Target Mass Corrections to Diffractive Scattering

Johannes Blümlein, DESY

in collaboration with B. Geyer (U. Leipzig) and D. Robaschik (BTU Cottbus)

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- The Case $M^2, t = 0$
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1. The Formalism

Kinematic variables:

\[ Q^2 := -q^2, \quad W := (p + q)^2, \quad x := \frac{Q^2}{Q + W^2 - M^2} \]
\[ t := (p_2 - p_1)^2, \quad x_P := -\frac{2\eta}{2 - \eta} = \frac{Q^2 + M_X^2 - t}{Q^2 + W^2 - M^2} \geq x \]

Hadronic Tensor for diffractive scattering via single photon exchange:

\[
W_{\mu\nu}^{\text{unp}} = \left( -g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^2} \right) W_1 + \left( p_{1\mu} - q_{\mu} \frac{p_{1\nu} q}{q^2} \right) \left( p_{1\nu} - q_{\nu} \frac{p_{1\mu} q}{q^2} \right) \frac{W_3}{M^2} + \left( p_{2\mu} - q_{\mu} \frac{p_{2\nu} q}{q^2} \right) \left( p_{2\nu} - q_{\nu} \frac{p_{2\mu} q}{q^2} \right) \frac{W_4}{M^2} \\
+ \left[ \left( p_{1\mu} - q_{\mu} \frac{p_{1\nu} q}{q^2} \right) \left( p_{2\nu} - q_{\nu} \frac{p_{2\mu} q}{q^2} \right) + \left( p_{2\mu} - q_{\mu} \frac{p_{2\nu} q}{q^2} \right) \left( p_{1\nu} - q_{\nu} \frac{p_{1\mu} q}{q^2} \right) \right] \frac{W_5}{M^2},
\]

\[
W_{\mu\nu}^{\text{pol}} = i \left[ \hat{p}_{1\mu} \hat{p}_{2\nu} - \hat{p}_{1\nu} \hat{p}_{2\mu} \right] \varepsilon_{p_1, p_2, q, S} \frac{\hat{W}_1}{M^6} + i \left[ \hat{p}_{1\mu} \varepsilon_{\nu S p_1 q} - \hat{p}_{1\nu} \varepsilon_{\mu S p_1 q} \right] \frac{\hat{W}_2}{M^4} \\
+ i \left[ \hat{p}_{2\mu} \varepsilon_{\nu S p_1 q} - \hat{p}_{2\nu} \varepsilon_{\mu S p_1 q} \right] \frac{\hat{W}_3}{M^4} + i \left[ \hat{p}_{1\mu} \varepsilon_{\nu S p_1 q} - \hat{p}_{1\nu} \varepsilon_{\mu S p_1 q} \right] \frac{\hat{W}_4}{M^4} \\
+ i \left[ \hat{p}_{2\mu} \varepsilon_{\nu S p_2 q} - \hat{p}_{2\nu} \varepsilon_{\mu S p_2 q} \right] \frac{\hat{W}_5}{M^4} + i \left[ \hat{p}_{1\mu} \varepsilon_{\nu S p_2 q} - \hat{p}_{1\nu} \varepsilon_{\mu S p_2 q} \right] \frac{\hat{W}_6}{M^4} \\
+ i \left[ \hat{p}_{2\mu} \varepsilon_{\nu S p_1 p} - \hat{p}_{2\nu} \varepsilon_{\mu S p_1 p} \right] \frac{\hat{W}_7}{M^4} + i \varepsilon_{\mu \nu q S} \frac{\hat{W}_8}{M^2}.
\]
1. The Formalism

- The physical condition for diffraction: rapidity gap

\[ \hat{\eta} \simeq \ln(1/x_P) \gg 1 \]
\[ 1 \gg x_p \simeq -\eta \gtrsim x \]

The Method:
- We deal with the twist\(_{2}\) contributions only.
- Factorization for diffraction
- Due to the rapidity gap, we may apply A. Mueller’s generalized optical theorem and turn the outgoing nucleon into the initial state.
- We consider the Compton Operator \( \hat{T}_{\mu\nu} \) \( (\gamma^* + P \to \gamma^* + P) \)
- This operator is finally evaluated between the states:

\[ \langle p_1, -p_2; t | \cdots \cdots | p_1, -p_2; t \rangle \]

- Diffractive structure functions & parton densities arise in this way.
- The notion of “pomeron” is not referred to at all.

We only consider physical objects.
2. The Case $M^2 = t = 0$

Lorentz Structure

\[
W_2 = W_3 + (1 - x_P)W_5 + (1 - x_P)^2W_4 \\
\hat{W}_9 = \hat{W}_2 + (1 - x_P)[\hat{W}_3 + \hat{W}_4] + (1 - x_P)^2\hat{W}_5
\]

I.e.: same structure as for inclusive DIS: 2 unpolarized and 2 polarized structure functions.

\[
\hat{T}_{\mu\nu}(x) \approx -e^2 \frac{\tilde{x}}{2i\pi^2(x^2 - i\epsilon)^2} \left[ S_{\mu\nu}^{\alpha\lambda}O_\alpha \left( \frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2} \right) - \epsilon_{\mu\nu}^{\alpha\lambda}O_{5\alpha} \left( \frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2} \right) \right]
\]

\[
O_\alpha(\kappa_1\tilde{x}, \kappa_2\tilde{x}) = i(\Omega_\alpha(\kappa_1\tilde{x}, \kappa_2\tilde{x}) - \Omega_\alpha(\kappa_1\tilde{x}, \kappa_2\tilde{x})) \\
O_{\alpha,5}(\kappa_1\tilde{x}, \kappa_2\tilde{x}) = (\Omega_{\alpha,5}(\kappa_1\tilde{x}, \kappa_2\tilde{x}) + \Omega_{\alpha,5}(\kappa_1\tilde{x}, \kappa_2\tilde{x}))
\]
\[ \Omega^{\text{tw}2}_n(\kappa x, -\kappa x) = \partial_\alpha \int_0^1 d\tau \int \frac{d^4 u}{(2\pi)^4} \Omega_n(\mu)(u) \left\{ x^\mu (2 + x \partial) - \frac{1}{2} i \kappa \tau u^\mu x^2 \right\} (3 + x \partial) \mathcal{H}_2(u, \kappa \tau x) \]

\[ \mathcal{H}_\nu(u, \kappa x) = \sqrt{\pi} \left( \kappa \sqrt{(ux)^2 - u^2 x^2} \right)^{1/2 - \nu} J_{\nu - 1/2} \left( \frac{\kappa}{2} \sqrt{(ux)^2 - u^2 x^2} \right) e^{i\kappa (xu)/2} \]

\[
\langle p_1, p_2 | e^2 i (\Omega_\mu(u) - \Omega_\mu(-u)) | p_1, -p_2 \rangle = \sum_a \mathcal{K}_\mu^a(p_\pm) \int DZ \delta^{(4)}(u - p_- z_- - p_+ z_+) f_a(z_-, z_+; t) \\
\langle p_1, p_2 | e^2 (\Omega_{5\mu}(u) + \Omega_{5\mu}(-u)) | p_1, -p_2 \rangle = \sum_a \mathcal{K}_\mu^a(p_\pm, S) \int DZ \delta^{(4)}(u - p_- z_- - p_+ z_+) f_{(5)a}(z_-, z_+; t)
\]

Kinematic factors:

\[ \mathcal{K}_1^\mu = p_-^\mu, \quad \mathcal{K}_2^\mu = \pi_-^\mu = p_+^\mu - p_-^\mu/\eta, \quad \mathcal{K}_5^\mu = S^\mu, \quad \mathcal{K}_5^{2\mu} = p_- p_2 \cdot S M^2, \quad \mathcal{K}_5^{3\mu} = \pi_- p_2 \cdot S M^2 \]
The absorptive part of:

$$T_{\mu\nu}(p_1, p_2, q) = \int_{1/\eta}^{-1/\eta} d\vartheta \left[ \left( -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) + \frac{2x}{q.p_1} \left( p_{1\mu} - q_\mu \frac{p_1.q}{q^2} \right) \left( p_{1\nu} - q_\nu \frac{p_1.q}{q^2} \right) \right]$$

yields a modified Callan-Gross relation (at leading order)

$$F_2(\beta, \eta, Q^2) = 2x F_1(\beta, \eta, Q^2)$$

Likewise one obtains the Wandzura-Wilczek relation:

$$G_2(\beta, \eta, Q^2) = -G_1(\beta, \eta, Q^2) + \int_{\beta}^{1} \frac{d\beta'}{\beta'} G_1(\beta', \eta, Q^2)$$
Applying again the non-forward formalism, one may derive the scaling violations of the Compton-Operator $\hat{T}$.

$$\mu^2 \frac{d}{d\mu^2} O^A(\kappa_+ \tilde{x}, \kappa_- \tilde{x}; \mu^2) = \int D\kappa' \gamma^{AB}(\kappa_+, \kappa_-, \kappa'_+, \kappa'_-; \mu^2) O^B(\kappa'_+ \tilde{x}, \kappa'_- \tilde{x}; \mu^2)$$

$$\langle p_1, -p_2; t | O^A(\kappa_+ \tilde{x}, \kappa_- \tilde{x}; \mu^2) | p_1, -p_2; t \rangle (\tilde{x} p_-)^{1-d_A} = \int_{-1/\eta}^{1/\eta} \exp[-\kappa'_- \tilde{x} p_- \vartheta'] f^A(\vartheta', \eta; t)$$

$$\tilde{O}^{AB}(u \vartheta' - \vartheta) = \begin{cases} 
\delta(u \vartheta' - \vartheta) & \text{for } A = B = q, G \\
\partial_u \delta(u \vartheta' - \vartheta) & \text{for } A = q; B = G \\
\vartheta(u \vartheta' - \vartheta) & \text{for } A = G; B = q 
\end{cases}$$

$$\vartheta' \int_0^1 du \tilde{O}^{AB}(u \vartheta' - \vartheta) \tilde{K}^{AB}(u, \mu^2) \equiv P^{AB} \left( \frac{\vartheta}{\vartheta'}, \mu^2 \right)$$

$$\mu^2 \frac{d}{d\mu^2} f^A(\vartheta, \eta; \mu^2) = \int_{\vartheta}^{-\text{sign}(\vartheta)/\eta} \frac{d\vartheta'}{\vartheta'} P^{AB} \left( \frac{\vartheta}{\vartheta'}, \mu^2 \right) f_B(\vartheta', \eta; \mu^2)$$

The value of $\vartheta$ is determined by the absorptive condition. In case of $t, M^2 \rightarrow 0$: $\vartheta = 2\beta$. 
\* \* \* Target Mass Corrections \* \* \*

- \( t \) and \( M^2 \) are of the same order and have to be resummed together.
- Target mass corrections are effective in the region of lower values of \( Q^2 \) and large values of \( \beta \).

Relevant Hadronic Momentum:

\[
\begin{align*}
\mathcal{P}^\mu &= p^\mu_-(1 - \zeta/\eta) + p^\mu_+ \zeta \\
\mathcal{P}^2 &= t(1 - \zeta/\eta)^2 + (4M^2 - t)\zeta^2 \geq 0 \\
q\mathcal{P} &= q^2/(2\beta) < 0
\end{align*}
\]

Correction of the type :

\[
\begin{align*}
F_1^a(\xi, \zeta) &= \Phi_a(\xi, \zeta) + \frac{\kappa \mathcal{P}^2}{[(q\mathcal{P})^2 - q^2 \mathcal{P}^2]^{1/2}} \Phi_1^a(\xi, \zeta) + \frac{\kappa^2[\mathcal{P}^2]^2}{(q\mathcal{P})^2 - q^2 \mathcal{P}^2} \Phi_2^a(\xi, \zeta) \\
F_2^{a}(\xi, \zeta) &= \Phi_a(\xi, \zeta) + \frac{3\kappa \mathcal{P}^2}{[(q\mathcal{P})^2 - q^2 \mathcal{P}^2]^{1/2}} \Phi_1^a(\xi, \zeta) + \frac{3\kappa^2[\mathcal{P}^2]^2}{(q\mathcal{P})^2 - q^2 \mathcal{P}^2} \Phi_2^a(\xi, \zeta)
\end{align*}
\]

(non-integrated: \( \zeta \)) structure functions

\( \Phi_a^{(0)}(\xi, \zeta) = f_a(\xi, \zeta), \Phi_a^k(\xi, \zeta) \) - iterated integrals.
The absorptive condition: actually 4 δ-functions; only one is physical.

\[ \xi \equiv 1 = -\frac{2\beta}{\kappa \nu} \frac{1}{1 + \sqrt{1 + 4\beta^2 P^2 / Q^2}} \]

In the corrections \( P^2(t, M^2, \zeta, \eta) \) in the diffractive case plays the same role as \( M^2 \) in DIS. Likewise, \( x \to \beta \).

**General Problem:** \( \zeta \) emerges as internal variable, describing transverse degrees of freedom between \( p_1 \) and \( p_2 \). This variable is not really accessible through the process kinematics and has to be integrated out.

This implies both new structure functions and generally destroys the notion of diffractive parton densities. This is valid whenever these corrections are non-negligible.
\[
\frac{1}{2\pi} \text{Im} T_{\{\mu \nu\}}(q) = -g_{\mu \nu} U_1(\beta, \eta) + \frac{p_T^\mu p_T^\nu}{M^2} U_2(\beta, \eta) + \frac{p_T^- \pi^- + p_T^- \pi^-}{M^2} U_3(\beta, \eta) + \frac{\pi^- \pi^-}{M^2} U_4(\beta, \eta)
\]

\[
U_1(\beta, \eta) = \int d\zeta \left[ T_1^1(\xi, \zeta) + \frac{p_T^-}{\mathcal{P}} T_2^1(\xi, \zeta) + \frac{p_T^-}{\mathcal{P}} T_2^1(\xi, \zeta) \right],
\]

\[
U_2(\beta, \eta) = \int d\zeta \left\{ \frac{M^2}{(\mathcal{P}_T)^2} \left[ T_3^1(\xi, \zeta) + \frac{p_T^-}{\mathcal{P}} T_4^1(\xi, \zeta) + \frac{p_T^-}{\mathcal{P}} T_4^1(\xi, \zeta) \right] + \frac{M^2}{\mathcal{P}} T_5^1(\xi, \zeta) \right\},
\]

\[
U_3(\beta, \eta) = \int d\zeta \zeta \left\{ \frac{M^2}{(\mathcal{P}_T)^2} \left[ T_3^1(\xi, \zeta) + \frac{p_T^-}{\mathcal{P}} T_4^1(\xi, \zeta) + \frac{p_T^-}{\mathcal{P}} T_4^1(\xi, \zeta) \right] + \frac{M^2}{\mathcal{P}} \left[ T_5^1(\xi, \zeta) + \frac{1}{\zeta} T_5^2(\xi, \zeta) \right] \right\},
\]

\[
U_4(\beta, \eta) = \int d\zeta \zeta^2 \left\{ \frac{M^2}{(\mathcal{P}_T)^2} \left[ T_3^1(\xi, \zeta) + \frac{p_T^-}{\mathcal{P}} T_4^1(\xi, \zeta) + \frac{p_T^-}{\mathcal{P}} T_4^1(\xi, \zeta) \right] + \frac{M^2}{\mathcal{P}} \frac{1}{\zeta} T_5^2(\xi, \zeta) \right\}.
\]

The (definite) $\zeta$-integrals have to be carried out.

\[\implies\] 4 structure functions.
\[\text{Im } T_{[\mu\nu]}^{\text{tw2}}(q) = -\pi \epsilon_{\mu\nu}^{\alpha\beta} \left\{ \frac{q_\alpha S_\beta^T}{q p_-} (G_{11} + G_{12}) + \frac{q_\alpha p_-^{T\beta}}{q p_-} \left[ -\frac{q S}{q p_-} G_{12} + \frac{p_2 S}{M^2} G_{21} + \frac{1}{2Q^2} \left( p_-^2 G_{20} + p_- \pi_-(G_{30} + H_{20}) + \pi_-^2 H_{30} \right) + M^2 G_{10} + M^2 \frac{\eta - 1}{\eta} H_{10} \right] \right\} + \frac{q_\alpha \pi_-^{T\beta}}{q p_-} \left[ -\frac{q S}{q p_-} H_{12} + \frac{p_2 S}{M^2} \left( G_{31} + G_{32} - H_{22} \right) + \frac{1}{2Q^2} \left( p_-^2 H_{20} + p_- \pi_-(H_{30} + K_{20}) + \pi_-^2 K_{30} + M^2 H_{10} + M^2 \frac{\eta - 1}{\eta} K_{10} \right) \right] \right\} \}

p_-^2 = t, \quad p_- \pi_- = -t/\eta, \quad \pi_-^2 = 4M^2 - t(1 - 1/\eta^2), \quad \frac{\eta - 1}{\eta} = \frac{2\beta}{x}.

\implies 8 \text{ structure functions.}
The Polarized Hadronic Tensor

Definition of structure functions

\[ g_{a1}(\beta, \eta, \zeta) = \beta \frac{\partial}{\partial \beta} \beta \frac{\partial}{\partial \beta} F_a(\beta, \eta, \zeta) \]
\[ g_{a2}(\beta, \eta, \zeta) = -\beta \frac{\partial}{\partial \beta} (\beta \frac{\partial}{\partial \beta} + 1) F_a(\beta, \eta, \zeta) \]
\[ g_{a0}(\beta, \eta, \zeta) = -2 \beta \frac{\partial}{\partial \beta} (\beta \frac{\partial}{\partial \beta} - 1) \beta \xi F_a(\beta, \eta, \zeta) . \]

\[ G_{aj}(\beta, \eta) = \int d\zeta g_{aj}(\beta, \eta, \zeta), \]
\[ H_{aj}(\beta, \eta) = \int d\zeta \zeta g_{aj}(\beta, \eta, \zeta), \]
\[ K_{aj}(\beta, \eta) = \int d\zeta \zeta^2 g_{aj}(\beta, \eta, \zeta), \]

Wandzura-Wilczek Relations:

\[ G_{a2}^{tw}(\beta, \eta) = -G_{a1}^{tw}(\beta, \eta) + \int_{\beta}^1 \frac{dy}{y} G_{a1}^{tw}(y, \eta) \]
\[ H_{a2}^{tw}(\beta, \eta) = -H_{a1}^{tw}(\beta, \eta) + \int_{\beta}^1 \frac{dy}{y} H_{a1}^{tw}(y, \eta) \]
Below the $\zeta$-integral several relations exist. Not all of them hold after this integral on the level of Observables.

\[
W_{a_1}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) + W_{a_L}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) = \frac{(1 + 4\beta^2 \mathcal{P}/Q^2)}{(-4\beta)} \frac{2q\mathcal{P}}{\mathcal{P}^2} W_{a_2}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta)
\]

\[
= \frac{(\mathcal{P}^T)^2}{\mathcal{P}^2} W_{a_2}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta).
\]

\[
2W_{a_0}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) = W_{a_L}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) - 2W_{a_1}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta),
\]

\[
\frac{1}{2} W_{a_1}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) = \sqrt{1 + 4\beta^2 \mathcal{P}^2/Q^2} \frac{\partial}{\partial \beta} W_{a_L}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta)
\]

\[
+ \left(1 - \frac{2}{\sqrt{1 + 4\beta^2 \mathcal{P}^2/Q^2}}\right) W_{a_L}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta)
\]

\[
- \frac{4\beta^2 \mathcal{P}^2/Q^2}{[1 + 4\beta^2 \mathcal{P}^2/Q^2]^{3/2}} \int_\beta^1 \frac{d\rho}{\rho^2} W_{a_L}^{\text{diff}}(\xi(\beta\rho), \beta\rho, \eta; \zeta).
\]

- The Callan–Gross Relation is broken; No other relations are obtained on the level of structure functions.

$\implies$ 4 unpolarized diffractive structure functions.
When do diffractive parton densities exist?

- Twist-2 approximation $t, M^2 \rightarrow 0$

- They are universal as building blocks for all structure functions involved. (for the DIS process.)

- $M^2$ and $t$ corrections break this picture, due to the presence of the $\zeta$-integrals.

- Below the definite $\zeta$-integral (with no external variable in the boundaries) un-integrated pre-parton densities $f_a(\zeta, \eta, \beta, Q^2, t)$ exist.

- One still may decompose the different structure functions w.r.t.

$$F^k_c(\eta, \beta, Q^2; t) = \sum_a \int d\zeta \zeta^k f_{a,c}(\zeta, \eta, \beta, Q^2; t)$$

- In regions with only small $M^2, t$ corrections the partonic description does thoroughly apply.
4. Conclusions

- DIS Diffractive Scattering can be described taking the expectation value of the Compton Operator between the diffractive states \( \langle p_1, p_2; t \rangle \) obtained by applying A. Mueller’s generalized optical theorem.

- In the limit \( M^2, t \to 0 \), 2 polarized and 2 unpolarized structure functions contribute to the DIS diffractive scattering cross section at twist \( \tau = 2 \).

- They are related by a modified Callan-Gross relation (in lowest order), resp. the Wandzura-Wilzcek relation (all orders).

- Target Mass Corrections \( (M^2, t\text{-resummation}) \) is required in the region of large values of \( \beta \) and low values of \( Q^2 \).

- The set of genuine diffractive structure functions becomes larger due to these effects; 4 unpolarized SF’s and 8 polarized SF’s - with one relation.

- These structure functions can be decomposed into generally different diffractive parton densities due to the \( \zeta \)-integral.

- The scaling violations of the twist \( \tau = 2 \) contribution to the diffractive structure functions are described by the evolution equations for forward scattering replacing \( x \to \beta \).

- The present approach results into a thorough description demanding a rapidity gap without any need to invoke a “pomeron”.

Johannes Blümlein  
DIS 2007, Munich  
April 2007