Johannes Blümlein
DESY

- Introduction
- Theory of Scale Evolution
- QCD Analysis of Unpolarized Structure Functions
- A few Remarks on Polarized Structure Functions
- Moments of Parton Densities
- $\Lambda_{QCD}$ and $\alpha_s(M_Z^2)$
- Outlook
Deeply Inelastic Scattering

Space – like process: \[ q^2 = (l - l')^2 = -Q^2 < 0 \quad W^2 = (p+q)^2 \geq M_p^2 \]

\[ x = \frac{Q^2}{2p.q}, \quad y = \frac{p.q}{p.l} \quad 0 \leq x, y \leq 1 \]
Discovery of the Proton (1919)

"We must conclude that the nitrogen atom is disintegrated under the intense forces developed in a close collision with a swift alpha particle, and that the hydrogen atom which is liberated formed a constituent part of the nitrogen nucleus."

-Ernest Rutherford

particle zoo: $e^-$, $p$
Nucleons at rising spatial resolution

\[ Q^2 \approx 0.5 M_N^2 \] : Hofstadter’s Experiments 1950-1960

Olson, Schopper, Wilson (1961)

R. Hofstadter (1915-1990)
The SLAC-MIT Experiments

Discovery of Scaling

SLAC

SLAC-MIT detector

W. Panofsky (1919-2007)
The SLAC-MIT Experiments

An American Success Story:
Discovery of Scaling

\[ Q^2 \approx 3M_N^2 \]

J. Friedman *1930

H. Kendall (1926-1999)

R. Taylor *1929 (1968/69)

FIG. 13. An early observation of scaling: \( \nu W_2 \) for the proton as a function of \( q^2 \) for \( W > 2 \text{ GeV} \), at \( \omega = 4 \).

FIG. 18. The Callan-Gross relation: \( K_0 \) vs \( q^2 \), where \( K_0 \) is defined in the text. These results established the spin of the partons as \( 1/2 \).
precise measurements in a new kinematic region confirm a theoretical prediction

J. Bjorken
*1934

scaling:

\[ \lim_{Q^2, \nu \to \infty, x = \text{fixed}} F_i(\nu, Q^2) = F_i(x) \]

and find the constituents of hadrons, the partons.

\[ W_i(x, Q^2) = \sum_i dx_i \int_0^1 e_i^2 f(x_i) \delta \left( \frac{q \cdot p_i}{M^2} - \frac{Q^2}{M^2} \right) \]

\[ \implies \text{The measurement of } F_L \text{ was instrumental to rule out vector-meson dominance models etc.} \]

R. Feynman
(1918-1988)
Charge Distribution

\[ Q^2 \sim 0.5 \cdot M_p^2 \]

\[ Q^2 \sim 3 \cdot M_p^2 \]

\[ Q^2 \sim 10...500 \cdot M_p^2 \]

Scaling

Violation of Scaling

Today: \[ 1 < Q^2 < 50.000 \text{ GeV}^2 \] \[ \equiv 1/10.000 R_p \]
\[ F_j(x, Q^2) = \hat{f}_i(x, \mu^2) \otimes \sigma^i_j \left( \frac{Q^2}{\mu^2}, x \right) \]

↑ bare pdf  ↑ sub-system cross - sect.

\[ = \hat{f}_i(x, \mu^2) \otimes \Gamma^i_k \left( \alpha_s(R^2), \frac{M^2}{\mu^2}, \frac{M^2}{R^2} \right) \]

\[ \otimes C^k_j \left( \alpha_s(R^2), \frac{Q^2}{\mu^2}, \frac{M^2}{R^2}, x \right) \]

finite pdf \( \equiv f_k \)

finite Wilson coefficient

**Move to Mellin space:**

\[ F_j(N) = \int_0^1 \frac{dxx^{N-1}}{x} F_j(x) \]

Diagonalization of the convolutions \( \otimes \) into ordinary products.
**Evolution Equations**

\[
\left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} - 2\gamma_{\psi}(g) \right] F_i(N) = 0
\]

\[
\left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} + \gamma^N_\kappa(g) - 2\gamma_{\psi}(g) \right] f_k(N) = 0
\]

\[
\left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} - \gamma^N_\kappa(g) \right] C^k_j(N) = 0
\]

**Callan-Symnanzik** equations for mass factorization

≡ **Altarelli-Parisi** evolution equations

**x-space :**

\[
\frac{d}{d \log(\mu^2)} \begin{pmatrix} q^+(x, Q^2) \\ G(x, Q^2) \end{pmatrix} = \frac{\alpha_s}{2\pi} P(x, \alpha_s) \otimes \begin{pmatrix} q^+(x, Q^2) \\ G(x, Q^2) \end{pmatrix}
\]

\[
P(x, \alpha_s) = P^{(0)}(x) + \frac{\alpha_s}{2\pi} P^{(1)}(x) + \left( \frac{\alpha_s}{2\pi} \right)^2 P^{(2)}(x) + \ldots
\]
Evolution Equations

\[ \frac{da_s(\mu^2)}{d \ln \mu^2} = - \sum_{k=0}^{\infty} \beta_k a_s^{k+2}(\mu^2), \quad a_s(\mu^2) = \frac{\alpha_s(\mu^2)}{4\pi} \]

\[ a_s(\mu^2) = \frac{a_s(\mu_0^2)}{1 + a_s(\mu_0^2)\beta_0 \ln (\mu^2/\mu_0^2)} \]

\[ \beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_R N_F > 0 \implies \text{asymptotic freedom} \]

Solution in Mellin space:

\[ \frac{df_{NS}(\mu^2)}{d \ln \mu^2} = a_s(\mu^2) P_{NS}^{(0)}(N) f_{NS}(N) + O(a_s^2) \]

\[ f_{NS}(\mu^2, N) = f_{NS}(\mu_0^2, N) \left( \frac{a_s(\mu^2)}{a_s(\mu_0^2)} \right)^{-P_{NS}^{(0)}(N)/\beta_0} [1 + O(a_s)] \]

\[ F_{NS}(Q^2, N) = C_{NS}(Q^2/\mu^2, N) \cdot f_{NS}(\mu^2, N), \quad \mu^2 = \text{factorization scale} \]
LO splitting functions:

\[ P_{qq}^{(0)}(x) = P_{NS}^{(0)}(x) = 2C_F \left( \frac{1 + x^2}{1 - x} \right)_+ \]

\[ P_{NS}^{(0)}(N) = -2C_F \left[ 2S_1(N - 1) - \frac{(N - 1)(3N + 2)}{2N(N + 1)} \right] \]

\[ P_{qq}^{(0)}(N) = \int_0^1 dx x^{N-1} P_{qq}^{(0)}(x) \]

\[ \int_0^1 dx \left[ f(x) \right]_+ g(x) = \int_0^1 dx \left[ g(x) - g(1) \right] f(x) \]

- No distribution valued components in \( P_{NS}^{(0)}(N) \).
- Harmonic sums appear.
- More involved, but similar expressions also for Wilson coefficients and all HO corrections.
Theory of DIS

Parton Model

Light Cone Expansion \([f]\)

Twist 2

Higher Twist

Fixed Order PT: QCD

\(\alpha_s\)

\(O(\alpha_s^4)\)

\(O(\alpha_s^3)\)

\(O(\alpha_s^3)\)

Twist 3

Twist 4 ...

Sum Rules

More General View

Non-forw. scattering

Angular Momentum: q, G

Special Kinematics

Domain: Small x

\('75, '86

\('90 - '98

\('73,'74,'80,'97

\('73,'82, 2004

\('82,'92,2004

Higher Orders

\(\Rightarrow\)

New Algorithms

Novel Mathematics

Resummations?

\('69, QCD: '72

\('69 - 72

\('75, '86

\('90 - '98

\('60ies - now

\sim 77

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Status of Highest Order Calculations

- Running $\alpha_s$: $O(\alpha_s^4)$ Larin, van Ritbergen, Vermaseren 1997
- Unpol. anomalous dimensions and Wilson coefficients: $O(\alpha_s^3)$
  Moch, Vermaseren, Vogt 2004/05
- Unpol. NS anomalous dimension 2nd Moment: $O(\alpha_s^4)$ Baikov, Chetyrkin 2006
- Pol. anomalous dimension: $O(\alpha_s^2)$; Mertig, van Neerven, 1995; Vogelsang 1995;
  $\Delta P_{qq} \Delta P_{qG}$: $O(\alpha_s^3)$ Moch, Rogal, Vermaseren, Vogt 2008
- Pol. Wilson coefficients: $O(\alpha_s^2)$; $\Delta C_{NS}^{qq}, \Delta C_{qG}$: van Neerven, Zijlstra 1994
- Transversity: $O(\alpha_s^2)$, some moments anom. dim.: $O(\alpha_s^3)$, Hayashigaki, Kanazawa, Koike;
  Kumano, Miyama; Vogelsang; 1997; Gracey 2006
- Unpol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^2)$ Laenen, van Neerven, Riemersma, Smith, 1993
  Fast Mellin Space code: Blümlein & Alekhin, 2003
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient $F_L$: $O(\alpha_s^3)$
  Blümlein, De Freitas, van Neerven, S. Klein 2005
- $Q^2 \gg m^2$ Pol. Heavy Flavor Wilson Coefficient : $O(\alpha_s^2)$ van Neerven, Smith et al. 1996,
  Bierenbaum, Blümlein & Klein 2007
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient $F_2$: $O(\alpha_s^2 \varepsilon)$: all operators
  (also polarized), Bierenbaum, Blümlein, Klein, Schneider, 2008; $O(\alpha_s^3)$: First contributions to the moments
  of the operator matrix elements, Bierenbaum, Blümlein, Klein, 2008
Anomalous Dimensions and Wilson Coefficients

Vermaseren, Moch, Vogt 2004
Complex Analysis of these Functions

- Construct exact analytic continuations to complex $N$
- The functions are meromorphic (up to soft corrections, which have a simple structure)
- Asymptotic Representation
- Recursion $z + 1 \rightarrow z$
- Solve the Evolution Equations fully analytically and form an analytic expression for the Structure functions in Mellin Space at all $Q^2$
- Include the heavy flavor Wilson coefficients in Mellin Space
- Perform a single fast, numerical Mellin inversion (at high precision)

$\Rightarrow$ Fastest and most Precise Way of Analysis
2. QCD Analysis of Unpolarized Structure Functions

DIS range
Nucleon structure:

\[ 10^{-5} < x < 0.9, \]
\[ 1 < Q^2 < 50.000 \text{GeV}^2 \]
\begin{align*}
    F_2^{-1} &\sim 10^{1.6} \\
    Q^2 &\sim 10^{5} \\
    \log_{10}(x) &\sim 2
\end{align*}

\text{ZEUS} \rightarrow \text{NLO QCD fit}

\text{H1 Collaboration}

\text{Supplementary information}
New ZEUS + H1 averaged $F_2(x, Q^2)$
New ZEUS + H1 averaged $F_2(x, Q^2)$
Direct $F_L(x, Q^2)$ Measurement at HERA

H1 Preliminary $F_L(x, Q^2)$

- $Q^2 = 12$ GeV$^2$
- $Q^2 = 15$ GeV$^2$
- $Q^2 = 20$ GeV$^2$
- $Q^2 = 25$ GeV$^2$
- $Q^2 = 35$ GeV$^2$
- $Q^2 = 45$ GeV$^2$
- $Q^2 = 60$ GeV$^2$
- $Q^2 = 90$ GeV$^2$
- $Q^2 = 120$ GeV$^2$
- $Q^2 = 150$ GeV$^2$
- $Q^2 = 200$ GeV$^2$
- $Q^2 = 250$ GeV$^2$
- $Q^2 = 300$ GeV$^2$
- $Q^2 = 400$ GeV$^2$
- $Q^2 = 500$ GeV$^2$
- $Q^2 = 650$ GeV$^2$
- $Q^2 = 800$ GeV$^2$

Medium & High $Q^2$

$E_p = 460, 575, 920$ GeV

H1 PDF 2000

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Direct $F_L(x, Q^2)$ Measurement at HERA
Parton Distributions: Overview

H1 PDF 2000: $Q^2 = 4 \text{ GeV}^2$

- Fit to H1 data
- Experimental errors
- Model uncertainties
- Fit to H1 + BCDMS data
- BCDMS data

H1 Collaboration
Parton Distributions: Overview

MSTW 2008 NLO PDFs (68% C.L.)

\[ x f(x, Q^2) \]

\[ Q^2 = 10 \text{ GeV}^2 \]

\[ Q^2 = 10^4 \text{ GeV}^2 \]

\[ g/10 \]

\[ u, d, c, \bar{c}, s, \bar{s}, b, \bar{b} \]
World Data Analysis: Valence Distributions

World data: NS-analysis

$W^2 > 12.5 \text{ GeV}^2, Q^2 > 4 \text{ GeV}^2$

$N^3\text{LO}$:

$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$

Why an $O(\alpha_s^4)$ analysis can be performed?

assume an $\pm 100\%$ error on the Pade approximant $\rightarrow \pm 2$ MeV in $\Lambda_{QCD}$

$$\gamma_n^{approx:3} = \frac{\gamma_n^{(2)}}{\gamma_n^{(1)}}$$

Baikov & Chetyrkin, April 2006:

$$\gamma_2^{3;NS} = \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[ \frac{3936832}{6561} - \frac{10240}{81} \zeta_3 \right] a_s^3$$
$$+ \left[ \frac{1680283336}{1777147} - \frac{24873952}{6561} \zeta_3 + \frac{5120}{3} \zeta_4 - \frac{56969}{243} \zeta_5 \right] a_s^4$$

The results agree better than 20%.
Valence Distributions

\[ x_{u,v}(X) \]

- NNLO Fit
- MRST04
- A06

\[ x_{d,v}(X) \]

- NNLO Fit
- MRST04
- A06

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Valence Distributions: higher twist

PROTON

\[ C_{HT}(x) \text{ [GeV}^2] \]

\[ 4.0 \text{ GeV}^2 < W^2 < 12.5 \text{ GeV}^2 \]

DEUTERON

\[ C_{HT}(x) \text{ [GeV}^2] \]

\[ 4.0 \text{ GeV}^2 < W^2 < 12.5 \text{ GeV}^2 \]

- agreement between \( p \) and \( d \) analysis, J.B., H. Böttcher, 2008
- LGT determination of interest

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Flavor distributions: light quarks

More work needed.

HERMES probably could measure $s(x, Q^2)$ in an independent way.
Flavor distributions: light quarks
$F_2^{c\bar{c}}(x, Q^2)$ will be very well measured at HERA.
Slope of $F_2$ at low $x$

Very likely, that the $\overline{\text{MS}}$–gluon is remains positive!
Perturbative or non-perturbative growth?

\[ F_2 = 0.5 \text{ GeV}^2 \]

\[ F_2 = 2 \text{ GeV}^2 \]

\[ F_2 = 7 \text{ GeV}^2 \]

- H1 QEDC 1997
- E665
- H1 1997
- NMC
- H1 SV 1995
- SLAC
- ZEUS BPT
- ALLM97
Gluon Density

More work needed; MS– vs scheme-invariant evolution.

\( F_L(x, Q^2) \) could be decisive.
Not both distributions can be correct.

$F_L(x, Q^2)$ could be decisive.

MRST06 vs Alekhin: 2006

More work needed! BB Analysis in progress.
Gluon Density

\[ \frac{g(x, Q^2)}{xg(x, Q^0)} = \begin{cases} 2 \text{ GeV}^2 \\ 5 \text{ GeV}^2 \\ 20 \text{ GeV}^2 \\ 100 \text{ GeV}^2 \end{cases} \]

MSTW 2008

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Gluon Density

Recent Fits based on H1 + ZEUS combined data sets
3. Polarized Structure Functions

High Luminosity is most important: Various precision measurements.
Polarized Parton Densities at Present

\[ x\Delta u_s(x) \sim \text{NLO} \]

\[ x\Delta d_s(x) \sim \text{NLO} \]

\[ x\Delta G(x) \sim \text{NLO} \]

\[ x\Delta \bar{q}(x) \sim \text{NLO} \]

J.B., H. Böttcher (2002)
Unfolding the Sea Quarks

\( \frac{\Delta \chi^2}{\chi^2} = 2\% \)

\[ Q^2 = 10 \, \text{GeV}^2 \]

De Florian, Sassot, Stratmann, Vogelsang, 2008
$g_2(x, Q^2) - a$ Window to Higher Twist

Accurate measurement highly desired.

How big is the $\tau = 3$ contribution?
4. Moments of PDF’s: PT + data

<table>
<thead>
<tr>
<th>$f$</th>
<th>$n$</th>
<th>This Fit</th>
<th>MRST04</th>
<th>A02</th>
</tr>
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<tbody>
<tr>
<td></td>
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<td>$N^3$LO</td>
<td>NNLO</td>
<td>NNLO</td>
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<tr>
<td>$u_v$</td>
<td>2</td>
<td>0.3006 ± 0.0031</td>
<td>0.285</td>
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<tr>
<td></td>
<td>3</td>
<td>0.0877 ± 0.0012</td>
<td>0.082</td>
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<tr>
<td></td>
<td>4</td>
<td>0.0335 ± 0.0006</td>
<td>0.032</td>
<td>0.033</td>
</tr>
<tr>
<td>$d_v$</td>
<td>2</td>
<td>0.1252 ± 0.0027</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0318 ± 0.0009</td>
<td>0.028</td>
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<tr>
<td></td>
<td>4</td>
<td>0.0106 ± 0.0004</td>
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<td>0.010</td>
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<tr>
<td>$u_v - d_v$</td>
<td>2</td>
<td>0.1754 ± 0.0041</td>
<td>0.171</td>
<td>0.184</td>
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<tr>
<td></td>
<td>3</td>
<td>0.0559 ± 0.0015</td>
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<tr>
<td></td>
<td>4</td>
<td>0.0229 ± 0.0007</td>
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<table>
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<tr>
<th>Moment</th>
<th>BB, NLO</th>
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<td>$\Delta u_v$</td>
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<td>$\Delta d_v$</td>
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<tr>
<td>$\Delta u_v - \Delta d_v$</td>
<td>0</td>
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<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

Lattice Results: developing; different fermion-types studied.
Low values of $m_\pi$ crucial; values approach 270 MeV now.

J.B., H. Böttcher, A. Guffanti, 2006

J.B., H. Böttcher, 2002
W and Z total cross sections at the LHC

\[ \sigma_{Z^0} \cdot B(Z^0 \rightarrow l^+\ell^-) \text{ (nb)} \]

\[ \sigma_{W^\pm} \cdot B(W^\pm \rightarrow l^+\nu) \text{ (nb)} \]

\[ R(W/Z) = 10.5, 10.6, 10.7 \]

MSTW08 NNLO
MRST06 NNLO
MRST04 NNLO
Alekhin02 NNLO
MSTW08 NLO
MRST06 NLO
MRST04 NLO
Alekhin02 NLO
CTEQ6.6 NLO
Parton Luminosities at LHC

ratios of parton luminosities at 10 TeV LHC and 14 TeV LHC

luminosity ratio vs. \( M_X \) (GeV)

- \( \Sigma q\bar{q} \)
- \( gg \)

MSTW2008NLO

MSTW08
5. $\Lambda_{QCD}$ and $\alpha_s(M^2_Z)$

\[
\frac{\delta \alpha_{em}(0)}{\alpha_{em}(0)} \sim 3 \cdot 10^{-11} \quad \frac{\delta \alpha_{weak}}{\alpha_{weak}} \sim 7 \cdot 10^{-4} \quad \frac{\delta \alpha_s(M^2_Z)}{\alpha_s(M^2_Z)} > 2 \cdot 10^{-2}
\]

P. Zerwas, 2004
Overview of the Analyses

- Various NLO analyses; \[\Rightarrow\] Precision requires NNLO analysis and higher!
- Mixed S- and NS-NNLO analyses \( e(\mu)N \) world data
- S- and NS-NNLO moment analyses \( \nu N \) world data
- NS-\( \n^3 \)LO analysis \( e(\mu)N \) world data
- NLO analyses polarized \( e(\mu)N \) world data
- Lattice measurements
<table>
<thead>
<tr>
<th>NLO</th>
<th>$\alpha_s(M_Z^2)$</th>
<th>expt</th>
<th>theory</th>
<th>Ref.</th>
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<tbody>
<tr>
<td>CTEQ6</td>
<td>0.1165 $\pm 0.0065$</td>
<td></td>
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<td>[1]</td>
</tr>
<tr>
<td>MRST03</td>
<td>0.1165 $\pm 0.0020$</td>
<td>$\pm 0.0030$</td>
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<td>[2]</td>
</tr>
<tr>
<td>A02</td>
<td>0.1171 $\pm 0.0015$</td>
<td>$\pm 0.0033$</td>
<td></td>
<td>[3]</td>
</tr>
<tr>
<td>ZEUS</td>
<td>0.1166 $\pm 0.0049$</td>
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<td></td>
<td>[4]</td>
</tr>
<tr>
<td>H1</td>
<td>0.1150 $\pm 0.0017$</td>
<td>$\pm 0.0050$</td>
<td></td>
<td>[5]</td>
</tr>
<tr>
<td>BCDMS</td>
<td>0.110 $\pm 0.006$</td>
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<td>[6]</td>
</tr>
<tr>
<td>GRS</td>
<td>0.112</td>
<td></td>
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<td>[10]</td>
</tr>
<tr>
<td>BBG</td>
<td>0.1148 $\pm 0.0019$</td>
<td></td>
<td></td>
<td>[9]</td>
</tr>
<tr>
<td>BB (pol)</td>
<td>0.113 $\pm 0.004$</td>
<td>$^{+0.009}_{-0.006}$</td>
<td></td>
<td>[7]</td>
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### NNLO

<table>
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<td>MRST03</td>
<td>0.1153 $\pm 0.0020$</td>
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<td>$\pm 0.0030$</td>
<td>[2]</td>
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<tr>
<td>A02</td>
<td>0.1143 $\pm 0.0014$</td>
<td></td>
<td>$\pm 0.0009$</td>
<td>[3]</td>
</tr>
<tr>
<td>SY01(ep)</td>
<td>0.1166 $\pm 0.0013$</td>
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<td>[8]</td>
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<tr>
<td>SY01($\nu$N)</td>
<td>0.1153 $\pm 0.0063$</td>
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<td>[8]</td>
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<tr>
<td>GRS</td>
<td>0.111</td>
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<td>[10]</td>
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<tr>
<td>A06</td>
<td>0.1128 $\pm 0.0015$</td>
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<td>[11]</td>
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<tr>
<td>BBG</td>
<td>0.1134 $^{+0.0019}_{-0.0021}$</td>
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### N$^3$LO

<table>
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<th>$\alpha_s(M_Z^2)$</th>
<th>expt</th>
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<tbody>
<tr>
<td>BBG</td>
<td>0.1141 $^{+0.0020}_{-0.0022}$</td>
<td></td>
<td></td>
<td>[9]</td>
</tr>
</tbody>
</table>

**NLO and N$^3$LO**

BBG: $N_f = 4$: non-singlet data-analysis at $O(\alpha_s^4)$: $\Lambda = 234 \pm 26$ MeV

**Lattice results:**

Alpha Collab: $N_f = 2$ Lattice; non-pert. renormalization $\Lambda = 245 \pm 16 \pm 16$ MeV

QCDSF Collab: $N_f = 2$ Lattice, pert. reno. $\Lambda = 261 \pm 17 \pm 26$ MeV

Lepage et al.: Larger Values, to be discussed.
\[ \alpha_s(M_Z^2) \]

![Graph showing\( \alpha_s(M_Z^2) \) values from various experiments and models.](image)

- **NLO**
- **NNLO**
- **N3LO (unp)**

**World Average**

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J.B., H. Böttcher, A. Guffanti, 2006
• $\alpha_s(M_Z^2)$ for different data sets included are too different!
⇒ applies also to HERA: IS vs FS; and also DIS vs TEVATRON-jet

M. Cooper-Sarkar, 2005
6. What would we like to know?

**HERA:**
- Analyze complete collected luminosity for $F_2(x, Q^2)$, $F_2^{car{c}}(x, Q^2)$, $g_2^{car{c}}(x, Q^2)$, and measure $h_1(x, Q^2)$.

**RHIC & LHC:**
- Improve constraints on gluon and sea–quarks: polarized and unpolarized. DIS PDF’s $\leftrightarrow$ Collider PDF’s

**JLAB:**
- High precision measurements in the large $x$ domain at unpolarized and polarized targets; supplements HERA’s high precision measurements at small $x$. 
HERA and JLAB: Improve DVCS data

Theory widely developed, cf. rev. Belitsky & Radyushkin, 2005

Expected DVCS asymmetry \( A_{UT}^{\sin(\phi - \phi_S) \cos \phi} \) with \( b_v = 1, b_s = \infty, J_u = 0.4(0.2, 0.0), J_d = 0.0 \) in the Regge (left panel) and factorized (right panel) ansatz, at the average kinematics of the full measurement. \( E = 0 \) denotes zero effective contribution from the GPD E. The projected statistical error for 8M DIS events is shown. The systematic error is expected to not exceed the statistical one.

F. Ellinghaus et al. 2005

The measurement of \( L_q \) off data is model-dependent at the moment.

Lattice calculations at low pion masses are needed to complete the picture.

J. Blümlein

RECAPP, Allahabad

February 2009
Further study of proposed mechanisms needed: RHIC, LHC for nucleus-nucleus collisions.

**ep scattering:** partly different mechanisms

more studies would be welcome; link to higher twist contributions in gluon–dynamics

How do the non-perturbative and perturbative parts factorize?

Conservation laws and interplay between the small $x$ and medium $x$ range behaviour
New DIS Machines

Where to go?

- High energies: small $x$, large $Q^2$ desirable.
- High luminosities: ELIC/EIC: $\sqrt{s}$ between CERN and HERA energies

R. Ent, 2004
high precision physics
polarized and unpolarized

Would be an important extension of the present programmes in many respects.
What is the correct value of $\alpha_s(M_Z^2)$? $\overline{\text{MS}}$-analysis vs. scheme-invariant evolution helps. Compare non–singlet and singlet analysis; careful treatment of heavy flavor. (Theory & Experiment)

Flavor Structure of Sea-Quarks: More studies needed. (All Experiments)

Revisit polarized data upon completion of the 3-loop anomalous dimensions; NLO heavy flavor contributions needed. (Theory)

QCD at Twist 3: $g_2(x, Q^2)$, semi–exclusive Reactions, Transversity, diffraction in polarized scattering (HERMES, High Precision polarized experiments, JLAB, ELIC)

Comparison with Lattice Results: $\alpha_s$, Moments of Parton Distributions, Angular Momentum.