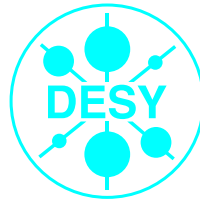


Target Mass Corrections to Diffractive Scattering

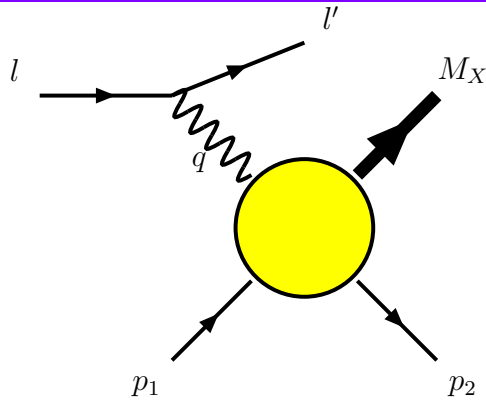
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- The Formalism
- The Case $M^2, t = 0$
- Target Mass Corrections
- Conclusions

Refs.: J. Blümlein, B. Geyer and D. Robaschik, Nucl. Phys. **B755** (2006) 112;
J. Blümlein and D. Robaschik, Phys. Lett. **B517** (2001) 222; Phys. Rev. **D65** (2002) 096002.



Kinematic variables:

$$Q^2 := -q^2, \quad W := (p + q)^2, \quad x := \frac{Q^2}{Q^2 + W^2 - M^2}$$

$$t := (p_2 - p_1)^2, \quad x_P := -\frac{2\eta}{2 - \eta} = \frac{Q^2 + M_X^2 - t}{Q^2 + W^2 - M^2} \geq x$$

Hadronic Tensor for **diffractive scattering** via **single photon exchange**:

$$W_{\mu\nu}^{unp} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) W_1 + \left(p_{1\mu} - q_\mu \frac{p_1 \cdot q}{q^2} \right) \left(p_{1\nu} - q_\nu \frac{p_1 \cdot q}{q^2} \right) \frac{W_3}{M^2} + \left(p_{2\mu} - q_\mu \frac{p_2 \cdot q}{q^2} \right) \left(p_{2\nu} - q_\nu \frac{p_2 \cdot q}{q^2} \right) \frac{W_4}{M^2} \\ + \left[\left(p_{1\mu} - q_\mu \frac{p_1 \cdot q}{q^2} \right) \left(p_{2\nu} - q_\nu \frac{p_2 \cdot q}{q^2} \right) + \left(p_{2\mu} - q_\mu \frac{p_2 \cdot q}{q^2} \right) \left(p_{1\nu} - q_\nu \frac{p_1 \cdot q}{q^2} \right) \right] \frac{W_5}{M^2},$$

$$W_{\mu\nu}^{pol} = i [\hat{p}_{1\mu} \hat{p}_{2\nu} - \hat{p}_{1\nu} \hat{p}_{2\mu}] \varepsilon_{p_1, p_2, q, S} \frac{\hat{W}_1}{M^6} + i [\hat{p}_{1\mu} \varepsilon_{\nu S p_1 q} - \hat{p}_{1\nu} \varepsilon_{\mu S p_1 q}] \frac{\hat{W}_2}{M^4} \\ + i [\hat{p}_{2\mu} \varepsilon_{\nu S p_1 q} - \hat{p}_{2\nu} \varepsilon_{\mu S p_1 q}] \frac{\hat{W}_3}{M^4} + i [\hat{p}_{1\mu} \varepsilon_{\nu S p_2 q} - \hat{p}_{1\nu} \varepsilon_{\mu S p_2 q}] \frac{\hat{W}_4}{M^4} \\ + i [\hat{p}_{2\mu} \varepsilon_{\nu S p_2 q} - \hat{p}_{2\nu} \varepsilon_{\mu S p_2 q}] \frac{\hat{W}_5}{M^4} + i [\hat{p}_{1\mu} \hat{\varepsilon}_{\nu p_1 p_2 S} - \hat{p}_{1\nu} \hat{\varepsilon}_{\mu p_1 p_2 S}] \frac{\hat{W}_6}{M^4} \\ + i [\hat{p}_{2\mu} \hat{\varepsilon}_{\nu p_1 p_2 S} - \hat{p}_{2\nu} \hat{\varepsilon}_{\mu p_1 p_2 S}] \frac{\hat{W}_7}{M^4} + i \varepsilon_{\mu\nu q S} \frac{\hat{W}_8}{M^2}.$$

- The physical condition for diffraction: rapidity gap

$$\hat{\eta} \simeq \ln(1/x_P) \gg 1$$

$$1 \gg x_p \simeq -\eta \gtrsim x$$

The Method :

- We deal with the **twist-2** contributions only.
- Factorization for diffraction
- Due to the rapidity gap, we may apply **A. Mueller's** generalized optical theorem and turn the outgoing nucleon into the initial state.
- We consider the **Compton Operator** $\hat{T}_{\mu\nu}$ ($\gamma^* + P \rightarrow \gamma^* + P$)
- This operator is finally evaluated between the states:

$$\langle p_1, -p_2; t | \dots \dots | p_1, -p_2; t \rangle$$

- Diffractive structure functions & parton densities arise in this way.
- The notion of “**pomeron**” is not referred to at all.

We only consider physical objects.

Lorentz Structure

$$\begin{aligned}
 W_2 &= W_3 + (1 - x_P)W_5 + (1 - x_P)^2W_4 \\
 \hat{W}_9 &= \hat{W}_2 + (1 - x_P)[\hat{W}_3 + \hat{W}_4] + (1 - x_P)^2\hat{W}_5
 \end{aligned}$$

I.e.: same structure as for inclusive DIS: 2 unpolarized and 2 polarized structure functions.

$$\hat{T}_{\mu\nu}(x) \approx -e^2 \frac{\tilde{x}}{2i\pi^2(x^2 - i\epsilon)^2} \left[S_{\mu\nu}^{\alpha\lambda} O_\alpha \left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2} \right) - \epsilon_{\mu\nu}^{\alpha\lambda} O_{5\alpha} \left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2} \right) \right]$$

$$\begin{aligned}
 O_\alpha(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) &= i(\Omega_\alpha(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) - \Omega_\alpha(\kappa_1 \tilde{x}, \kappa_2 \tilde{x})) \\
 O_{\alpha,5}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) &= (\Omega_{\alpha,5}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}) + \Omega_{\alpha,5}(\kappa_1 \tilde{x}, \kappa_2 \tilde{x}))
 \end{aligned}$$

$$\Omega_{(5)\alpha}^{\text{tw}2}(\kappa x, -\kappa x) = \partial_\alpha \int_0^1 d\tau \int \frac{d^4 u}{(2\pi)^4} \Omega_{(5)\mu}(u) \left\{ x^\mu (2 + x\partial) - \frac{1}{2} i\kappa\tau u^\mu x^2 \right\} (3 + x\partial) \mathcal{H}_2(u, \kappa\tau x)$$

$$\mathcal{H}_\nu(u, \kappa x) = \sqrt{\pi} \left(\kappa \sqrt{(ux)^2 - u^2 x^2} \right)^{1/2-\nu} J_{\nu-1/2} \left(\frac{\kappa}{2} \sqrt{(ux)^2 - u^2 x^2} \right) e^{i\kappa(xu)/2}$$

$$\langle p_1, p_2 | e^2 i(\Omega_\mu(u) - \Omega_\mu(-u)) | p_1, -p_2 \rangle = \sum_a \mathcal{K}_\mu^a(p_\pm) \int DZ \delta^{(4)}(u - p_- z_- - p_+ z_+) f_a(z_-, z_+; t)$$

$$\langle p_1, p_2 | e^2 (\Omega_{5\mu}(u) + \Omega_{5\mu}(-u)) | p_1, -p_2 \rangle = \sum_a \mathcal{K}_\mu^a(p_\pm, S) \int DZ \delta^{(4)}(u - p_- z_- - p_+ z_+) f_{(5)a}(z_-, z_+; t)$$

Kinematic factors :

$$\mathcal{K}^{1\mu} = p_-^\mu, \quad \mathcal{K}^{2\mu} = \pi_-^\mu = p_+^\mu - p_-^\mu/\eta, \quad \mathcal{K}_5^{1\mu} = S^\mu, \quad \mathcal{K}_5^{2\mu} = p_-^\mu \frac{p_2 \cdot S}{M^2}, \quad \mathcal{K}_5^{3\mu} = \pi_-^\mu \frac{p_2 \cdot S}{M^2}$$

The absorptive part of :

$$\begin{aligned}
 T_{\mu\nu}(p_1, p_2, q) &= \int_{1/\eta}^{-1/\eta} d\vartheta \left[\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) + \frac{2x}{q \cdot p_1} \left(p_{1\mu} - q_\mu \frac{p_1 \cdot q}{q^2} \right) \left(p_{1\nu} - q_\nu \frac{p_1 \cdot q}{q^2} \right) \right. \\
 &\quad \times \left. \left[\frac{\hat{f}(\vartheta, \eta)}{\vartheta - 2\beta + i\varepsilon} - \frac{\hat{f}(-\vartheta, \eta)}{\vartheta - 2\beta + i\varepsilon} \right] \right. \\
 &\quad \left. + 2 \frac{p_{1\mu} p_{1\nu}}{q \cdot p_1} \int_{1/\eta}^{-1/\eta} d\vartheta (\vartheta x_P - 2x) \frac{\hat{f}(\vartheta, \eta)}{\vartheta - 2\beta + i\varepsilon} \right]
 \end{aligned}$$

yields a modified Callan-Gross relation (at leading order)

$$F_2(\beta, \eta, Q^2) = 2x F_1(\beta, \eta, Q^2)$$

Likewise one obtains the Wandzura-Wilczek relation :

$$G_2(\beta, \eta, Q^2) = -G_1(\beta, \eta, Q^2) + \int_{\beta}^1 \frac{d\beta'}{\beta'} G_1(\beta', \eta, Q^2)$$

Applying again the non-forward formalism, one may derive the scaling violations of the Compton-Operator \hat{T} .

$$\mu^2 \frac{d}{d\mu^2} O^A(\kappa_+ \tilde{x}, \kappa_- \tilde{x}; \mu^2) = \int D\kappa' \gamma^{AB}(\kappa_+, \kappa_-, \kappa'_+, \kappa'_-; \mu^2) O^B(\kappa'_+ \tilde{x}, \kappa'_- \tilde{x}; \mu^2)$$

$$\langle p_1, -p_2; t | O^A(\kappa_+ \tilde{x}, \kappa_- \tilde{x}; \mu^2) | p_1, -p_2; t \rangle (\tilde{x} p_-)^{1-d_A} = \int_{1/\eta}^{-1/\eta} \exp[-i\kappa'_- \tilde{x} p_- \vartheta'] f^A(\vartheta', \eta; t)$$

$$\tilde{O}^{AB}(u\vartheta' - \vartheta) = \begin{cases} \delta(u\vartheta' - \vartheta) & \text{for } A = B = q, G \\ \partial_u \delta(u\vartheta' - \vartheta) & \text{for } A = q; B = G \\ \theta(u\vartheta' - \vartheta) & \text{for } A = G; B = q \end{cases}$$

$$\vartheta' \int_0^1 du \tilde{O}^{AB}(u\vartheta' - \vartheta) \hat{K}^{AB}(u, \mu^2) \equiv P^{AB} \left(\frac{\vartheta}{\vartheta'}; \mu^2 \right)$$

$$\mu^2 \frac{d}{d\mu^2} f^A(\vartheta, \eta; \mu^2) = \int_{\vartheta}^{-\text{sign}(\vartheta)/\eta} \frac{d\vartheta'}{\vartheta'} P^{AB} \left(\frac{\vartheta}{\vartheta'}, \mu^2 \right) f_B(\vartheta', \eta; \mu^2)$$

The value of ϑ is determined by the absorptive condition. In case of $t, M^2 \rightarrow 0$: $\vartheta = 2\beta$.

- t and M^2 are of the same order and have to be resummed together.
- Target mass corrections are effective in the region of lower values of Q^2 and large values of β .

Relevant Hadronic Momentum:

$$\begin{aligned}\mathcal{P}^\mu &= p_-^\mu(1 - \zeta/\eta) + p_+^\mu\zeta \\ \mathcal{P}^2 &= t(1 - \zeta/\eta)^2 + (4M^2 - t)\zeta^2 \geq 0 \\ q\mathcal{P} &= q^2/(2\beta) < 0\end{aligned}$$

Correction of the type :

$$\begin{aligned}F_1^a(\xi, \zeta) &\equiv \Phi_a(\xi, \zeta) + \frac{\kappa\mathcal{P}^2}{[(q\mathcal{P})^2 - q^2\mathcal{P}^2]^{1/2}}\Phi_a^1(\xi, \zeta) + \frac{\kappa^2[\mathcal{P}^2]^2}{(q\mathcal{P})^2 - q^2\mathcal{P}^2}\Phi_a^2(\xi, \zeta) \\ F_2^a(\xi, \zeta) &\equiv \Phi_a(\xi, \zeta) + \frac{3\kappa\mathcal{P}^2}{[(q\mathcal{P})^2 - q^2\mathcal{P}^2]^{1/2}}\Phi_a^1(\xi, \zeta) + \frac{3\kappa^2[\mathcal{P}^2]^2}{(q\mathcal{P})^2 - q^2\mathcal{P}^2}\Phi_a^2(\xi, \zeta)\end{aligned}$$

(non-integrated: ζ) structure functions

$\Phi_a^{(0)}(\xi, \zeta) = f_a(\xi, \zeta)$, $\Phi_a^k(\xi, \zeta)$ - iterated integrals.

The absorptive condition: actually 4 δ -functions; only one is physical.

$$\xi \equiv 1 = -\frac{2\beta}{\kappa\vartheta} \frac{1}{1 + \sqrt{1 + 4\beta^2\mathcal{P}^2/Q^2}}$$

In the corrections $\mathcal{P}^2(t, M^2, \zeta, \eta)$ in the diffractive case plays the same role as M^2 in DIS.

Likewise, $x \rightarrow \beta$.

General Problem: ζ emerges as internal variable, describing transverse degrees of freedom between p_1 and p_2 . This variable is not really accessible through the process kinematics and has to be integrated out.

This implies both new structure functions and generally **destroys** the notion of **diffractive parton densities**. This is valid whenever these corrections are non-negligible.

$$\begin{aligned} \frac{1}{2\pi} \text{Im} T_{\{\mu\nu\}}(q) &= -g_{\mu\nu}^T U_1(\beta, \eta) + \frac{p_{-\mu}^T p_{-\nu}^T}{M^2} U_2(\beta, \eta) \\ &\quad + \frac{p_{-\mu}^T \pi_{-\nu}^T + p_{-\nu}^T \pi_{-\mu}^T}{M^2} U_3(\beta, \eta) + \frac{\pi_{-\mu}^T \pi_{-\nu}^T}{M^2} U_4(\beta, \eta) \end{aligned}$$

$$\begin{aligned} U_1(\beta, \eta) &= \int d\zeta \left[T_1^1(\xi, \zeta) + \frac{\mathcal{P}p_-}{\mathcal{P}^2} T_2^1(\xi, \zeta) + \frac{\mathcal{P}\pi_-}{\mathcal{P}^2} T_2^2(\xi, \zeta) \right], \\ U_2(\beta, \eta) &= \int d\zeta \left\{ \frac{M^2}{(\mathcal{P}^T)^2} \left[T_3^1(\xi, \zeta) + \frac{\mathcal{P}p_-}{\mathcal{P}^2} T_4^1(\xi, \zeta) + \frac{\mathcal{P}\pi_-}{\mathcal{P}^2} T_4^2(\xi, \zeta) \right] + \frac{M^2}{\mathcal{P}^2} T_5^1(\xi, \zeta) \right\}, \\ U_3(\beta, \eta) &= \int d\zeta \zeta \left\{ \frac{M^2}{(\mathcal{P}^T)^2} \left[T_3^1(\xi, \zeta) + \frac{\mathcal{P}p_-}{\mathcal{P}^2} T_4^1(\xi, \zeta) + \frac{\mathcal{P}\pi_-}{\mathcal{P}^2} T_4^2(\xi, \zeta) \right] \right. \\ &\quad \left. + \frac{M^2}{\mathcal{P}^2} \left[T_5^1(\xi, \zeta) + \frac{1}{\zeta} T_5^2(\xi, \zeta) \right] \right\}, \\ U_4(\beta, \eta) &= \int d\zeta \zeta^2 \left\{ \frac{M^2}{(\mathcal{P}^T)^2} \left[T_3^1(\xi, \zeta) + \frac{\mathcal{P}p_-}{\mathcal{P}^2} T_4^1(\xi, \zeta) + \frac{\mathcal{P}\pi_-}{\mathcal{P}^2} T_4^2(\xi, \zeta) \right] + \frac{M^2}{\mathcal{P}^2} \frac{1}{\zeta} T_5^2(\xi, \zeta) \right\}. \end{aligned}$$

The (definite) ζ -integrals have to be carried out.

\implies 4 structure functions.

$$\begin{aligned}
 \text{Im } T_{[\mu\nu]}^{\text{tw}2}(q) = & -\pi \epsilon_{\mu\nu}^{\alpha\beta} \left\{ \frac{q_\alpha S_\beta^{\text{T}}}{qp_-} (G_{11} + G_{12}) \right. \\
 & + \frac{q_\alpha p_{-\beta}^{\text{T}}}{qp_-} \left[-\frac{qS}{qp_-} G_{12} + \frac{p_2 S}{M^2} \left[G_{21} \right. \right. \\
 & \quad \left. \left. + \frac{1}{2Q^2} \left(p_-^2 G_{20} + p_- \pi_- (G_{30} + H_{20}) + \pi_-^2 H_{30} \right) + M^2 G_{10} + M^2 \frac{\eta-1}{\eta} H_{10} \right] \right] \\
 & + \frac{q_\alpha \pi_{-\beta}^{\text{T}}}{qp_-} \left[-\frac{qS}{qp_-} H_{12} + \frac{p_2 S}{M^2} \left[G_{31} + G_{32} - H_{22} \right. \right. \\
 & \quad \left. \left. + \frac{1}{2Q^2} \left(p_-^2 H_{20} + p_- \pi_- (H_{30} + K_{20}) + \pi_-^2 K_{30} + M^2 H_{10} + M^2 \frac{\eta-1}{\eta} K_{10} \right) \right] \right] \left. \right\},
 \end{aligned}$$

$$p_-^2 = t, \quad p_- \pi_- = -t/\eta, \quad \pi_-^2 = 4M^2 - t(1 - 1/\eta^2), \quad \frac{\eta-1}{\eta} = \frac{2\beta}{x}.$$

\implies 8 structure functions.

Definition of structure functions

$$\begin{aligned}
 g_{a1}(\beta, \eta, \zeta) &= \beta \frac{\partial}{\partial \beta} \beta \frac{\partial}{\partial \beta} \mathcal{F}_a(\beta, \eta, \zeta) \\
 g_{a2}(\beta, \eta, \zeta) &= -\beta \frac{\partial}{\partial \beta} \left(\beta \frac{\partial}{\partial \beta} + 1 \right) \mathcal{F}_a(\beta, \eta, \zeta) \\
 g_{a0}(\beta, \eta, \zeta) &= -2 \beta \frac{\partial}{\partial \beta} \left(\beta \frac{\partial}{\partial \beta} - 1 \right) \beta \xi \mathcal{F}_a(\beta, \eta, \zeta).
 \end{aligned}$$

$$\begin{aligned}
 G_{aj}(\beta, \eta) &= \int d\zeta g_{aj}(\beta, \eta, \zeta), \\
 H_{aj}(\beta, \eta) &= \int d\zeta \zeta g_{aj}(\beta, \eta, \zeta), \\
 K_{aj}(\beta, \eta) &= \int d\zeta \zeta^2 g_{aj}(\beta, \eta, \zeta),
 \end{aligned}$$

Wandzura-Wilczek Relations :

$$\begin{aligned}
 G_{a2}^{\text{tw}2}(\beta, \eta) &= -G_{a1}^{\text{tw}2}(\beta, \eta) + \int_{\beta}^1 \frac{dy}{y} G_{a1}^{\text{tw}2}(y, \eta) \\
 H_{a2}^{\text{tw}2}(\beta, \eta) &= -H_{a1}^{\text{tw}2}(\beta, \eta) + \int_{\beta}^1 \frac{dy}{y} H_{a1}^{\text{tw}2}(y, \eta)
 \end{aligned}$$

Below the ζ -integral several relations exist. Not all of them hold after this integral on the level of Observables.

$$\begin{aligned} W_{a1}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) + W_{aL}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) &= \frac{(1 + 4\beta^2 \mathcal{P}^2 / Q^2)}{(-4\beta)} \frac{2q\mathcal{P}}{\mathcal{P}^2} W_{a2}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) \\ &= \frac{(\mathcal{P}^T)^2}{\mathcal{P}^2} W_{a2}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) . \end{aligned}$$

$$\begin{aligned} 2V_{a0}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) &= W_{aL}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) - 2W_{a1}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) , \\ \frac{1}{2} V_{a1}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) &= \sqrt{1 + 4\beta^2 \mathcal{P}^2 / Q^2} \beta \frac{\partial}{\partial \beta} W_{aL}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) \\ &\quad + \left(1 - \frac{2}{\sqrt{1 + 4\beta^2 \mathcal{P}^2 / Q^2}} \right) W_{aL}^{\text{diff}}(\xi(\beta), \beta, \eta; \zeta) \\ &\quad - \frac{4\beta^2 \mathcal{P}^2 / Q^2}{[1 + 4\beta^2 \mathcal{P}^2 / Q^2]^{3/2}} \int_{\beta}^1 \frac{d\rho}{\rho^2} W_{aL}^{\text{diff}}(\xi(\beta\rho), \beta\rho, \eta; \zeta) . \end{aligned}$$

• The Callan–Gross Relation is broken; No other relations are obtained on the level of structure functions.

⇒ 4 unpolarized diffractive structure functions.

When do diffractive parton densities exist ?

- Twist-2 approximation $t, M^2 \rightarrow 0$
- They are universal as building blocks for all structure functions involved. (for the DIS process.)
- M^2 and t corrections break this picture, due to the presence of the ζ -integrals.
- Below the definite ζ -integral (with no external variable in the boundaries) un-integrated pre-parton densities $f_a(\zeta, \eta, \beta, Q^2, t)$ exist.
- One still may decompose the different structure functions w.r.t.

$$F_c^k(\eta, \beta, Q^2; t) = \sum_a \int d\zeta \zeta^k f_{a,c}(\zeta, \eta, \beta, Q^2; t)$$

- In regions with only small M^2, t corrections the partonic description does thoroughly apply.

- DIS Diffractive Scattering can be described taking the expectation value of the **Compton Operator** between the diffractive states $\langle p_1, p_2; t |$ obtained by applying A. Mueller's generalized optical theorem.
- In the limit $M^2, t \rightarrow 0$, **2 polarized and 2 unpolarized structure functions** contribute to the DIS diffractive scattering cross section **at twist $\tau = 2$** .
- They are related by a **modified Callan-Gross relation** (in lowest order), resp. the **Wandzura-Wilzcek relation** (all orders).
- Target Mass Corrections (M^2, t -resummation) is required in the region of large values of β and low values of Q^2 .
- The set of genuine diffractive structure functions becomes larger due to these effects; **4 unpolarized SF's and 8 polarized SF's - with one relation**.
- These structure functions can be decomposed into **generally different diffractive parton densities due to the ζ -integral**.
- The **scaling violations** of the twist $\tau = 2$ contribution to the diffractive structure functions are described by the **evolution equations for forward scattering** replacing $x \rightarrow \beta$.
- The present approach results into a thorough description demanding a **rapidity gap** without any need to invoke a "pomeron".