Humboldt-Universität zu Berlin
Institut für Physik

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Statistical Methods of Data Analysis
Problem Set #5

Due Date: Thursday, December 9, 2010, during the lecture

Problem 1: (10 points)

For Gaussian distributions the square root of the sample variance s can be corrected with a factor $c(n)$ such that $c(n)E[s] = \sigma$, i.e. $c(n)s$ is an unbiased estimator of σ. Use the the fact that the estimator

$$\sqrt{\hat{z}} = \sqrt{\sum_i \frac{(x_i - \overline{x})^2}{\sigma^2}}$$

follows a χ distribution with $n-1$ degrees of freedom, where the PDF of the χ distribution with n degrees of freedom is given by

$$f(z;n) = \frac{2^{1-\frac{n}{2}}}{\Gamma\left(\frac{n}{2}\right)} z^{n-1} \exp\left[-\frac{z^2}{2}\right],$$

to show that $c(n)$ is given by

$$c(n) = \sqrt{\frac{n-1}{2}} \frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}.$$

Which sample size is required for the correction to be below 5%?

Problem 2: (10 points)

Determine the number π using Monte Carlo methods:

i) Write a computer program that uses the hit-or-miss Monte Carlo method to determine the value of π from the area of a circle. How many iterations are required to achieve a precision of 1%? Which empirical precision does your algorithm achive after 10^6 iterations?

ii) Suppose that the floor of your living room is made of parallel strips, each with the same width w. You drop a toothpick with length $l \leq w$. Show that the probability for the toothpick to lie across the line between two strips is given by $P = 2l/(w\pi)$. Write a computer program to determine the value of π that makes use of this process.

Problem 3: (10 points)

Use Monte Carlo methods to solve the following integral:

$$\int_{-1}^{1} \mathrm{d}x\, f(x) \text{ with } f(x) = \exp[-|x|].$$

i) Write a program to generate random numbers distributed according to $f(x)$ using the transformation method.

ii) Write a program to solve the integral using rejection sampling.

iii) Write a program to solve the integral by evaluating the value of the integrand for random values of $x \in [-1,1]$.