Good Morning to Seattle!
Status of Unpolarized PDFs and $\alpha_s(M_Z^2)$

The Major Goals

DIS Theory Status

Unpolarized Parton Distribution Functions

Predictions for TEVATRON and the LHC

$\Lambda_{QCD}$ and $\alpha_s(M_Z^2)$

Advanced Technologies to Evaluate Feynman Diagrams @ 3 Loops

Outlook
1. The Major Goals

- Precision Measurement of the Strong Coupling Constant $\alpha_s(M_Z^2)$
- Precision Measurement of the Unpolarized Parton Densities
- Higher Twist Effects
- Is there Saturation in DIS at small $x$? \(\Rightarrow\) answered by experiment.
- PDFs for TEVATRON, the LHC, and the EIC
Theory of DIS

Parton Model

Light Cone Expansion [f]

Twist 2

Higher Twist

Twist 3
Twist 4 ...

Sum Rules

Fixed Order PT: QCD

Splitting functions

Coefficient functions

\(O(\alpha_s^4)\) '73, '74, '80, '97

\(O(\alpha_s^3)\) '73, '82, '04

\(O(\alpha_s^3)\) '82, '92, '05, '09

Special Kinematics
Domain: Small x

\(O(\alpha_s^4)\) '75, '86

\(O(\alpha_s^4)\) '90 - '98

Resummations ?

Diffractive Scattering

Angular Momentum: q, G

More General View
Non-forw. scattering

Higher Orders

New Algorithms
Novel Mathematics

'69, QCD: '72

'69 - 72

\'60ies - now

\'77

\'75, '86

\'90 - '98

\'72

= 1
Status of Highest Order Calculations

- Running $\alpha_s$: $O(\alpha_s^4)$ Larin, van Ritbergen, Vermaseren 1997
- Unpol. anomalous dimensions and Wilson coefficients: $O(\alpha_s^3)$
  - Moch, Vermaseren, Vogt 2004/05
- Unpol. NS anomalous dimension 2nd Moment: $O(\alpha_s^4)$ Baikov, Chetyrkin 2006
- Pol. anomalous dimension: $O(\alpha_s^2)$; Mertig, van Neerven, 1995; Vogelsang 1995;
  $\Delta P^{qq} \Delta P_{qG}$: $O(\alpha_s^3)$ Moch, Rogal, Vermaseren, Vogt 2008
- Pol. Wilson coefficients: $O(\alpha_s^2)$; $\Delta C_{NS}^{qq}$, $\Delta C_{qG}$: van Neerven, Zijlstra 1994
- Transversity: $O(\alpha_s^2)$, some moments anom. dim.: $O(\alpha_s^3)$, Hayashigaki, Kanazawa, Kole; Kumano, Miyama; Vogelsang; 1997; Gracey 2006, HQ: JB, S.Kle in, B. Tödtli 2008
- Unpol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^2)$ Laenen, van Neerven, Riemersma, Smith, 1993
  - Fast Mellin Space code: Blümlein & Alekhin, 2003
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient $F_L$: $O(\alpha_s^3)$
  - Blümlein, De Freitas, van Neerven, S. Klein 2005
- $Q^2 \gg m^2$ Pol. Heavy Flavor Wilson Coefficient: $O(\alpha_s^2)$ van Neerven, Smith et al. 1996,
  Bierenbaum, Blümlein & Klein 2007
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient $F_2$: $O(\alpha_s^2 \epsilon)$: all operators
  (also polarized), Bierenbaum, Blümlein, Klein, Schneider, 2008; $O(\alpha_s^3)$: Moments 2–10(12,14)
  of the operator matrix elements, HQ Wilson coeff. Bierenbaum, Blümlein, Klein, 2008

emplaced at DESY (or in DESY collab.).
\[ F_j(x, Q^2) = \hat{f}_i(x, \mu^2) \otimes \sigma^i_j \left( \frac{Q^2}{\mu^2}, x \right) \]

\[ = \hat{f}_i(x, \mu^2) \otimes \Gamma^i_k \left( \alpha_s(R^2), \frac{M^2}{\mu^2}, \frac{M^2}{R^2} \right) \]

\[ \otimes C^k_j \left( \alpha_s(R^2), \frac{Q^2}{\mu^2}, \frac{M^2}{R^2}, x \right) \]

\[ \text{Move to Mellin space:} \]

\[ F_j(N) = \int_0^1 dx x^{N-1} F_j(x) \]

Diagonalization of the convolutions \( \otimes \) into ordinary products.
\[
\left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} - 2\gamma_\psi(g) \right] F_i(N) = 0
\]

\[
\left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} + \gamma_N^\kappa(g) - 2\gamma_\psi(g) \right] f_k(N) = 0
\]

\[
\left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} - \gamma_N^\kappa(g) \right] C_j^k(N) = 0
\]

**CALLAN–SYMANZIK** equations for mass factorization \( \equiv \)

**ALTARELLI–PARISI** evolution equations

**x-space :**

\[
\frac{d}{d \log(\mu^2)} \left( \begin{array}{c} q^+(x, Q^2) \\ G(x, Q^2) \end{array} \right) = \frac{\alpha_s}{2\pi} P(x, \alpha_s) \otimes \left( \begin{array}{c} q^+(x, Q^2) \\ G(x, Q^2) \end{array} \right)
\]

\[
P(x, \alpha_s) = P^{(0)}(x) + \frac{\alpha_s}{2\pi} P^{(1)}(x) + \left( \frac{\alpha_s}{2\pi} \right)^2 P^{(2)}(x) + \ldots
\]
Anomalous Dimensions and Wilson Coefficients

\[ \gamma_{qq}(N) \]

\[ \gamma_{gg}(N) \]

\[ \alpha_S = 0.2, N_f = 4 \]

Vermaseren, Moch, Vogt 2004
The Basic Functions of massless QCD to w=5: $\equiv$ 3 Loops

Representative: $S_1(N) = \psi(N+1) + \gamma_E$ and its derivatives.

**Weight w=3:**

\[ F_1(N) = M \left[ \frac{\ln(1+x)}{1+x} \right] (N) \]

\[ F_2(N) = M \left[ \frac{\text{Li}_2(x)}{1+x} \right] (N), \quad F_3(N) = M \left[ \left( \frac{\text{Li}_2(x)}{1-x} \right)_+ \right] (N) \]

Yndurain et al., 1981: $F_2(N)$

**Weight w=4:**

\[ F_4(N) = M \left[ \frac{S_{1,2}(x)}{1+x} \right] (N), \quad F_5(N) := M \left[ \left( \frac{S_{1,2}(x)}{1-x} \right)_+ \right] (N) \]

$F_3(N) - F_5(N)$: J.B., 2003; J.B., V. Ravindran, 2004
Weight w=5 :

\[ F_{6,7}(N) = M \left[ \frac{\text{Li}_4(x)}{1 \pm x} \right]^{(+)} (N), \quad F_8(N) = M \left[ \frac{S_{1,3}(x)}{1 + x} \right] (N), \]

\[ F_{9,10}(N) = M \left[ \frac{S_{2,2}(x)}{1 \pm x} \right]^{(+)} (N), \quad F_{11}(N) = M \left[ \frac{\text{Li}_2(x)}{1 + x} \right] (N), \]

\[ F_{12,13}(N) := M \left[ \left( \frac{\ln(x)S_{1,2}(-x) - \text{Li}_2^2(-x)/2}{1 \pm x} \right) \right]^{(+)} (N) \]

\[ F_6(N) - F_{13}(N) : J.B., S. Moch, 2004. \]

**Massless QCD to 3 Loops depends on 14 Functions.**

Weight w=6 :

Construct exact analytic continuations to complex $N$

The functions are meromorphic (up to soft corrections, which have a simple structure)

Asymptotic Representation

Recursion $z + 1 \rightarrow z$

Solve the Evolution Equations fully analytically and form an analytic expression for the Structure functions in Mellin Space at all $Q^2$

Include the heavy flavor Wilson coefficients in Mellin Space $\Rightarrow$ nearly accomplished to $O(a_s^3)$ I. Bierenbaum, JB, S. Klein (2009)

Perform a single fast, numerical Mellin inversion (at high precision)

$\Rightarrow$ Fastest and most Precise Way of Analysis
I will mainly discuss NNLO extractions in the following.
Direct $F_L(x, Q^2)$ Measurement at HERA

H1 Preliminary $F_L(x, Q^2)$

medium & high $Q^2$

$Q^2 = 12$ GeV$^2$
$Q^2 = 15$ GeV$^2$
$Q^2 = 20$ GeV$^2$
$Q^2 = 25$ GeV$^2$

$Q^2 = 35$ GeV$^2$
$Q^2 = 45$ GeV$^2$
$Q^2 = 60$ GeV$^2$
$Q^2 = 90$ GeV$^2$

$Q^2 = 120$ GeV$^2$
$Q^2 = 150$ GeV$^2$
$Q^2 = 200$ GeV$^2$
$Q^2 = 250$ GeV$^2$

$Q^2 = 300$ GeV$^2$
$Q^2 = 400$ GeV$^2$
$Q^2 = 500$ GeV$^2$
$Q^2 = 650$ GeV$^2$

$Q^2 = 800$ GeV$^2$

$F_L(x, Q^2)$

$10^{-3}$ $10^{-2}$ $10^{-1}$

$10^3$ $10^2$ $10^1$ $10^0$

$0.5$ $1.0$ $1.5$

○ H1 (Prelim.)

$E_p = 460, 575, 920$ GeV

H1 PDF 2000
Direct $F_L(x, Q^2)$ Measurement at HERA (H1-prel.)
Parton Distributions: Overview

H1 Collaboration

H1 PDF 2000: $Q^2 = 4 \text{ GeV}^2$

Fit to H1 data
- experimental errors
- model uncertainties

Fit to H1 + BCDMS data
- parton distribution
World Data Analysis: Valence Distributions

World data: NS-analysis

\[ W^2 > 12.5 \text{ GeV}^2, \, Q^2 > 4 \text{ GeV}^2 \]

N^{3}\text{LO}:

\[ \alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022} \]

Why an \( O(\alpha_s^4) \) analysis can be performed?

Assume an \( \pm 100\% \) error on the Pade approximant \( \rightarrow \pm 2 \text{ MeV} \) in \( \Lambda_{QCD} \)

\[
\gamma_{n}^{\text{approx}:3} = \frac{\gamma_n^{(2)}}{\gamma_n^{(1)}},
\]

Baikov & Chetyrkin, April 2006:

\[
\gamma_2^{3;NS} = \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[ \frac{3936832}{6561} - \frac{10240}{81} \zeta_3 \right] a_s^3 \\
+ \left[ \frac{1680283336}{1777147} - \frac{24873952}{6561} \zeta_3 + \frac{5120}{3} \zeta_4 - \frac{56969}{243} \zeta_5 \right] a_s^4
\]

The results agree better than 20\%. 

J. Blümlein

Status of Unpolarized PDFs and \( \alpha_s \left( M_Z^2 \right) \)

Seattle, WA, October 20th 2009
Valence Distributions

$F_2^p(x, Q^2) \times 2^i$

- H1
- ZEUS
- BCDMS

$W^2 > 12.5 \text{ GeV}^2$

$Q^2, \text{ GeV}^2$

$F_2^p(x, Q^2) \times 2^i$

- BCDMS
- NMC

- SLAC
- E665

$Q^2, \text{ GeV}^2$
Valence Distributions: higher twist

PROTON
\[ C_{HT}(x) \ [\text{GeV}^2] \]
\[ 4.0 \ \text{GeV}^2 < W^2 < 12.5 \ \text{GeV}^2 \]

DEUTERON
\[ C_{HT}(x) \ [\text{GeV}^2] \]
\[ 4.0 \ \text{GeV}^2 < W^2 < 12.5 \ \text{GeV}^2 \]

- agreement between \( p \) and \( d \) analysis, J.B., H. Böttcher, 2008
- LGT determination of interest
Slope of $F_2$ at low $x$

Very likely, that the $\overline{\text{MS}}$–gluon is remains positive!

J.B., A. Guffanti 2005
Flavor distributions: light quarks (NNLO)

Current Fitting Community (NNLO):
+ Many NLO analyses worldwide: CTEQ, NNPDF, H1, ZEUS, ...

S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102
Correct treatment of HQ very essential: FFNS, BSMN-schemes.
full lines: ABKM error band; dashed lines: MSTW08

J. Bl"umlein
Status of Unpolarized PDFs and $\alpha_s(M_Z^2)$
Seattle, WA, October 20th 2009 – p.21
Flavor distributions: strangeness

FIG. 3: The strange parton distribution $xS(x)$ from the measured HERMES multiplicity for charged kaons evolved to $Q_0^2 = 2.5$ GeV$^2$ assuming $\int D_S^K(z)dz = 1.27 \pm 0.13$. The solid curve is a 3-parameter fit for $S(x) = x^{-0.024}e^{-x/0.0404(1-x)}$, the dashed curve gives $xS(x)$ from CTEQ6L, and the dot-dash curve is the sum of light antiquarks from CTEQ6L.

Nice HERMES measurement (hep-ex/0803.2993); still to be understood.
Heavy quarks and gluon (NNLO)

S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102
full lines: ABKM error band; dashed lines: MSTW08
Jimenez-Delgado/ Reya (2008)
4. Some Predictions for Tevatron and the LHC

Drell-Yan Process (NNLO)

\[ L_{qq} V^2_{qq} \]

pp → W^+X
\[ \sqrt{s} = 14 \text{ TeV} \]

\[ \tilde{p}p \rightarrow W^+X \]
\[ \sqrt{s} = 1.96 \text{ TeV} \]

ABKM (2009)
Cross Section in $pp(\bar{p})$ scattering at $(\text{NNLO})$

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (TeV)</th>
<th>this paper</th>
<th>MSTW2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.96 (\bar{p}p)</td>
<td>$6.91 \pm 0.17$</td>
<td>7.04</td>
</tr>
<tr>
<td>7 (pp)</td>
<td>$131.3 \pm 7.5$</td>
<td>160.5</td>
</tr>
<tr>
<td>10 (pp)</td>
<td>$343 \pm 15$</td>
<td>403</td>
</tr>
<tr>
<td>14 (pp)</td>
<td>$780 \pm 28$</td>
<td>887</td>
</tr>
</tbody>
</table>

ABKM (2009) vs MSTW08
Higgs Cross Section in $pp(\bar{p})$ scattering at (NNLO)

\[ \sigma_H \text{ (pb)} \]

\[ M_H \text{ (GeV)} \]

bands: ABKM (2009); lines: MSTW08
Moments of PDF’s: PT + data

<table>
<thead>
<tr>
<th>( f )</th>
<th>( n )</th>
<th>This Fit ( N^3 ) LO</th>
<th>MRST04 NNLO</th>
<th>A02 NNLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_v )</td>
<td>2</td>
<td>0.3006 ± 0.0031</td>
<td>0.285</td>
<td>0.304</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0877 ± 0.0012</td>
<td>0.082</td>
<td>0.087</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0335 ± 0.0006</td>
<td>0.032</td>
<td>0.033</td>
</tr>
<tr>
<td>( d_v )</td>
<td>2</td>
<td>0.1252 ± 0.0027</td>
<td>0.115</td>
<td>0.120</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0318 ± 0.0009</td>
<td>0.028</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0106 ± 0.0004</td>
<td>0.009</td>
<td>0.010</td>
</tr>
<tr>
<td>( u_v - d_v )</td>
<td>2</td>
<td>0.1754 ± 0.0041</td>
<td>0.171</td>
<td>0.184</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0559 ± 0.0015</td>
<td>0.055</td>
<td>0.059</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0229 ± 0.0007</td>
<td>0.022</td>
<td>0.024</td>
</tr>
</tbody>
</table>

J.B., H. Böttcher, A. Guffanti, 2006

Lattice Results: developing; different fermion-types studied. Low values of \( m_\pi \) crucial; values approach 270 MeV now.
5. $\Lambda_{QCD}$ and $\alpha_s(M_Z^2)$

\[
\frac{\delta \alpha_{\text{em}}(0)}{\alpha_{\text{em}}(0)} \sim 3 \cdot 10^{-11} \quad \frac{\delta \alpha_{\text{weak}}}{\alpha_{\text{weak}}} \sim 7 \cdot 10^{-4} \quad \frac{\delta \alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} > 2 \cdot 10^{-2}
\]

(untill recently)

\[\text{P. Zerwas, 2004}\]
Overview of the Analyses

- Various NLO analyses; \(\Rightarrow\) Precision requires NNLO analysis and higher!
- Mixed S- and NS-NNLO analyses \(e(\mu)N\) world data
- S- and NS-NNLO moment analyses \(\nu N\) world data
- NS-N\(^3\)LO analysis \(e(\mu)N\) world data
- NLO analyses polarized \(e(\mu)N\) world data
- Lattice measurements
\[ \alpha_s(M_Z^2) \]

<table>
<thead>
<tr>
<th>NLO</th>
<th>( \alpha_s(M_Z^2) )</th>
<th>expt</th>
<th>theory</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTEQ6</td>
<td>0.1165 ±0.0065</td>
<td></td>
<td></td>
<td>[1]</td>
</tr>
<tr>
<td>MRST03</td>
<td>0.1165 ±0.0020 ±0.0030</td>
<td></td>
<td></td>
<td>[2]</td>
</tr>
<tr>
<td>A02</td>
<td>0.1171 ±0.0015 ±0.0033</td>
<td></td>
<td></td>
<td>[3]</td>
</tr>
<tr>
<td>ZEUS</td>
<td>0.1166 ±0.0049</td>
<td></td>
<td></td>
<td>[4]</td>
</tr>
<tr>
<td>H1</td>
<td>0.1150 ±0.0017 ±0.0050</td>
<td></td>
<td></td>
<td>[5]</td>
</tr>
<tr>
<td>BCDMS</td>
<td>0.110 ±0.006</td>
<td></td>
<td></td>
<td>[6]</td>
</tr>
<tr>
<td>GRS</td>
<td>0.112 ±0.0019</td>
<td></td>
<td></td>
<td>[10]</td>
</tr>
<tr>
<td>BBG</td>
<td>0.1148 ±0.0019</td>
<td></td>
<td></td>
<td>[9]</td>
</tr>
<tr>
<td>BB (pol)</td>
<td>0.113 ±0.004</td>
<td>+0.009</td>
<td>−0.006</td>
<td>[7]</td>
</tr>
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<table>
<thead>
<tr>
<th>NNLO</th>
<th>( \alpha_s(M_Z^2) )</th>
<th>expt</th>
<th>theory</th>
<th>Ref.</th>
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<tbody>
<tr>
<td>MRST03</td>
<td>0.1153 ±0.0020 ±0.0030</td>
<td></td>
<td></td>
<td>[2]</td>
</tr>
<tr>
<td>A02</td>
<td>0.1143 ±0.0014 ±0.0009</td>
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<td></td>
<td>[3]</td>
</tr>
<tr>
<td>SY01(ep)</td>
<td>0.1166 ±0.0013</td>
<td></td>
<td></td>
<td>[8]</td>
</tr>
<tr>
<td>SY01(\nuN)</td>
<td>0.1153 ±0.0063</td>
<td></td>
<td></td>
<td>[8]</td>
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<tr>
<td>GRS</td>
<td>0.111 ±0.0015</td>
<td></td>
<td></td>
<td>[10]</td>
</tr>
<tr>
<td>A06</td>
<td>0.1128 ±0.0015</td>
<td></td>
<td></td>
<td>[11]</td>
</tr>
<tr>
<td>BBG</td>
<td>0.1134 +0.0019/−0.0021</td>
<td></td>
<td></td>
<td>[9]</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>N^3LO</th>
<th>( \alpha_s(M_Z^2) )</th>
<th>expt</th>
<th>theory</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>BBG</td>
<td>0.1141 +0.0020/−0.0022</td>
<td></td>
<td></td>
<td>[9]</td>
</tr>
</tbody>
</table>

**NNLO and N^3LO**

**NLO**

- **BBG**: \( N_f = 4 \): non-singlet data-analysis at \( O(\alpha_s^4) \): \( \Lambda = 234 \pm 26 \text{ MeV} \)

**Lattice results**:

- **Alpha Collab**: \( N_f = 2 \) Lattice; non-pert. renormalization \( \Lambda = 245 \pm 16 \pm 16 \text{ MeV} \)
- **QCDSF Collab**: \( N_f = 2 \) Lattice, pert. reno. \( \Lambda = 261 \pm 17 \pm 26 \text{ MeV} \)

**Lepage et al.**: Larger, but no quenched result.
\[ \frac{\delta \alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} \approx 1.2\% \]

*(obtained by July 1st)*

<table>
<thead>
<tr>
<th></th>
<th>(\alpha_s(M_Z^2))</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABKM</td>
<td>(0.1135 \pm 0.0014)</td>
<td>HQ: FFS (N_f = 3)</td>
</tr>
<tr>
<td>ABKM</td>
<td>(0.1129 \pm 0.0014)</td>
<td>HQ: BSMN-approach</td>
</tr>
<tr>
<td>BBG (2006)</td>
<td>(0.1134 \pm 0.0019)</td>
<td>valence analysis, NNLO</td>
</tr>
<tr>
<td></td>
<td>(+0.0019)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.0021)</td>
<td></td>
</tr>
<tr>
<td>JR (2008)</td>
<td>(0.1124 \pm 0.0020)</td>
<td>dynamical approach</td>
</tr>
<tr>
<td>MSTW (2008)</td>
<td>(0.1171 \pm 0.0014)</td>
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<tr>
<td>BBG (2006)</td>
<td>(0.1141 \pm 0.0020)</td>
<td>valence analysis, (N^3)LO</td>
</tr>
<tr>
<td></td>
<td>(+0.0020)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(-0.0022)</td>
<td></td>
</tr>
</tbody>
</table>
\[ \alpha_s(M_Z^2) \]

**Diagram Description:**
- The graph shows the values of \( \alpha_s(M_Z^2) \) for different models and data sets.
- There are three sets of data points:
  - **NLO** (blue dots)
  - **NNLO** (green squares)
  - **N3LO (unp)** (purple triangles)

- The shaded region represents the world average of \( \alpha_s(M_Z^2) \).
- The horizontal dashed line indicates the average value of \( \alpha_s(M_Z^2) \).

**Data Sets:**
- CTEQ06
- MRST03
- A02
- ZEUS
- H1
- BCDMS
- SY01(ep)
- SY01(\nu/N)
- BBG
- MRST03
- A06
- SY01(ep)
- SY01(\nu/N)
- BBG
- BBG
- BSM
- ABR

**Reference:**
J.B., H. Böttcher, A. Guffanti, 2006

**Status of Unpolarized PDFs and \( \alpha_s(M_Z^2) \):**
Seattle, WA, October 20th 2009
• $\alpha_s(M_Z^2)$ for different data sets included are too different!

⇒ applies also to HERA: IS vs FS; and also DIS vs TEVATRON-jet

M. Cooper-Sarkar, 2005
6. Advanced Technologies to Evaluate Feynman Diagrams

in QED & QCD @ 3 loops and beyond

- Automatic diagram generation mandatory: QGRAF
  # 2500 - 15000 diagrams

- The ‘Only’ problem: Calculation of Feynman Parameter Integrals;
  everything else automated: FORM-codes

- Renormalization still not always trivial: \( \gamma_5 \), mass(es), ...

- Work with linguistic standards: Harmonic Sums, Harmonic Polylogarithms, Euler-Zagier
  values, etc. - Avoids the problem of Babel in analytic integration

- Generalized Hypergeometric Functions and their Generalizations are to the Heart of the Matter. M. Kalmykov et al., JB et al.

- Need: advanced Difference Equation Establishers & Solvers: Sigma

- Do not proliferate !, i.e. avoid IBP, MB, and other methods causing gigantic Zeroes.

- What remains is: Integrating the hard way.
Some Examples:

**Zero-scale Problems**: Euler-Zagier and Multiple Zeta Values
JB, D. Broadhurst, J. Vermaseren, DESY 09-03

- find all relations: **Tera-Terms** to be processed
  - alternating: all relations up to $w = 12$ (6-loop level);
  - non-alternating: all relations up to $w = 22$; determined.
- Interesting relations: to $w = 30$;

**Reconstructing recurrent quantities from Mellin Moments**
JB, M. Kauers, S. Klein, C. Schneider DESY 09-02
Can one find the anomalous dimensions and Wilson coefficients to 3-loops just from their moments? Yes - recurrent quantities in Mellin space.

- $\leq 5114$ Moments; difference equation fills 440 books
- Complete computation: 5 CPU Months

**Massive Wilson coefficients at 3 Loops**
I. Bierenbaum, JB, S. Klein, DESY 09-57
first analytic massive 1-scale calculation @ 3-loops

- Moments 2–10 (12/14) have been calculated for all unpolarized channels
- Complete computation: 300 CPU days, partly req. 32-64 Gbyte computers
7. Outlook

Theory:
- **Polarized** Anomalous Dimensions & massless Wilson coefficients @ 3 Loops
- **Unpolarized** Heavy Flavor Wilson coefficients @ 3 Loops: general $N$
- **Polarized** Heavy Flavor Wilson coefficients @ 3 Loops

Along with this: Development of efficient analytic calculation methods being suited for 3-Loops and higher

$ep$ & $pp$ jet cross sections at HO; progress in pdf Lattice calculations

Code:
- Creation of an Open Source Code for DIS and pp-hard scattering data for experimental precision analyzes to derive pdfs

Experiment:
- Precision Data from LHC, JLAB and EIC.

Can we get $\delta \alpha_s$ even smaller?
Improving the Accuracy of $\alpha_s(M_Z^2)$

Improving the Accuracy of Unpolarized and Polarized Parton Densities; Were SLAC & BCDMS right after all?

Measuring $F_L(x, Q^2)$ precisely

Unfolding the sea-quarks finally.

Theory: may need to go to 4 Loop level. This is within reach for the moments.

We envisage a bright future.