Humboldt-Universität zu Berlin
Institut für Physik

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Statistical Methods of Data Analysis
Problem Set #8

Due Date: Thursday, February 3, 2010, during the lecture

Problem 1: (8 points)

The data file `ProblemSet8.root` contains three histograms with 1,000 random numbers each (`huniform1`, `huniform2`, `huniform3`). Check the hypothesis H_0 that the random numbers were drawn from a uniform distribution between 0 and 1.

i) Which PDF does Pearson's χ^2 follow for the histograms? Plot the PDF for χ^2 values between 0 and 50.

ii) Perform χ^2 tests for the three histograms and calculate the p-values by integrating the PDF from i). Assuming a significance level of $\alpha = 0.05$, can H_0 be rejected for any of the histograms? In case H_0 is rejected, what could be the reason?

iii) Calculate the p-values for the three histograms using a Monte-Carlo simulation: Draw 10,000 distributions with 1,000 random numbers each from a uniform distribution and count how many of the χ^2 values are larger than the observed one. Compare the results to the solution from ii) and with 10,000 distributions drawn from each of the three histograms.

Problem 2: (10 points)

In an experiment a random variable which follows a Gaussian distribution with unknown mean μ but known standard deviation $\sigma = 2$ has been measured ten times.

i) Simulate the experiment by drawing the ten random variables from a Gaussian distribution with $\mu = 5$ and $\sigma = 2$.

ii) Compute the maximum likelihood estimator for μ and the corresponding 95% confidence level (CL) central interval from the simulated experiment.

iii) Repeat the simulated experiment 10,000 times to check the coverage of the confidence interval: How many times does the true value $\mu = 5$ lie in the confidence interval?

iv) Suppose you have to estimate σ from the experiment as well: In the calculation of the confidence interval, replace σ^2 with its estimator, the sample variance s^2, and check the coverage of the new interval. Discuss your observations.

(see reverse side)

Problem 3: (10 points)

An experiment is searching for a rare decay of a particle. The expected number of background events has been estimated to be 0.1. Two events are observed.

i) Evidence for the rare decay can usually be claimed if the observed effect is 3σ or larger, i.e. if the probability for a background fluctuation is 0.27% or less. Can the experimenters claim evidence for the decay?

ii) Compute the classical upper limit for the number of rare decays assuming Poisson probabilities:

$$P(n; \nu_s, \nu_b) = \frac{(\nu_s + \nu_b)^n}{n!} \exp[-\nu_s - \nu_b]$$

iii) Compare the limits with limits obtained from the Feldman-Cousins approach by using the ROOT class `TFeldmanCousins`.

If you feel uncomfortable with using the ROOT code "blindly" (you should!), have a look the original publication (G.J. Feldman, R.D. Cousins, Phys.Rev.D57:3873-3889,1998), where the case of "Poisson with background" is treated explicitly.

Problem 4: (6 points)

Read the article "Journal's Paper on ESP Expected to Prompt Outrage" published in the New York Times on January 5, 2011, as well as the cited original preprint by D. Bem and the rebuttal by E.-J. Wagenmakers et al. (copies are linked from the lecture web site) and answer the following questions:

i) Describe the experiments and the hypothesis tests performed by Bem to obtain evidence for extrasensory perception (ESP).

ii) What are the main points of criticism by Wagenmakers et al.?

iii) Explain the difference between "exploratory" and "confirmatory" studies. Can they be performed using the same data? Why (not)?