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$\mathcal{O}(\alpha)$ and $\mathcal{O}(\alpha^2)$ QED Radiative Corrections to Deep Inelastic ep Scattering

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DESY – Zeuthen

1. Introduction
2. Application of the Renormalization Group
3. Different Variables
4. The Corrections up to $\mathcal{O}(\alpha^2 L^2)$
5. Numerical Results
6. Conclusions

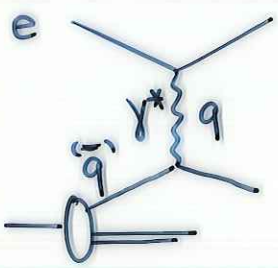
1. Introduction

AIM OF DEEP INELASTIC SCATTERING EXPERIMENTS:

- MEASUREMENT OF BORN-LEVEL STRUCTURE FUNCTIONS $F_i = F_i(x, Q^2)$
- QCD ANALYSIS

BORN CROSS SECTIONS:

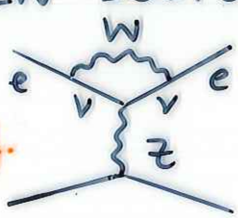
e.g.: $$\frac{d^2\sigma^{NC}}{dx\,dy}\gamma^* = \frac{2\pi\alpha^2}{x Q^4} S x \left\{ Y_+ F_2(x,Q^2) - y^2 F_L(x,Q^2) \right\}$$



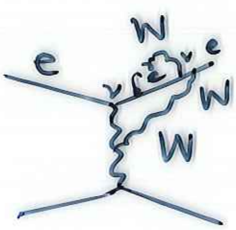
RADIATIVE CORRECTIONS:

- EW-LOOPS:

e.g.



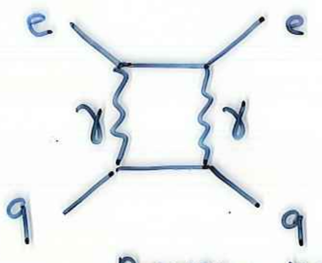
1LOOP



2LOOP

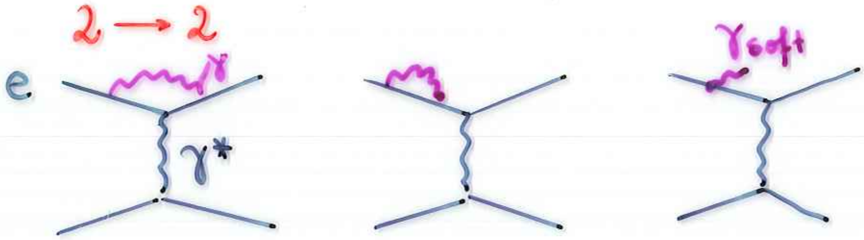
+ ...

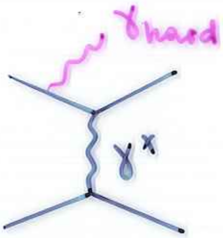
NORMALLY ONLY $2 \to 2$ PROCESSES WITH 'EFFECTIVE' VERTICES (FORMFACTORS)



Boxes; no effective vertex[1]!

QED CORRECTIONS:





$2 \longrightarrow 3$ VS BORN : $2 \longrightarrow 2$

$\mathcal{O}(\alpha)$

$\vdots$

$2 \longrightarrow 2+n$

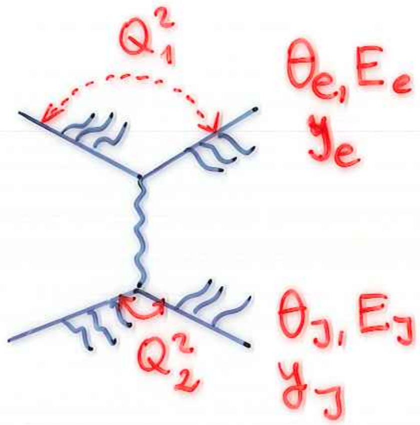
$\mathcal{O}(\alpha^n)$

MOST OF THE IRRADIATED PHOTONS CAN NOT BE TAGGED OR EVEN FULLY MEASURED.

$\Rightarrow$ CALCULATE K-FACTORS

$$\delta(x,y) = \frac{\dfrac{d^2\sigma^{(Born+RC)}}{dx\,dy}}{\dfrac{d^2\sigma^{Born}}{dx\,dy}}.$$

- PROBLEM: WHAT ARE x & y FOR $d^2\sigma^{RC}$?



MANY WAYS TO DEFINE x & y !?

EXPERIMENTAL CHOICE !

- DIFFERENT KINEMATICAL REGIONS AT HERA (E.G.) REQUIRE DIFFERENT CHOICES.
- COMPARE AT LEAST TWO VARIABLE'S SETS !

- 1ST ORDER CORRECTIONS ARE FOUND TO BE LARGE IN SOME RANGES OF THE PHASE SPACE

  → HOW LARGE ARE THE 2nd ORDER CORRECTIONS ?

- FULL CALCULATIONS: TIME CONSUMING

  → X · NUMBER OF DIFFERENT VARIABLE SETS!

  → DO DOMINANT CORRECTIONS SUFFICE ?

2. Application of the Renormalization Group

FACTORIZATION OF LOGARITHMIC CONTRIBUTIONS:
(LEPTON RAD.)

$$\frac{d\sigma^{RAD}}{dx\,dy} = \int_0^1 dz\; \mathbb{G}(z,\mu^2;Q^2;m_e^2)\,\frac{\widehat{d\sigma^{(0)}}}{dx\,dy}$$

SPLITTING FUNCTIONS

$$\mathbb{G}(z,\mu^2,Q^2,m_e^2) = \Gamma_{i\alpha}\left(z,\frac{\mu^2}{m_e^2},\alpha(\mu^2)\right)\otimes\Gamma_{f\beta}\left(z,\frac{\mu^2}{m_e^2},\alpha(\mu^2)\right)$$
$$\otimes\,\tilde{\sigma}_{\alpha\beta}\left(z,\frac{Q^2}{\mu^2},\alpha(\mu^2)\right)$$

WILSON COEFFICIENT

$$A(z)\otimes B(z) = \int_0^1 dx_1\int_0^1 dx_2\,\delta(x_1x_2-z)\,A(x_1)\,B(x_2)$$

μ^2 : FACTORIZATION SCALE & REN. SCALE.

$$\boxed{\frac{d}{d\mu^2}\,\mathbb{G}(z,\mu^2,Q^2,m_e^2)=0}$$

$$\Gamma_{ij} = \sum_{k=0}^{\infty}\alpha^k(\mu^2)\sum_{n=0}^{k}a_{nk}\log^n\left(\frac{m_e^2}{\mu^2}\right)$$

$$\tilde{\sigma}_{ij} = \sum_{k=0}^{\infty}\alpha^k(\mu^2)\sum_{n=0}^{k}b_{nk}\log^n\left(\frac{Q^2}{\mu^2}\right).$$

a_{nk} & b_{nk} ARE RELATED!

CONSIDER E.G. MOMENTS:

$$\Gamma_{ij}^{(n)} = \int_0^1 dz\, z^{n-1}\, \Gamma_{ij}(z)$$

$$\tilde{\sigma}_{ij}^{(n)} = \int_0^1 dz\, z^{n-1}\, \tilde{\sigma}_{ij}(z)$$

$$\left[\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right)\delta_{a\ell} + \gamma_{a\ell}^{(n)}(g)\right]\Gamma_{\ell i}^{(n)}\left(\frac{\mu^2}{m_e^2}, g(\mu)\right) = 0$$

$$\left[\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right)\delta_{\ell a}\delta_{kb} - \gamma_{\ell a}^{(n)}\delta_{kb} - \gamma_{kb}^{(n)}\delta_{\ell a}\right]\tilde{\sigma}_{\ell k}\left(\frac{Q^2}{\mu^2}, g(\mu)\right) = 0.$$

IN FACT ONE OBTAINS:

$$\left[\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right]\frac{d\sigma^{RAD}}{dx\,dy} = 0.$$

$$\beta(g) = -\beta_0\frac{g^3}{16\pi^2} - \beta_1\frac{g^5}{(16\pi^2)^2} + \cdots \quad ; \quad \beta_0 = -\frac{4}{3}, \quad \beta_1 = -4$$

$N_f = 1$ (e)

$$\gamma^{(n)}(g) = \begin{pmatrix} \gamma_{ee}^{(n)}(g) & \gamma_{\gamma e}^{(n)}(g) \\ \gamma_{e\gamma}^{(n)}(g) & \gamma_{\gamma\gamma}^{(n)}(g) \end{pmatrix}$$

$$\gamma_{ij}^{(n)}(g) = \gamma_{0,ij}^{(n)}\frac{g^2}{16\pi^2} + \gamma_{1,ij}^{(n)}\left(\frac{g^2}{16\pi^2}\right)^2 + \cdots$$

EXAMPLES:

1) LLA: INSPECT Γ_{ij}, $\tilde{\sigma}_{ij}$.

ONLY CONTRIBUTIONS CONTAINING $\gamma_{ij}^{(n)}$! $\quad \propto \alpha^n(\mu^2=\varphi^2) \log^n\left(\frac{Q^2}{m_e^2}\right)$.

i.E. NO 'NONTRIVIAL' CONTRIBUTIONS DUE TO THE WILSON COEFFICIENTS

$\longrightarrow$ SOLVE EVOLUTION EQU. WITH RUNNING α_{QED} TO SOME ORDER.

2) NON LEADING LOGS:

e.g. $\alpha^2 \log\left(\frac{Q^2}{m_e^2}\right)$: REQUIRES THE KNOWLEDGE OF THE COMPLETE $O(\alpha)$ CALCULATION (SAME KIN. VARIABLES).

$\longrightarrow$ WAY TO DETERMINE NON-TRIVIAL PARTS IN THE WILSON COEFFICIENTS.

3. Different Variables

- INTEGRATION OVER THE $PS^{(2+n)} \setminus PS^{(2)}$ PHASE SPACE ($n\gamma$ HARD BREMSSTRAHLUNG $f\bar{f}$, e^+e^- PAIR RADIATION).

CHOICES:

- (CLASSICAL): LEPTON MEASUREMENT: Q_e^2, y_e
- DOUBLE ANGEL METHOD θ_e, θ_J
- JET MEASUREMENT NC Q_J^2, y_J
- JET MEASUREMENT CC Q_J^2, y_J
  ← CC: ONLY WAY
- MIXED VARIABLES Q_e^2, y_J
- θ_e, y_J

ALL K-FACTORS TURN OUT TO BE DIFFERENT FUNCTIONS OF x & y

$\hat{V}$ SUBSYSTEM VARIABLE

V 'TREE LEVEL' VARIABLE

	$\hat{s}$	$\hat{Q}^2$	$\hat{y}$	$\mathcal{J}(x,y,z)$
lepton measurement	zs	$Q^2 z$	$(z+y-1)/z$	$y/(z+y-1)$
jet measurement	zs	$Q^2(1-y)/(1-y/z)$	y/z	$(1-y)/(z-y)$
mixed variables	zs	$Q^2 z$	y/z	1
double angle method	zs	$Q^2 z^2$	y	z
y_{JB} and θ_e	zs	$Q^2 z(z-y)/(1-y)$	y/z	$(z-y)/(1-y)$

Table 1: The shifted variables for different types of cross section measurement

- z_0 :

LEPTON MEASUREMENT
JET MEASUREMENT — $\hat{x}(z_0) = 1$

MIXED VARIABLES
θ_e , y_J — $z_0 = y$

DOUBLE ANGEL : $z_0 \equiv 0.$

$\hat{\mathcal{J}}(\hat{x} \to 0, \hat{Q}^2 \to 0)$! for $z \to z_0$!

BUT:

$$2E_e = E'_e(1-\cos\theta_e) + E_J(1-\cos\theta_J) \geq \mathcal{A} \quad ! \qquad (3)$$

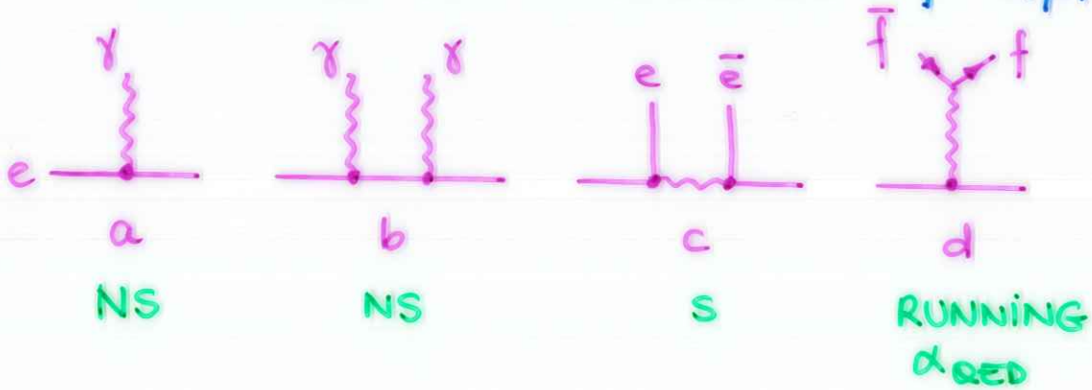
FORTUNATELY: $z_0 = \dfrac{\mathcal{A}}{2E_e}$.

$\Longrightarrow$ ZEUS: THIS HELPS ONLY IN THE CASE OF THE DOUBLE ANGEL METHOD !

4. The Corrections up to $\mathcal{O}(\alpha^2 L^2)$

CONTRIBUTIONS:

1. BREMSSTRAHLUNG : DIAGRAMS a, b
2. ELECTRON PAIR PRODUCTION : DIAGRAM c
3. FERMION PAIR PRODUCTION : $f = e, \mu, \tau, u, d, s, c, b$



'RADIATOR' : 'ABSORB' VIRTUAL CORRECTIONS [(PHYSICAL GAUGE), LADDER'S, QCD TECHNIQUES]

$\Rightarrow$ DERIVATION OF INDIVIDUAL CONTRIBUTIONS TO INITIAL AND FINAL STATE RADIATION IS POSSIBLE.

$\Rightarrow$ LEADING LOG'S: $O\left(\left(\frac{\alpha}{2\pi} L\right)^n\right)$

SUBLEADING LOG'S: $\left(\frac{\alpha}{2\pi}\right)^n L^{n-1}$, $n > 1$.

RGE – METHOD.

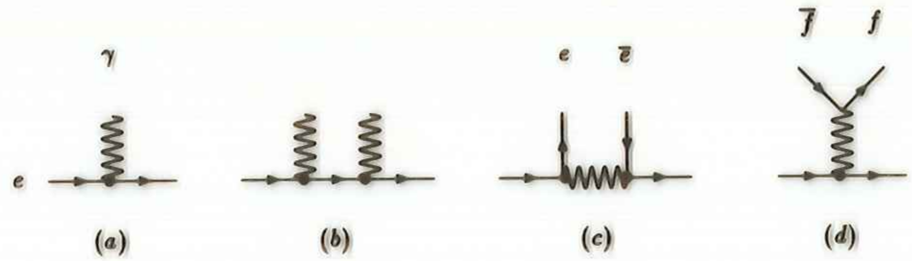
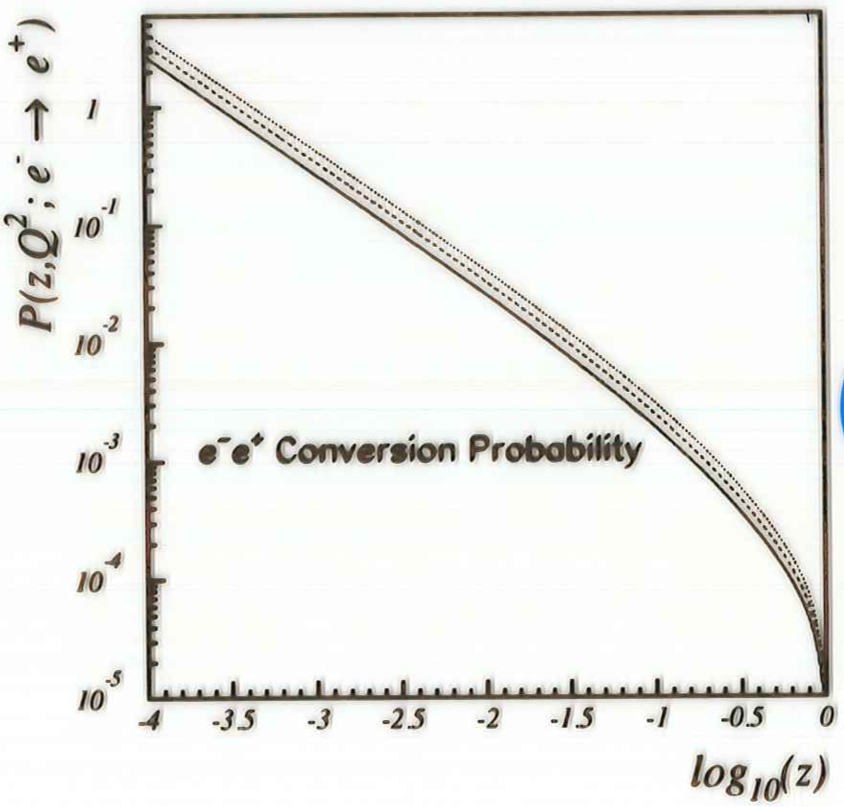


Figure 1: Diagrams contributing to the radiative corrections up to $\mathcal{O}(\alpha^2 L^2)$.



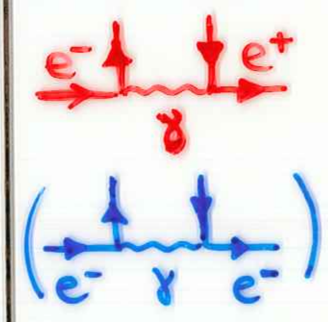


Figure 2: $e^- \to e^+$ transition probability for different values of Q^2. Full line: $Q^2 = 10\,\mathrm{GeV}^2$, dashed line: $Q^2 = 100\,\mathrm{GeV}^2$, and dotted line: $Q^2 = 1000\,\mathrm{GeV}^2$.

THE CONTRIBUTIONS:

$$\frac{d^2\sigma^{(2)}}{dx\,dy} = \frac{d^2\sigma^{(0)}}{dx\,dy} + \frac{d^2\sigma^{(1)}}{dx\,dy} + \frac{d^2\sigma^{(2)}}{dx\,dy}$$

$$\frac{d^2\sigma^{(1)}}{dx\,dy} = \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\int_0^1 dz\, P_{ee}^{(1)}(z)\left\{\theta(z-z_0)\,\mathbb{J}(x,y,s)\left.\frac{d^2\sigma^{(0)}}{dx\,dy}\right|_{\substack{\hat{x}=x\\ \hat{y}=y\\ \hat{s}=s}} - \frac{d^2\sigma^{(0)}}{dx\,dy}\right\}$$

BREMSSTRAHLUNG

$$\frac{d^2\sigma^{(2)}}{dx\,dy} = \left(\frac{\alpha}{2\pi}\right)^2\ln^2\left(\frac{Q^2}{m_e^2}\right)\int_0^1 dz\, P_{ee}^{(2,1)}(z)\left\{\theta(z-z_0)\,\mathbb{J}(x,y,s)\,\widehat{\frac{d^2\sigma^{(0)}}{dx\,dy}} - \frac{d^2\sigma^{(0)}}{dx\,dy}\right\}$$

BREMSSTRAHLUNG

$$+\left(\frac{\alpha}{2\pi}\right)^2\int_{z_0}^1 dz\left\{\ln^2\left(\frac{Q^2}{m_e^2}\right)P_{ee}^{(2,2)}(z) + \sum_{f=\ell,q}\ln^2\left(\frac{Q^2}{m_f^2}\right)P_{ee,f}^{(2,3)}(z)\right\}$$

e^+e^- PAIRS "S"

FERMION PAIRS RUNNING α_{QED}.

$$\times\ \mathbb{J}(x,y,s)\,\widehat{\frac{d^2\sigma^{(0)}}{dx\,dy}}$$

$$\mathbb{J}(x,y,s) = \begin{vmatrix} \partial\hat{x}/\partial x & \partial\hat{y}/\partial x \\ \partial\hat{x}/\partial y & \partial\hat{y}/\partial y \end{vmatrix}$$

RESCALING: SEE TABLE ABOVE.

SPLITTING FUNCTIONS:

$z < 1$:

$O(\alpha)$: $$P_{ee}(z) = \frac{1+z^2}{1-z}$$

$O(\alpha^2)$:

$$\begin{cases} P_{ee}^{(2,1)}(z) = \frac{1}{2}\left[P_{ee}(z) \otimes P_{ee}(z)\right] \\ \quad = \frac{1+z^2}{1-z}\left[2\ln(1-z) - \ln z + \frac{3}{2}\right] + \frac{1}{2}(1+z)\ln z \\ \qquad - (1-z) \\ P_{ee}^{(2,2)}(z) = \frac{1}{2}\left[P_{e\gamma}(z) \otimes P_{\gamma e}(z)\right] \\ \quad = (1+z)\ln z + \frac{1}{2}(1-z) + \frac{3}{2}\frac{1}{z}(1-z^3) \\ P_{ee}^{(2,3)}(z) = N_c(f)\, e_f^2\, \frac{1}{3} P_{ee}(z)\, \theta\left(1 - z - \frac{2m_f}{E_e}\right) \end{cases}$$

OTHER CONTRIBUTIONS: (UNIVERSAL, LARGE)

$\sum_f$  $\times\ \delta_{vac\text{-}pol}(Q^2)$

SOFT EXPONENTIATION :

SOLVE : LO - GRIBOV LIPATOV eq. (NS) FOR $z \to 1$

$$D_{NS}(z,Q^2) = \zeta(1-z)^{\zeta-1}\frac{\exp\left[\frac{1}{2}\zeta\left(\frac{3}{2}-2\gamma_E\right)\right]}{\Gamma(1+\zeta)} \tag{8}$$

with

$$\zeta = -3\ln\left[1-(\alpha/3\pi)\ln(Q^2/m_e^2)\right] \tag{9}$$

(RUNNING α_{QED} !)

THESE TERMS WERE TAKEN INTO ACC. ALREADY !

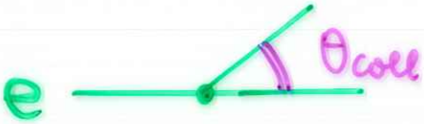
$$P_{ee}^{>2,\ soft}(z,Q^2) = D_{NS}(z,Q^2) - \frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\frac{2}{1-z}\left\{1+\frac{\alpha}{2\pi}\ln\left(\frac{Q^2}{m_e^2}\right)\left[\frac{11}{6}+2\ln(1-z)\right]\right\} \tag{10}$$

and[6]

$$\frac{d^2\sigma^{(>2,soft)}}{dx\,dy} = \int_0^1 dz\,P_{ee}^{(>2)}(z)\left\{\theta(z-z_0)\mathcal{J}(x,y,z)\frac{d^2\sigma^{(0)}}{dx\,dy}\Big|_{x=\hat{x},y=\hat{y},s=\hat{s}} - \frac{d^2\sigma^{(0)}}{dx\,dy}\right\} \tag{11}$$

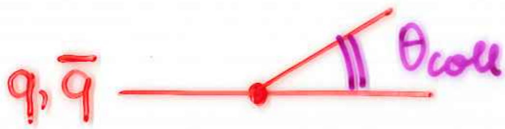
→ NOTE: NO 'UNIQUE' EXPONENTIATION EXISTS !

COLLINEAR SITUATIONS



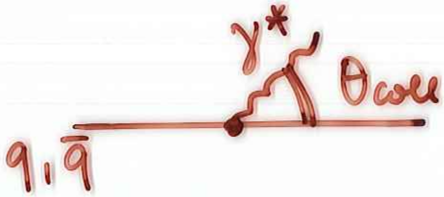
e : INITIAL STATE ELECTRON

e : FINAL STATE ELECTRON



$\overset{(-)}{q}$: INITIAL STATE QUARK (ANTIQUARK)

$\overset{(-)}{q}$: FINAL STATE QUARK (ANTIQUARK).



COMPTON PEAK.

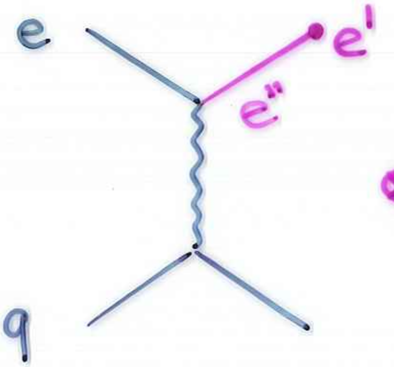
$q^2(\gamma^*) \lesssim M_p^2$.

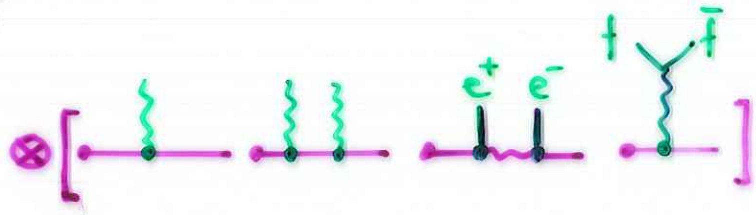
⟹ LEADING LOG CORRECTIONS

RELEVANCE ? OF ALL THESE CONTRIBUTIONS IN THE K-FACTORS.

- ISR LEPTON RADIATION REMAINS (BEAM-HOLE).

FINAL STATE ELECTRON RADIATION





COLLINEAR SITUATION $\theta_{rad} \approx \theta_{e'}$

CALORIMETRIC MEASUREMENT OF THE FINAL STATE, FINAL RESOLUTION

↳ INTEGRATE RADIATED γ MOMENTA & FERMION MOMENTA

↳ MEASURE e'' KINEMATICS

KLN-THEOREM FOR RAD. FINAL STATE!

$\boxed{\text{NO LOGARITHMIC CONTRIBUTIONS!}}$

⟶ LEPTONIC FINAL STATE

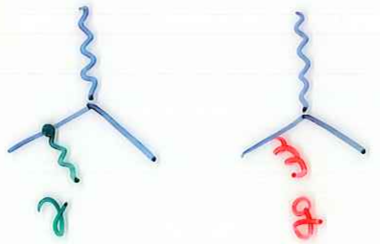
⟶ QUARK FINAL STATE.

QED Corrections: Radiation from Quarks

CONSIDER ISR NOW.

- EVERY PHOTON EMISSION CAN BE SUBSTITUTED BY A GLUON EMISSION !
- $\alpha_s \gg \alpha$!
- QED CORRECTION TO SCALING VIOLATIONS

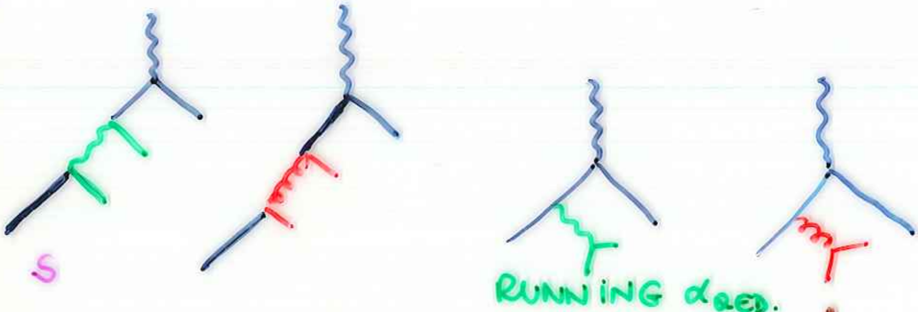
IN MORE DETAIL: $(m_q \to 0)$



$$\tilde{P}_{ff}(z) = C_F \frac{\alpha_s}{2\pi} \frac{1+z^2}{1-z} \left[1 + e_f^2 \frac{\alpha}{\alpha_s} \frac{1}{C_F} \right]$$

& SIM. OTHER TERMS.

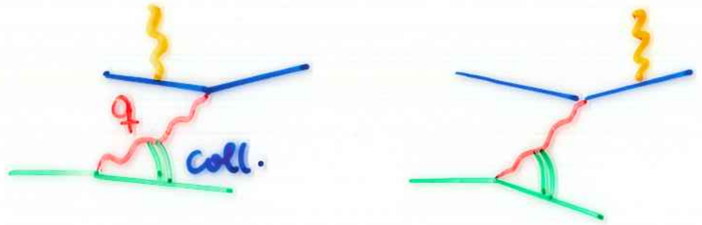
NS



MODIFY EVOLUTION EQUATIONS!

$\rightarrow$ $O(1\%)$ effects.

The Compton Peak

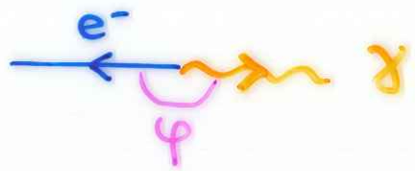


$|q^2| \lesssim M_p^2$

JB, G. LEVMAN, H. SPIESBERGER '93.

THE SIGNATURE:

- φ : ~BACK TO BACK



- ~BOOSTED (e, γ) PAIR AT FINITE $p_\perp$
- LITTLE HADRONIC ACTIVITY (SEEN ?)

$W^2 \lesssim M_p^2 \ldots$ few GeV^2.

$\boxed{\text{NO REAL DIS SIGNATURE !}}$

ADVANTAGE: LEAVE IT OUT FOR DIS RC'S (LEPTONIC VARIABLES)

- $\delta_{RC}^{NC,\,lept}$ DIMINUISHES !

USE ?!

WINDOW ! TO $F_{2,L}(x \to 0, Q^2 \to 0)$

NONPERTURBATIVE RANGE !

WHY DO WE WANT TO KNOW $F_i(x \to 0, Q^2 \to 0)$ AT ALL ?

- WE DO NOT KNOW HOW TO PREDICT IT WITHIN QCD (SMALL x!).

- 1° : PRACTICAL REASON:

(OTHER) DIS RC'S : e.g. INITIAL STATE RADIATION

$$\frac{d\hat{\sigma}}{dx\, dQ^2} \longrightarrow \frac{d\sigma^{(0)}}{dx\, dQ^2} \left(\hat{Q}^2 \to z\, Q^2\right)!$$

↑ small AT SMALL x !

↑ INPUT : STANDARD RC'S.

$\boxed{\text{FIG}}$

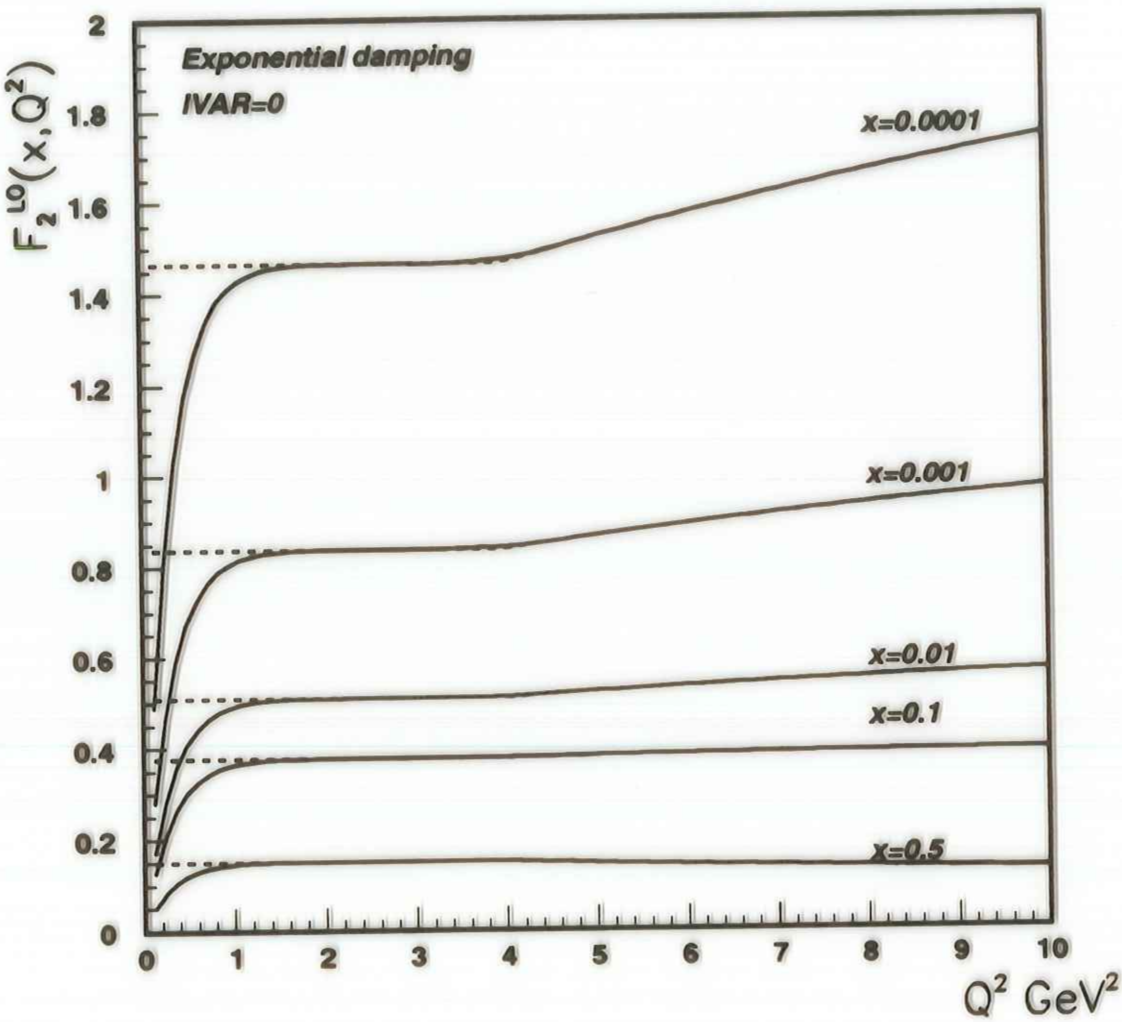
- WE WANT TO SEE THE TRANSITION IN

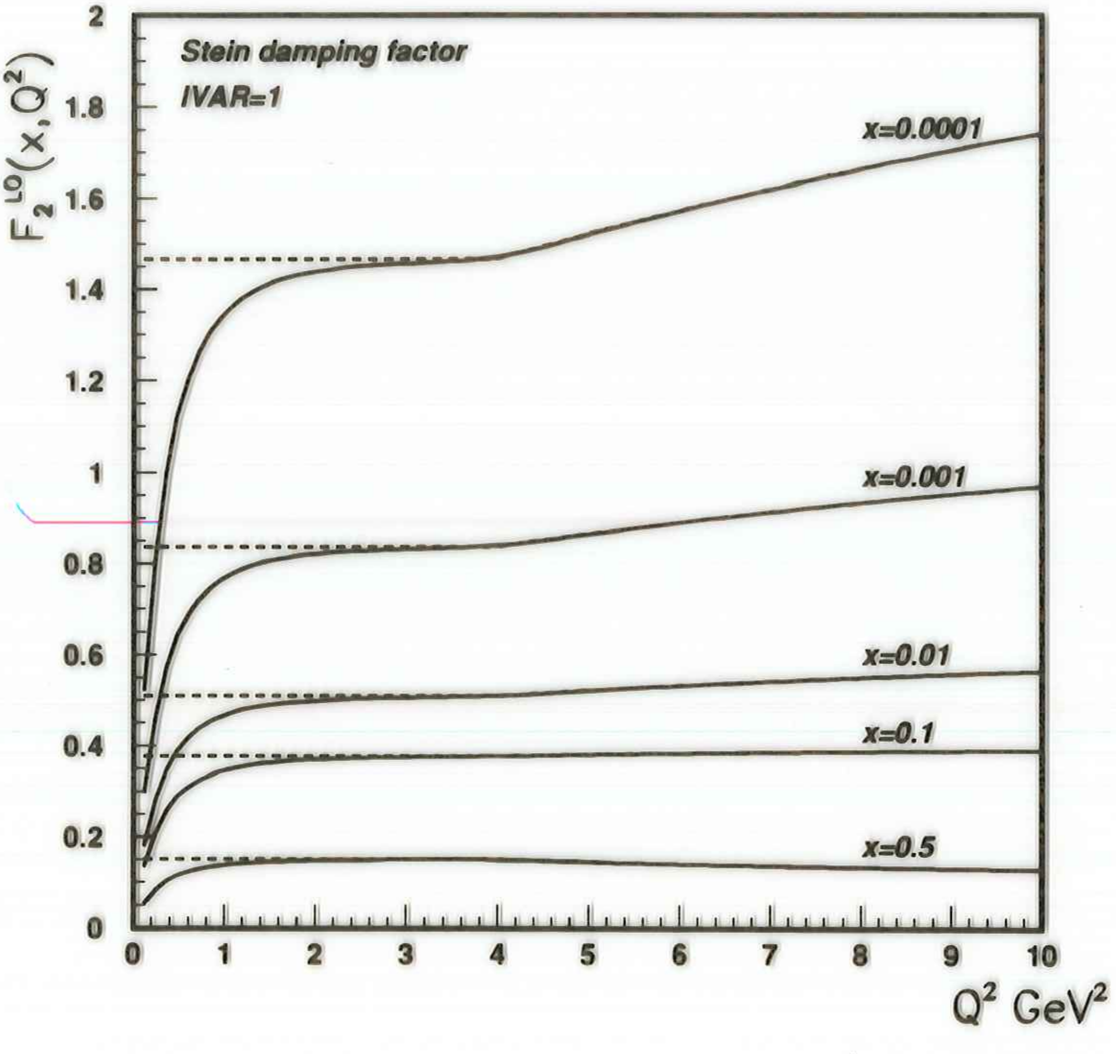
$d\sigma(\gamma^* p)$ FROM $Q^2 > 0$ TO $Q^2 \to 0$.

DIS $\longrightarrow$ PHOTOPRODUCTION

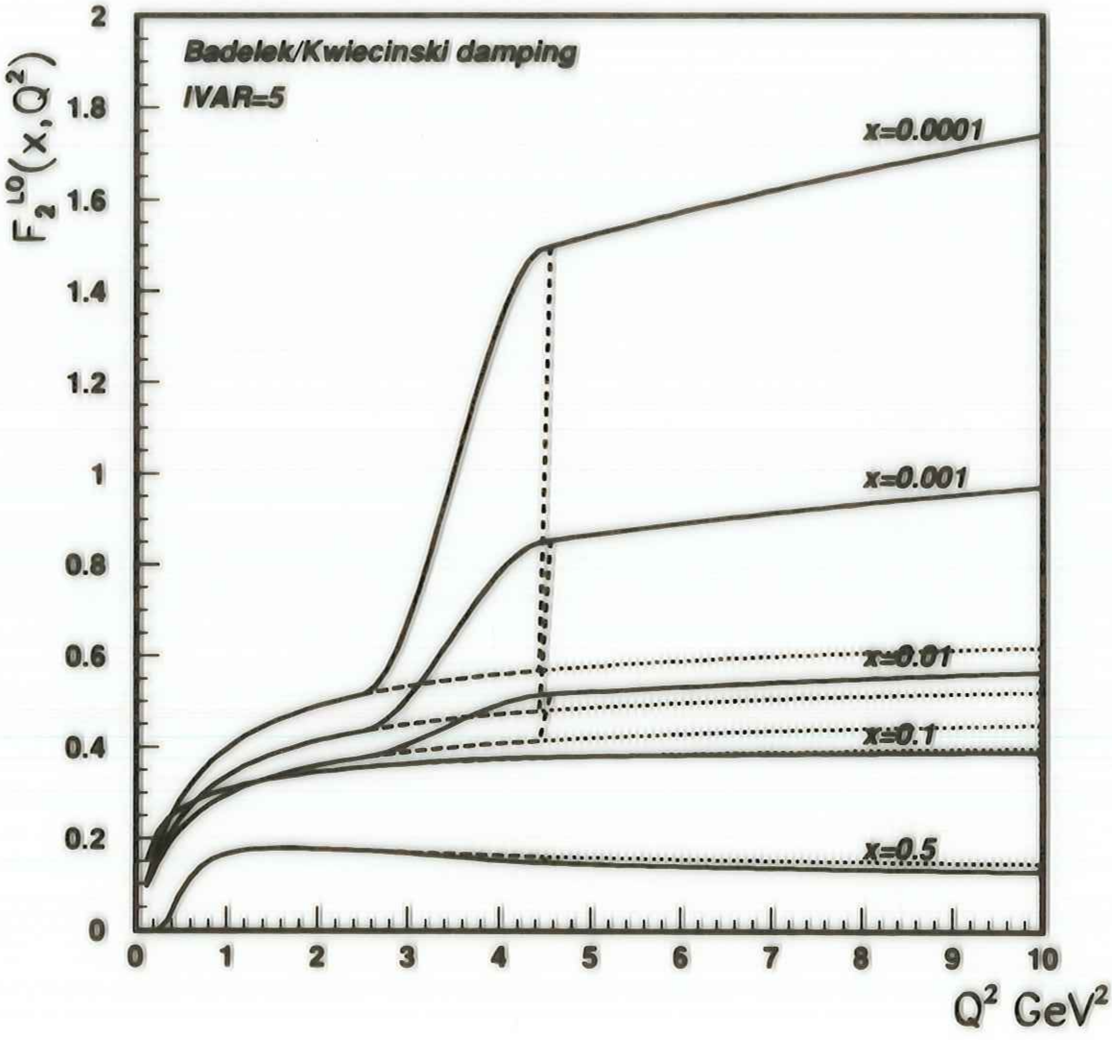
ALSO AT SMALL x.

$F_2(x, Q^2)$ At Low Q^2

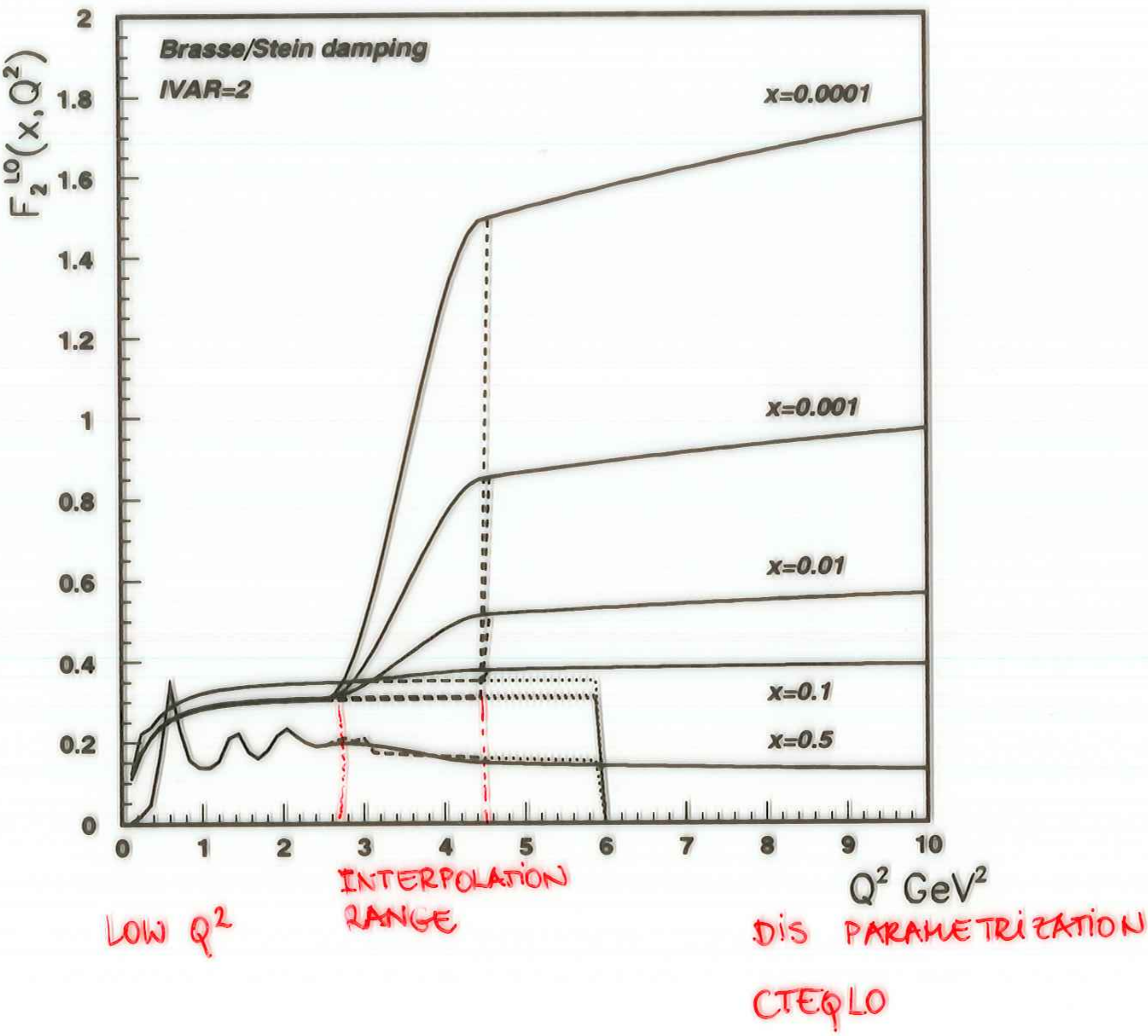




Stein damping factor
IVAR=1
x=0.0001
x=0.001
x=0.01
x=0.1
x=0.5
$F_2^{LO}(x,Q^2)$
Q^2 GeV^2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0
1
2
3
4
5
6
7
8
9
10



Badelek/Kwiecinski damping
IVAR=5
x=0.0001
x=0.001
x=0.01
x=0.1
x=0.5
$F_2^{LO}(x,Q^2)$
Q^2 GeV^2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0 1 2 3 4 5 6 7 8 9 10



Brasse/Stein damping
IVAR=2
x=0.0001
x=0.001
x=0.01
x=0.1
x=0.5
$F_2^{LO}(x,Q^2)$
Q^2 GeV^2
INTERPOLATION RANGE
LOW Q^2
DIS PARAMETRIZATION
CTEQ LO

THE CROSS SECTION:

$$\left.\frac{d^2\sigma}{dx_\ell\, dy_\ell}\right|^{C} = \frac{\alpha^3}{x_\ell s}\,\frac{1+(1-y_\ell)^2}{1-y_\ell}\int_{x_\ell}^{1} dz \int_{Q^2_{h,\min}}^{Q_\ell^2} \frac{dQ_h^2}{Q_h^2}\,\frac{z}{x_\ell}\,\cdot$$

$$\cdot\left\{\frac{1+(1-z)^2}{z^2}\,F_2\!\left(\frac{x_\ell}{z}, Q_h^2\right) - F_L\!\left(\frac{x_\ell}{z}, Q_h^2\right)\right\}$$

no y-separation possible!

$$\frac{d\sigma}{dx_\ell\, dy_\ell\, dz} \rightarrow \text{difficult to measure.}$$

$$\frac{d^2\sigma^C}{dx_\ell\, dy_\ell} = \int_0^1 \frac{dz}{z}\, D_{\gamma/p}(z, Q_\ell^2)\,\left.\frac{d^2\hat{\sigma}(e\gamma\to e\gamma)}{d\hat{x}\, dy_\ell}\right|_{\substack{\hat{s}=sz\\ \hat{x}=x_\ell/z}}$$

$$\frac{d^2\hat{\sigma}}{d\hat{x}\, dy_\ell}(e\gamma\to e\gamma) = \frac{2\pi\alpha^2}{\hat{s}}\,\frac{1+(1-y_\ell)^2}{1-y_\ell}\,\delta(1-\hat{x})$$

TRANSV. PHOTONS : CALLEN-GROSS REL.: (APPR.) $F_L = 0$

$$D_{\gamma/p}(x_\ell, Q_\ell^2) = \frac{\alpha}{2\pi}\sum_{f,\bar{f}}\int_{Q_0^2}^{Q_\ell^2}\frac{dQ_h^2}{Q_h^2}\int_{x_\ell}^{1}\frac{dz}{z}\, P_{\gamma/q_f}(z, Q_h^2)$$

$$\times \underbrace{\sum_{f,\bar{f}} e_f^2\,\frac{x_\ell}{z}\, q_f\!\left(\frac{x_\ell}{z}, Q_h^2\right)}_{F_2\left(\frac{x_\ell}{z},\, Q_h^2\right).}$$

$Q_h^2 \sim O(M_p^2)$ or smaller!

$$P_{\gamma/q_f}(z) = \frac{1+(1-z)^2}{z}$$

$\Longrightarrow$ ONE CAN UNFOLD :

$$D_{\gamma/\text{proton}}(x, Q^2) \propto D_{\gamma/\text{quark}}(x, Q^2).$$

$\longrightarrow$ THIS IS A COMBINATION OF F_2 & F_L AT SMALL x & Q^2.

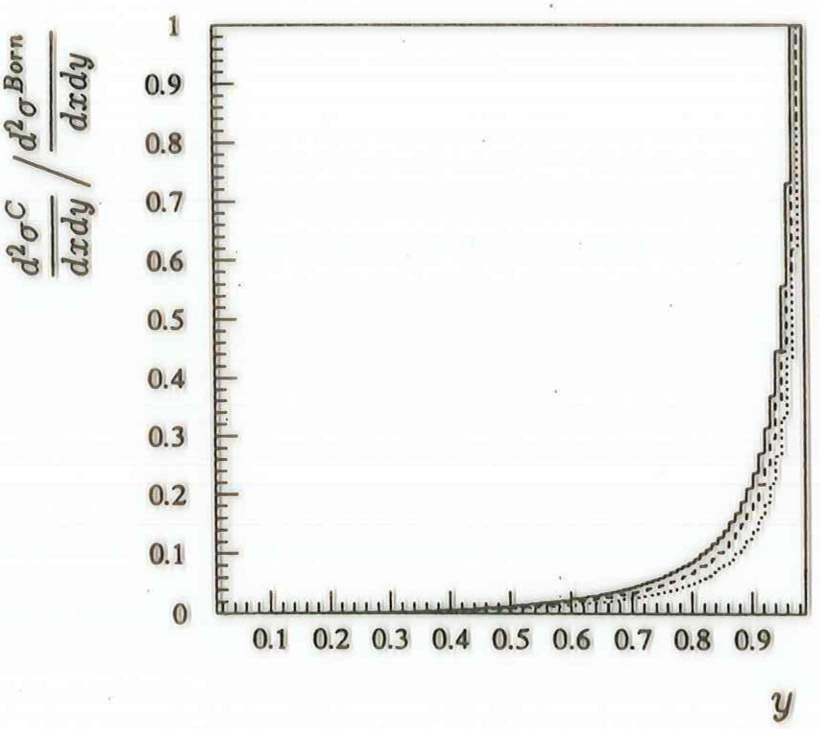


Figure 1: Differential Compton cross section Eq. (8) as a function of y_l for $x_l = 10^{-4}$ (dotted line), 10^{-3} (dashed line), and 10^{-2} (full line).

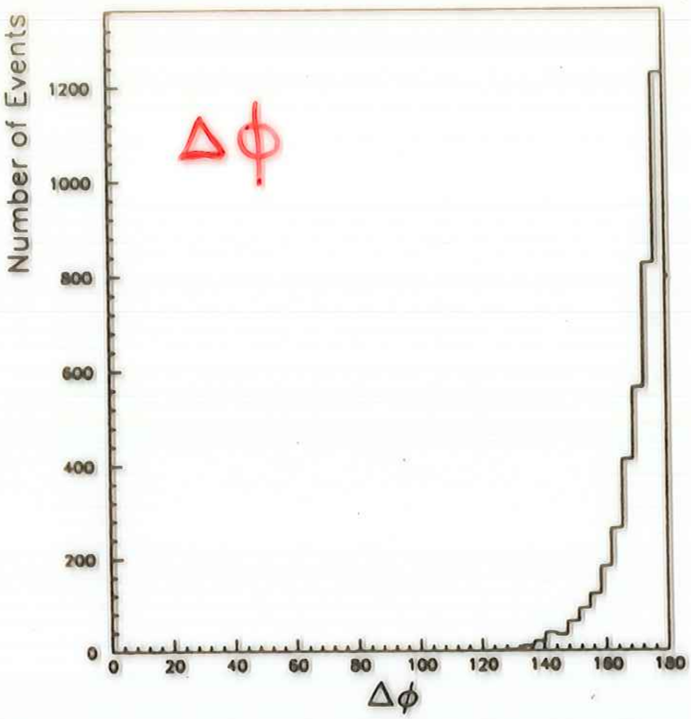


Figure 2: The difference in azimuth of the photon and electron for accepted Compton events.

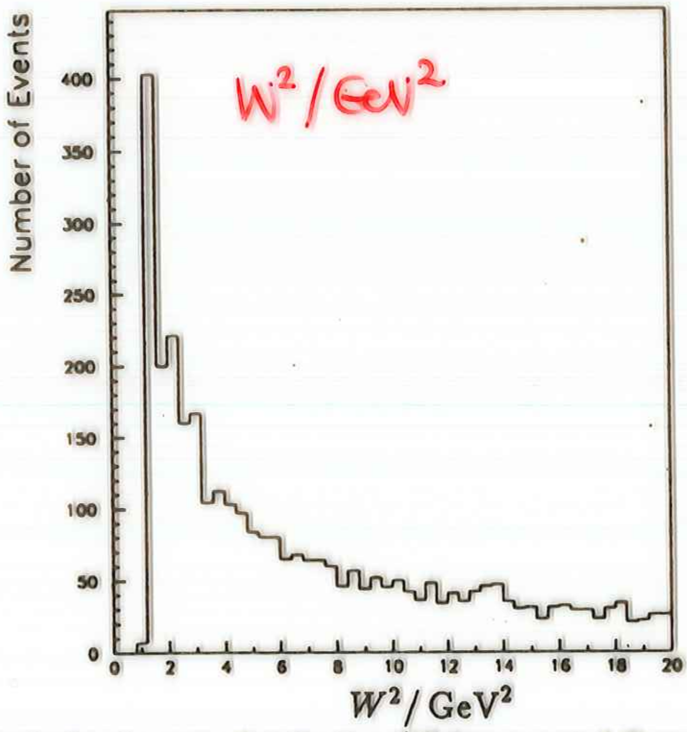


Figure 3: The hadronic mass distribution W^2 for accepted Compton events.

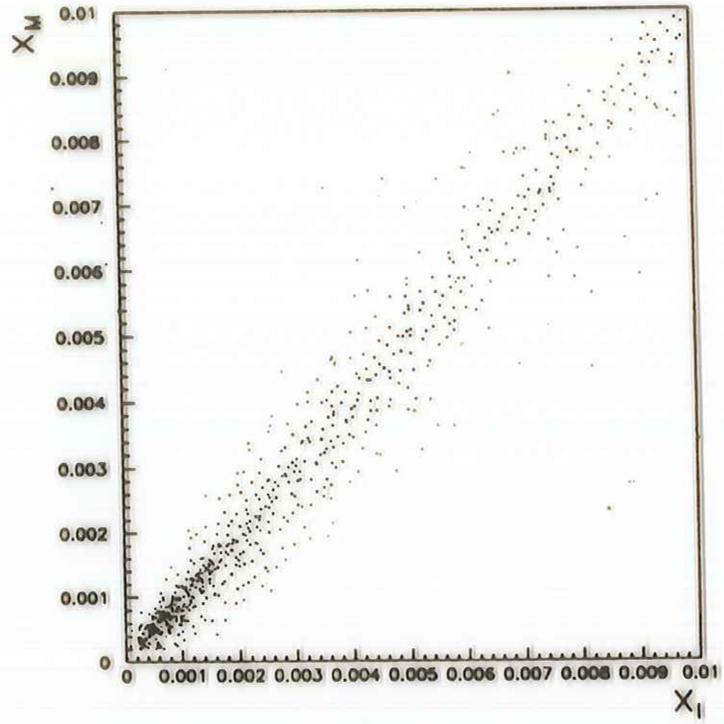


Figure 4: A scatter plot of $x_l = Q_l^2/2p \cdot (l - l')$ and $x_M = M_{e\gamma}^2/s$.

x - reconstruction

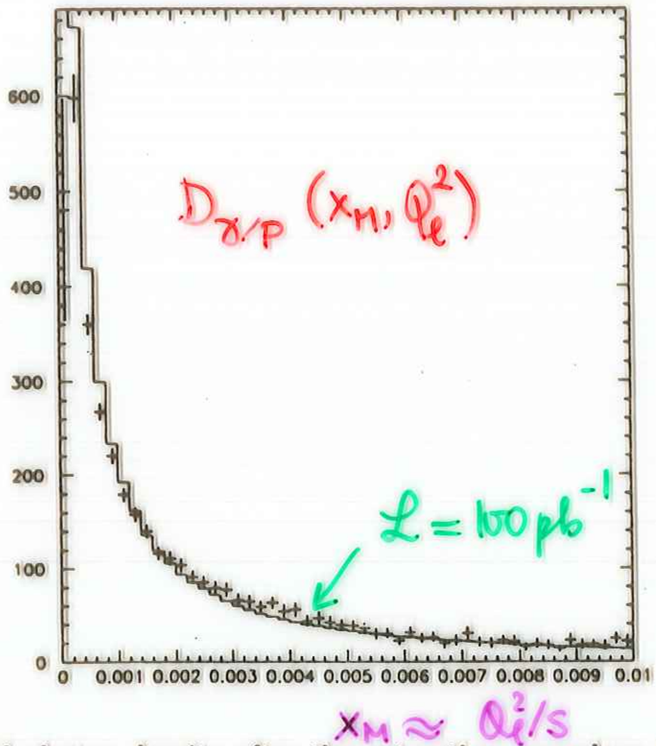


Figure 5: The expected photon density after the extraction procedure described in the text. The errors represent the statistical errors for an integrated luminosity of 100 pb^{-1}. The solid histogram is the prediction of Eq. (13) for HMRSB [21]. The units of the vertical scale are arbitrary.

5. Numerical Results

PDF'S : MRS D^-, SIMILAR RESULTS

- GRV
- CTEQ2
- MRSA-NEW JUNE '94 (AFTER MRSH, MRSA).

$\mathcal{O}(\alpha L)$, $\mathcal{O}(\alpha^2 L^2)$.

CONSIDER : ISR - LEPTONS

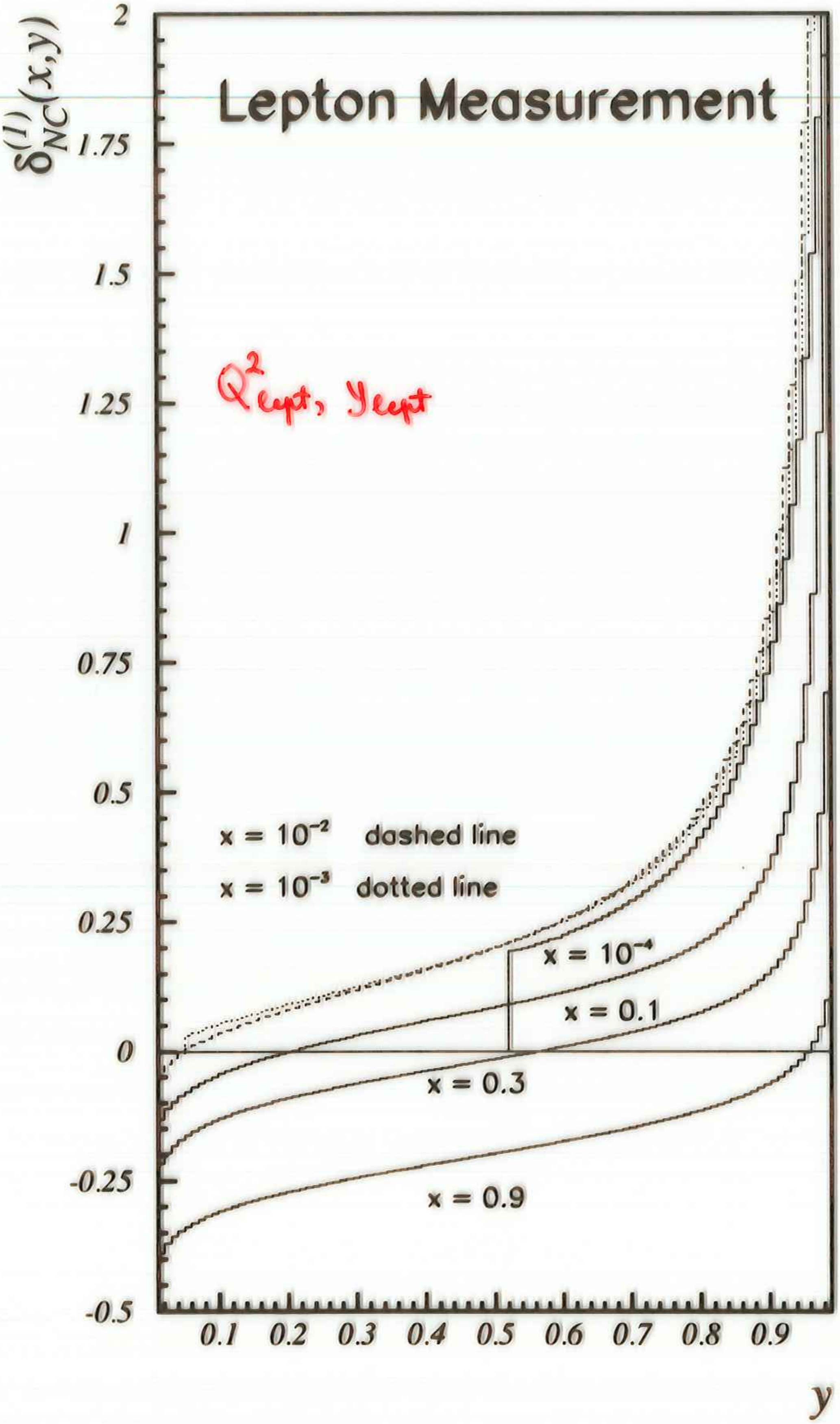
- FSR INTEGRATED, CALOR. MEASUREMENT
- QUARKS : $O(\frac{\alpha_s}{\alpha})$ CORR. TO SCALING VIOLATIONS
- COMPTON : SEPARATE TREATMENT.

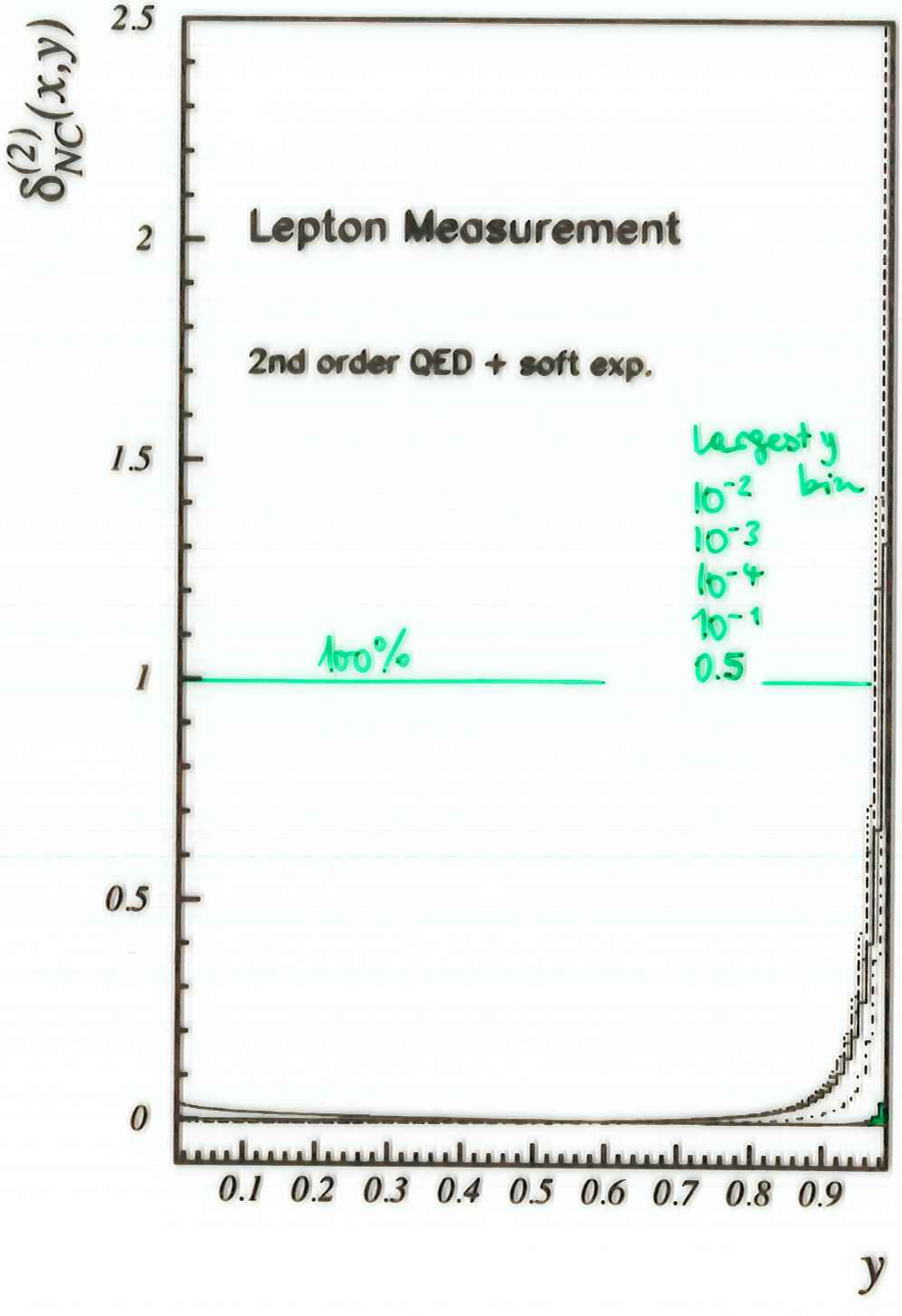
- $O(\alpha)$: COMPARISON WITH FULL $O(\alpha)$ CALCULATION.

$\Longrightarrow$ LOW Q^2 BEHAVIOUR OF STRUCT. FCT.

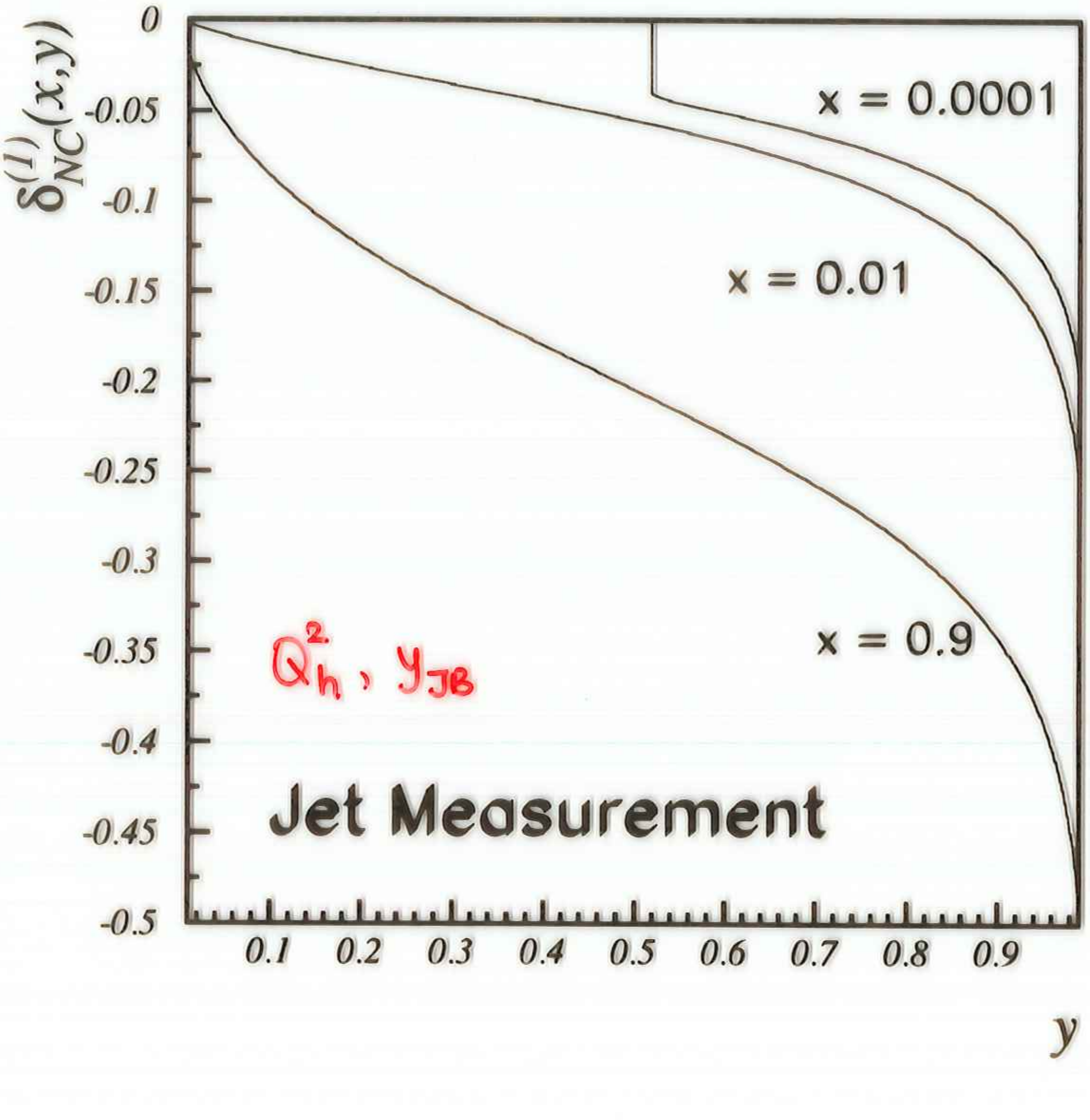
$\Longleftrightarrow$ RC'S. MEASUREMENT REQUIRED !

$O(\alpha)$


Lepton Measurement
$\delta^{(1)}_{NC}(x,y)$
Q^2_{lept}, y_{lept}
$x = 10^{-2}$ dashed line
$x = 10^{-3}$ dotted line
$x = 10^{-4}$
$x = 0.1$
$x = 0.3$
$x = 0.9$
2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
-0.25
-0.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y



$\delta^{(2)}_{NC}(x,y)$
2.5
2
1.5
1
0.5
0
Lepton Measurement
2nd order QED + soft exp.
Largest y bin
10^{-2}
10^{-3}
10^{-4}
10^{-1}
0.5
100%
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
y



$\delta_{NC}^{(1)}(x,y)$
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
-0.4
-0.45
-0.5
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y
x = 0.0001
x = 0.01
x = 0.9
Q_h^2, y_{JB}
Jet Measurement

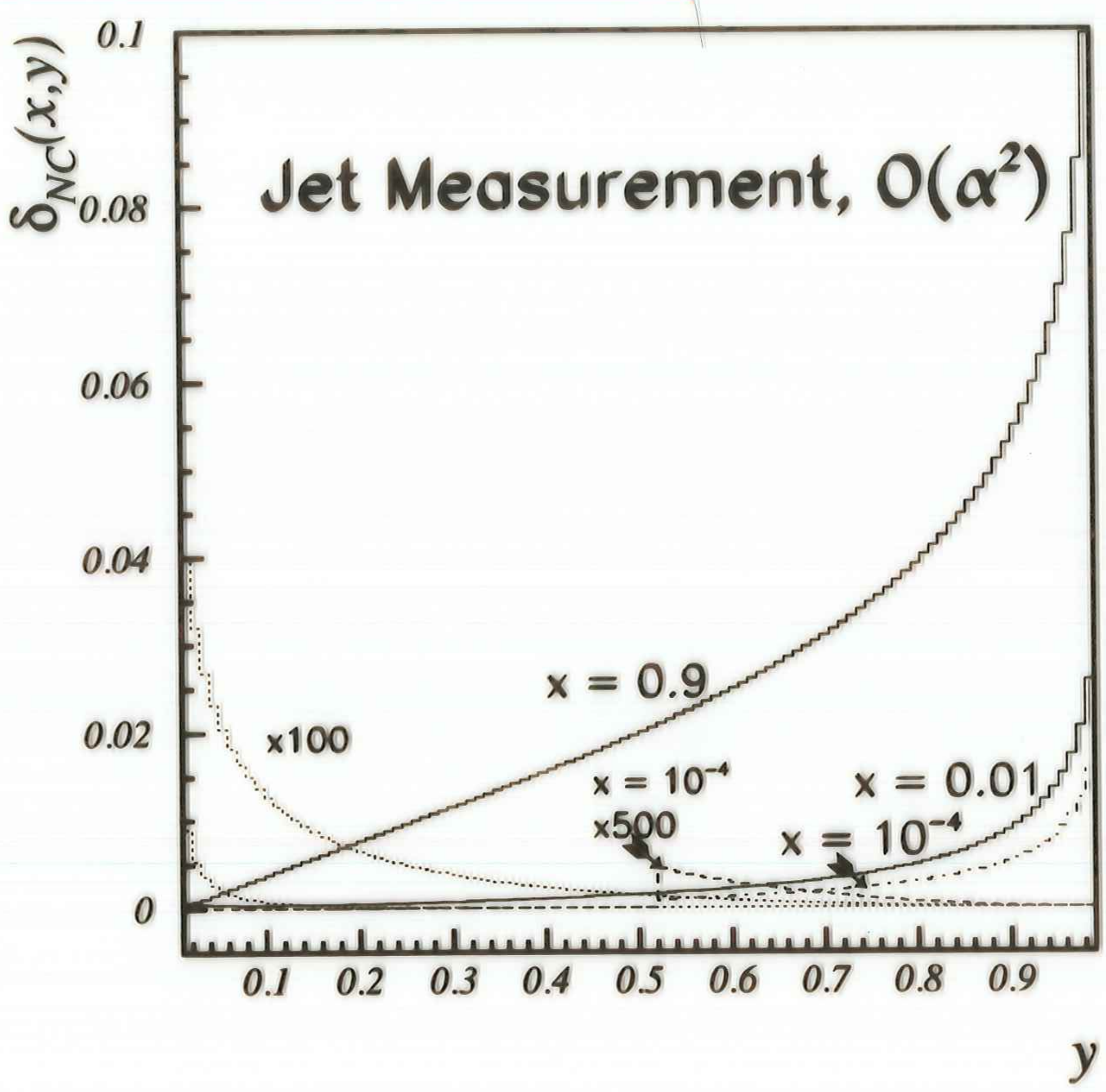


Figure 3: Leptonic initial state radiative corrections $\delta_{NC}(x,y) = (d\sigma^{(2+>2,soft)}/dx\,dy)/(d\sigma^0/dx\,dy)$ in LLA for e^-p deep inelastic scattering in the case of jet measurement for $\sqrt{s} = 314\,\mathrm{GeV}$, $\mathcal{A} = 0$, and $Q^2 \geq 5\,\mathrm{GeV}^2$. $\mathcal{O}(\alpha^2)$ corrections: full lines: $x = 0.01$ and $x = 0.9$; dash–dotted line: $x = 10^{-4}$. Contributions due to $e^- \rightarrow e^+$ conversion eq. (13), $\delta_{NC}^{e^-\rightarrow e^+}(x,y) = (d\sigma^{(2,e^-\rightarrow e^+)}/dx\,dy)/(d\sigma^0/dx\,dy)$: upper dotted line: $x = 0.01$, lower dotted line: $x = 0.9$. Both graphs are scaled by $\times 100$; dashed line: $x = 10^{-4}$, scaled by $\times 500$.

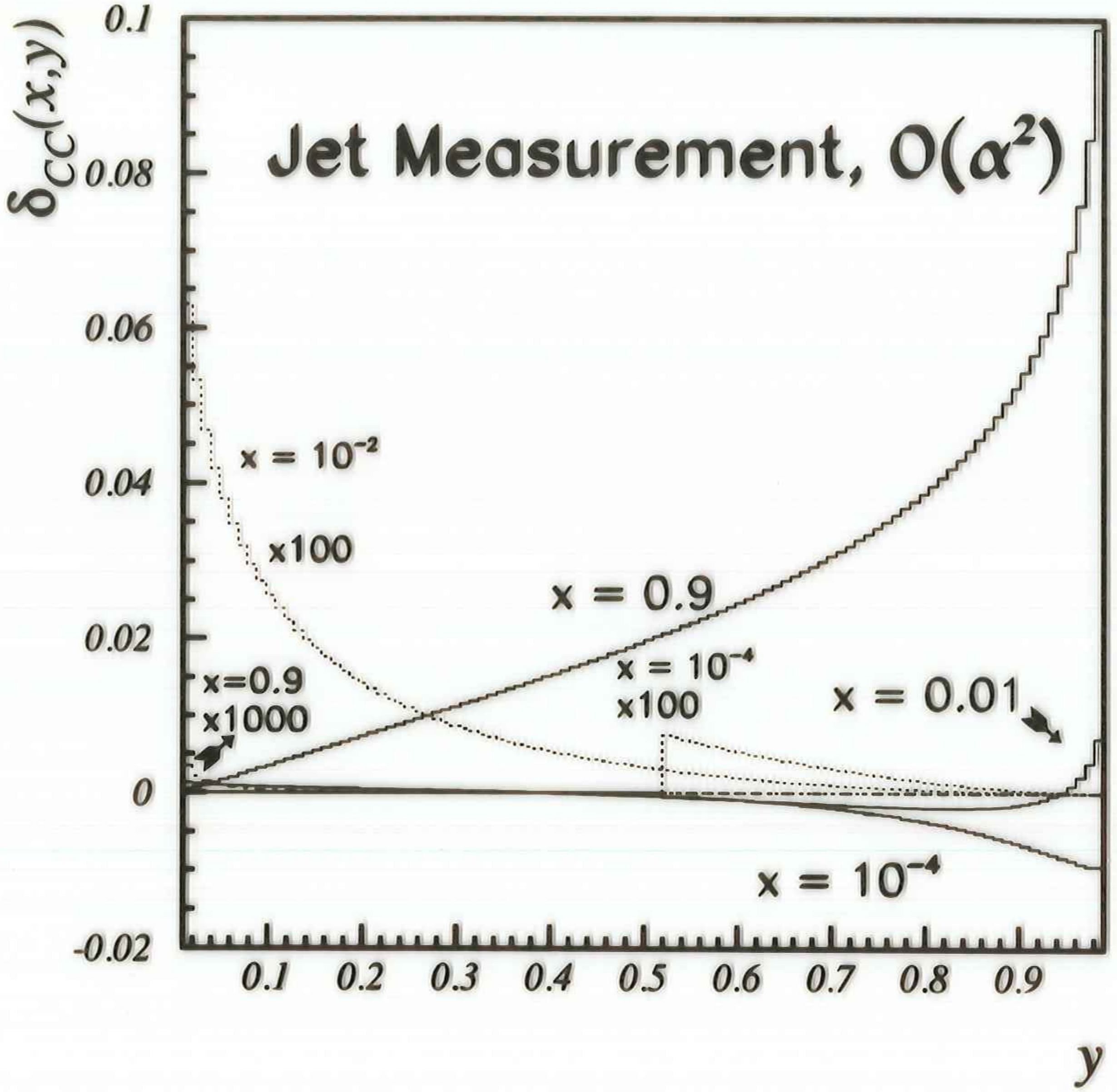
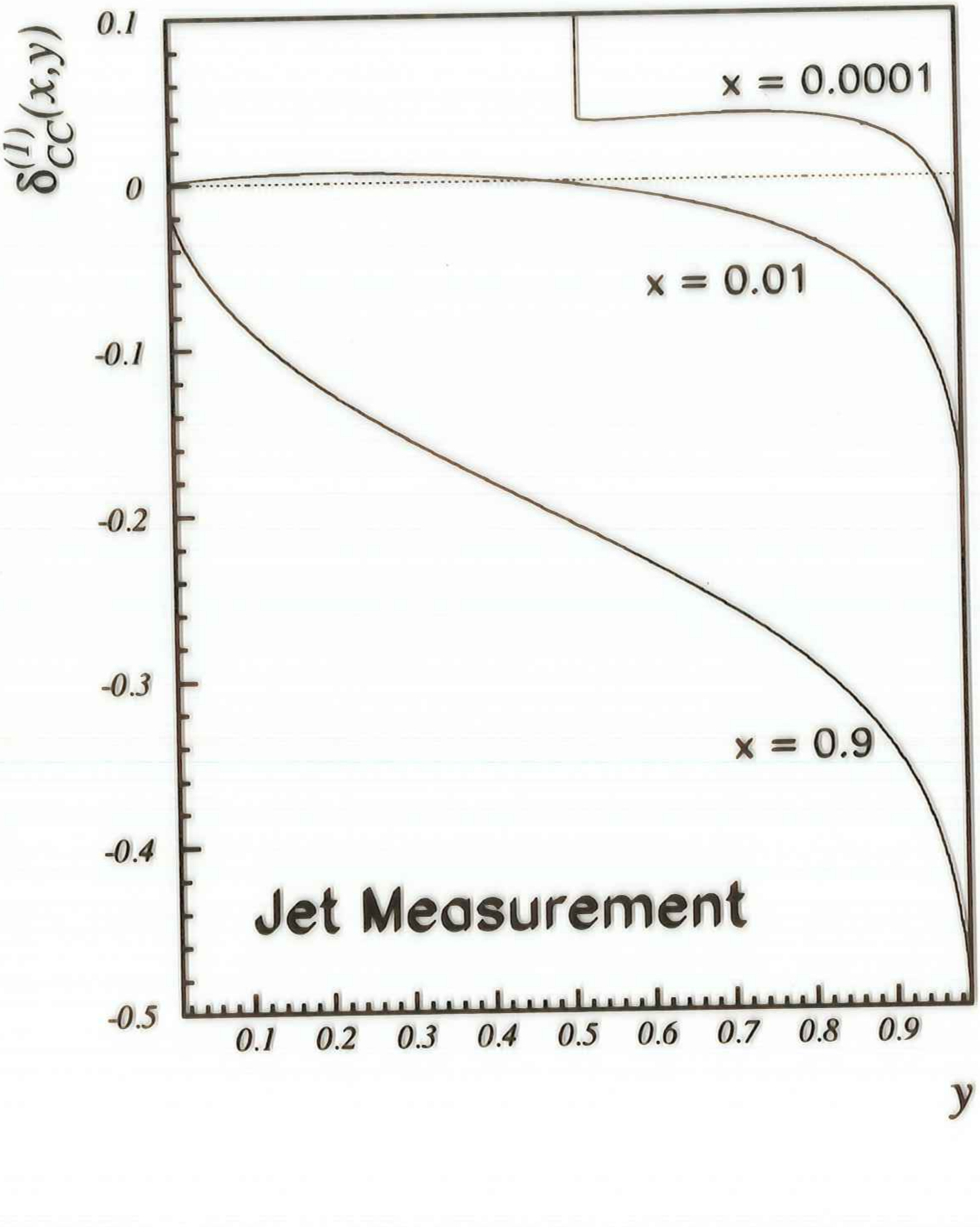
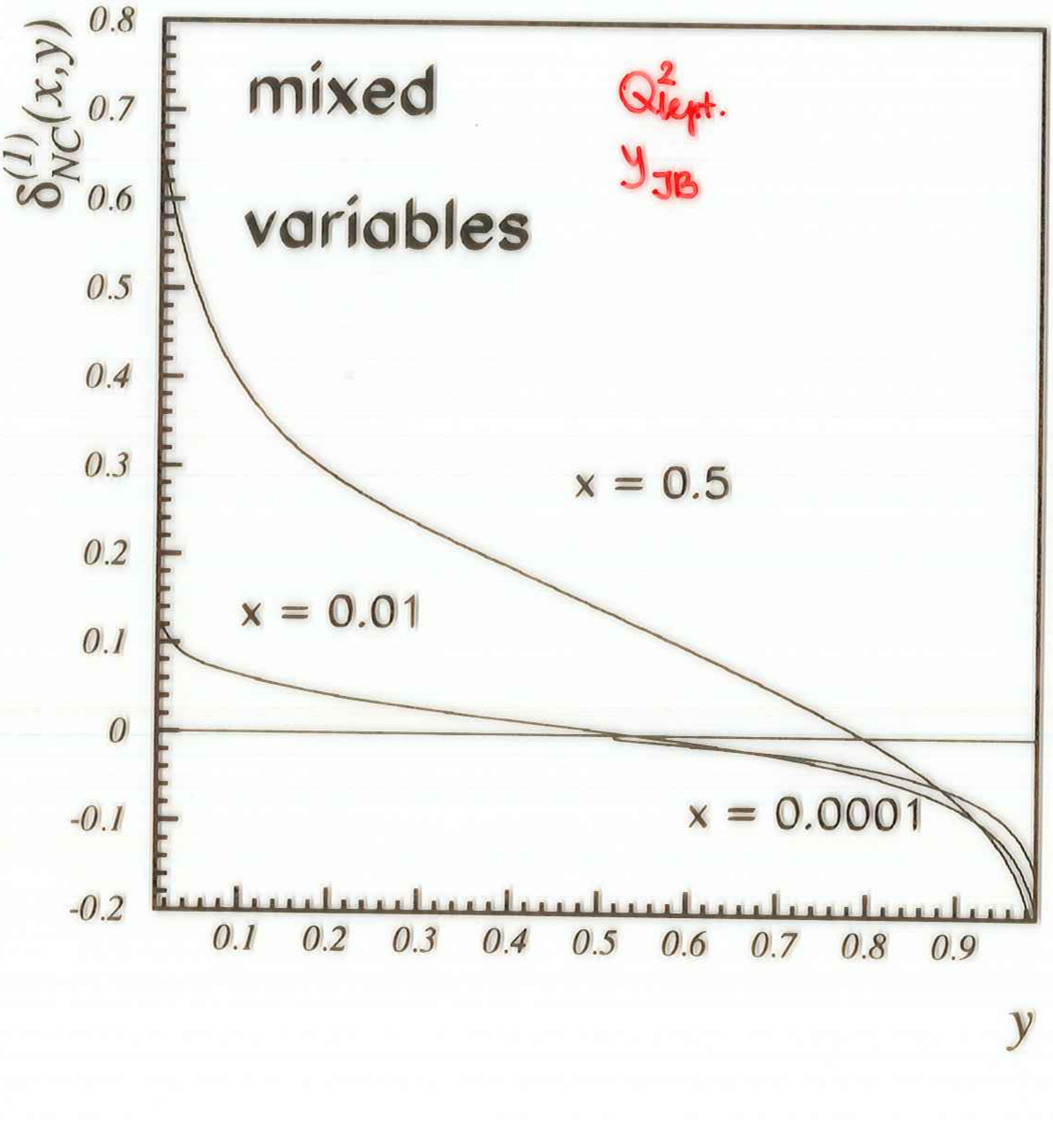


Figure 4: $\delta_{CC}(x,y) = (d\sigma_{CC}^{(2+>2,soft)}/dxdy)/(d\sigma_{CC}^0/dxdy)$ for deep inelastic e^-p scattering in the case of jet measurement. Full lines: $\mathcal{O}(\alpha^2)$ corrections for $x = 0.9, 0.01$ and 10^{-4}. $\delta_{CC}^{e^-\to e^+}(x,y)$: dotted lines: $x = 10^{-2}$ and $x = 10^{-4}$. Both graphs are scaled by $\times 100$. Dashed line: $x = 0.9$ scaled by $\times 1000$. The other parameters are the same as in figure 3.



$\delta^{(1)}_{CC}(x,y)$
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
x = 0.0001
x = 0.01
x = 0.9
Jet Measurement
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
y


$\delta_{NC}^{(1)}(x,y)$
mixed
variables
$Q^2_{lept.}$
y_{JB}
x = 0.5
x = 0.01
x = 0.0001
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
y

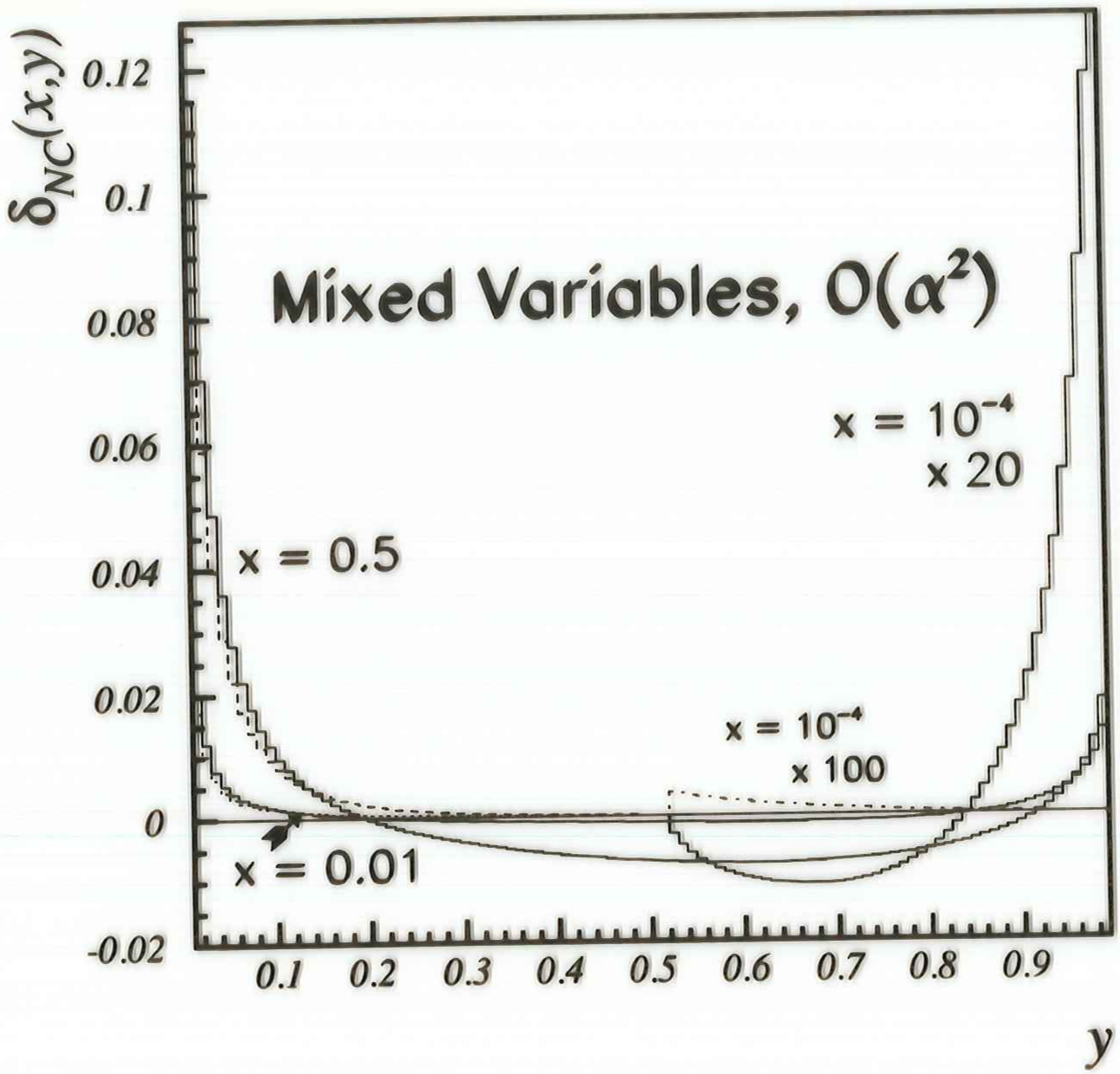


Figure 5: $\delta_{NC}(x, y)$ for the case of mixed variables. Full lines: $\mathcal{O}(\alpha^2)$ corrections for $x = 0.5$, $x = 0.01$ and $x = 10^{-4}$. The latter graph is scaled by $\times 20$. $\delta_{NC}^{e^- \to e^+}(x, y)$: upper dotted line: $x = 0.5$, lower dotted line: $x = 0.01$, dash-dotted line: $x = 10^{-4}$ scaled by $\times 100$. The other parameters are the same as in figure 3.

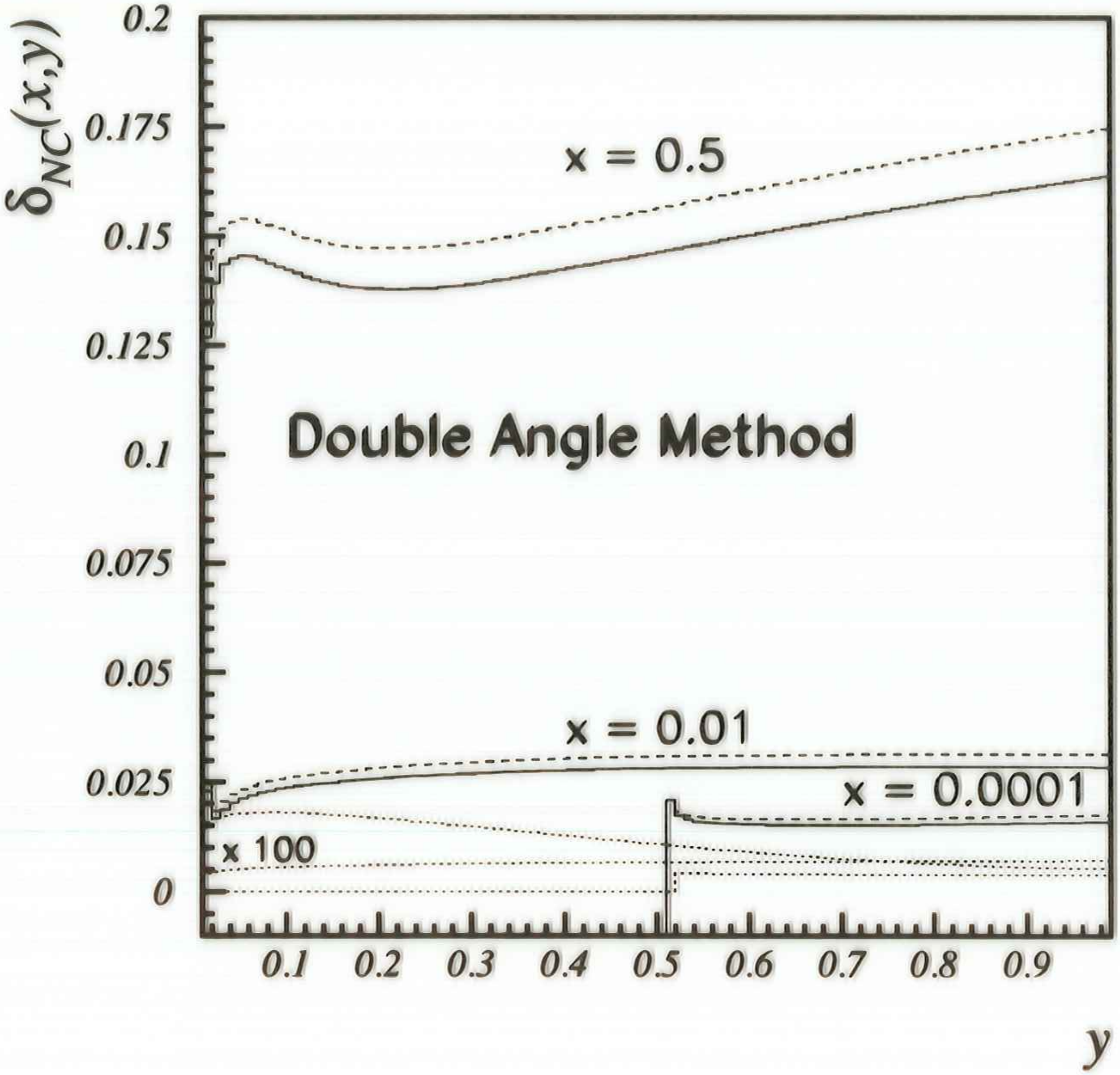


Figure 6: $\delta_{NC}(x,y)$ for the case of the double angle method for $\mathcal{A} = 35\,\mathrm{GeV}$. Full lines: $\delta_{NC}^{(1+2+>2,soft)}(x,y)$, dashed lines: $\delta_{NC}^{(1)}(x,y)$. Dotted lines: $\delta_{NC}^{e^- \to e^+}(x,y)$ scaled by $\times 100$; upper line: $x = 0.5$, middle line: $x = 0.01$, lower line: $x = 0.0001$. The other parameters are the same as in figure 3.

A DANGEROUS CASE:

θ_e & y_J

RESCALING: ISR

$$\hat{Q}^2 = Q^2 z \frac{z-y}{1-y}$$

$$\hat{x} = x \frac{z(z-y)}{1-y}$$

$$\boxed{z_0 = y}$$

ZEUS: $\boxed{z_0 = \max\left\{\frac{35 \text{ GeV}}{2E_e}, y\right\}}$

$\delta_{NC}(x,y)$ JUMPS! AT $y \gtrsim \frac{\mathcal{A}}{2E_e}$, $\mathcal{A} = 35$ GeV.

$$\frac{\sigma(Q^2, x \to 0)}{\sigma(Q^2, x)}$$!

NO CONTROL ON INPUT AT ALL !

$\longrightarrow$ UNFORTUNATE CHOICE OF VARIABLES.

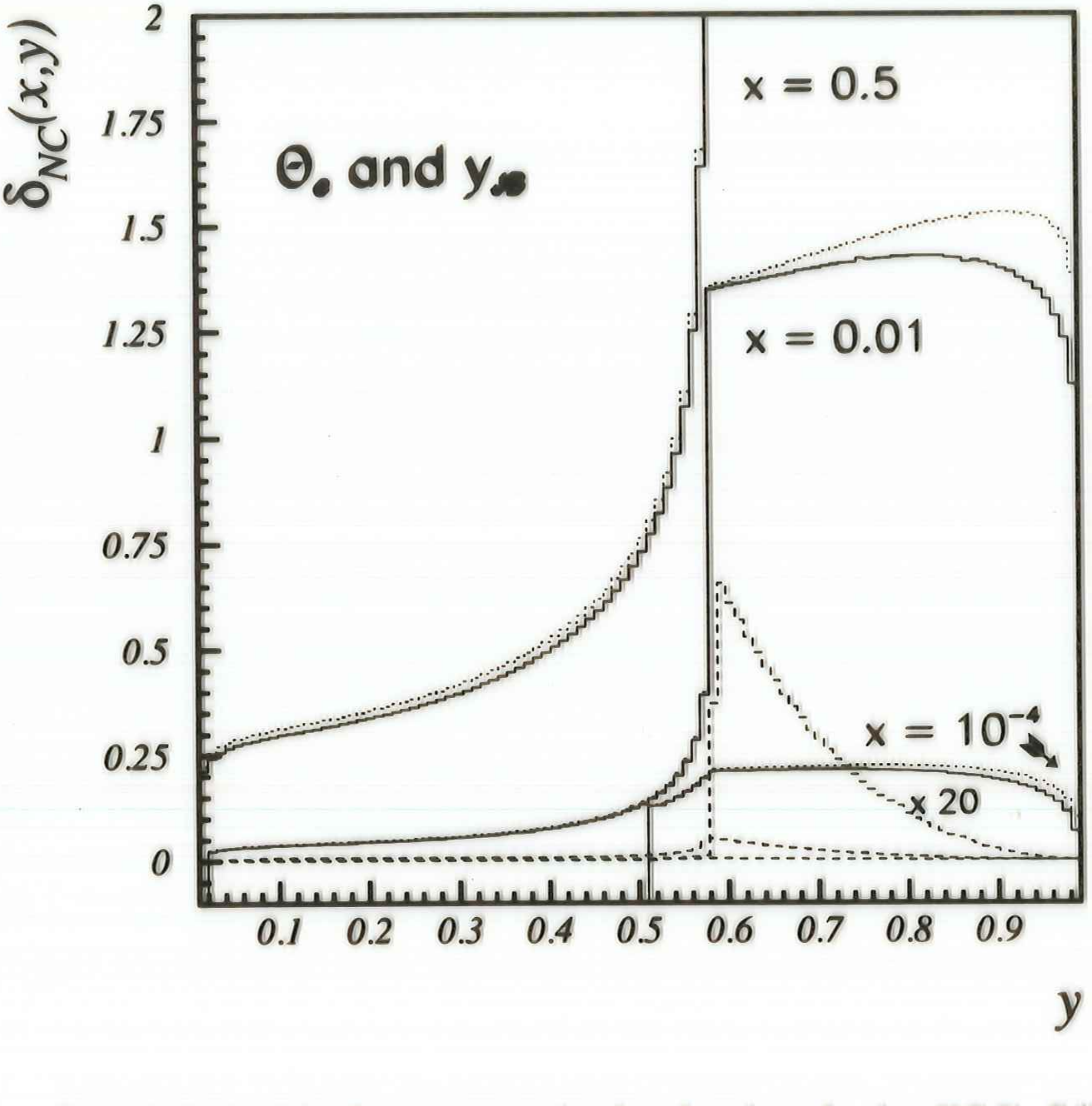


Figure 7: $\delta_{NC}(x,y)$ for the measurement based on θ_e and y_{JB} for $\mathcal{A} = 35\,\mathrm{GeV}$. Full lines: $\delta_{NC}^{(1+2+>2,soft)}(x,y)$, dotted lines: $\delta_{NC}^{(1)}(x,y)$. Dashed lines: $\delta_{NC}^{e^-\to e^+}(x,y)$ scaled by $\times 20$; upper line: $x = 0.5$, middle line: $x = 10^{-2}$, lower line: $x = 10^{-4}$. The other parameters are the same as in figure 3.

COMPARISON IN $O(\alpha)$

LLA vs COMPLETE CALCULATION (TERAD 91).

Bardin et al.

Q_e, y_e

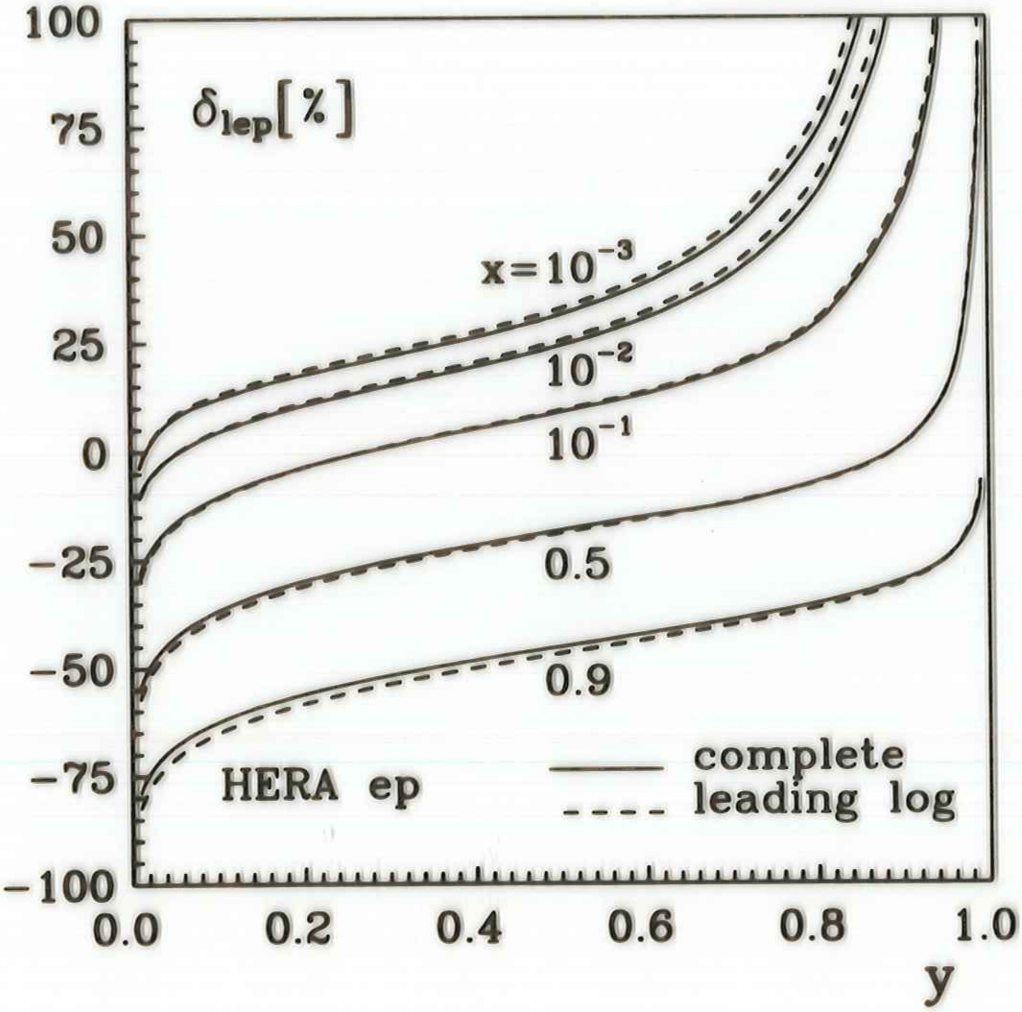


Figure 1: Comparison of complete $\mathcal{O}(\alpha)$ and LLA calculation for leptonic variables

Q_J, y_J

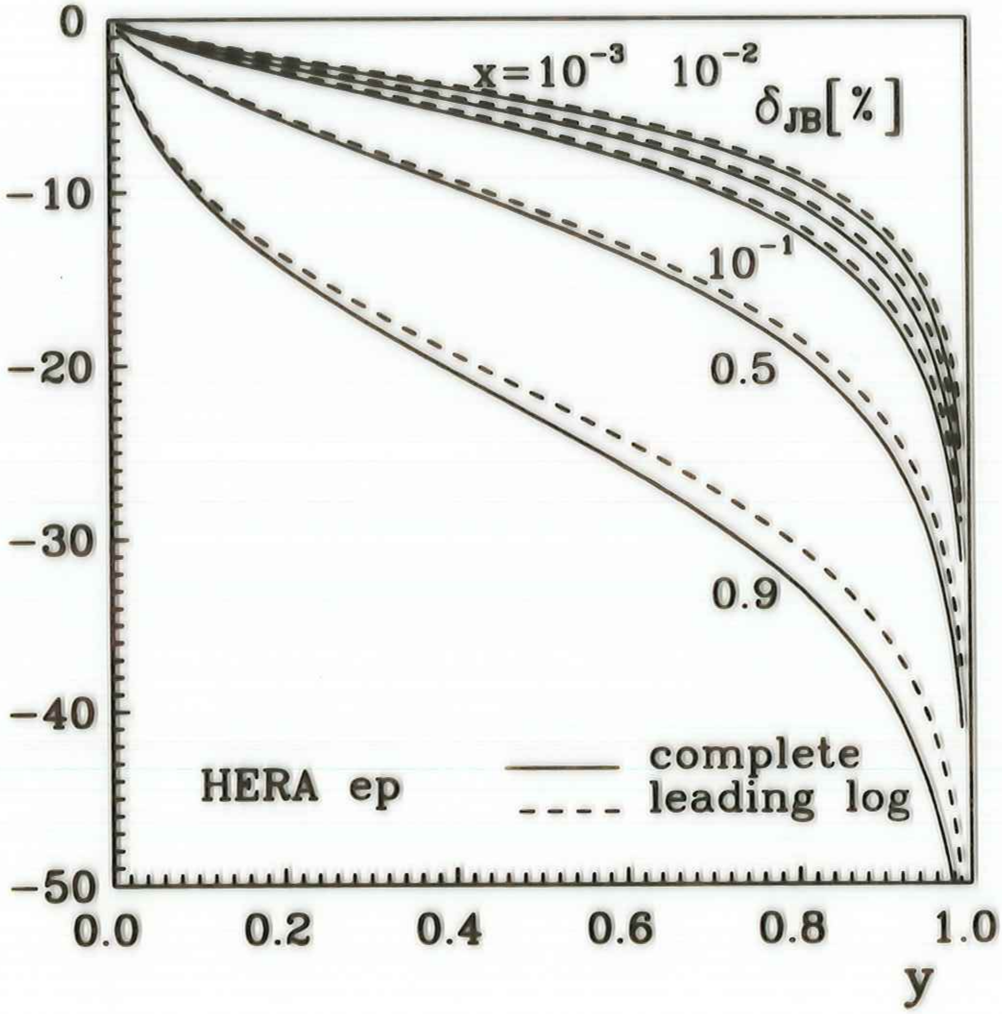


Figure 2: Comparison of complete $\mathcal{O}(\alpha)$ and LLA calculation for jet variables

Q_e, y_J

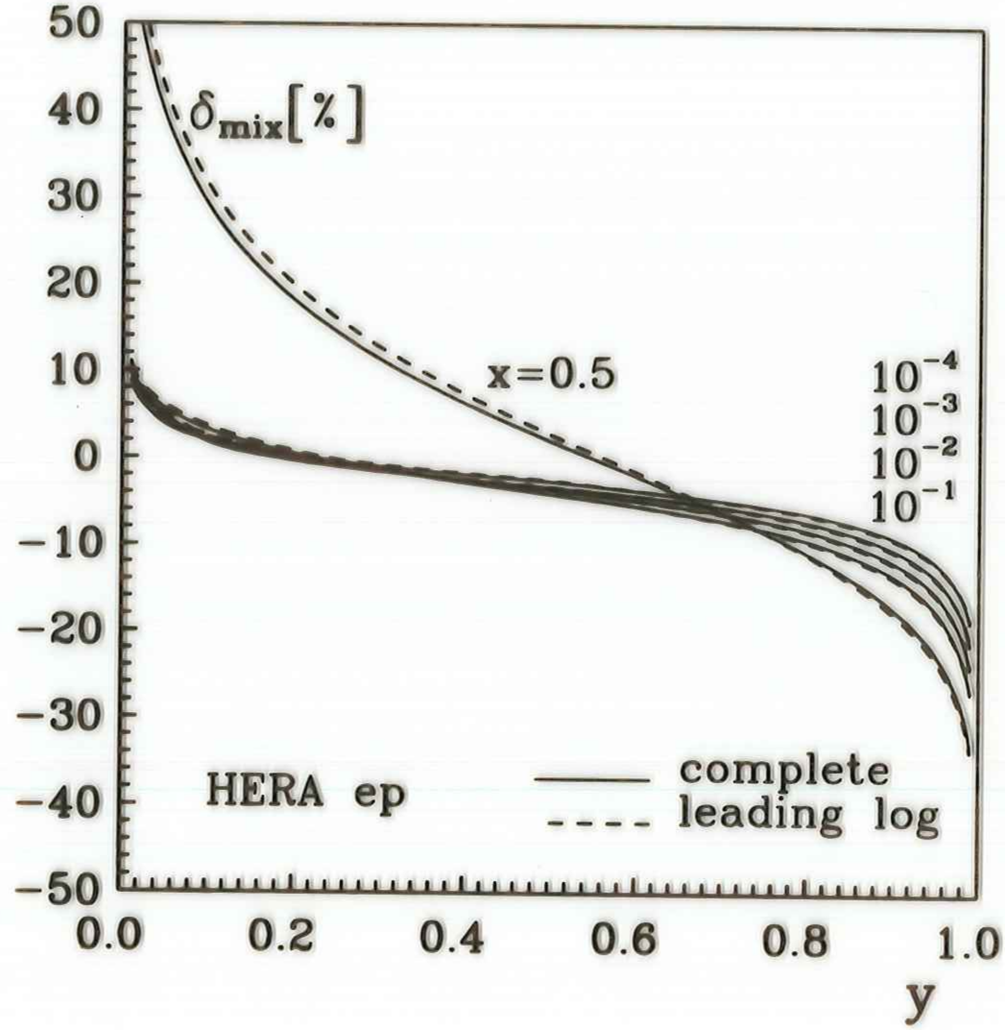


Figure 3: Comparison of complete $\mathcal{O}(\alpha)$ and LLA calculation for mixed variables

HECTOR – a program to calculate QED and electroweak corrections to ep and $l^{\pm}N$ deep inelastic NC and CC scattering

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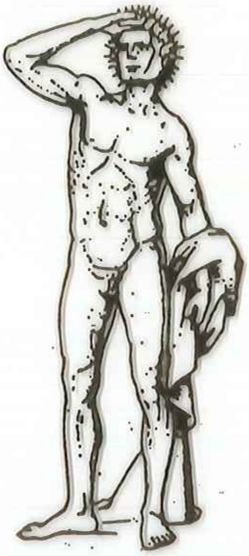
[3] *Theoretical Physics Division, CERN, CH-1211 Geneva 23, Switzerland*

ABSTRACT

A description of the Fortran program HECTOR for a variety of semi-analytical calculations of radiative QED, QCD, and electroweak corrections to the double-differential cross sections of NC and CC deep-inelastic charged lepton–proton (or –deuteron) scattering is presented. HECTOR originates from the substantially improved and extended earlier programs HELIOS and TERAD91. It is mainly intended for the calculations at HERA or other ep–colliders, but may be also used for similar processes like muon–proton scattering in fixed–target experiments. The QED corrections may be calculated in several different sets of variables: leptonic, hadronic, mixed, Jaquet-Blondel, double angle etc. Besides the leading-logarithmic approximation up to order $\mathcal{O}(\alpha^2)$, the exact $\mathcal{O}(\alpha)$corrections and soft-photon exponentiation are taken into account. The photoproduction region is also covered.

† Supported by the Heisenberg-Landau fund.

HELIOS



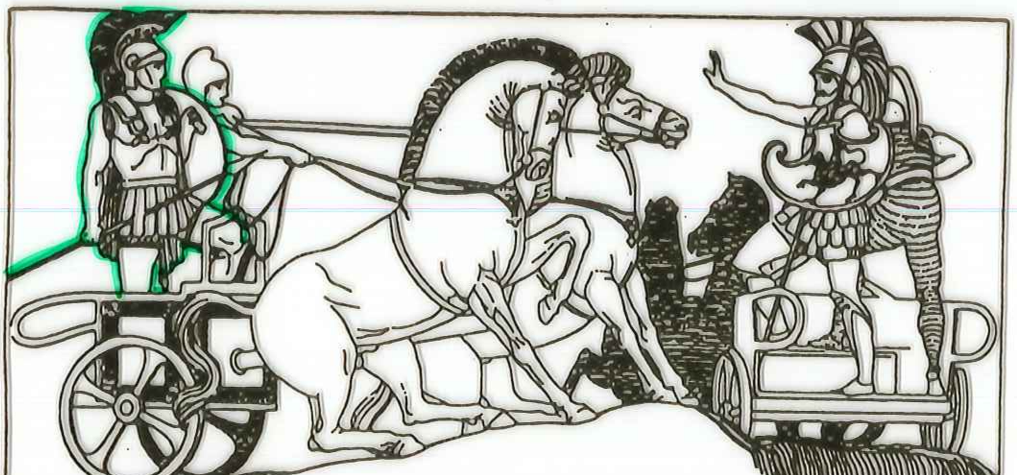
TERAD 91

JB. + BARDIN et al.

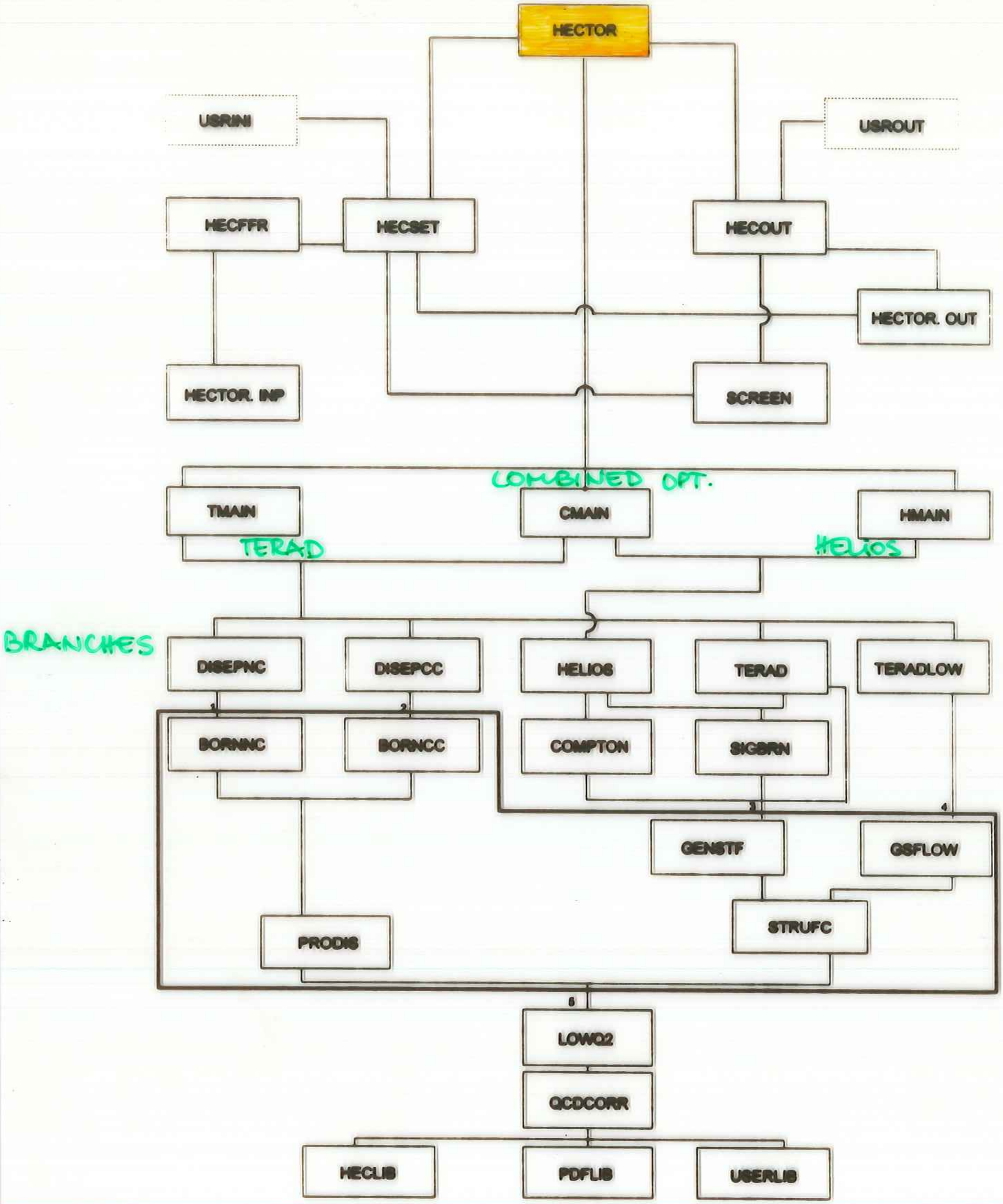
- UPGRADES & IMPROVEMENTS
  QED, QCD, new Variables, ...

HECTOR

HADRON
ELECTRON
CODE
TO CALCULATE 1ST AND HIGHER
ORDER
RADIATIVE CORRECT-IONS.



ENGRAVING BY J. FLAXMAN 1790ies:
: POLYDAMAS ADVICES HECTOR TO MAKE THE ASSAULT ONTO THE CAMP OF THE GREEKS ON FOOT:


HECTOR
USRINI
USROUT
HECFFR
HECSET
HECOUT
HECTOR. OUT
HECTOR. INP
SCREEN
COMBINED OPT.
TMAIN
CMAIN
HMAIN
TERAD
HELIOS
BRANCHES
DISEPNC
DISEPCC
HELIOS
TERAD
TERADLOW
1
2
BORNNC
BORNCC
COMPTON
SIGBRN
3
4
GENSTF
GSFLOW
STRUFC
PRODIS
5
LOWQ2
QCDCORR
HECLIB
PDFLIB
USERLIB

6. Conclusions

1) LEADING AND NONLEADING LOGARITHMIC CONTRIBUTIONS TO THE RADIATIVE CORRECTIONS TO DEEP INELASTIC SCATTERING CAN BE DETERMINED STUDYING THE RGE-BEHAVIOUR FOR THE DIFFERENTIAL CROSS SECTIONS.

2) THE LLA TERMS RESULT FROM THE EVOLUTION EQUATIONS ONLY. NLA TERMS IN HIGHER ORDER REQ. THE KOWLEDGE OF COMPLETE LOWER ORDER CALCULATIONS.

3) K-FACTORS ARE VERY DIFFERENT FOR DIFFERENT CHOICES OF BORN LEVEL VARIABLES

4) LLA TERMS TO $O(\alpha^2 L^2)$ HAVE BEEN STUDIED QUANTITATIVELY FOR 6 CHOICES USED AT HERA CURRENTLY.

5) LO TERMS CAN BE FACTORIZED IN TERMS OF RADIATING FERMION LINES.

6) THE COMPTON TERM HAS BEEN ANALYSED; IT IS RATHER TO BE CONSIDERED AS AN EXCLUSIVE CHANNEL YIELDING A WINDOW TO STRUCTURE FUNCTIONS AT SMALL Q^2 & SMALL x THAN A CONTRIBUTION TO THE RC'S.

7) NOT EVERY CHOICE OF BORN LEVEL VARIABLES LEADS TO STABLE RC'S (θ_e, y_J).

8) LEPTONIC VARIABLES REQUIRE A COMPLETE SOLUTION OF THE EVOLUTION EQUS. $\rightarrow$ HIGH y.

9) THE DOUBLE ANGEL METHOD IS MOST CONVENIENT FROM THE POINT OF VIEW OF THE RC'S: THEY ARE FLAT IN y FOR $x =$ const.
AND THEY ARE SMALL. $\rightarrow$ EFFECTIVE BORN LEVEL MEASUREMENT.

10) $O(\alpha)$ COMPLETE & $O(\alpha L)$ RESULTS ARE VERY CLOSE TO EACH OTHER NUMERICALLY.