Humboldt-Universität zu Berlin
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Statistical Methods of Data Analysis
Problem Set #3

Due Date: Thursday, November 11, 2010, during the lecture

Problem 1: (10 points)

You measure two angles α and β five times each and you get the following measurements.

$\alpha\,[^\circ]$	35	31	33	32	34
$\beta\,[^\circ]$	50	55	51	53	51

1) Evaluate the best value of the angle $\gamma = 180^\circ - \alpha - \beta$ and its standard deviation supposing that α and β are independent.

2) Evaluate the effect on the standard deviation of γ of a correlation between α and β supposing that α and β are not independent.

3) Prove that $\sigma_\gamma \leq \left|\frac{\partial\gamma}{\partial\alpha}\right| \sigma_\alpha + \left|\frac{\partial\gamma}{\partial\beta}\right| \sigma_\beta$.

   Tip: exploit the Schwarz inequality $|\sigma_{xy}| \leq \sigma_x \sigma_y$.

Problem 2: (12 points)

1) Calculate the expectation value $E[x]$ and the variance $V[x]$ of:

   (i) a random variable x uniformly distributed in the interval $[a,b]$. Consider the variable $y = \sin(x)$. What is its PDF $g(y)$ in the case of $[a,b] = [-\pi,\pi]$?

   (ii) an exponentially distributed random variable with parameter λ.

2) The distribution of the decay time (in arbitrary units) of a certain particle is displayed in the histogram `DecayTimes` in the file `ProblemSet3.root` that you can download from the course web page. Evaluate the mean lifetime of this particle.

   Tip: The decay time of an unstable particle is described by an exponential distribution. The mean lifetime is the inverse of the parameter λ. In ROOT you can define the single-parameter function `[0]*exp(-[0]*x)` where `[0]` represent the parameter λ.

Problem 3: (12 points)

The Weibull distribution is often used in industry to describe the lifetime of products. The PDF of the Weibull distribution is given by

$$f(x;\alpha,\beta) = \begin{cases} \alpha\beta x^{\beta-1}\exp[-\alpha x^{\beta}] & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

1) Calculate the expectation value $E[x]$ and the variance $V[x]$ of $f(x;\alpha,\beta)$.
   Tip: The Gamma function is defined as $\Gamma(x) = \int_0^\infty \mathrm{d}t\, t^{x-1}e^{-t}$.

2) Assume that lifetime (in hours) of an emergency backup battery follows a Weibull distribution with $\alpha = 0.1$ and $\beta = 0.5$.

   (i) Plot the PDF given these parameters.

   (ii) Calculate the mean lifetime of the batteries.

   (iii) What is the probability that such a battery will last more than 300 hours?

Problem 4: (8 points)

Let r and ϕ be two invertible functions of the two random variables x and y with PDF

$$f(x,y) = \frac{1}{2\pi}\mathrm{e}^{-\frac{x^2+y^2}{2}} \tag{1}$$

What is the form of the PDF $g(r,\phi)$ for $x = r\cos(\phi)$ and $y = r\sin(\phi)$?