QCD CORRECTIONS TO STRUCTURE FUNCTIONS AT SMALL X

JB 12/92

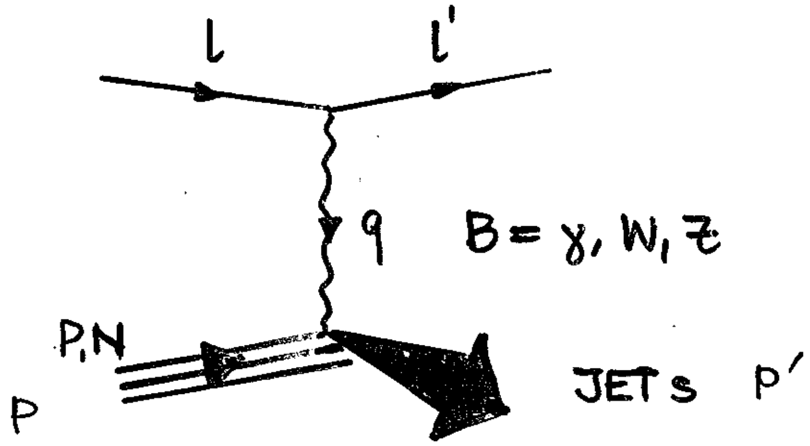
Zeuthen

Thy seminar

1. INTRODUCTION
2. EXTRAPOLATION OF THE AP EQUATIONS TO SMALL X, $O(\alpha_s, \alpha_s^2)$ RESULTS
3. BFKL EQUATION
4. GLR EQUATION & NUMERICAL STUDY
5. EFFECTS DUE TO GLUON VIRTUALITY (IS)
6. CONCLUSION & OUTLOOK

1. INTRODUCTION

DEEP INELASTIC SCATTERING :



$$Q^2 = -(l-l')^2 \quad , \quad x = \frac{Q^2}{2Pq} \quad , \quad y = Q^2/sx$$

$$x \ll 1$$

$$\frac{d^2\sigma^{e^\pm p}}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[2xF_1(x,Q^2)\,y^2 + F_2(x,Q^2)\,2(1-y)\right]$$

EVOLUTION:

$x = 0.01 \ldots 1$ AP-EQU.

$$\begin{pmatrix} q_i(x,Q^2) \\ G(x,Q^2) \end{pmatrix} = \left(A_{kj}(Q^2, Q_0^2, x; \Lambda^2)\right) \otimes \begin{pmatrix} q_i(x,Q_0^2) \\ G(x,Q_0^2) \end{pmatrix}$$

$$(A \otimes B)(x) = \int_0^1 dx_1 \int_0^1 dx_2\, \delta(x - x_1 x_2)\, A(x_1)\, B(x_2)$$

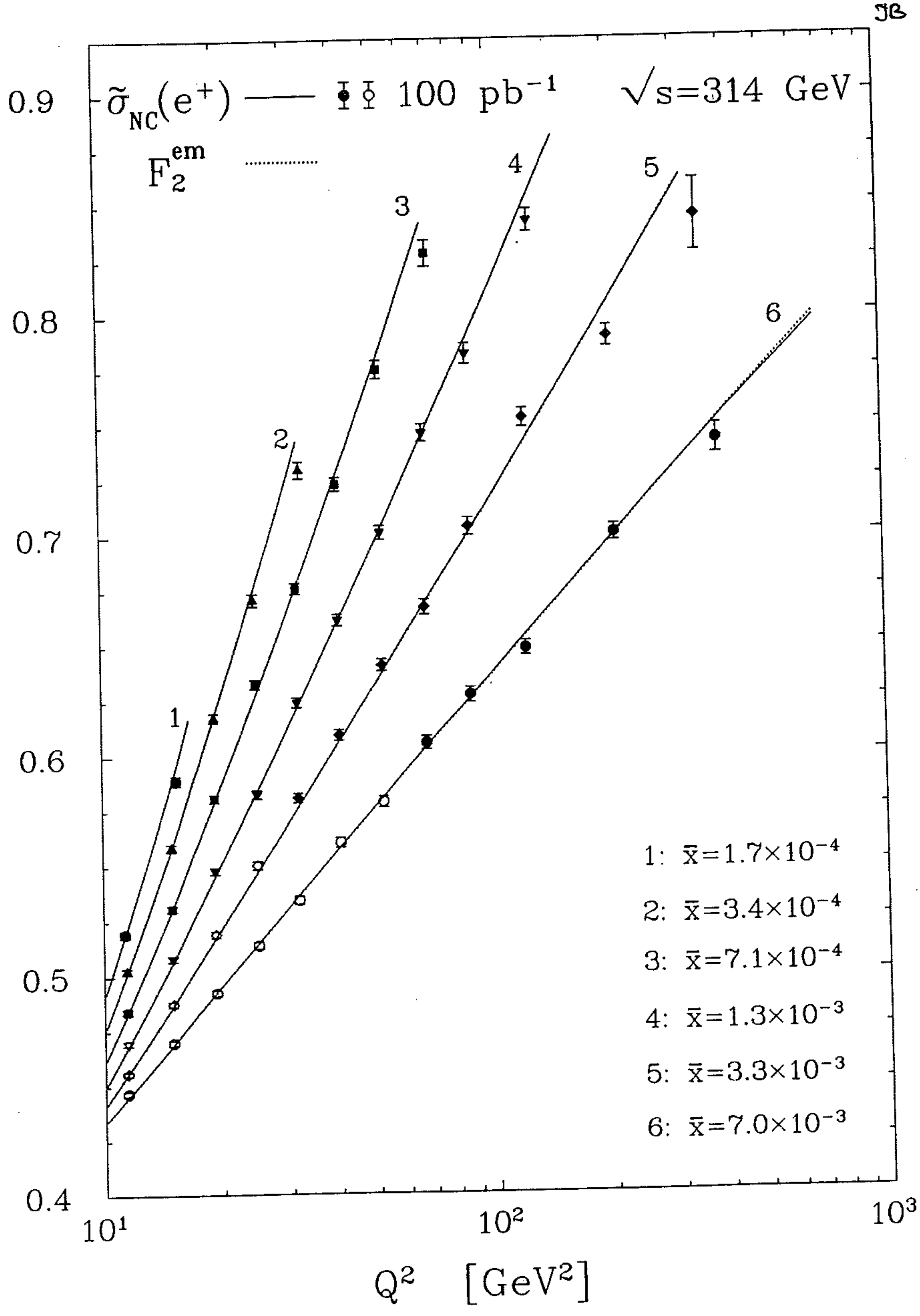
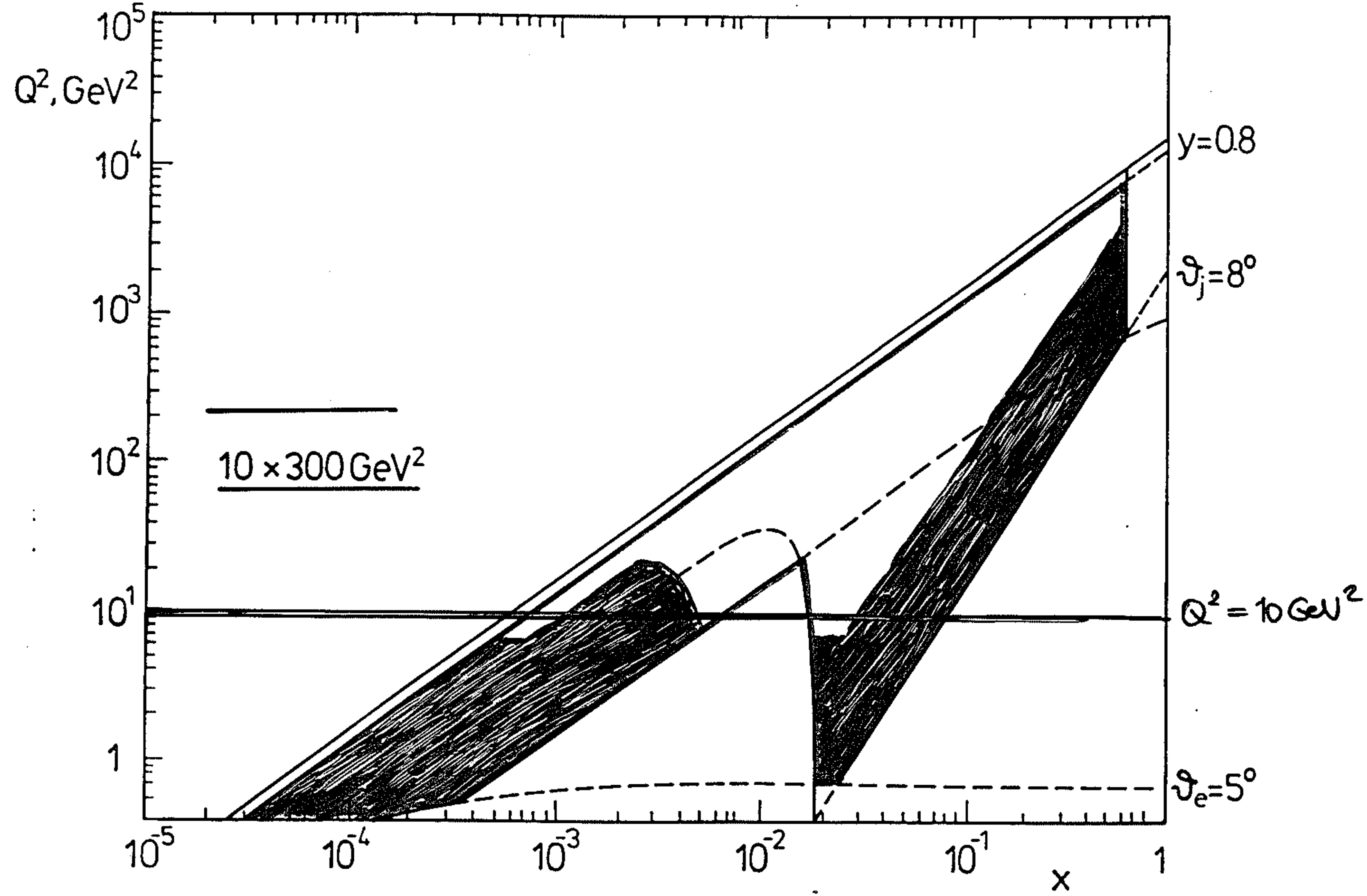
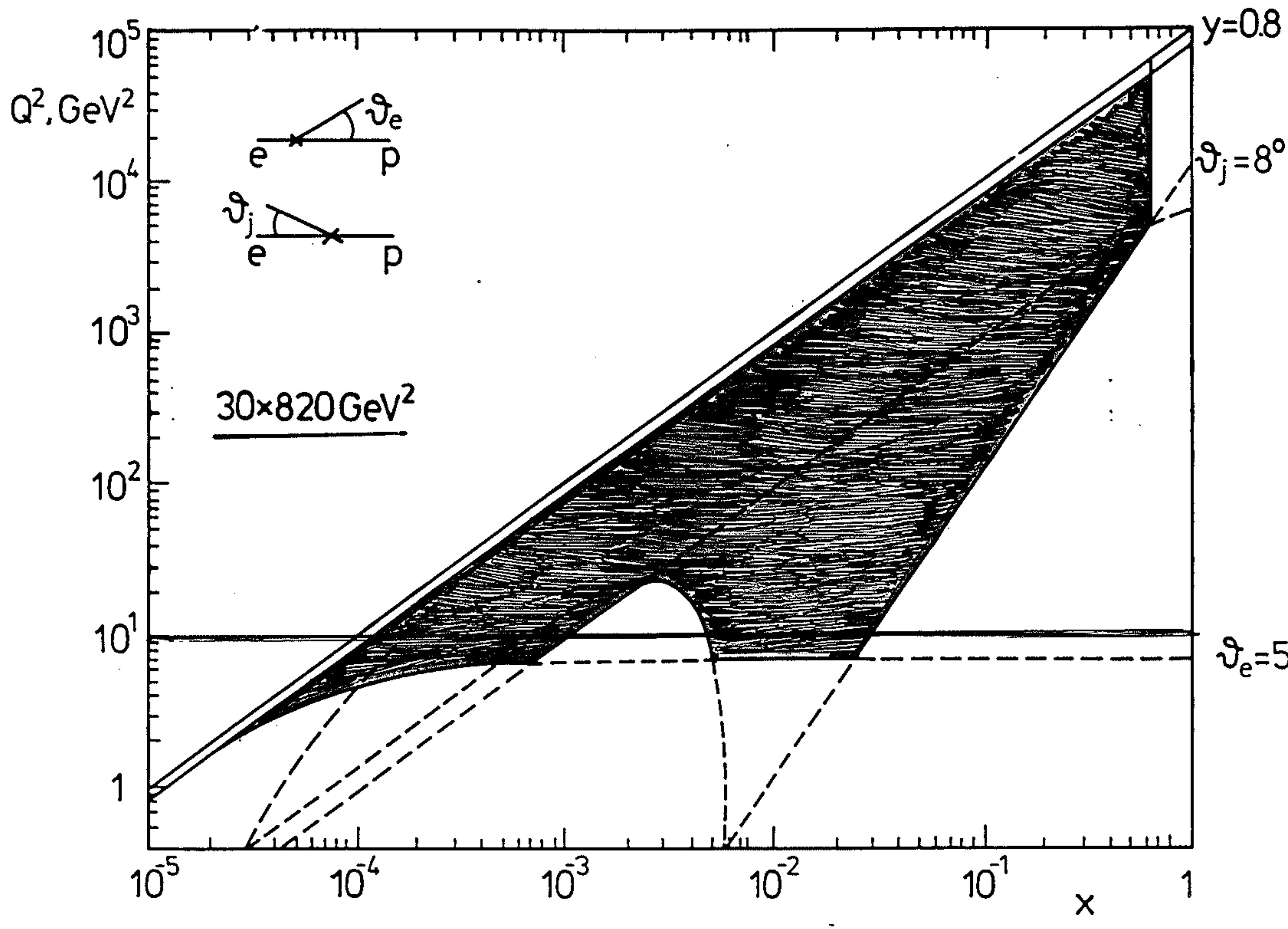

JB
$\tilde{\sigma}_{NC}(e^+)$ 100 pb^{-1}
$\sqrt{s}$=314 GeV
F_2^{em}
1
2
3
4
5
6
1: $\bar{x}=1.7\times10^{-4}$
2: $\bar{x}=3.4\times10^{-4}$
3: $\bar{x}=7.1\times10^{-4}$
4: $\bar{x}=1.3\times10^{-3}$
5: $\bar{x}=3.3\times10^{-3}$
6: $\bar{x}=7.0\times10^{-3}$
0.9
0.8
0.7
0.6
0.5
0.4
10^1
10^2
10^3
Q^2 [GeV^2]

Fig. 6


Q^2, GeV^2
10^5
10^4
10^3
10^2
10^1
1
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x
$10 \times 300 GeV^2$
y=0.8
$\vartheta_j=8°$
$Q^2 = 10 GeV^2$
$\vartheta_e=5°$



y=0.8
$\vartheta_j=8^{\circ}$
$\vartheta_e=5^{\circ}$
Q^2, GeV^2
ϑ_e
e
p
ϑ_j
e
p
30×820 GeV^2
10^5
10^4
10^3
10^2
10^1
1
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

PROBLEM:

$$F_i(x,Q^2) \underset{\substack{x \to 0 \\ AP}}{\sim} \exp\sqrt{\ln \frac{1}{x}}$$

$\longrightarrow$ HO TWIST 2

$\longrightarrow$ NEW DYNAMICS TO GET

$$\sigma \sim \pi R^2(s)$$

$\longrightarrow$ SUMMATION OF LARGE EFFECTS AT SMALL X

AP EQUATIONS:

$$\frac{d f^a(x, Q^2)}{d \ln Q^2} = P\left(x, \frac{\alpha_s(Q^2)}{2\pi}\right)_{ab} \otimes f_b(x, Q^2)$$

$$P\left(x, \frac{\alpha_s}{2\pi}\right)_{ab} = \frac{\alpha_s}{2\pi} \left\{ P^0_{ab}(x) + \frac{\alpha_s}{2\pi} P^1_{ab}(x) + \dots \right\}$$

$$x \ll 1$$

	1 ST ORDER	2nd ORDER
FF	$C_F \frac{1+x^2}{1-x}$	$\frac{1}{x} 2N_f T_R C_F \frac{20}{9}$
FG	$2N_f T_R [x^2 + (1-x)^2]$	$\frac{1}{x} 2N_f T_R C_G \frac{20}{9}$
GF	$C_F \frac{1}{x} [1 + (1-x)^2]$	$\frac{1}{x} 2N_f T_R \left(-\frac{20}{9}\right) + C_F G$
GG	$2C_G \left[\frac{1}{x} + \frac{1}{1-x} - 2 + x - x^2\right]$	$\frac{1}{x} 2N_f T_R \left(-\frac{23}{9} C_G + \frac{2}{3} C_F\right)$

2. EXTRAPOLATION OF AP TO SMALL x & $O(\alpha_s, \alpha_s^2)$ RESULTS

- CONSIDER ONLY GLUONS (DOMINATING, $x \ll 1$)

$$\mathbb{G}(x, Q^2) := x\, G(x, Q^2)$$

$$\frac{d\mathbb{G}(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dx'}{x'} \Big[\underset{\text{LO}}{6} - \frac{61}{9} N_f \underset{\text{NTLO}}{\frac{\alpha_s}{2\pi}} \Big] \frac{x^2}{x'^2}\, \mathbb{G}(x', Q^2)$$

!

DF: $\quad y = \dfrac{8N_c}{\beta_0} \ln \dfrac{1}{x}, \quad \xi = \ln \ln \left(\dfrac{Q^2}{\Lambda^2} \right)$

$$\frac{\partial^2 \mathbb{G}(y,\xi)}{\partial y\, \partial \xi} = \frac{1}{2}\, \mathbb{G}(y,\xi) \qquad \text{LO}$$

$$\frac{\partial^2 \mathbb{G}(y,\hat{\xi})}{\partial y\, \partial \hat{\xi}} = \frac{1}{2}\, \mathbb{G}(y,\hat{\xi}) \qquad \text{NTLO}$$

$$\hat{\xi} = \xi + f(\xi)\ ; \quad f'(\xi) = -\left[\frac{\beta_1}{\beta_0} \xi e^{-\xi} + \frac{61}{63} \frac{2N_f}{\beta_0} e^{-\xi} \left(1 - \frac{\beta_1}{\beta_0} \xi e^{-\xi}\right)^2 \right.$$

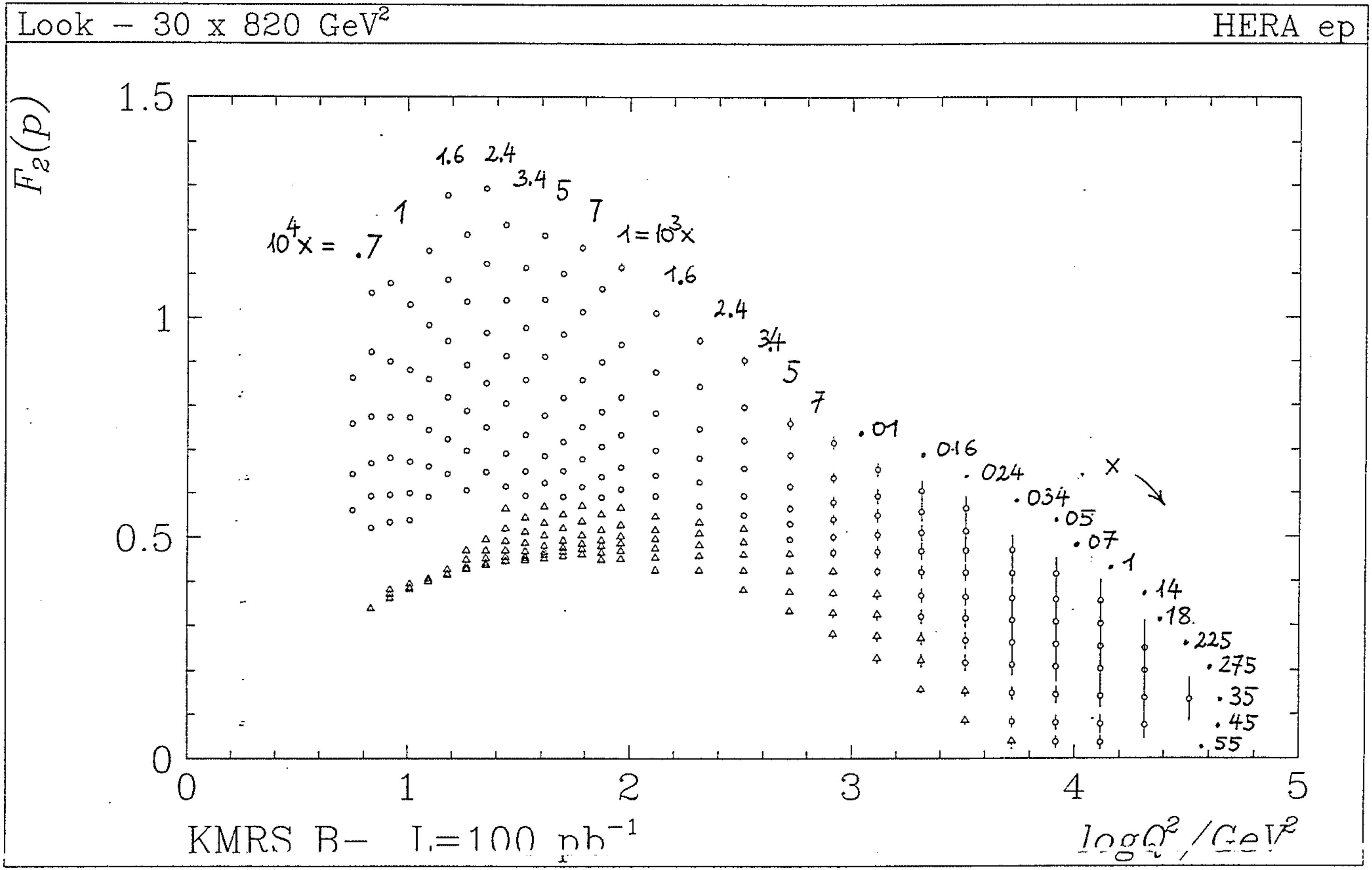
SOLUTIONS:

$$\mathbb{G}(y, \overset{(\wedge)}{\xi}) = \sum_{\nu=0}^{\infty} \left\{ A_\nu \left(\frac{2\overset{(\wedge)}{\xi}}{y} \right)^{\nu/2} + B_\nu \left(\frac{y}{2\overset{(\wedge)}{\xi}} \right)^{\nu/2} \right\}$$

$$\cdot\, I_\nu\left(\sqrt{2\xi y}\right)$$

$\mathbb{G}(y,\xi)_{\,y \to \infty}$ GROWS FASTER THAN A POWER OF $\ln(1/x)$.

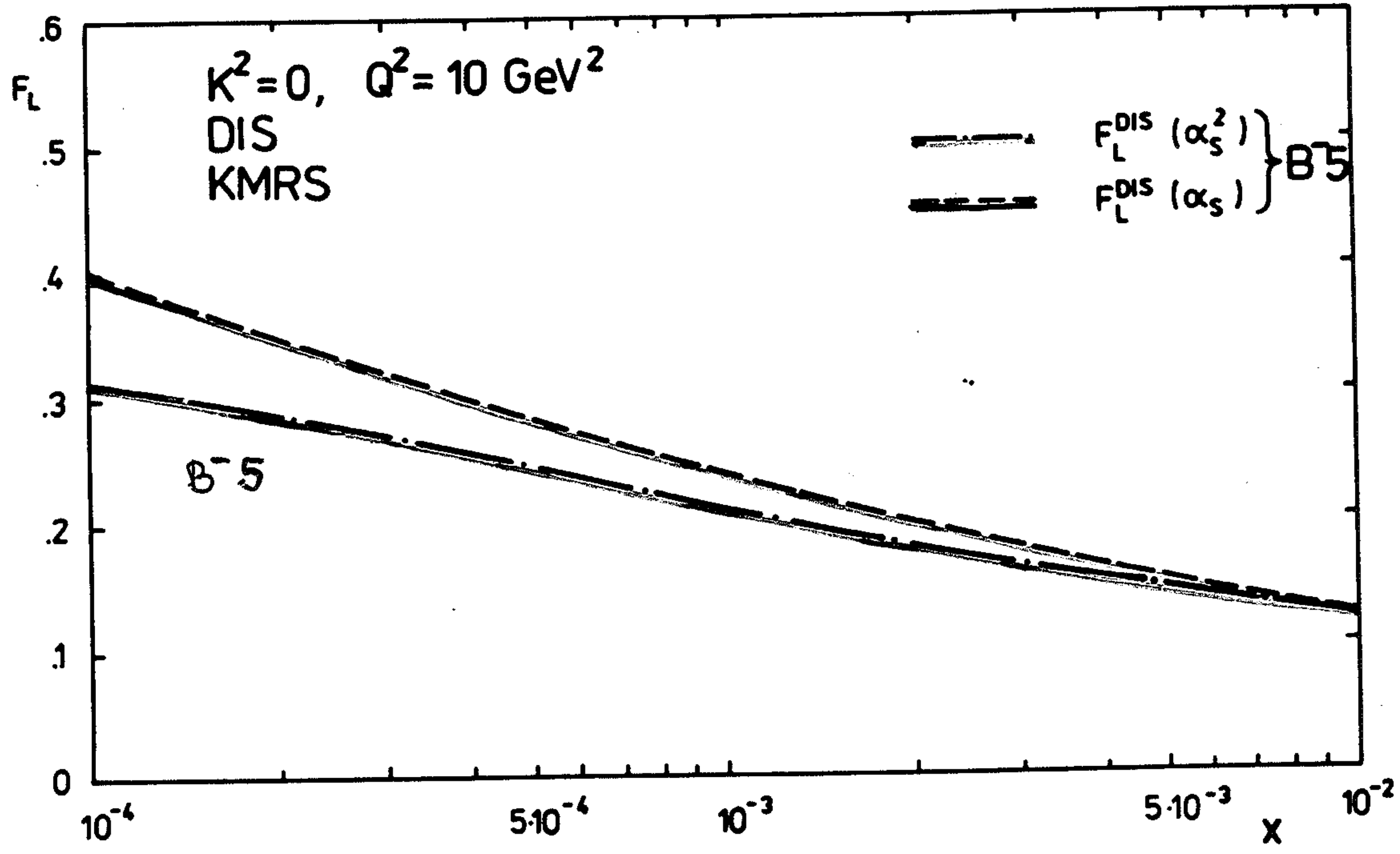
JB, M. KLEIN



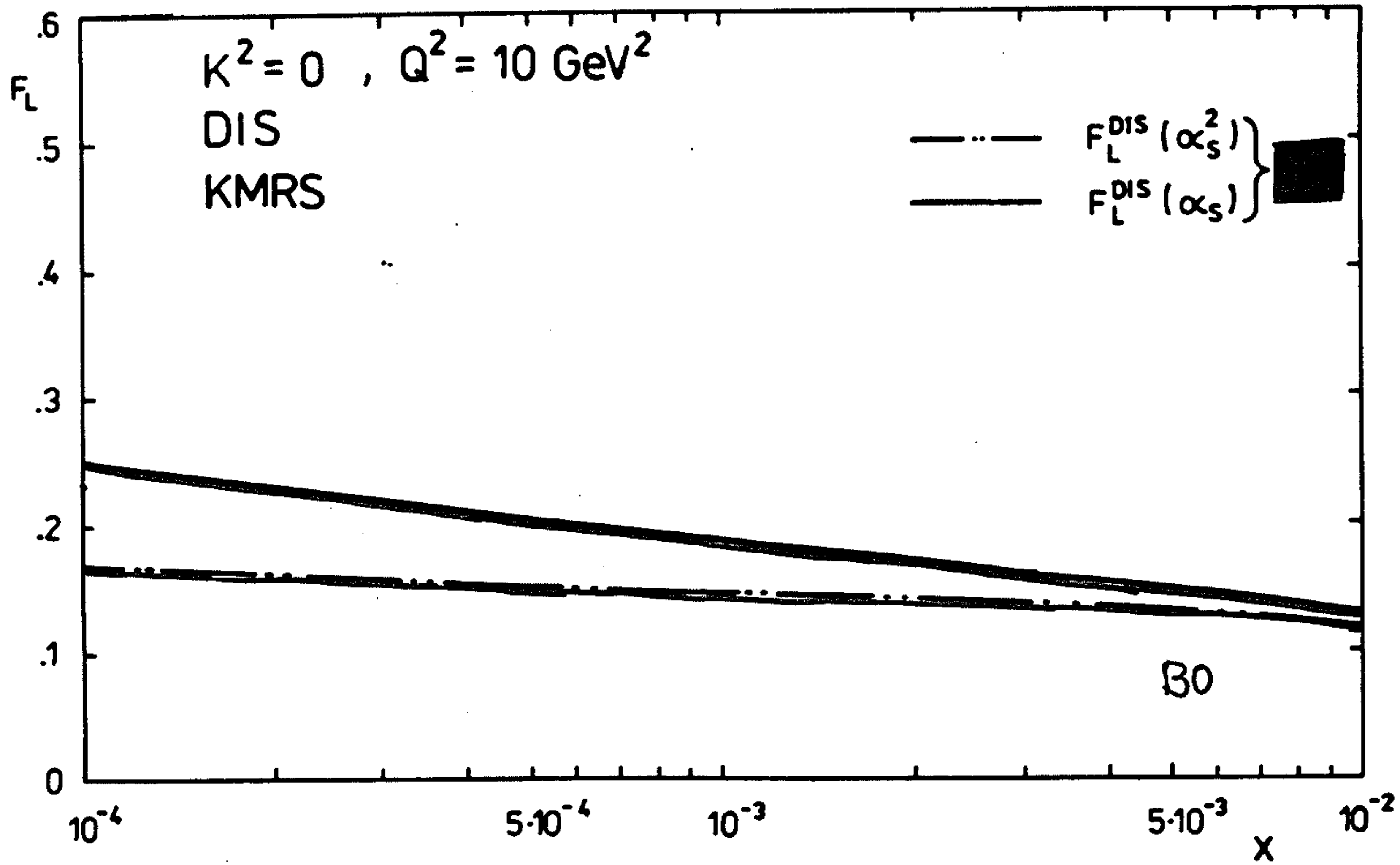
Look – 30 x 820 GeV^2
HERA ep
$F_2(p)$
1.5
1
0.5
0
$10^4 x =$ .7
1
1.6
2.4
3.4
5
7
$1 = 10^3 x$
1.6
2.4
3.4
5
7
.01
.016
.024
.034
.05
.07
.1
.14
.18
.225
.275
.35
.45
.55
x
0
1
2
3
4
5
KMRS B– L=100 pb^{-1}
$\log Q^2 / GeV^2$

IMPORTANCE OF HIGHER ORDER CORRECTIONS

ZIJLSTRA, VAN NEERVEN



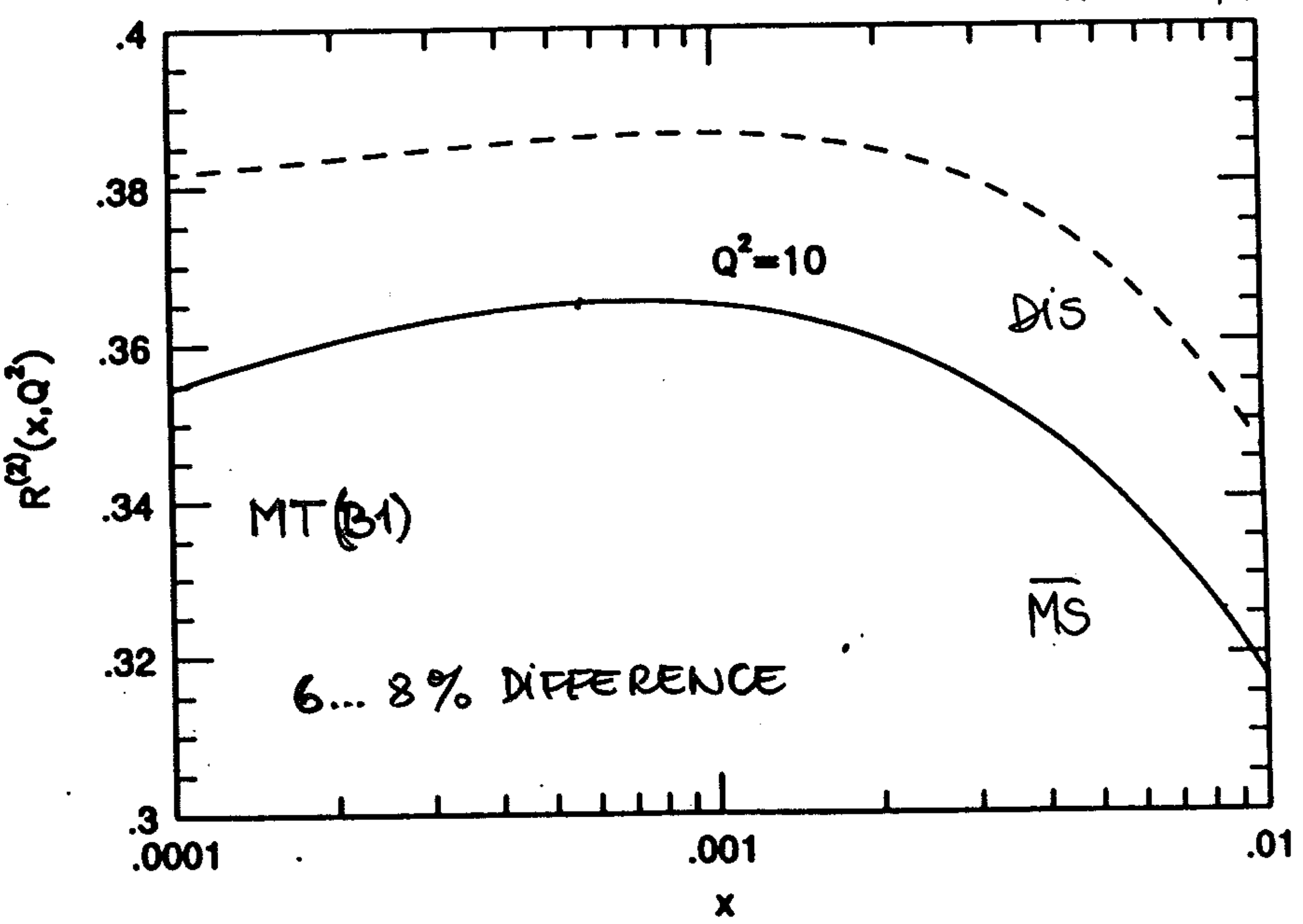
ZIJLSTRA, VAN NEERVEN



$$R^{(2)}(x,Q^2) = \frac{F_L^{(2)}(x,Q^2)}{\left(1+\frac{4M_p^2x^2}{Q^2}\right)F_2^{(1)}(x,Q^2) - F_L^{(2)}(x,Q^2)}$$

$\mathcal{O}(\alpha_s^2)$

ZIJLSTRA, VAN NEERVEN



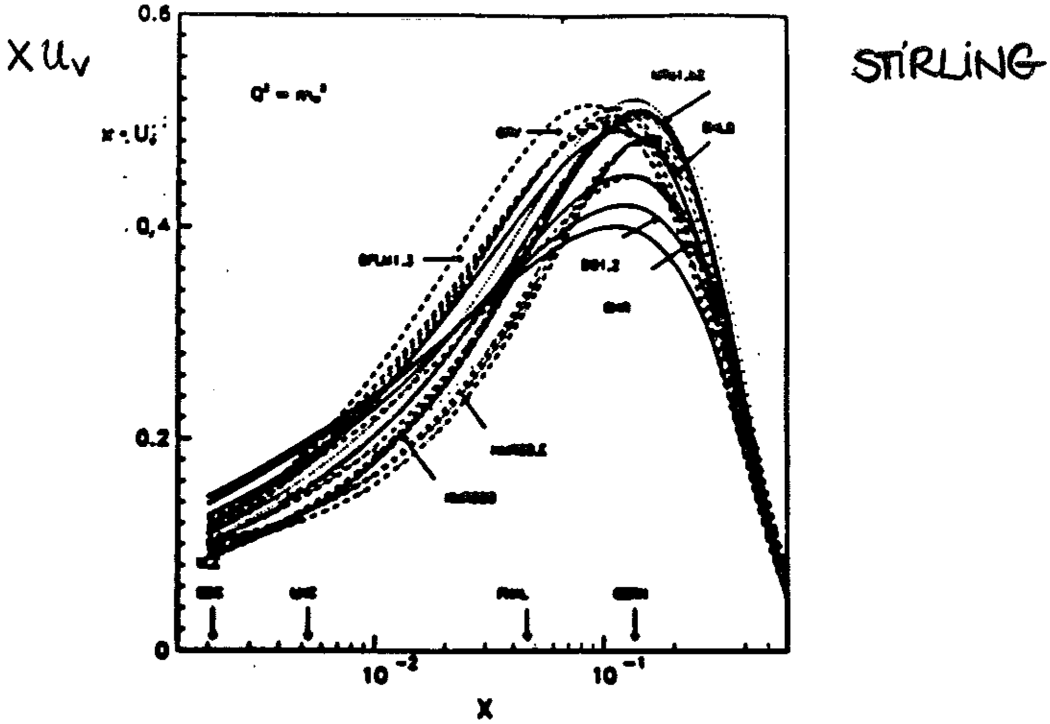


Figure 1: The valence u-quark distribution at $Q^2 = M_W^2$ [4].

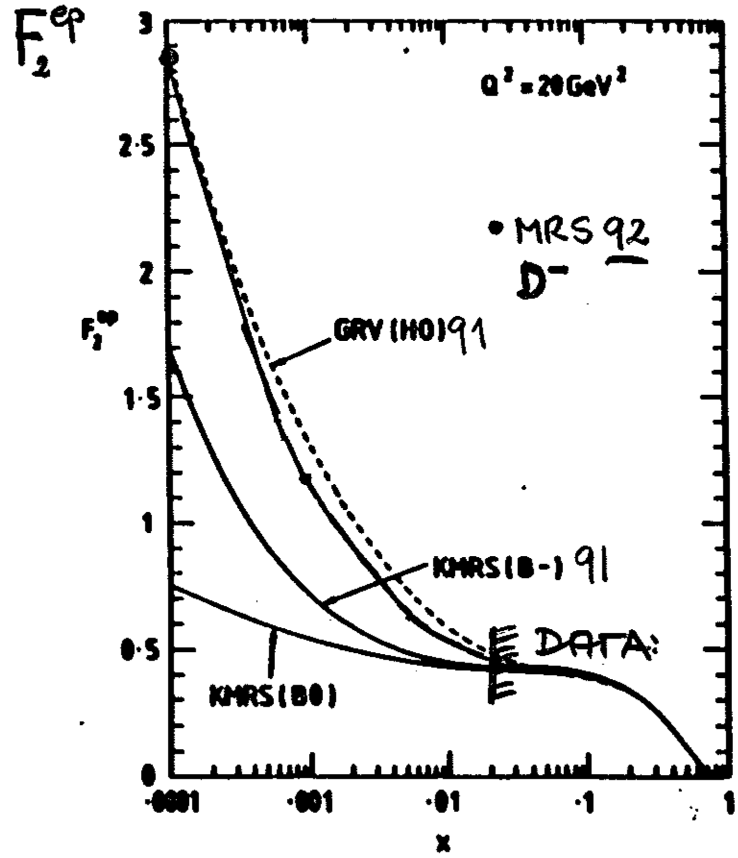


Figure 3: F_2^{ep} structure function predictions.

3. THE BFKL - EQUATION

FADIN, KURAEV, LIPATOV; BALITZKII

$$f(n,k^2) = \frac{1}{n-1} f_0(n,k^2) + \frac{3}{\pi}(\mathcal{L} \otimes f)(n,k^2)$$

i) $\alpha_s = \text{const.}$ $\quad \mathcal{L} = L_1$

$$L_1 \otimes f_n = \frac{\alpha_s}{n-1} \int_{k_0^2}^{\infty} \frac{d\bar{k}^2}{\bar{k}^2} \left\{ \frac{k^2}{|\bar{k}^2 - k^2|} \left[f(n,\bar{k}^2) - f(n,k^2) \right] + \frac{f(n,k^2)\, k^2}{\sqrt{k^4 + 4\bar{k}^2}} \right\}$$

$k_0^2 = 0$

$\longrightarrow$ EIGENFUNCTIONS :

$$e(n,\omega) = \frac{e(n,\omega^0)}{n - \left(1 + \frac{3\alpha_s}{\pi} K(\omega)\right)}$$

n_0 - POLE of $e(n,\omega)$ FOR $\omega = 0$

$$n_0 = 1 + 2.64\, \alpha_s$$

$$G(x) \longrightarrow \sim \frac{1}{x^{n_0}}$$

COLLINS
KWIECINSKI

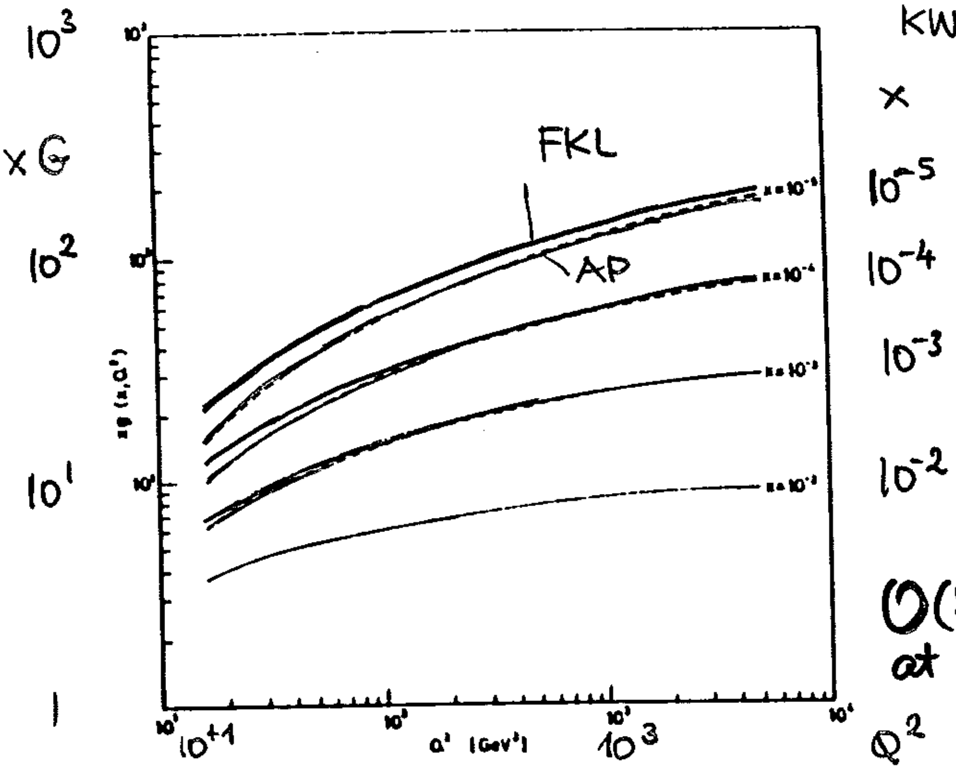
ii) α_s running: $\quad \mathcal{L} = L_2$

$$L_2 \otimes f_n = L_1(\alpha_s \to \alpha_s(k^2),\ k_0^2 > 0) \otimes f_n$$

$$1 + \frac{3.6}{\pi} \alpha_s(k_0^2) \lesssim n_0 \lesssim 1 + 4 \ln 2 \cdot \left(\frac{3}{\pi}\right) \alpha_s(k_0^2)$$

$$1.31 \lesssim n_0 \lesssim 1.72$$

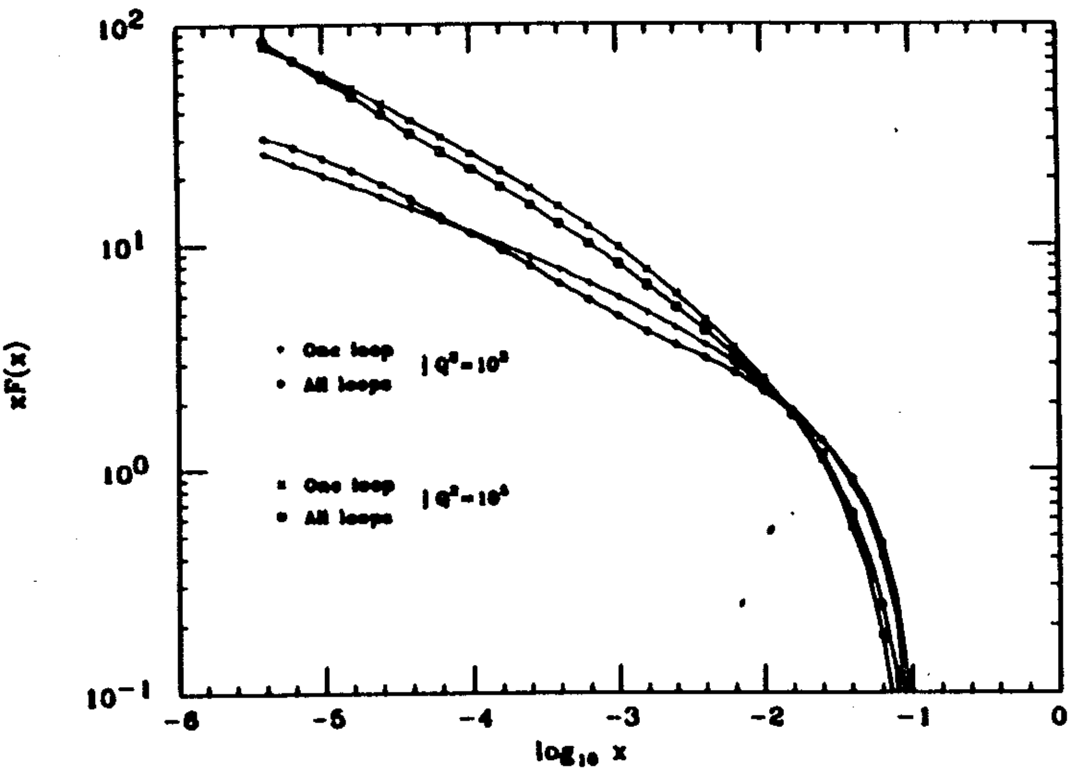
KWIECINSKI



O(5%)
at HERA.

Fig. 3. The Q^2 evolution of gluon distributions beyond the leading ln $1/x$ approximation after corrections due to the constant term in the product $A_{gg}(n) \cdot \Delta A_{gg}(n)$ are included. The full line corresponds to the solution of the appropriately modified (3.6) and the dotted line to its leading log Q^2 counterpart

LEVIN
MARCHESINI
RYSKIN
WEBBER



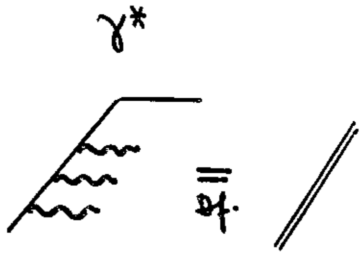
(MC)

Fig. 6. Monte Carlo structure functions for $Q_0 = 0.4$ GeV with starting condition $xF(x, Q_0^2) = \delta(x - 0.1)$ at $Q_0^2 = 5$ GeV^2.

4. THE GLR EQUATION

a) TWIST - 2 : EVOLUTION OF INDIVIDUAL PARTONS

$x \gtrsim 10^{-2} \quad Q^2 > 10\ \mathrm{GeV}^2$



b) SEMI - HARD RANGE : x - GETTING SMALLER

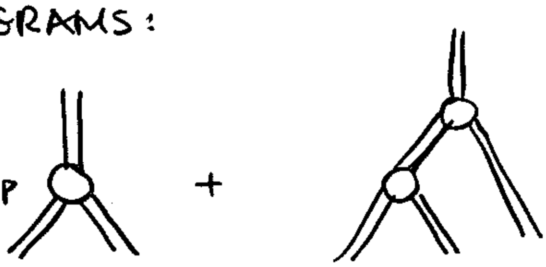
$Q^2 \gtrsim 10 \dots 50\ \mathrm{GeV}^2$

PARTONS RECOMBINE – FINITE 'PROTON' RADIUS

\+ HIGH PARTONIC DENSITIES

DIAGRAMS :

G_{3P}



etc.

- GRIBOV, LEVIN, RYSKIN, 1982: LEADING PART AT EACH NODE
  $G + G \to G$ (Ladders)

- BARTELS 1992 (FURTHER CONTRIBUTIONS)

1^{ST} FAN - DIAGRAM ~ 1% CORR.

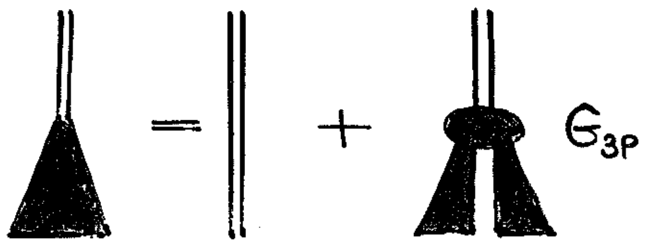
MUELLER, QIU: DLA - EXPRESSION FOR G_{3P}.

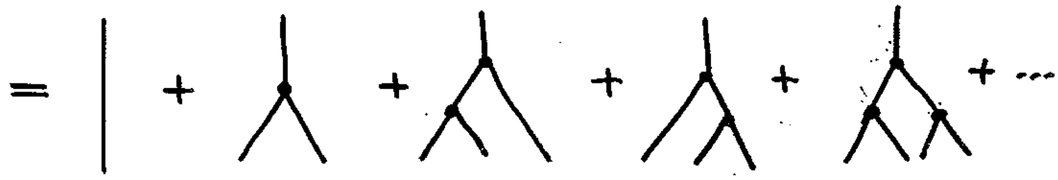
$$G_{3P}(\xi) = \frac{3}{4}\pi^2 \frac{1}{\beta_0} e^{e^{\xi_0}} \exp\left[-(e^{\xi}+\xi)\right]$$

$$= \frac{3}{4}\pi^2 \frac{1}{\beta_0}\left(\frac{Q_0^2}{Q^2}\right)\frac{1}{\ln\left(\frac{Q^2}{\Lambda^2}\right)} \quad ; \text{ HT}$$

$$= C \cdot \frac{\Lambda^2}{Q^2}$$

SUMMATION OF FAN DIAGRAMS:





DLA

GLR: AGK CUTTING RULES (NOT YET FOUND IN QCD!)

$$\underbrace{\frac{\partial^2 F(\xi,y)}{\partial\xi\,\partial y} = \frac{1}{2}}_{\text{DLA}} F(\xi,y) - C\exp\left[-(e^{\xi}+\xi)\right] F^2(\xi,y)$$

ASSUMPTION: $G_n(x_1,\ldots,x_n) \simeq G(x_1)\ldots G(x_n)$

IMPROVE WITH RESPECT TO REALISTIC INITIAL CONDITIONS:
(BARTELS, JB, SCHULER)

$$F(\xi,y) = G(\xi_0,y) + \int_{\xi_0}^{\xi} d\xi' \int_0^{y} dy'\, F(y',\xi')\left[\frac{1}{2} - C\exp\left[-(e^{\xi'}+\xi')\right] \times F(\xi',y')\right]$$

↑

INPUT AT Q_0^2.

PROPERTIES OF THE EQUATION

i) SOLUTIONS ARE BOUNDED FROM ABOVE:

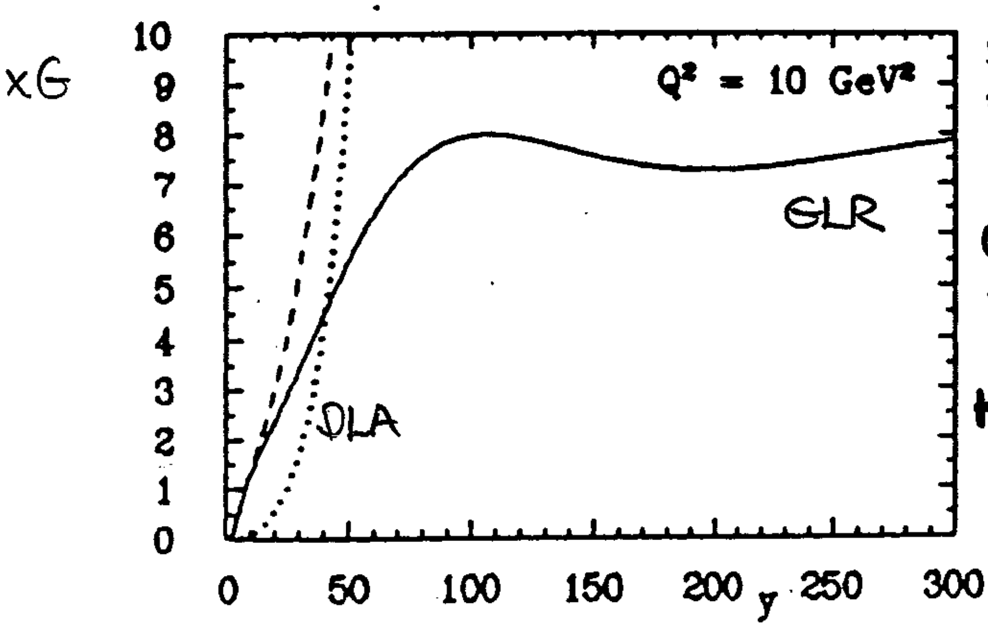
$F \leq F_0(\xi, y)$ (DLA) (AND BELOW $F_{Plup} \geq 0$!)

ii) $$\lim_{y \to \infty} F(y, \xi) = \frac{1}{2C} \exp\left[e^{\xi} + \xi\right]$$
$$= \frac{1}{2C} \frac{Q^2}{\Lambda^2} \cdot \ln\left(\frac{Q^2}{\Lambda^2}\right) - \mathrm{const}_{\xi \text{ or } Q^2}.$$

INDEPENDENT OF THE PARTICULAR CHOICE OF $G(y, \xi_0)$!

iii) $\exists!\ F(y, \xi)$

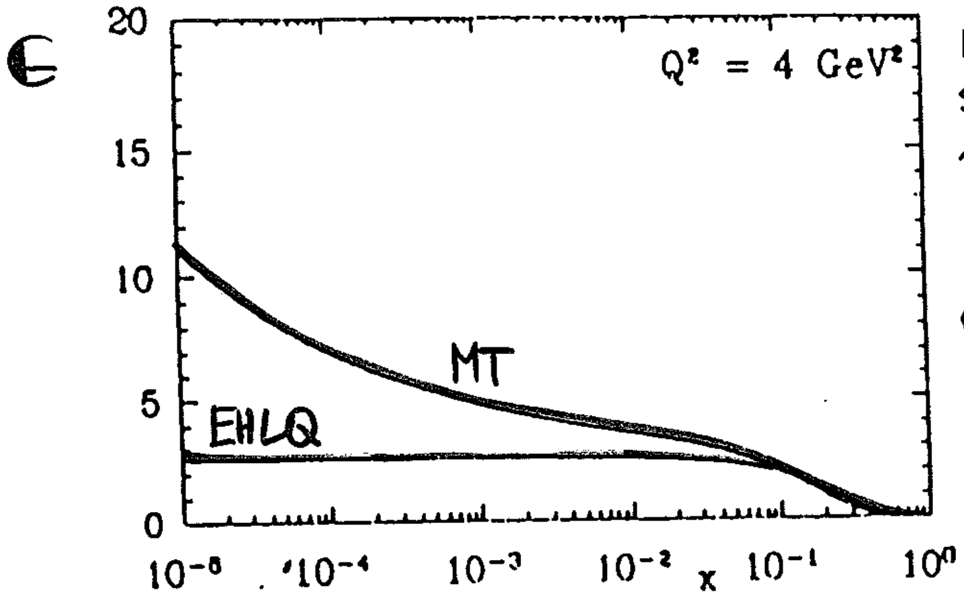
SOLUTION: QUADR. EQU. (LOCALLY) AT A SUFF. FINE GRID IN (ξ, y)
'VOLTERRA-TYPE' EQ. BOTH IN ξ & y.



BARTELS, JB, SCHULER

OSCILLATIONS 'FEED BACK' $\mapsto$ SATURATION

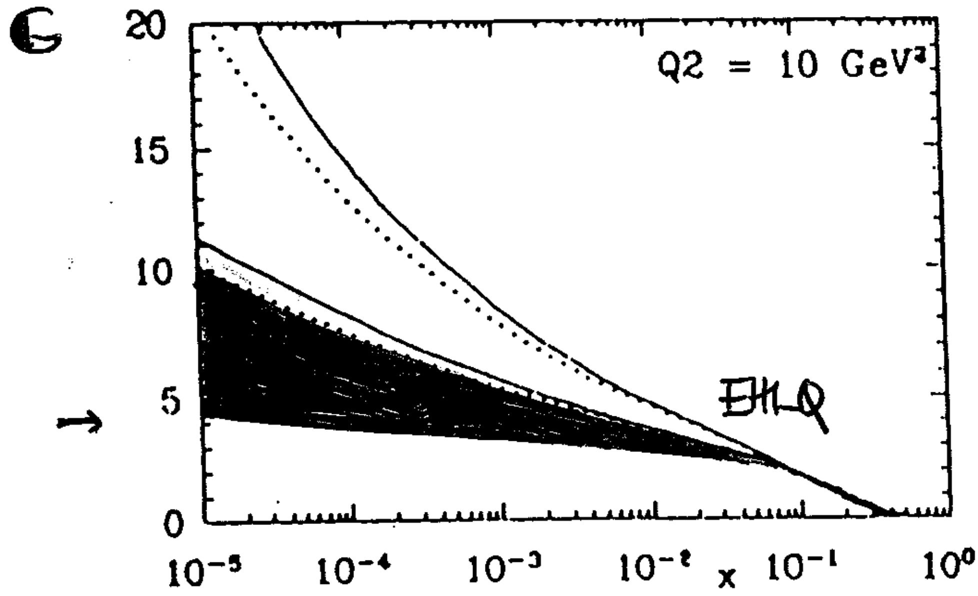
EXTREME y



a

BARTELS, JB, SCHULER 1990

(cf. COLLINS, KWIECINSKI 1990; ALTMANN, GLÜCK, REYA, 1992 FOR SIMILAR INVESTIG.)

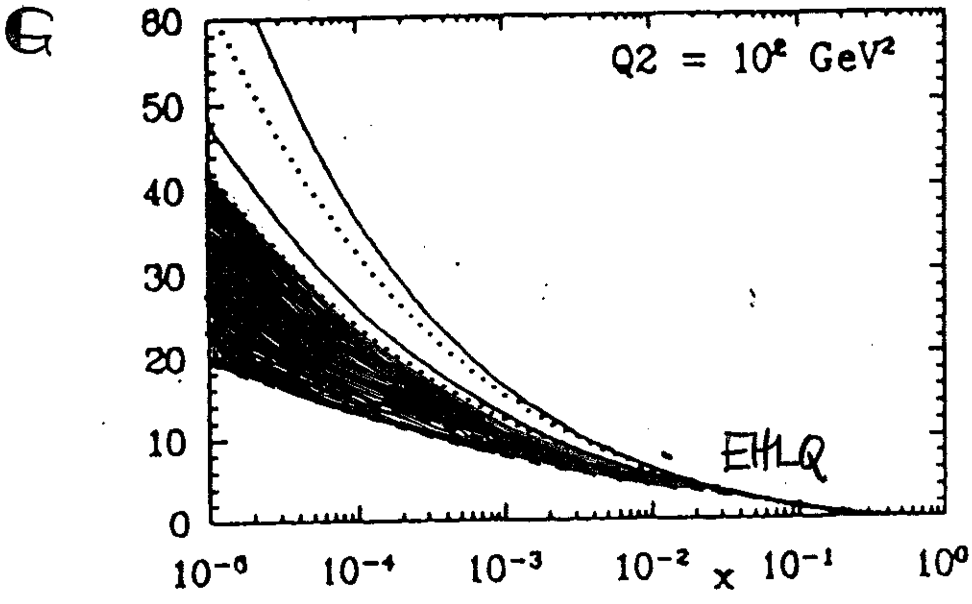


C →

$C = C(\hat{Q}_0^2)$

$\hat{Q}_0 = \hat{Q}_0(R_{sc}$

b



c

SEMICLASSICAL SOLUTIONS

BARTELS, JB, SCHULER ; COLLINS, KWIECINSKI

$$F(y,\xi) \underset{Df}{=} \exp\left(S(y,\xi)\right)$$

$$S_{|y} S_{|\xi} = \frac{1}{2} - C \exp\left[S - e^{\xi} - \xi\right]$$

$$S_{|y} S_{|\xi} \gg S_{|y|\xi} \qquad (\xi \cdot y \gg 1)$$

$$\dot{y} = S_{|\xi}$$

$$\dot{\xi} = S_{|y}$$

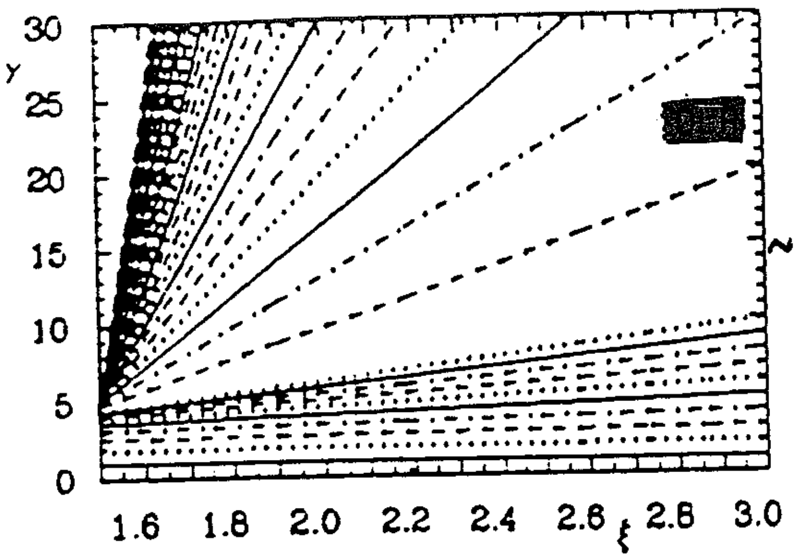
$$\dot{S} = 2 S_{|y} S_{|\xi}$$

$$\dot{S}_{|\xi} = - C\exp\left(S - e^{\xi} - \xi\right) S_{|y}$$

$$\dot{S}_{|\xi} = - C\exp\left(S - e^{\xi} - \xi\right)\left(S_{|\xi} - 1 - e^{\xi}\right)$$

$C=0$ $\qquad \ddot{y} = \ddot{\xi} = 0$: STRAIGHT LINES IN ξ & y.

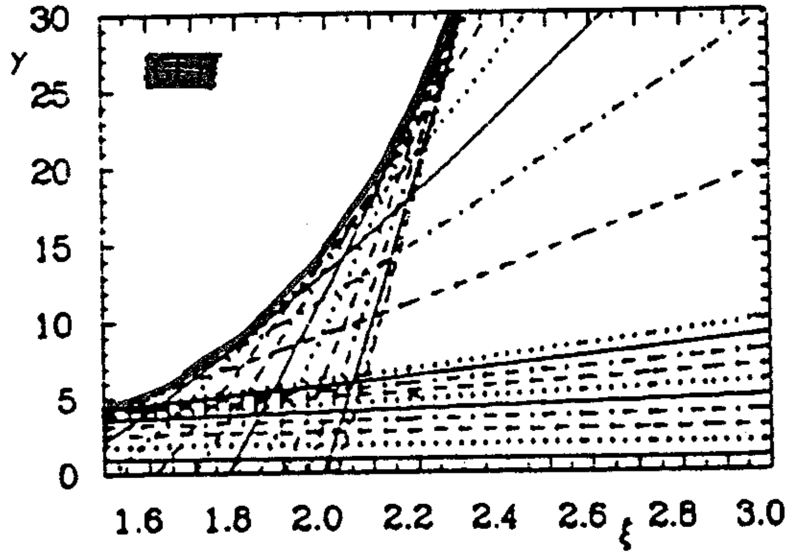
BARTELS, JB, SCHULER



~AP

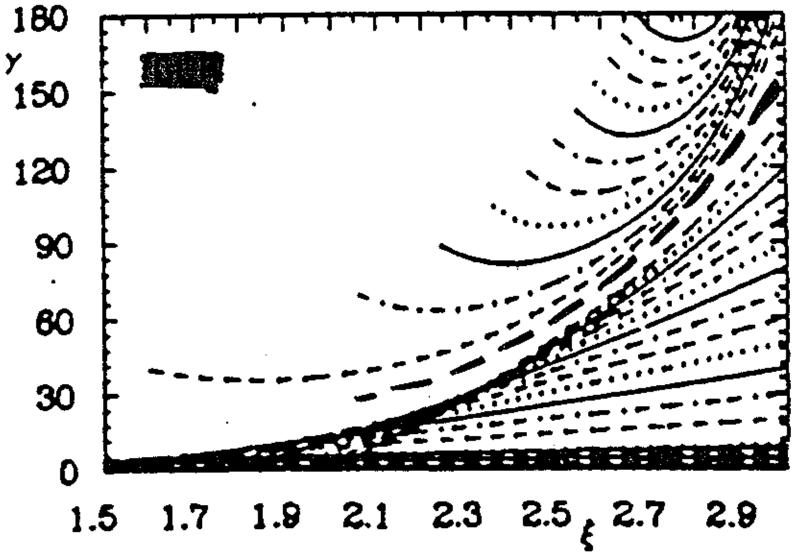
a

TRAJECTORIES STARTING BELOW



b

ABOVE THE 'CRITICAL' LINE



AP + FAN-DIAGRAMS (xG) : MUELLER, QIU

$$\frac{d\,x q_s(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\left[P_{qg}\otimes xG \quad P_{qq}\otimes x q_s\right]$$

$$- \frac{27\,\alpha_s^2}{160\,R^2 Q^2}\left(xG(x,Q^2)\right)^2$$

$$+ \frac{\alpha_s}{\pi Q^2}\,\theta(x_0-x)\int_x^{x_0}\frac{dx'}{x'}\,\gamma\left(\frac{x}{x'}\right)xG_H(x',Q^2)$$

$$\gamma(y) = -2y + 15y^2 - 30y^3 + 18y^4$$

$$\frac{d\,xG_H(x,Q^2)}{d\ln Q^2} = -\frac{81\,\alpha_s^2}{16\,R^2}\,\theta(x_0-x)\int_x^{x_0}\frac{dx'}{x'}\left[x'G(x',Q^2)\right]^2$$

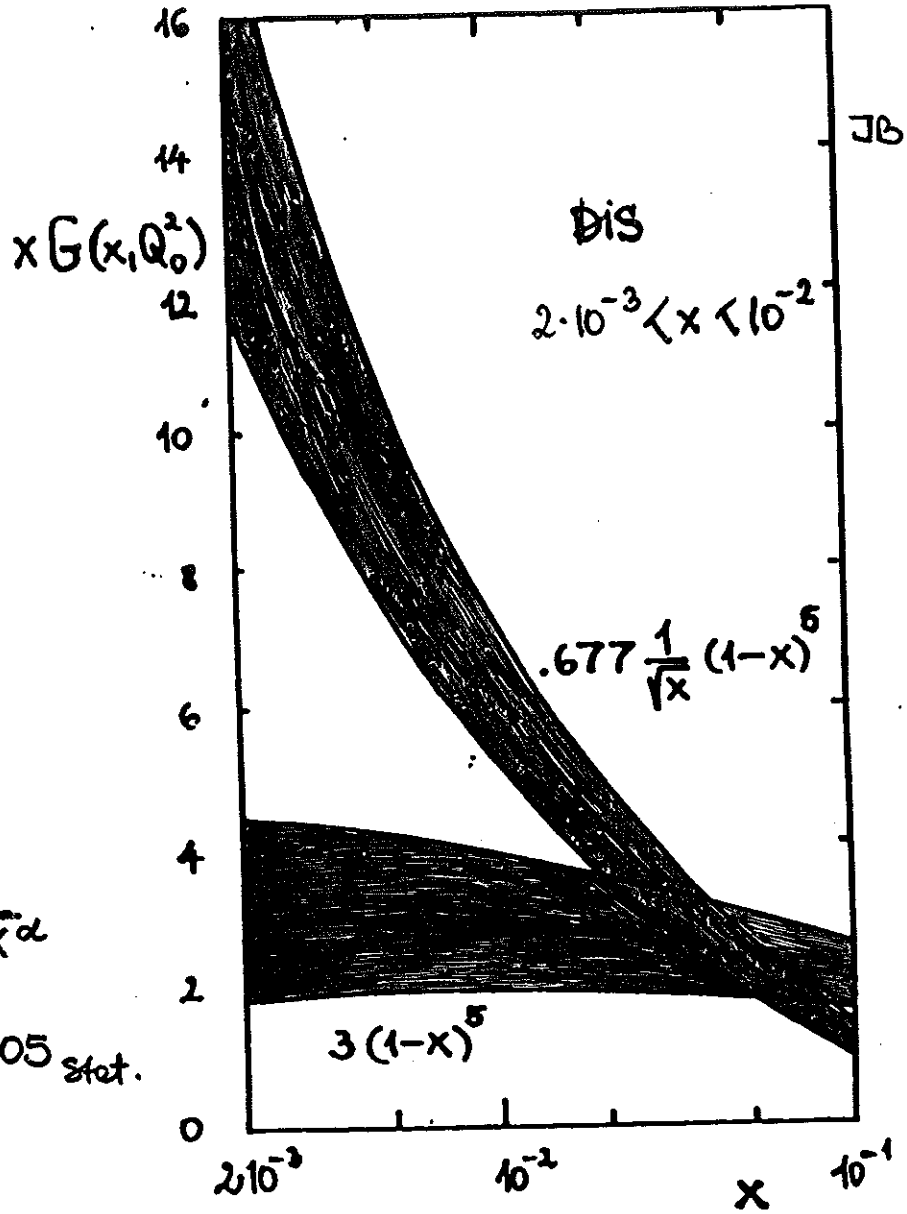
$$\frac{d\,xG(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\left[P_{gg}\otimes xG + P_{gq}\otimes xq\right]$$

$$- \frac{81\,\alpha_s}{16\,R^2 Q^2}\,\theta(x_0-x)\int_x^{x_0}\frac{dx'}{x'}\left[x'G(x',Q^2)\right]^2$$

USED IN: KMRS

→ MODIFICATIONS CURRENTLY WORKED OUT !

JB, M. KLEIN



$xG \propto \frac{1}{x^\alpha}$

$\delta\alpha \simeq \pm 0.05_{stat.}$

Figure 8: Possible determination of $xG(x, Q_0^2)$ in a QCD fit for $x < 0.1$, see text. The upper error band corresponds to the choice $\alpha = -0.5$ and the lower band to $\alpha = 0$, see eq. 15. The inner error denotes the statistical error for $\mathcal{L} = 100\ pb^{-1}$ for both the low and high s option.

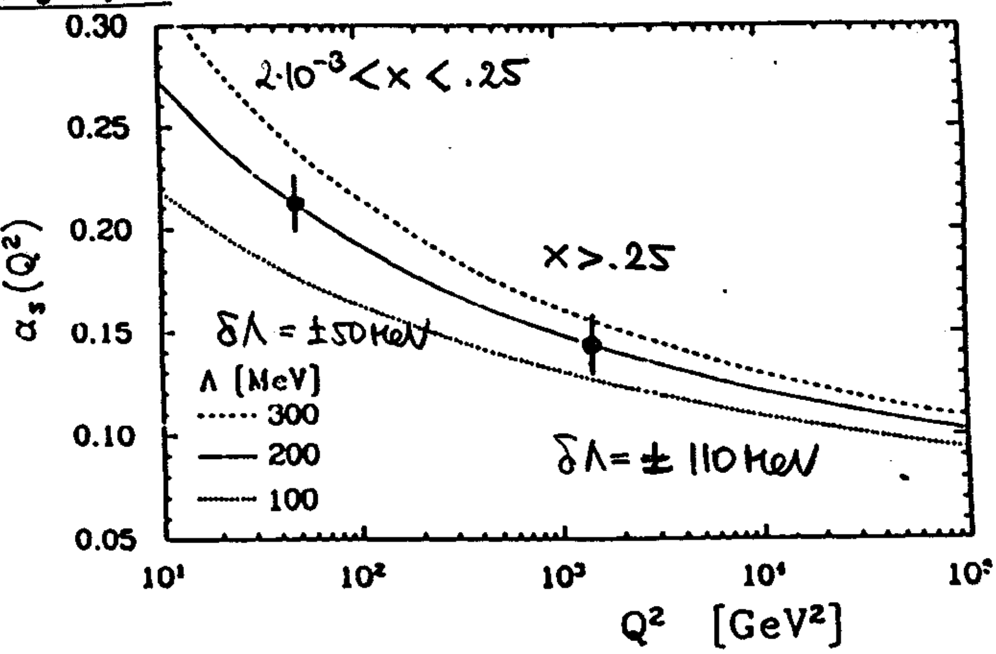
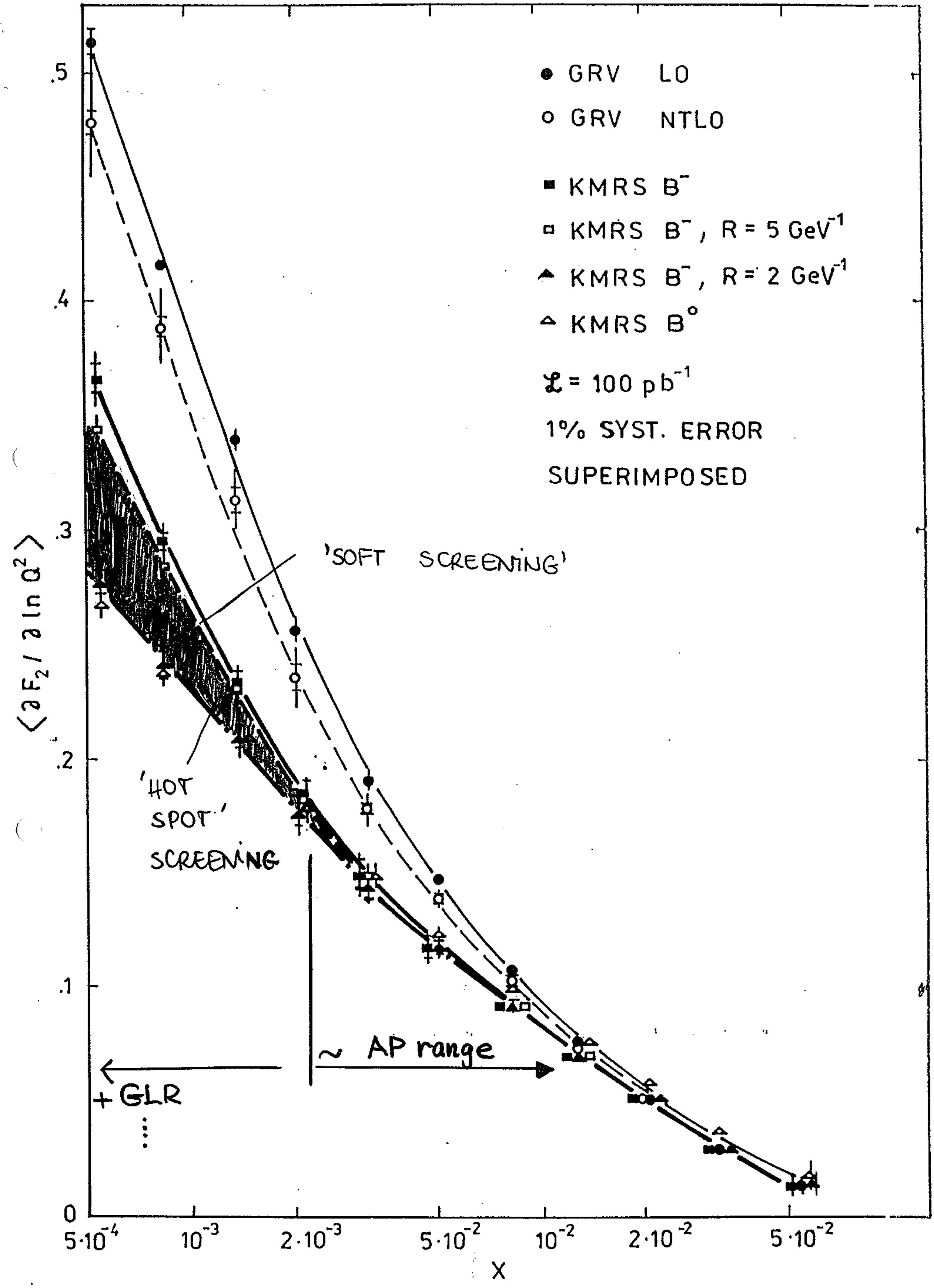
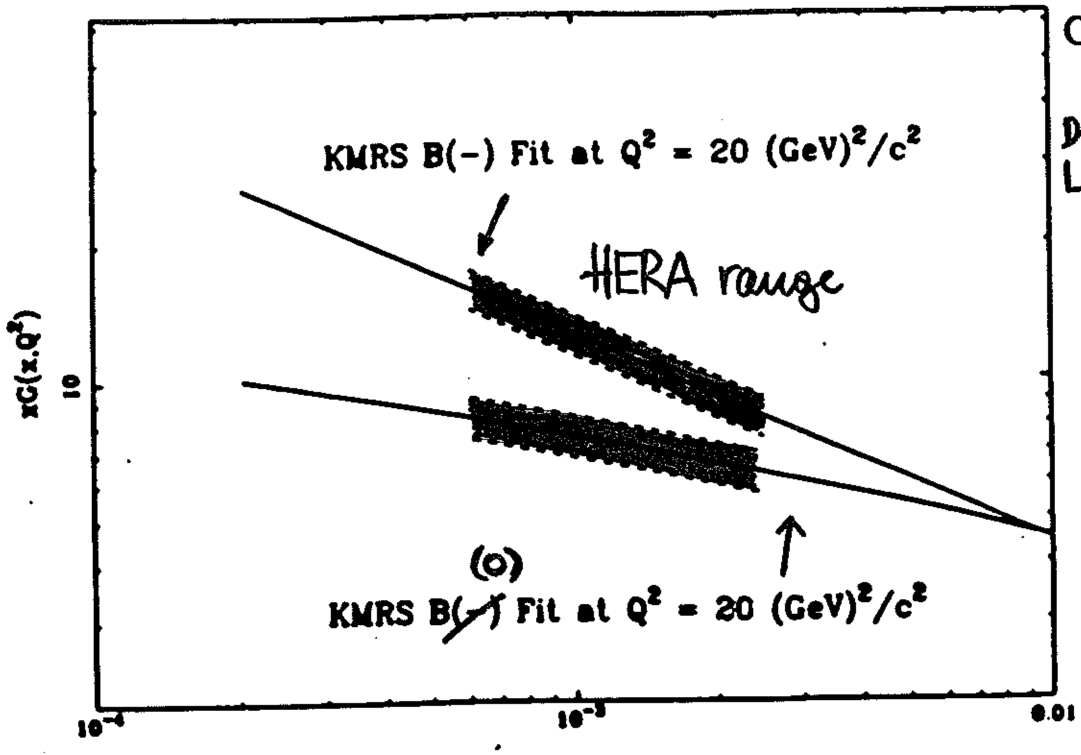


Figure 7: Dependence of α_s on Q^2 from a combined fit using two samples of $\sqrt{s} = 314$ GeV and $\sqrt{s} = 110$ GeV with $\mathcal{L} = 100\ pb^{-1}$ each. The upper point corresponds to a nonsinglet fit for $\theta_e > 5°$ and $x > 0.25$. The lower point at $Q^2 \sim 50$ GeV^2 corresponds to a fit in the



GRV LO
GRV NTLO
KMRS B⁻
KMRS B⁻, R = 5 GeV⁻¹
KMRS B⁻, R = 2 GeV⁻¹
KMRS B°
ℒ = 100 pb⁻¹
1% SYST. ERROR
SUPERIMPOSED
'SOFT SCREENING'
'HOT SPOT' SCREENING
~ AP range
+ GLR
⟨∂F₂ / ∂ ln Q²⟩
.5
.4
.3
.2
.1
0
5·10⁻⁴
10⁻³
2·10⁻³
5·10⁻²
10⁻²
2·10⁻²
5·10⁻²
X



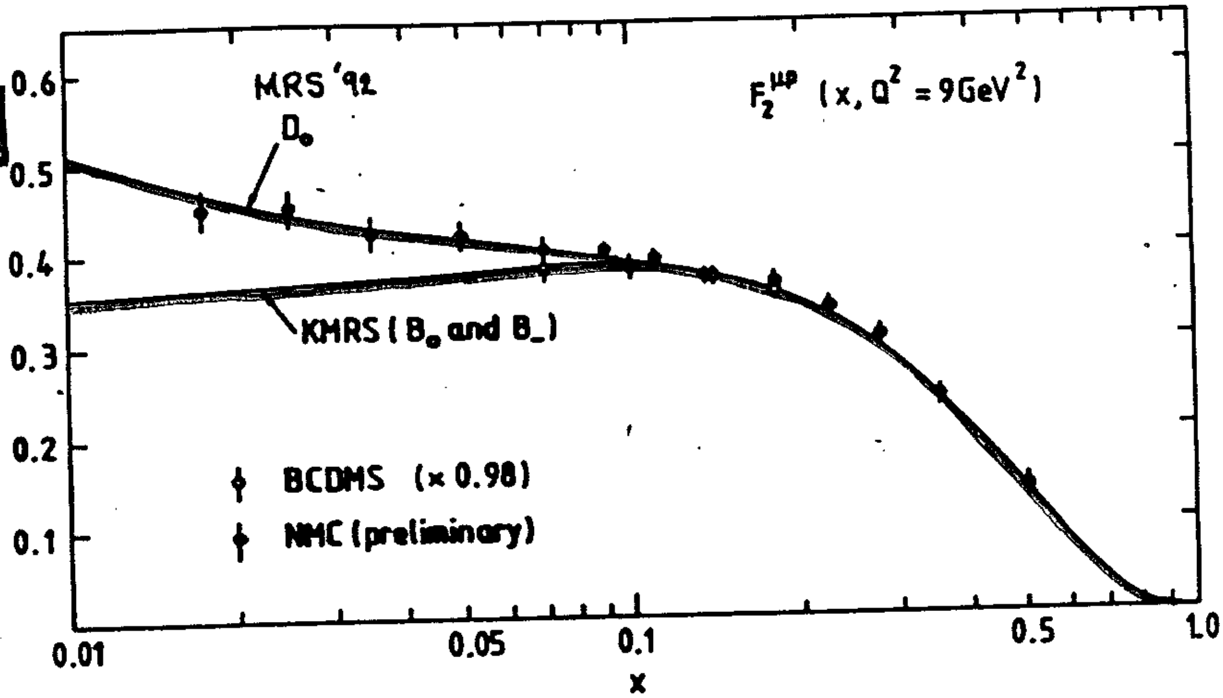
COOPER-SARKAR, DEVENISH, LANCASTER.

Fig 11 - Gluon at Low-x & measurable domain (with errors)

(FROM $F_L^{AP} - O(\alpha_s)$.)

MARTIN, ROBERTS, STIRLING

[redacted]



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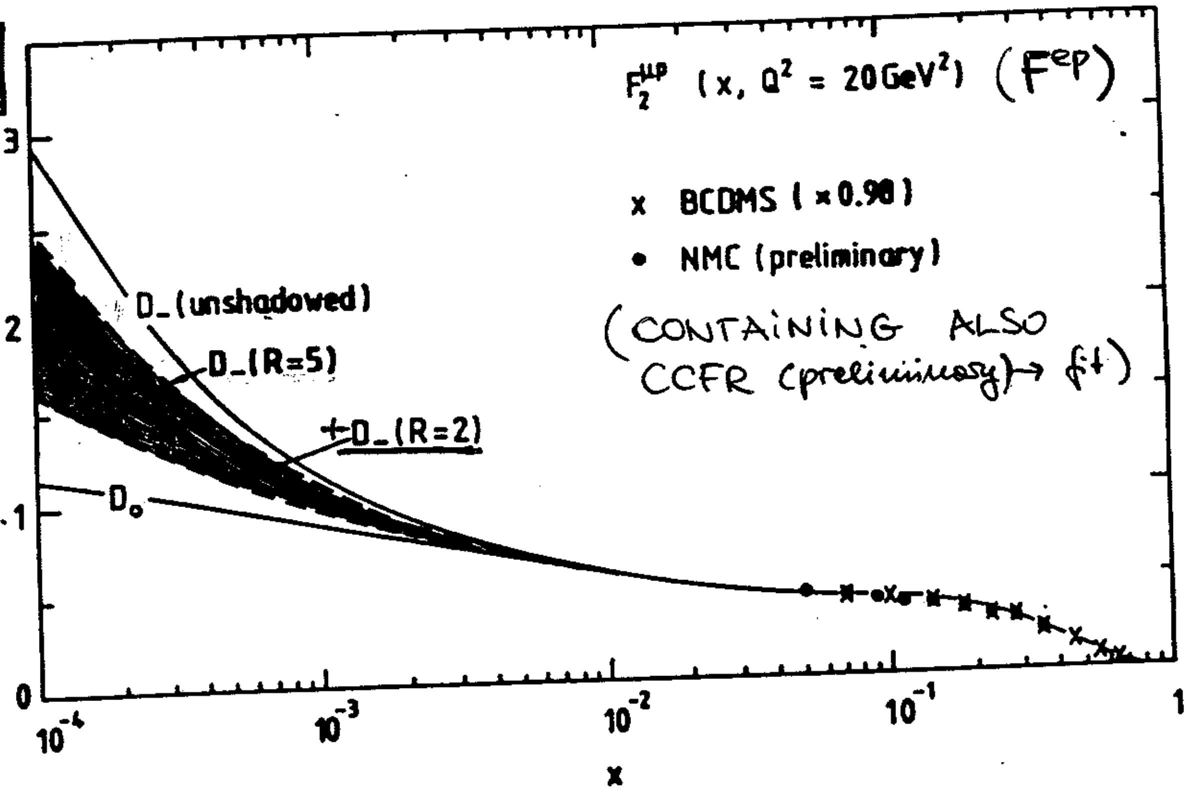
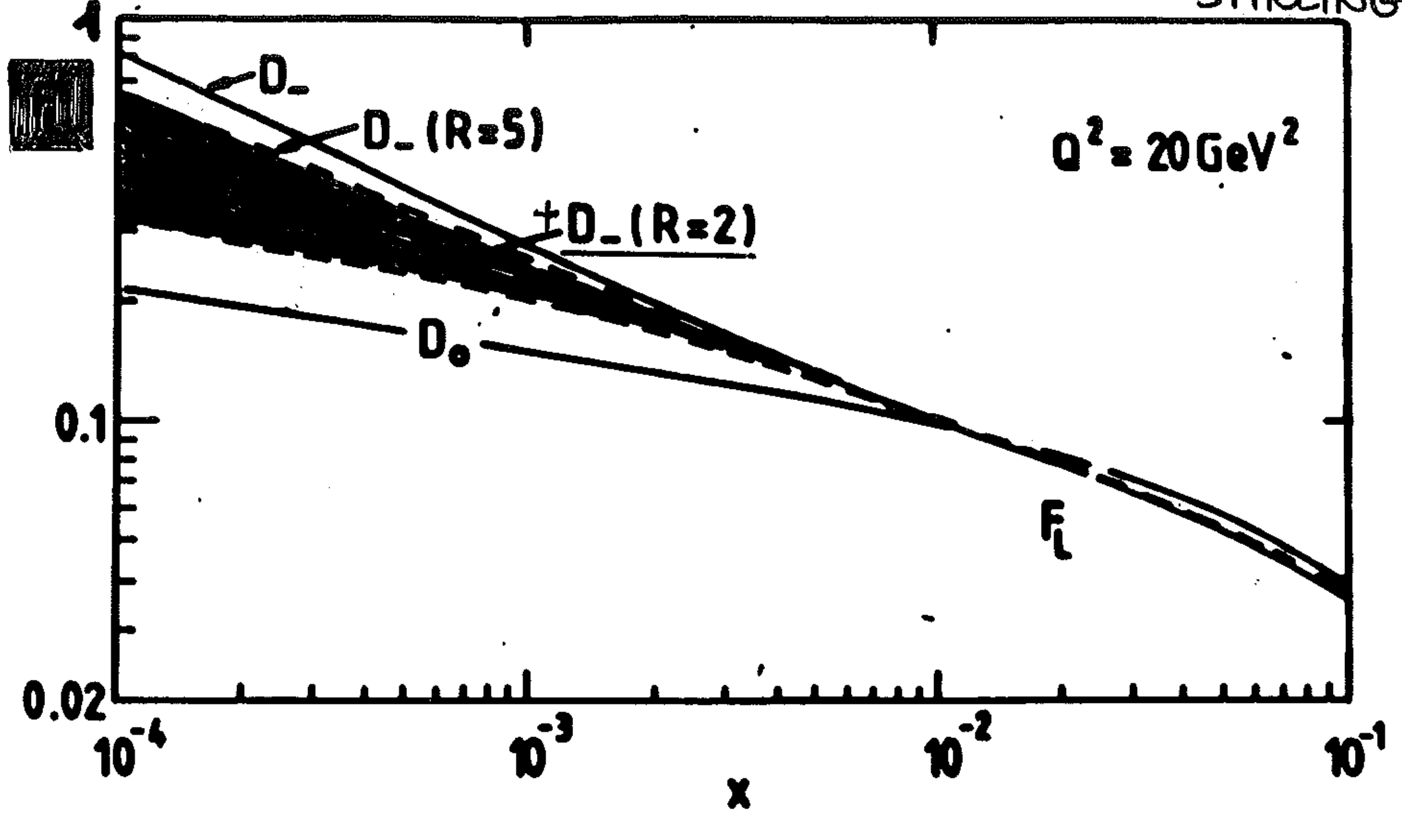


MARTIN, ROBERTS, STIRLING



$O(\alpha_s)$:

$$F_L(x,Q^2) = \frac{\alpha_s}{2\pi}\left\{ \frac{8}{3}\int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2 F_2(y,Q^2) \right.$$

$$\left. + 2\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^2\left(1-\frac{x}{y}\right) y\,\underline{G(y,Q^2)} \right\}$$

ROBERTS:

$$xG(x,Q^2) \simeq \frac{3}{5} \times 5.85 \left\{ \frac{3\pi}{4\alpha_s} F_L(0.4x,Q^2) \right.$$

$$\left. - \frac{1}{2} F_2(0.8x,Q^2 \right\}$$

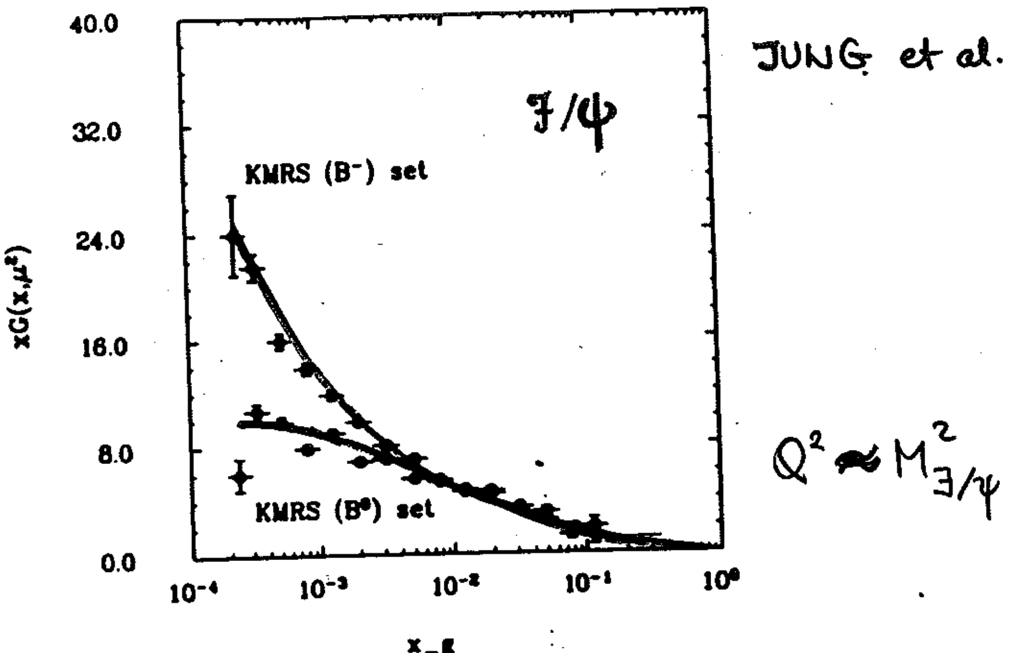


Figure 16: The gluon density reconstructed from inelastic J/ψ production for the input function of KMRS. The statistical error bars correspond to an integrated luminosity of 100 pb^{-1} The curves show the input gluon density.

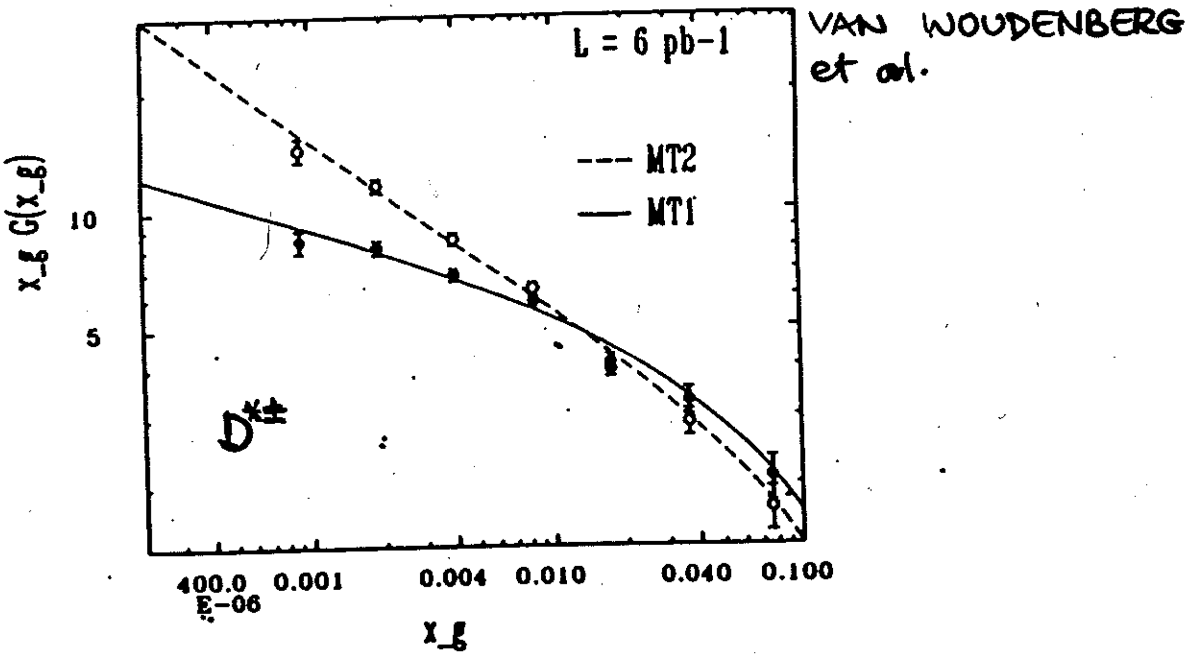
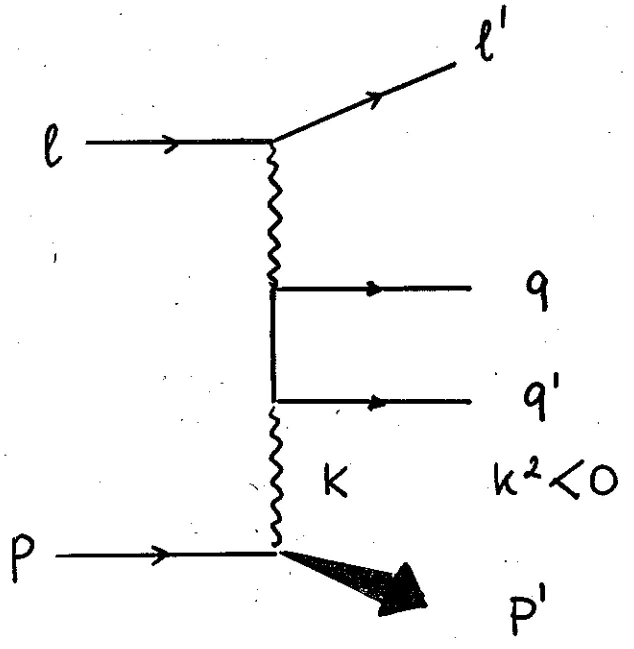


Figure 18

Reconstructed gluon densities from inclusive $D^{*\pm}$ production. The curves show the input gluon functions taken from Morfin and Tung [36]. The error bars include statistical errors for an integrated luminosity of 6 pb^{-1}.

5. EFFECTS DUE TO 'IS' GLUON VIRTUALITY



$+$ CROSSED

SO FAR: $k^2 = 0$ $\longrightarrow$ AP KERNELS

F_2, F_3, F_L

$\longrightarrow$ AT SMALL x THE $k^2 \neq 0$ TERMS MODIFY THE COEFFICIENT FUNCTIONS & THE ANOMALOUS DIMENSIONS

'HIGH s' : LEVIN, RYSKIN (γ^*)

FULL CALC.: JB

PROBLEM OF HIGH 'S' APPROX:

i) CONTACT TO THE USUAL $K^2=0$ PDF's ?!
NEEDED: WORK IN A WELL DEFINED FACTORIZATION SCHEME !
THE DERIVED TERMS ARE NON-SINGULAR !

ii) CONVOLUTION IN X REQUIRES ALSO THE LARGE (i.E. FULL) X BEHAVIOUR.

$P_{ij}(x) \qquad x \in [x_{min}, 1]$

iii) PHASE SPACE REQUIREMENTS HAVE TO BE MET : E.G. $0 \leq k^2 \leq Q^2 \frac{1}{x}(1-x)$
$\neq \infty$ etc.

iv) OTHER CONSTANT TERMS CAN NOT BE IGNORED

e.g. : $\left|24 - 2 \ln \frac{1}{x}\right| \not\approx \left|2\ln\frac{1}{x}\right|$ etc.

NOT ONLY THE LOG'S COUNT. $\qquad x \approx 10^{-4}$

KINEMATICS

$$q + k = p_1 + p_2 \tag{1}$$

$$k_\mu = \xi q' + \eta P_\mu + k_{\perp\mu} \tag{4}$$

with $k_\perp . q' = k_\perp . P = 0$ and

$$x = \frac{Q^2}{2P.q} \tag{5}$$

$$\xi = \frac{2k.P}{2q'.P} \tag{6}$$

$$\eta = \frac{2k.q'}{2q'.P} \tag{7}$$

and

$$K^2 = k^2 - \xi\eta\frac{Q^2}{x} \tag{8}$$

$$\xi \ll \eta \tag{9}$$

which implies

$$\xi \approx 0 \qquad P.k \approx 0 \qquad k_\mu \approx \eta P_\mu + k_{\perp\mu} \qquad K^2 \approx k^2 \tag{10}$$

CMS TURNED OUT TO BE CRUCIAL !

OTHER CHOICES (CIAFALONI et al.) (HEAVY FLAV.) DO NOT ALLOW TO GO TO $n \neq 0$ MOMENTS, I.E. TO <u>ALL</u> X.

The representation of the particle momenta is thus given by:

$$
\begin{aligned}
k &= (K_0, 0, 0, |\vec{k}|) \\
q &= (Q_0, 0, 0, -|\vec{k}|) \\
P &= E_p(1, \sin\beta, 0, \cos\beta) \\
p_1 &= (E_1, q_1 \sin\theta\cos\varphi, q_1 \sin\theta\sin\varphi, q_1\cos\theta) \\
p_2 &= (E_2, -q_1 \sin\theta\cos\varphi, -q_1 \sin\theta\sin\varphi, -q_1\cos\theta)
\end{aligned} \tag{16}
$$

with

$$
\begin{aligned}
K_0 &= \mathcal{E}(\hat{s}, -K^2, -Q^2) \\
Q_0 &= \mathcal{E}(\hat{s}, -Q^2, -K^2) \\
E_1 &= \mathcal{E}(\hat{s}, m_1^2, m_2^2) \\
E_2 &= \mathcal{E}(\hat{s}, m_2^2, m_1^2) \\
|\vec{k}| &= \mathcal{P}(\hat{s}, -Q^2, -K^2) \\
q_1 &= \mathcal{P}(\hat{s}, m_1^2, m_2^2) \\
\cos\theta &= \frac{2K_0E_1 + K^2 - m_1^2 + \hat{t}}{2|\vec{k}|q_1} \\
E_p &= \mathcal{E}(\hat{s}, 0, t_{qP'}) = \frac{Q^2}{2x\sqrt{\hat{s}}} \\
\cos\beta &= \frac{K_0}{|\vec{k}|} \quad = \frac{\hat{S} - K^2 + Q^2}{\left[(\hat{S} - K^2 + Q^2)^2 + 4\hat{S}K^2\right]^{1/2}}
\end{aligned} \tag{17}
$$

where

$$
\begin{aligned}
\mathcal{E}(a,b,c) &= \frac{a+b-c}{2\sqrt{a}} \\
\mathcal{P}(a,b,c) &= \sqrt{\frac{\lambda(a,b,c)}{4a}}
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
dPS^{(3)} &= \frac{1}{128\pi^3}\frac{d\varphi_P}{2\pi}\frac{d\hat{s}\,d\hat{t}\,dK^2}{\lambda^{1/2}(\hat{s}, -K^2, -Q^2)\lambda^{1/2}(s_\gamma, 0, -Q^2)}\frac{d\varphi}{2\pi} \\
&\times\ \Theta\{-\mathcal{G}(s_\gamma, -K^2, \hat{s}, 0, -Q^2, 0)\}\Theta\{-\mathcal{G}(\hat{s}, \hat{t}, 0, -K^2, -Q^2, 0)\}\Theta\{\hat{s}\}
\end{aligned} \tag{21}
$$

$$
\int dPS^{(3)} = \frac{1}{128\pi^3}\int_{K^2_{min}}^{K^2_{max}} dK^2 \int_{\eta_{min}(K^2)}^{\eta_{max}(K^2)} d\eta \int_0^{2\pi}\frac{d\varphi_P}{2\pi}\frac{1}{2}\int_{-1}^{1} d\cos\theta \int_0^{2\pi}\frac{d\varphi}{2\pi} \tag{22}
$$

$$
\begin{aligned}
K^2_{min} &= 0 \\
K^2_{max} &= sy(1-x) \; = \; Q^2\,\frac{1-x}{x} \gg \underline{Q^2}\,! \\
\eta_{min} &= x\left(1 + \frac{K^2}{Q^2}\right) \\
\eta_{max} &= 1
\end{aligned} \tag{24}
$$

FACTORIZATION

$$H(x,\mu^2) = \int_0^1 dx_1 dx_2 \delta(x - x_1 x_2) G(x_1, \mu^2, Q_0^2) \sigma_H^{pt}(x_2, \mu^2) \tag{25}$$

'AP' $\quad K^2 \equiv 0$

$$H(x,\mu^2) = \int d^2\boldsymbol{k} \int_0^1 dx_1 dx_2 \delta(x - x_1 x_2) \mathcal{F}(x_1, \boldsymbol{k}, Q_0^2) \sigma_H^{pt}(x_2, \boldsymbol{k}) \tag{26}$$

Here $\mathcal{F}(x, \boldsymbol{k}, Q_0^2)$ is defined by (cf. [20, 3])

$$G(x,\mu^2,Q_0^2) = \int_0^{\mu^2} d\,\boldsymbol{k}^2 \mathcal{F}(x, \boldsymbol{k}, Q_0^2) \tag{27}$$

$K_\perp$ - SCHEME. $\longrightarrow$ MOST GENERAL SCHEME KNOWN SO FAR.

DERIVATION OF THE COEFF. FUNCTION

$$\frac{d^2\sigma}{dQ^2dy} = 2\pi\alpha^2 \frac{Ms}{(s-M^2)^2}\frac{1}{Q^4}L_{\mu\nu}W^{\mu\nu} \tag{29}$$

$$\begin{aligned} L_{\mu\nu} &= 2\left[l_\mu l'_\nu + l'_\mu l_\nu - g_{\mu\nu} l.l'\right] \\ W_{\mu\nu} &= \frac{1}{4\pi}\sum_n \langle P|J_\mu^{em\dagger}(0)|n\rangle\langle n|J_\nu^{em}(0)|P\rangle(2\pi)^4\delta^{(4)}(P+q-p_n) \end{aligned} \tag{30}$$

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)W_1(x,Q^2) + \frac{1}{M^2}\left[\left(P_\mu - \frac{P.q}{q^2}q_\mu\right)\left(P_\nu - \frac{P.q}{q^2}q_\nu\right)\right]W_2(x,Q^2) \tag{31}$$

$$\begin{aligned} F_2(x,Q^2) &= xT^1_{\mu\nu}W^{\mu\nu} = x\left(-g_{\mu\nu} + \frac{12x^2}{Q^2}P_\mu P_\nu\right)W^{\mu\nu} \\ F_L(x,Q^2) &= xT^2_{\mu\nu}W^{\mu\nu} = \frac{8x^3}{Q^2}P_\mu P_\nu W^{\mu\nu} \end{aligned} \tag{32}$$

$$-g^{\mu\nu}\widehat{W}_{\mu\nu} = 32\pi\alpha_s e_q^2\left\{\frac{(p_1.P)^2+(p_2.P)^2}{\hat{t}\hat{u}} - \frac{Q^2}{K^2}\left[\frac{p_1.P}{\hat{t}} - \frac{p_2.P}{\hat{u}}\right]^2\right\} \tag{34}$$

$$P^\mu P^\nu \widehat{W}_{\mu\nu} = 64\pi\alpha_s e_q^2\frac{1}{K^2}\left\{-2\frac{(p_1.P)^2(p_2.P)^2}{\hat{t}\hat{u}} + \frac{(p_1.P)^3(p_2.P)}{\hat{t}^2} + \frac{(p_1.P)(p_2.P)^3}{\hat{u}^2}\right\} \tag{35}$$

$$-g^{\mu\nu}\widehat{W}_{\mu\nu} = 8\pi\alpha_s e_q^2\left\{\frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}} - 2\frac{sQ^2}{tu}\right\} \tag{36}$$

$$P_\mu P_\nu \widehat{W}_{\mu\nu} = 8\pi\alpha_s e_q^2\hat{s} \tag{37}$$

$$
\begin{aligned}
f_L^{qG}(K^2,x,Q^2) &= -\frac{1}{4\pi}\int dPS^{(2)} x T^2_{\mu\nu}\widehat{W}^{\mu\nu} \\
&= \frac{\alpha_s e_q^2}{4\pi}\left\{\frac{4Q^4}{K^4 x}G_{1L}(\beta,\zeta) + \frac{xQ^2}{K^2}\frac{1}{\sqrt{1-\zeta}}\log\left|\frac{1+\sqrt{1-\zeta}}{1-\sqrt{1-\zeta}}\right| G_{2L}(\beta,\zeta)\right. \\
&\qquad \left. + \frac{2xQ^2}{K^2}G_{3L}(\beta,\zeta)\right\} \qquad (39)
\end{aligned}
$$

with

$$\zeta = \frac{4K^2x^2}{Q^2} \qquad (40)$$

$$G_{i(2,L)}(\beta,\zeta) = \sum_{j=0}^{4} g_{ji}^{(2,L)}(\beta)\left(\frac{\zeta}{W(\zeta)}\right)^j \qquad (41)$$

where

$$W(\zeta) = 1-\zeta+\sqrt{1-\zeta} \qquad (42)$$

$$\cos\beta = \frac{K_0}{|\vec{K}|}$$

$$
\begin{aligned}
g_{01}^{(L)}(\beta) &= -\frac{1}{8}+\frac{1}{4}\cos\beta-\frac{1}{4}\cos^3\beta+\frac{1}{8}\cos^4\beta \\
g_{02}^{(L)}(\beta) &= -\frac{1}{4}+2\cos\beta-\cos^2\beta-3\cos^3\beta+\frac{9}{4}\cos^4\beta \\
g_{03}^{(L)}(\beta) &= -\frac{1}{4}+6\cos\beta-\frac{9}{2}\cos^2\beta-10\cos^3\beta+\frac{35}{4}\cos^4\beta \\
g_{11}^{(L)}(\beta) &= \cos\beta-\frac{3}{4}\cos^2\beta-\frac{3}{2}\cos^3\beta+\frac{5}{4}\cos^4\beta \\
g_{12}^{(L)}(\beta) &= \frac{1}{4}+\frac{13}{2}\cos\beta-\frac{15}{2}\cos^2\beta-\frac{21}{2}\cos^3\beta+\frac{45}{4}\cos^4\beta \\
g_{13}^{(L)}(\beta) &= 1+18\cos\beta-24\cos^2\beta-30\cos^3\beta+35\cos^4\beta \\
g_{21}^{(L)}(\beta) &= \frac{3}{16}+\frac{9}{8}\cos\beta-\frac{9}{4}\cos^2\beta-\frac{15}{8}\cos^3\beta+\frac{45}{16}\cos^4\beta \\
g_{22}^{(L)}(\beta) &= \frac{5}{4}+\frac{27}{4}\cos\beta-15\cos^2\beta-\frac{45}{4}\cos^3\beta+\frac{75}{4}\cos^4\beta \\
g_{23}^{(L)}(\beta) &= \frac{7}{2}+18\cos\beta-42\cos^2\beta-30\cos^3\beta+\frac{105}{2}\cos^4\beta \\
g_{31}^{(L)}(\beta) &= \frac{3}{16}+\frac{6}{16}\cos\beta-\frac{15}{8}\cos^2\beta-\frac{5}{2}\cos^3\beta+\frac{35}{4}\cos^4\beta \\
g_{32}^{(L)}(\beta) &= \frac{9}{8}+\frac{9}{4}\cos\beta-\frac{45}{4}\cos^2\beta-\frac{15}{4}\cos^3\beta+\frac{105}{8}\cos^4\beta \\
g_{33}^{(L)}(\beta) &= 3+6\cos\beta-30\cos^2\beta-10\cos^3\beta+35\cos^4\beta \\
g_{41}^{(L)}(\beta) &= \frac{3}{64}-\frac{15}{32}\cos^2\beta+\frac{35}{64}\cos^4\beta \\
g_{42}^{(L)}(\beta) &= \frac{9}{32}-\frac{45}{16}\cos^2\beta+\frac{105}{32}\cos^4\beta \\
g_{43}^{(L)}(\beta) &= \frac{3}{4}-\frac{15}{2}\cos^2\beta+\frac{35}{4}\cos^4\beta \qquad (57)
\end{aligned}
$$

$K^2 \rightarrow 0$ RESULT:

$$f_L^{qG(0)}(x,Q^2) = \frac{8x^3}{Q^2}\frac{1}{4\pi}\int dPS^{(2,0)}P_\mu P_\nu \widehat{W}^{\mu\nu} = \frac{2}{\pi}e_q^2\alpha_s x^2(1-x) \qquad (43)$$

$$F_i^G(x,Q^2) = \int_x^1 \frac{d\eta}{\eta} \int_{\Lambda^2}^{\mathbb{k}^2_{\perp max}} d\mathbb{k}_\perp^2 \; \frac{\partial G(\eta, \mathbb{k}_\perp^2)}{\partial \mathbb{k}_\perp^2} \times \theta(\mathbb{k}^2_{\perp max} - \Lambda^2)$$

$$\times \left[\sigma^G_{i(0)}\left(\frac{x}{\eta}, Q^2\right) + \sigma^G_{i(1)}\left(\frac{x}{\eta}, Q^2, x\, \mathbb{k}_\perp^2\right) \right]$$

$$\mathbb{k}^2_{\perp max} = \frac{1}{x}\left(\frac{1-\eta}{1-x}\right)(\eta - x)\, Q^2 \quad , \quad \mathbb{K}^2 = \frac{\mathbb{k}_\perp^2}{1-\eta}$$

$$\sup k^2_{\perp max} = \frac{1-x}{4x} Q^2$$

Contact to the $\mathbb{K}^2 = 0$ representations of $F_i^G(x,Q^2)$:

$$\lim_{\mathbb{K}^2 \to 0} \sigma^G_{i(1)}\left(z, Q^2, x, \mathbb{k}_\perp^2\right) = 0$$

DIS scheme expr. derived ($\mathbb{K}^2 = 0$) (transition to other schemes as usual).

$\mathbb{K}^2/Q^2 \ll 1$: $\sigma^G_{i(1)} \sim A \cdot \frac{\mathbb{K}^2}{Q^2}\left(1 + \ldots\right)$

Numerical work in progress:

1st estimates: · little effect for $x \gtrsim 10^{-2}$

· non-negligible contribution for $x \sim 10^{-4}$

6. CONCLUSIONS & OUTLOOK

1) FOR $x \to 0$ THE CONTRIBUTIONS IN LEADING TWIST YIELD A STRONG GROTH IN LO & NTLO; NTLO DAMPS A BIT, BUT DOES NOT STOP THE GROTH.

2) THE $O(\alpha_s^2)$ ARE VERY IMPORTANT FOR HERA MEASUREMENTS. $\sim O(25\%)$

3) THE BFKL TERMS (LO) YIELD $\sim 5\%$.

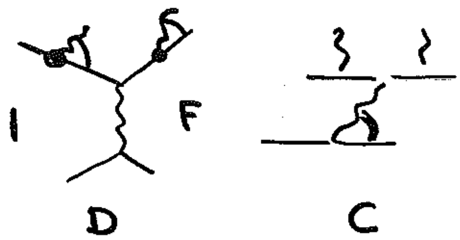
4) GLR 'MECHANISM' : FIRST ATTEMPT TO DESCRIBE SHADOWING
STILL MORE WORK NEEDED:
- − SIGN FROM QCD
- FULL x FORMULAE

6 POINT FCT ! WILL LOOK DIFFERENT. (JB, WKN)

5) VIRTUAL IS − PARTON AMPLITUDES
WORK IN PROGRESS: νN, $Q\bar{Q}$, J/ψ
→ EFFECT ON COEFF. FCT (AND ANOM. DIM)

6) EFFECT OF $k_\perp^2$ − SCHEME IN q&G − EVOLUTION.
→ FURTHER STUDY

7) QUANTITATIVE UNDERSTANDING OF F_2, F_L etc. AT <u>%</u> LEVEL REQU.
FOR $Q^2 \sim 10\ \mathrm{GeV}^2$, $x \gtrsim 10^{-4}$. HERA



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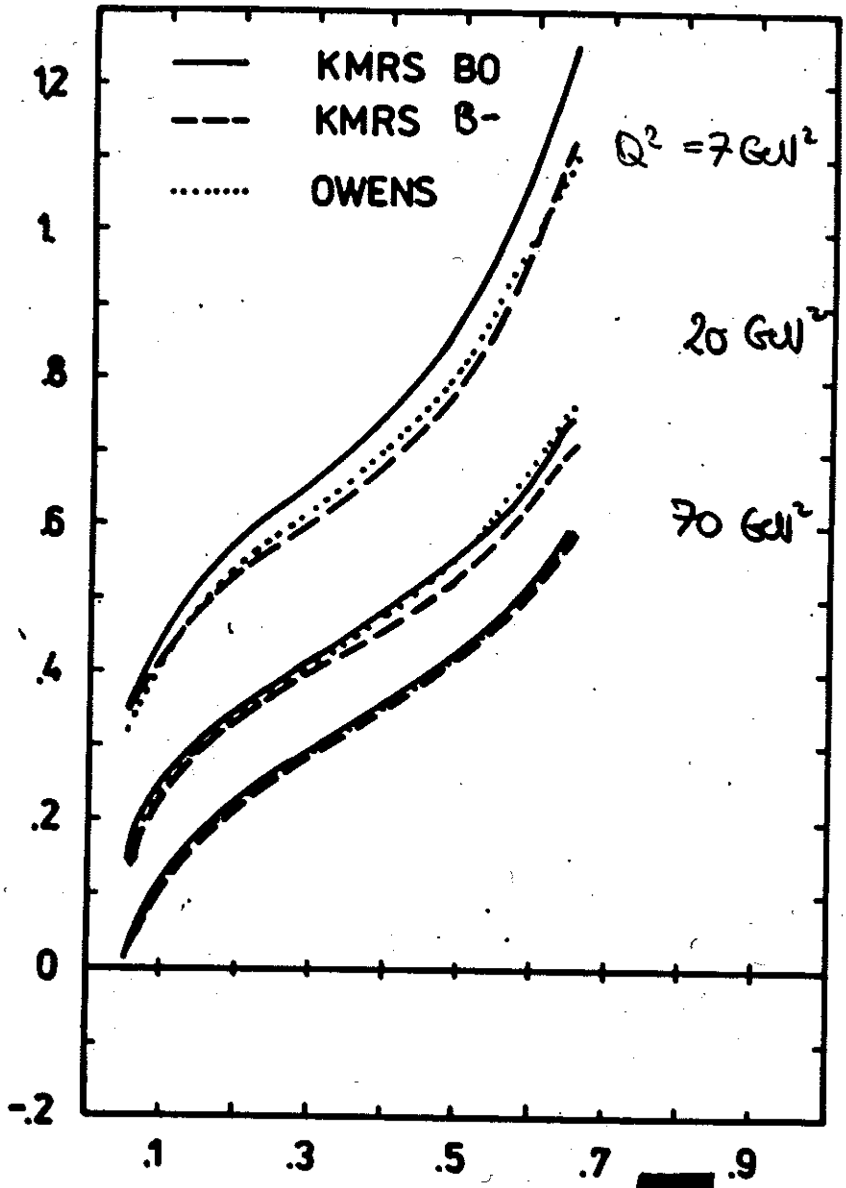

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JB

cf. also
RC WG
DESY '91

$$\frac{d^2\sigma^{e^-p}}{dy\,dQ^2} = \frac{d^2\sigma_0^{e^-p}}{dy\,dQ^2}\left(1 + \delta^{e^-p}(y, Q^2)\right)$$

$\mathcal{O}(\alpha)$ QED.