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2) \quad =$
$$= \left|\frac{N(\tilde{P}_-) \quad N(\tilde{S}) \quad \langle \tilde{P}_-(T_0)|(\bar{Q}\gamma_5\gamma_3 D_3 Q)(T_1)|\tilde{S}(T_2)\rangle}{(m(\tilde{P}_-) - m(\tilde{S})) \quad \langle \tilde{P}_-(T_0)|\tilde{P}_-(T_1)\rangle \quad \langle \tilde{S}(T_1)|\tilde{S}(T_2)\rangle}\right| \quad , \quad \ldots$$

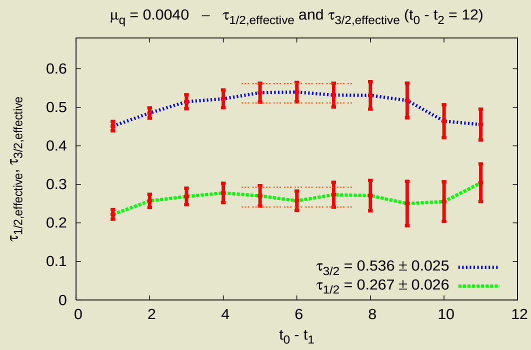
- Three-point functions $\langle \tilde{P}_-(T_0)|(\bar{Q}\gamma_5\gamma_3 D_3 Q)(T_1)|\tilde{S}(T_2)\rangle$:

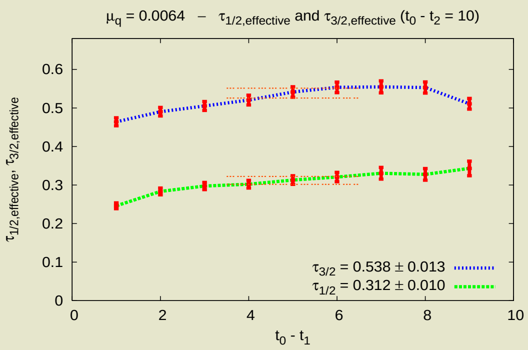


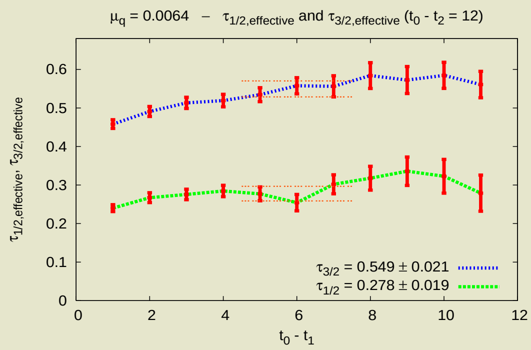
- Analogously for the other three-point functions.

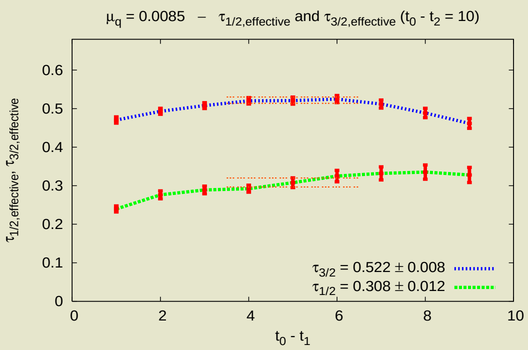
- Mass differences $m(P_-) - m(S)$ and $m(P_+) - m(S)$: cf.K. Jansen, et al.[ETM Collaboration], PoS **LATTICE2008**, 122 [hep-lat]].

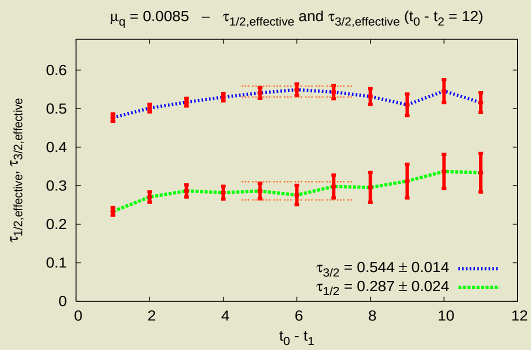

μq = 0.0040 – τ1/2,effective and τ3/2,effective (t0 - t2 = 10)
τ1/2,effective, τ3/2,effective
t0 - t1
τ3/2 = 0.519 ± 0.013
τ1/2 = 0.299 ± 0.014


μq = 0.0040 – τ1/2,effective and τ3/2,effective (t0 - t2 = 12)
τ1/2,effective, τ3/2,effective
t0 - t1
τ3/2 = 0.536 ± 0.025
τ1/2 = 0.267 ± 0.026


μq = 0.0064 – τ1/2,effective and τ3/2,effective (t0 - t2 = 10)
τ1/2,effective, τ3/2,effective
t0 - t1
τ3/2 = 0.538 ± 0.013
τ1/2 = 0.312 ± 0.010


μq = 0.0064 – τ1/2,effective and τ3/2,effective (t0 - t2 = 12)
τ1/2,effective, τ3/2,effective
t0 - t1
τ3/2 = 0.549 ± 0.021
τ1/2 = 0.278 ± 0.019


μq = 0.0085 – τ1/2,effective and τ3/2,effective (t0 - t2 = 10)
τ1/2,effective, τ3/2,effective
t0 - t1
τ3/2 = 0.522 ± 0.008
τ1/2 = 0.308 ± 0.012


μq = 0.0085 – τ1/2,effective and τ3/2,effective (t0 - t2 = 12)
τ1/2,effective, τ3/2,effective
t0 - t1
τ3/2 = 0.544 ± 0.014
τ1/2 = 0.287 ± 0.024

Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (5)

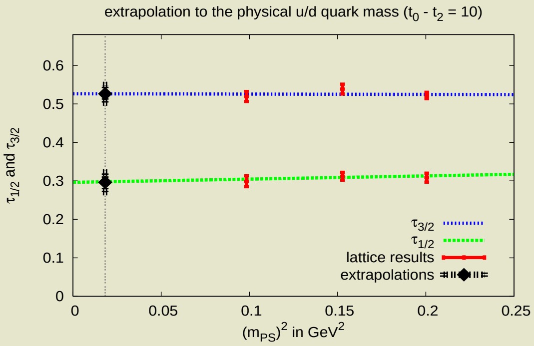
	$t_0 - t_2 = 10$			$t_0 - t_2 = 12$		
μ_q	$\tau_{1/2}$	$\tau_{3/2}$	$(\tau_{3/2})^2 - (\tau_{1/2})^2$	$\tau_{1/2}$	$\tau_{3/2}$	$(\tau_{3/2})^2 - (\tau_{1/2})^2$
0.0040	0.299(14)	0.519(13)	0.180(16)	0.267(26)	0.536(25)	0.216(30)
0.0064	0.312(10)	0.538(13)	0.193(13)	0.278(19)	0.549(21)	0.225(23)
0.0085	0.308(12)	0.522(8)	0.177(9)	0.287(24)	0.544(14)	0.214(21)

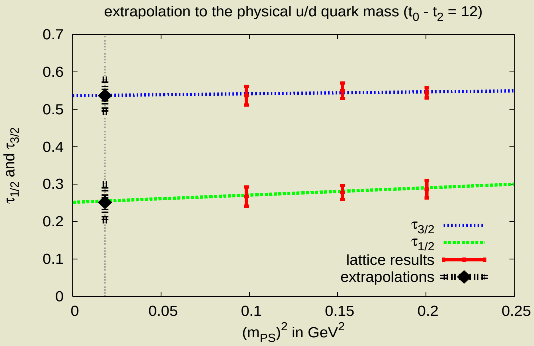
- $\tau_{3/2} > \tau_{1/2}$, i.e. theoretical expectation confirmed.
- "Consistent" with Uraltsev sum rule:

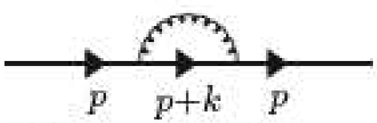
$$\sum_n |\tau_{3/2}^{(n)}|^2 - |\tau_{1/2}^{(n)}|^2 = \frac{1}{4}.$$

- We have used perturbative renormalisation. The correction is small

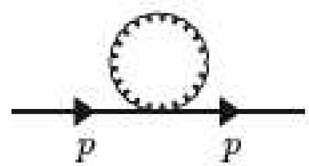
$$Z_{\mathcal{D}}(\mathrm{tlSym}, \mathrm{HYP2}) = 0.976$$



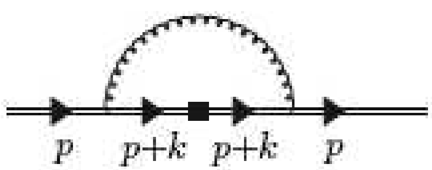




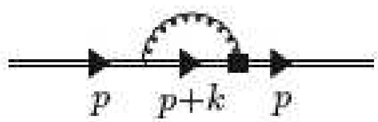
(a): sunset diagram



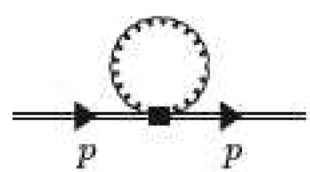
(b): tadpole diagram



(a)



(b)



(c)

Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (6)

Our final result:

$$\tau_{1/2}(1) = 0.296(26), \quad \tau_{3/2}(1) = 0.526(23)$$
$$|\tau_{3/2}(1)|^2 - |\tau_{3/2}(1)|^2 \approx 0.17 \ldots 0.21$$

- Comparison with the only existing lattice study (quenched, exploratory):
  - D. Becirevic *et al.*, "Lattice measurement of the Isgur-Wise functions $\tau_{1/2}$ and $\tau_{3/2}$," Phys. Lett. B **609**, 298 (2005) [arXiv:hep-lat/0406031].
  - $16^3 \times 40$ lattice, $m_{\mathrm{sea}} = \infty$, $m_{\mathrm{PS}} = 800\,\mathrm{MeV}$.
  - $\tau_{1/2} = 0.38(4)$, $\tau_{3/2} = 0.53(8)$.
- Comparison with a relativsitic quark model, V. Morenas et al. Phys. Rev. D **56** (1997) 5668 [arXiv:hep-ph/9706265]:
  $\tau_{1/2} = 0.22$ and $\tau_{3/2} = 0.54$.
- Comparison with BELLE experiment D. Liventsev *et al.* Phys. Rev. D **77**, 091503 (2008) [arXiv:0711.3252 [hep-ex]].
  $\tau_{1/2} = 1.28$ (????) and $\tau_{3/2} = 0.75$.
  But only one of the two expected $1/2$ states.

Conclusions

- $\tau_{1/2}$ and $\tau_{3/2}$ have been computed on dynamical ETMC gauge field configurations.
- This clearly indicates that in the static limit

  $$\Gamma(B \rightarrow D_{0,1}^{1/2}\, l\, \nu) \quad < \quad \Gamma(B \rightarrow D_{1,2}^{3/2}\, l\, \nu)$$

  (as expected from OPE and phenomenological models). The theoretical answer in the static limit seems to be settled. But the "puzzle" is still alive.
- "To do list":
  - Perform the continuum limit.
  - Compute HQET $1/m_Q$ corrections. **
  - A closer scrutiny on the experimental side thanks to the **SuperB factory**.

Wait for my backup slide

A word about the events in France: a nightmare

The present situation in this country deserves some explanation. There is a large protest going on in universities and research labs. 100 000 poeple marching in the streets.

It happens that our president, nanoleon, has submitted our universities and research to a brutal, rough and stubborn attack.

The major ideology gouverning these poeple is that universities should be managed as corporations, that science should only aim at producing technological goods. That research should be embodied in a strict pyramidal structure governed by the state. They deny the accademic freedom. To the standard qualitative evaluation of research by peers they want to substitute an automatic quantitative evaluation system which will kill creativity.

They are treating us as ennemies, one day insulting us, the next day claiming they heard us, the next day attacking on another front.
I believe that this line is being developed more or less in all Europe, our italian colleagues can testify, and indeed worldwide.