Humboldt-Universität zu Berlin
Institut für Physik

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Statistical Methods of Data Analysis
Problem Set #6

Due Date: Thursday, January 6, 2010, during the lecture

Problem 1: (5 points)

In a particle decay one of the decay products is emitted under an angle θ with respect to the z axis in some coordinate system. The decay angle distribution is predicted to follow the PDF

$$f(\cos\theta;\lambda) = \frac{1}{2}(1 + \lambda\cos\theta)$$

A measurement has resulted in the following values of $\cos\theta$:

−0.612	−0.235	0.919	−0.735	0.328
0.928	0.975	−0.827	0.104	0.442
0.203	0.267	0.462	0.025	0.243
−0.822	0.363	−0.126	−0.481	−0.994

Estimate the parameter λ and its uncertainty using the maximum likelihood method.

Tip: Carry out the minimization required for the solution numerically. Use e.g. the ROOT class `TF1`, which contains the method `TF1::GetMinimumX()` for finding local minima.

Problem 2: (10 points)

Consider the exponential PDF $f(x;\lambda) = \lambda\exp[-\lambda x]$.

i) Derive the maximum likelihood (ML) estimator $\hat{\lambda}$ of the parameter λ. What is the bias of $\hat{\lambda}$?

ii) Substitute $\lambda = 1/\tau$ in the exponential PDF. Compare value and bias of the ML estimator $\hat{\tau}$ with $\hat{\lambda}$.

Problem 3: (10 points)

The binomial random variable y can take on the value 1 with probability P and 0 with probability $1 - P$.

i) Prove that the mean and the variance of y are equal to P and $P(1-P)$ respectively.

ii) Find the maximum likelihood estimator $\hat{P}$ for a random sample of binomial random variables $\vec{y} = (y_1, \ldots, y_n)$.

iii) Find the variance of this estimator.

Problem 4: (5 points)

Generate and plot random numbers using ROOT:

i) Uniformly distributed on a circle.

ii) Uniformly distributed on the surface of a ball (Tip: use the `TGraph2D` class to plot these numbers).

Note: write some comments into your ROOT macro and hand it in with the solutions.