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# On the Evolution Kernels of Twist 2 Light-Ray Operators for Unpolarized and Polarized Deep Inelastic Scattering

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# 1. INTRODUCTION

- NON-FORWARD COMPTON SCATTERING :

  A NEW QCD - LABORATORY.

  CELEBRATED FORWARD CASE :

  GPGWGLDAPKS - EQUS. (WHO IS WHO ?)

  $\longleftrightarrow$ RGE IN THE FORWARD CASE.
  !

- DO WE NEED STATES IN THE FIRST PLACE ?

  $\langle p_1 | \longrightarrow | p_2 \rangle$ ?

  NO !

- RGE FOR OPERATORS (NON-FORWARD).

- INTRODUCE DIFFERENT APPLICATIONS TROUGH STATES : $\langle p_1 | , \langle p_2 |$

  $$\langle p_1 | \infty \langle p_2 | \quad \left\{ \begin{array}{ll} \langle p | , \langle 0 | & \bullet \text{ BLER} \\ \langle p | , \langle p | & \bullet \text{ Georgi, Politzer, Gross, Wilczek et al.} \\ \vdots & \\ \text{etc.} & \end{array} \right.$$

OLDER & RECENT DEVELOPMENTS:

- Brodsky, Lepage (LO)
- Dittes, Radyushkin (NLO) $P_{qq}$ Loewe, Bartels
- Efremov, Radyushkin,
- Geyer, Horejsi, Dittes, D. Müller, Robaschik, Braunschweig
- Balitsky, Braun

1996 →

- Radyushkin
- X. Ji
- Collins et al.
- JB, Geyer, Robaschik
- Frankfurt, Freund, Guzey, Strikman

⋮

more to come.

# One-Variable Partition Functions

$$\left.\frac{\langle p_1|O^q(-\kappa_-\tilde{x},\kappa_-\tilde{x})|p_2\rangle}{(i\tilde{x}p_+)}\right|_{\tilde{x}p_-=\tau\tilde{x}p_+} = \int_{-\infty}^{+\infty} dt e^{-i\kappa_-\tilde{x}p_+t} F_q(t)$$

$$\left.\frac{\langle p_1|O^G(-\kappa_-\tilde{x},\kappa_-\tilde{x})|p_2\rangle}{(i\tilde{x}p_+)^2}\right|_{\tilde{x}p_-=\tau\tilde{x}p_+} = \int_{-\infty}^{+\infty} dt e^{-i\kappa_-\tilde{x}p_+t}\, tF_G(t)\,.$$

$\tau = \frac{\tilde{x}p_-}{\tilde{x}p_+}$

$$\mu^2\frac{d}{d\mu^2}F^{\rm NS}(t) = \frac{\alpha_s(\mu^2)}{2\pi}\int_{-\infty}^{+\infty} dt' V_{ext}^{\rm NS}(t,t',\tau)F^{\rm NS}(t')$$

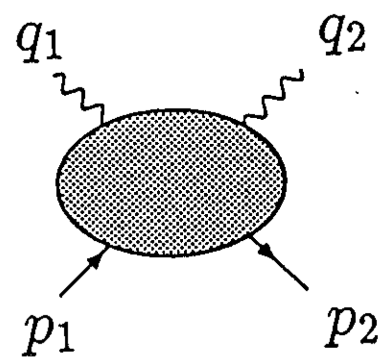
$$\mu^2\frac{d}{d\mu^2}\begin{pmatrix} F^q(t) \\ F^G(t)\end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi}\int_{-\infty}^{+\infty} dt' \mathbf{V}_{ext}(t,t',\tau)\begin{pmatrix} F^q(t') \\ F^G(t')\end{pmatrix}$$

2-dim Kernels.

$$V_{ext}^{ij}(t,t',\tau) = \int_0^1 d\alpha_1\int_0^{1-\alpha_1} d\alpha_2 K^{ij}(\alpha_1,\alpha_2)\frac{1}{2\pi}\int_{-\infty}^{+\infty} d(p_+\tilde{x}\kappa_-)\underline{(\kappa_-)^{-a_{ij}}}$$
$$\times\underline{[ip_+\tilde{x}(\kappa_-)]^{a_{ij}}}\,\underline{\frac{t'^{a_j}}{t^{a_i}}}\exp\left\{ip_+\kappa_-\tilde{x}\left[t-(1-\alpha_1-\alpha_2)t'+\tau(\alpha_1-\alpha_2)\right]\right\},$$

# Compton Amplitude

$$T_{\mu\nu}(p_+,p_-,Q) = i\int d^4x e^{iqx}\langle p_2|T(J_\mu(x/2)J_\nu(-x/2))|p_1\rangle,$$




$p_1 + q_1 = p_2 + q_2$

$p_+ = p_1 + p_2$
$p_- = p_2 - p_1$
$Q = \frac{1}{2}(q_1+q_2)$

OPE: (no states)

$$T(J_\mu(x/2)J_\nu(-x/2)) \approx \int_{-\infty}^{+\infty} d\kappa_- \int_{-\infty}^{+\infty} d\kappa_+ \Big[C_a(x^2,\kappa_-,\kappa_+,\mu^2)S_{\mu\nu}{}^{\rho\sigma}\tilde{x}_\rho O^a_\sigma(\kappa_+\tilde{x},\kappa_-\tilde{x},\mu^2) + C_{a,5}(x^2,\kappa_-,\kappa_+,\mu^2)\varepsilon_{\mu\nu}{}^{\rho\sigma}\tilde{x}_\rho O^a_{5,\sigma}(\kappa_+\tilde{x},\kappa_-\tilde{x},\mu^2)\Big] \quad (2)$$

Anikin, Zavialov.

with $S_{\mu\nu\rho\sigma} = g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}$ and $\varepsilon_{\mu\nu\rho\sigma}$ denoting the Levi–Civita symbol. The light–like vector

$$\tilde{x} = x + r(x.r/r.r)\left[\sqrt{1 - x.xr.r/(x.r)^2} - 1\right] \quad (3)$$

# Twist 2 Light-Ray Operators

$$O^{\rm NS}(\kappa_1,\kappa_2) = \frac{i}{2}\left[\overline{\psi_a}(\kappa_1\tilde{x})\lambda_f\gamma_\mu\tilde{x}^\mu\psi_a(\kappa_2\tilde{x}) - \overline{\psi_a}(\kappa_2\tilde{x})\lambda_f\gamma_\mu\tilde{x}^\mu\psi_a(\kappa_1\tilde{x})\right] \tag{4}$$

$$O_5^{\rm NS}(\kappa_1,\kappa_2) = \frac{i}{2}\left[\overline{\psi_a}(\kappa_1\tilde{x})\gamma_5\lambda_f\gamma_\mu\tilde{x}^\mu\psi_a(\kappa_2\tilde{x}) + \overline{\psi_a}(\kappa_2\tilde{x})\gamma_5\lambda_f\gamma_\mu\tilde{x}^\mu\psi_a(\kappa_1\tilde{x})\right] \tag{5}$$

$$O^q(\kappa_1,\kappa_2) = \frac{i}{2}\left[\overline{\psi_a}(\kappa_1\tilde{x})\gamma_\mu\tilde{x}^\mu\psi_a(\kappa_2\tilde{x}) - \overline{\psi_a}(\kappa_2\tilde{x})\gamma_\mu\tilde{x}^\mu\psi_a(\kappa_1\tilde{x})\right] \tag{6}$$

$$O_5^q(\kappa_1,\kappa_2) = \frac{i}{2}\left[\overline{\psi_a}(\kappa_1\tilde{x})\gamma_5\gamma_\mu\tilde{x}^\mu\psi_a(\kappa_2\tilde{x}) + \overline{\psi_a}(\kappa_2\tilde{x})\gamma_5\gamma_\mu\tilde{x}^\mu\psi_a(\kappa_1\tilde{x})\right] \tag{7}$$

$$O^G(\kappa_1,\kappa_2) = \tilde{x}^\mu F_{a\mu}{}^{\nu}(\kappa_1\tilde{x})\tilde{x}^{\mu'}F^a{}_{\mu'\nu}(\kappa_2\tilde{x}) \tag{8}$$

$$O_5^G(\kappa_1,\kappa_2) = \frac{1}{2}\left[\tilde{x}^\mu F_{a\mu}{}^{\nu}(\kappa_1\tilde{x})\tilde{x}^{\mu'}\tilde{F}^a{}_{\mu'\nu}(\kappa_2\tilde{x}) - \tilde{x}^\mu F^a{}_{\mu\nu}(\kappa_2\tilde{x})\tilde{x}^{\mu'}\tilde{F}^a{}_{\mu'}{}^{\nu}(\kappa_1\tilde{x})\right], \tag{9}$$

UNPOLA-
RIZED

POLARIZED

RGE : FOR OPERATORS

$$\mu^2\frac{d}{d\mu^2}O^{\rm NS}_{(5)}(\kappa_1,\kappa_2) = \frac{\alpha_s(\mu^2)}{2\pi}\int_0^1 d\alpha_1\int_0^1 d\alpha_2\theta(1-\alpha_1-\alpha_2)K^{\rm NS}(\alpha_1,\alpha_2)O^{\rm NS}_{(5)}(\kappa_1',\kappa_2'),$$

$$\mu^2\frac{d}{d\mu^2}\begin{pmatrix} O^q(\kappa_1,\kappa_2)\\ O^G(\kappa_1,\kappa_2)\end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi}\int_0^1 d\alpha_1\int_0^1 d\alpha_2\theta(1-\alpha_1-\alpha_2)\boldsymbol{K}(\alpha_1,\alpha_2)\begin{pmatrix} O^q(\kappa_1',\kappa_2')\\ O^G(\kappa_1',\kappa_2')\end{pmatrix}$$

(ALPHA - REPRESENTATION)

KERNELS : $\boldsymbol{K} = \begin{pmatrix} K^{qq} & K^{qG}\\ K^{Gq} & K^{GG}\end{pmatrix}$ and $\Delta\boldsymbol{K} = \begin{pmatrix} \Delta K^{qq} & \Delta K^{qG}\\ \Delta K^{Gq} & \Delta K^{GG}\end{pmatrix}$,

UNPOLARIZED

POLARIZED

$$k_1 = k_+ + k_- \qquad k_1' = k_1 (1-\alpha_1) + k_2 \alpha_1$$

$$k_2 = k_+ - k_- \qquad k_2' = k_1 \alpha_2 + k_2 (1-\alpha_2)$$

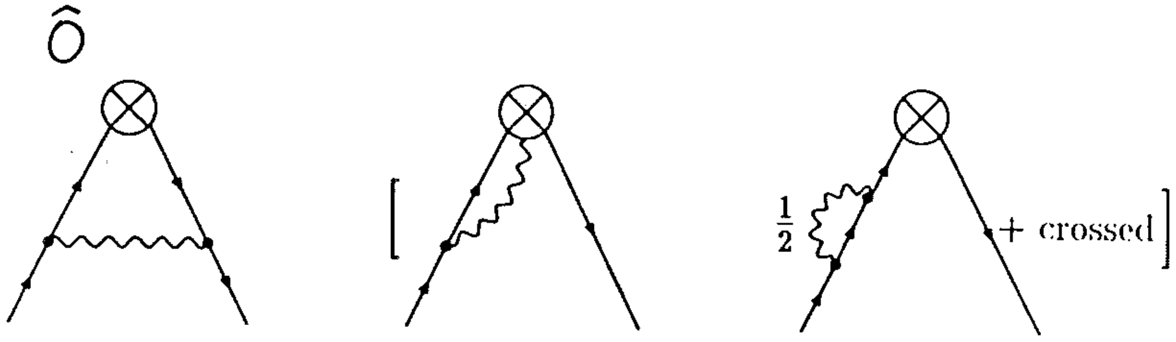
$$k_+ = \frac{1}{2}(k_1 + k_2) \qquad k_+' = \frac{1}{2}(k_1' + k_2')$$

$$k_- = \frac{1}{2}(k_1 - k_2) \qquad k_-' = \frac{1}{2}(k_1' - k_2')$$

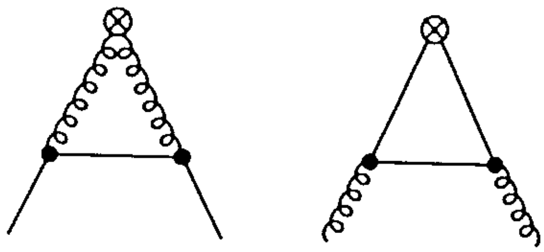
$$\equiv k_- (1 - \alpha_1 - \alpha_2).$$

DIAGRAMS IN $O(\alpha_s)$ : (AXIAL GAUGE)

qq
(NS)

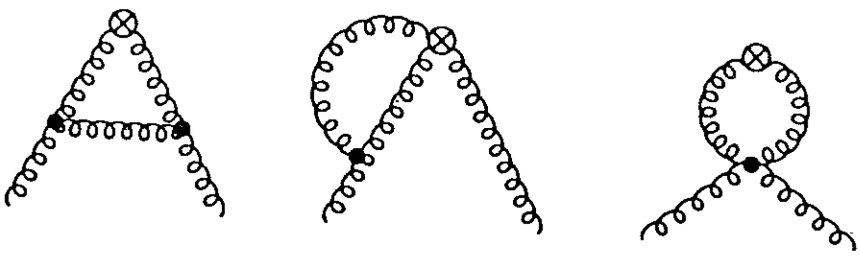



Gq

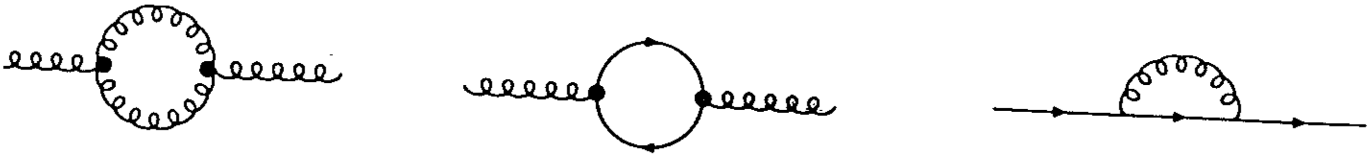


qG

GG



Z - FACTORS.



# Anomalous Dimensions

UNPOLARIZED:

$$
\begin{aligned}
K^{qq}(\alpha_1,\alpha_2) &= C_F\left\{1-\delta(\alpha_1)-\delta(\alpha_2)+\delta(\alpha_1)\left[\frac{1}{\alpha_2}\right]_+ +\delta(\alpha_2)\left[\frac{1}{\alpha_1}\right]_+ +\frac{3}{2}\delta(\alpha_1)\delta(\alpha_2)\right\}\\
K^{qG}(\alpha_1,\alpha_2) &= -2N_fT_R\underline{\kappa_-}\left\{1-\alpha_1-\alpha_2+4\alpha_1\alpha_2\right\}\\
K^{Gq}(\alpha_1,\alpha_2) &= -C_F\underline{\frac{1}{\kappa_-}}\left\{\delta(\alpha_1)\delta(\alpha_2)+2\right\}\\
K^{GG}(\alpha_1,\alpha_2) &= C_A\Big\{4(1-\alpha_1-\alpha_2)+12\alpha_1\alpha_2\\
&\quad +\delta(\alpha_1)\left(\left[\frac{1}{\alpha_2}\right]_+ -2+\alpha_2\right)+\delta(\alpha_2)\left(\left[\frac{1}{\alpha_1}\right]_+ -2+\alpha_1\right)\Big\}\\
&\quad +\frac{\beta_0}{2}\delta(\alpha_1)\delta(\alpha_2),
\end{aligned}
$$

· BALITSKY, BRAUN
· RADYUSHKIN
· JB, GEYER, ROBASCHIK

POLARIZED CASE:

$$
\begin{aligned}
\Delta K^{qq}(\alpha_1,\alpha_2) &= K^{qq}(\alpha_1,\alpha_2) \equiv K^{NS}(\alpha_1,\alpha_2)\\
\Delta K^{qG}(\alpha_1,\alpha_2) &= -2N_fT_R\underline{\kappa_-}\left\{1-\alpha_1-\alpha_2\right\}\\
\Delta K^{Gq}(\alpha_1,\alpha_2) &= -C_F\underline{\frac{1}{\kappa_-}}\left\{\delta(\alpha_1)\delta(\alpha_2)-2\right\}\\
\Delta K^{GG}(\alpha_1,\alpha_2) &= K^{GG}(\alpha_1,\alpha_2)-12C_A\alpha_1\alpha_2 .
\end{aligned}
$$

(COEFFICIENT FCTS. BECOME NONTRIVIAL ONLY FOR NLO NF-PDF's)

· JB, GEYER, ROBASCHIK
· BALITSKY, RADYUSHKIN

$T_R=\frac{1}{2}$, $C_A=N_c\equiv 3$

$C_F=\frac{N_c^2-1}{2N_c}=\frac{4}{3}$.

$\beta_0=\frac{11}{3}C_A-\frac{4}{3}T_RN_f$

$$\int_0^1 dx\,[f(x,y)]_+\,\varphi(x)=\int_0^1 dx\,f(x,y)\,[\varphi(x)-\varphi(y)]\ ,$$

# Two-Variable Partition Functions

$$\frac{\langle p_1|O^q|p_2\rangle}{(i\tilde{x}p_+)} = e^{-i\kappa_+\tilde{x}p_-}\int_{-\infty}^{+\infty} dz_+ \int_{-\infty}^{+\infty} dz_- e^{-i\kappa_-(\tilde{x}p_+z_+ + \tilde{x}p_-z_-)} F_q(z_-, z_+)$$

$$\frac{\langle p_1|O^G|p_2\rangle}{(i\tilde{x}p_+)^2} = e^{-i\kappa_+\tilde{x}p_-}\int_{-\infty}^{+\infty} dz_+ \int_{-\infty}^{+\infty} dz_- e^{-i\kappa_-(\tilde{x}p_+z_+ + \tilde{x}p_-z_-)} F_G(z_-, z_+)\,.$$

(FOURIER-REPRESENT.)

The evolution equations for the partition functions read

$$\mu^2\frac{d}{d\mu^2}F^{\rm NS}(z_+,z_-) = \frac{\alpha_s(\mu^2)}{2\pi}\int_{-\infty}^{+\infty}\frac{dz'_+}{|z'_+|}\int_{-\infty}^{+\infty} dz'_-\widetilde{K}^{\rm NS}\overset{(z_+,z_-;\,z'_+,z'_-)}{\cancel{(\alpha_1,\alpha_2)}}F^{\rm NS}(z'_+,z'_-)$$

$$\mu^2\frac{d}{d\mu^2}\begin{pmatrix} F^q(z_+,z_-)\\ F^G(z_+,z_-)\end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi}\int_{-\infty}^{+\infty}\frac{dz'_+}{|z'_+|}\int_{-\infty}^{+\infty} dz'_-\widetilde{\boldsymbol{K}}(z_+,z_-;z'_+,z'_-)\begin{pmatrix} F^q(z'_+,z'_-)\\ F^G(z'_+,z'_-)\end{pmatrix},$$

where $F^{\rm NS}(z_+,z_-) = F^{q_i}(z_+,z_-) - F^{\bar{q}_j}(z_+,z_-)$, and

$$\widetilde{K}^{ij}(\alpha_1,\alpha_2) = \frac{1}{2}\int_0^1 dz''_+\underline{\tilde{O}^{ij}(z_+,z''_+)}\theta(1-\alpha'_+)\theta(\alpha'_+)\theta(\alpha'_+ + \alpha'_-)\theta(\alpha'_+ - \alpha'_-)K^{ij}(\alpha'_1,\alpha'_2)\,,$$

with $\widetilde{K}^{\rm NS} = \widetilde{K}^{qq}$ and $\alpha'_\rho = \alpha_\rho(z_+ \to z''_+)$,

$$\tilde{O}^{ij}(z_+,z''_+) = \begin{pmatrix} \delta(z_+ - z''_+) & \partial_{z_+}\delta(z_+ - z''_+)\\ -\theta(z_+ - z''_+) & \delta(z_+ - z''_+)\end{pmatrix}$$

$$\alpha_{1,2} \equiv \alpha_{1,2}(z_+,z_-;\,z'_+,z'_-)$$

$$\alpha_1 = \frac{\alpha_+ + \alpha_-}{2} \qquad \alpha_2 = \frac{\alpha_+ - \alpha_-}{2},$$

$$\alpha_+ = 1 - \frac{z_+}{z'_+} \qquad -\alpha_- = \frac{z_+z'_- - z_-z'_+}{z'_+},$$

# The Evolution Kernels

$$
\begin{aligned}
V^{qq}_{ext}(t,t',\tau) &= \frac{1}{2}\left\{V^{qq}(x,y)\rho(x,y) + V^{qq}(\overline{x},\overline{y})\rho(\overline{x},\overline{y}) + \frac{3}{2}C_F\delta(x-y)\right\}\frac{1}{\tau} \equiv V^{NS}_{ext}(t,t',\tau) \\
V^{qG}_{ext}(t,t',\tau) &= \frac{1}{2}\left(\frac{2y-1}{2}\right)\left\{V^{qG}(x,y)\rho(x,y) - V^{qG}(\overline{x},\overline{y})\rho(\overline{x},\overline{y})\right\}\frac{1}{\tau} \\
V^{Gq}_{ext}(t,t',\tau) &= \frac{1}{2}\left(\frac{2}{2x-1}\right)\left\{V^{Gq}(x,y)\rho(x,y) - \overline{V}^{Gq}(\overline{x},\overline{y})\rho(\overline{x},\overline{y})\right\}\frac{1}{\tau}
\end{aligned}
$$

UNPOLARIZED

$$
\begin{aligned}
V^{GG}_{ext}(t,t',\tau) &= \frac{1}{2}\left(\frac{2y-1}{2x-1}\right)\left\{V^{GG}(x,y)\rho(x,y) + V^{GG}(\overline{x},\overline{y})\rho(\overline{x},\overline{y})\right\}\frac{1}{\tau} \\
&\quad + \frac{1}{2}\frac{1}{2}\beta_0\delta(x-y)\frac{1}{\tau} \\
\Delta V^{qq}_{ext}(t,t',\tau) &= V^{qq}_{ext}(t,t',\tau) \\
\Delta V^{qG}_{ext}(t,t',\tau) &= \frac{1}{2}\left(\frac{2y-1}{2}\right)\left\{\Delta V^{qG}(x,y)\rho(x,y) - \Delta V^{qG}(\overline{x},\overline{y})\rho(\overline{x},\overline{y})\right\}\frac{1}{\tau} \\
\Delta V^{Gq}_{ext}(t,t',\tau) &= \frac{1}{2}\left(\frac{2}{2x-1}\right)\left\{\Delta V^{Gq}(x,y)\rho(x,y) - \Delta\overline{V}^{Gq}(\overline{x},\overline{y})\rho(\overline{x},\overline{y})\right\}\frac{1}{\tau} \\
\Delta V^{GG}_{ext}(t,t',\tau) &= \frac{1}{2}\left(\frac{2y-1}{2x-1}\right)\left\{\Delta V^{GG}(x,y)\rho(x,y) + \Delta V^{GG}(\overline{x},\overline{y})\rho(\overline{x},\overline{y})\right\}\frac{1}{\tau} \\
&\quad + \frac{1}{2}\frac{1}{2}\beta_0\delta(x-y)\frac{1}{\tau}
\end{aligned}
$$

POLARIZED

$$
V^{ij}_{ext}(t,t',\tau) = \frac{1}{\tau}V^{ij}_{ext}\left(\frac{t}{\tau},\frac{t'}{\tau},1\right) \qquad \underline{x = \frac{1}{2}\left(1+\frac{t}{\tau}\right), \quad y = \frac{1}{2}\left(1+\frac{t'}{\tau}\right)} \qquad \rho(x,y) = \theta\left(1-\frac{x}{y}\right)\theta\left(\frac{x}{y}\right)\ \mathrm{sign}(y)\ ,
$$

UNPOLARIZED

$$
\begin{aligned}
V^{qq}(x,y) &= C_F\left[\frac{x}{y}-\frac{1}{y}+\frac{1}{(y-x)_+}\right] \\
V^{qG}(x,y) &= -2N_fT_R\frac{x}{y}\left[4(1-x)+\frac{1-2x}{y}\right] \\
V^{Gq}(x,y) &= C_F\left[1-\frac{x^2}{y}\right] \\
V^{GG}(x,y) &= C_A\left[2\frac{x^2}{y}\left(3-2x+\frac{1-x}{y}\right)+\frac{1}{(y-x)_+}-\frac{y+x}{y^2}\right]
\end{aligned}
$$

POLARIZED

$$
\begin{aligned}
\Delta V^{qq}(x,y) &= V^{qq}(x,y) \\
\Delta V^{qG}(x,y) &= -2N_fT_R\frac{x}{y^2} \\
\Delta V^{Gq}(x,y) &= C_F\left[\frac{x^2}{y}\right] \\
\Delta V^{GG}(x,y) &= C_A\left[2\frac{x^2}{y^2}+\frac{1}{(y-x)_+}-\frac{y+x}{y^2}\right]
\end{aligned}
$$

## Special Case, X· ∃i $(t' = 1)$

$$
\begin{aligned}
K^{qq}(t,t',\xi) &= C_F \frac{t^2 + t'^2 - \xi^2/2}{(t'^2 - \xi^2/4)(t'-t)_+} + \frac{3}{2}\delta(t'-t) \\
K^{qG}(t,t',\xi) &= T_R N_f \frac{t^2 + (t'-t)^2 - \xi^2/4}{(t'^2 - \xi^2/4)^2} t' \\
K^{Gq}(t,t',\xi) &= C_F \frac{t'^2 + (t'-t)^2 - \xi^2/4}{t(t'^2 - \xi^2/4)} \\
K^{GG}(t,t',\xi) &= 2C_A \left(\frac{t'}{t}\right) \frac{1}{(t'^2 - \xi^2/4)^2} \left[ \frac{(t'^2 - \xi^2/4)^2}{(t'-t)_+} + t'(t'^2 + \xi^2/4) \right. \\
&\quad \left. - t(3t'^2 - \xi^2/4) - (t'+t)(t'-t)^2 \right] + \frac{\beta_0}{2}\delta(t'-t), \\
\Delta K^{qq}(t,t',\xi) &= K^{qq}(t,t',\xi) \\
\Delta K^{qG}(t,t',\xi) &= T_R N_f \frac{t^2 - (t'-t)^2 - \xi^2/4}{(t'^2 - \xi^2/4)^2} t' \\
\Delta K^{Gq}(x,\xi) &= C_F \frac{t' - (t'-t)^2 - \xi^2/4}{t(t'^2 - \xi^2/4)} \\
\Delta K^{GG}(x,\xi) &= 2C_A \left(\frac{t'}{t}\right) \frac{1}{(t'^2 - \xi^2/4)^2} \left[ \frac{(t'^2 - \xi^2/4)^2}{(t'-t)_+} + t'(t'^2 + \xi^2/4) \right. \\
&\quad \left. - t(3t'^2 - \xi^2/4) - 2t'(t'-t)^2 \right] + \frac{\beta_0}{2}\delta(t'-t) \ .
\end{aligned}
$$

## The Brodsky–Lepage Limit

$\tau \to \pm 1$ : $\langle p_2| \to \langle p|$, $\langle p_1| \to \langle 0|$

BRODSKY-LEPAGE-EFREMOV-RADYUSHKIN Kernels.

$$V^{qq}(x,y) = C_F\{\Theta(y-x)[\frac{x}{y} - \frac{1}{y} + \frac{1}{(y-x)_+}] + \Theta(x-y)[\frac{1-x}{1-y} - \frac{1}{1-y} + \frac{1}{(x-y)_+}\}.$$

etc.

## The Altarelli–Parisi Limit

$\tau \to 0$.

(ONE POSSIBILITY).

In the case $t > \tau, t' > \tau$ we obtain another representation. First note :sign$\overline{y} = -$sign$y$, $\Theta(1-\frac{\overline{x}}{\overline{y}}) = \Theta(y-x)$. Using these changes we obtain (For simplicity, we dropped here the +-prescriptions.)

$$\begin{aligned} V^{qq}(x,y) &= C_F\Theta(y-x)\{\frac{x}{y}[1+\frac{1}{y-x}] - \frac{1-x}{1-y}[1+\frac{1}{x-y}]\} \\ &= C_F\Theta(y-x)\frac{1}{y-x}[1+\frac{x\overline{x}}{y\overline{y}}], \boxed{x = \frac{1}{2\tau}(\tau+t),\ y = \frac{1}{2\tau}(\tau+t').} \end{aligned}$$

$x = x(\tau)$
$y = y(\tau)$

Note, that the Altarelli-Parisi Limit is obtained by

$$\underline{lim_{\tau\to 0}}\, V^{qq}(x,y) = \frac{1}{|t'|}P^{qq}(\frac{t}{t'}) = \underline{\underline{\frac{1}{|t'|}C_F\frac{z^2+1}{1-z}}}.$$

SIMILARLY FOR THE OTHER KERNELS.

SECOND POSSIBILITY: $\langle P_2| \to \langle p_1| \equiv \langle p|$ RIGHT FROM THE BEGINNING.

$$
\begin{aligned}
f^q(z,\mu) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} d(2p\tilde{x}\kappa_-)\langle p|O^q|p\rangle(\kappa_-,\mu)\frac{e^{2ip\tilde{x}\kappa_-}}{2ip\tilde{x}} \\
zf^G(z,\mu) &= \frac{1}{2\pi}\int_{-\infty}^{+\infty} d(2p\tilde{x}\kappa_-)\langle p|O^G|p\rangle(\kappa_-,\mu)\frac{e^{2ip\tilde{x}\kappa_-}}{(2ip\tilde{x})^2}\,.
\end{aligned}
$$

$$
\begin{aligned}
\mu^2\frac{d}{d\mu^2}f^{\rm NS}(z,\mu) &= \frac{\alpha_s(\mu^2)}{2\pi}\int_{-\infty}^{+\infty}\frac{dz'}{|z'|}\widehat{P}^{\rm NS}\left(\frac{z}{z'}\right)f^{\rm NS}(z,\mu), \\
\mu^2\frac{d}{d\mu^2}\begin{pmatrix} f^q(z,\mu) \\ f^G(z,\mu)\end{pmatrix} &= \frac{\alpha_s(\mu^2)}{2\pi}\int_{-\infty}^{+\infty}\frac{dz'}{|z'|}\widehat{\boldsymbol{P}}\left(\frac{z}{z'}\right)\begin{pmatrix} f^q(z,\mu) \\ f^G(z,\mu)\end{pmatrix}\,.
\end{aligned}
$$

$$
\begin{aligned}
\widehat{P}^{ij}(z) &= P^{ij}(z)\theta(z)\theta(1-z) \\
P^{ij}(z) &= \int_{-\infty}^{+\infty} du\,\underline{\widehat{O}^{ij}(u,z)}\int_0^1 d\xi(1-u)\widehat{K}^{ij}(\alpha_1,\alpha_2)\theta(1-u)\theta(u),
\end{aligned}
$$

$$
\widehat{\boldsymbol{K}} = \begin{pmatrix} K^{qq} & (1/\kappa_-)K^{qG} \\ (\kappa_- - i\varepsilon)K^{Gq} & K^{GG}\end{pmatrix},
$$

$$
\widehat{O}^{ij}(u,z) = \begin{pmatrix} \delta(z-u) & \partial_z\delta(z-u) \\ -\theta(z-u)/z & \delta(z-u)/z\end{pmatrix}
$$

$$
\begin{aligned}
P^{qq}(z) &= C_F\left(\frac{1+z^2}{1-z}\right)_+ \\
P^{qG}(z) &= 2N_fT_R\left[z^2+(1-z)^2\right] \\
P^{Gq}(z) &= C_F\frac{1+(1-z)^2}{z} \\
P^{GG}(z) &= 2C_A\left[\frac{1}{z}+\frac{1}{(1-z)_+}-2+z(1-z)\right]+\frac{\beta_0}{2}\delta(1-z), \\
\Delta P^{qq}(z) &= P^{qq}(z) \\
\Delta P^{qG}(z) &= 2N_fT_R\left[z^2-(1-z)^2\right] \\
\Delta P^{Gq}(z) &= C_F\frac{1-(1-z)^2}{z} \\
\Delta P^{GG}(z) &= 2C_A\left[1-2z+\frac{1}{(1-z)_+}\right]+\frac{\beta_0}{2}\delta(1-z).
\end{aligned}
$$

In deriving eq. (70) it is useful to apply the relations

$$
\theta(x)=\lim_{\varepsilon\to 0^+}\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\xi\frac{e^{ix\xi}}{i\xi+\varepsilon}, \qquad \delta^{(k)}(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\xi\;(i\xi)^k e^{ix\xi},
$$

which are valid for tempered distributions [16].

## 4 Conclusions

- NON - FORWARD COMPTON SCATTERING: PDF'S DEPEND ON 2 VARIABLES

$\longrightarrow$ LO SCALING VIOLATIONS EVALUATED (PREDICTION)

- CALL FOR EXP. TEST FOR:
  - UNPOLARIZED
  - POLARIZED TARGETS
  - NS & SINGLET.

- CHECK SPECIAL CASES!

$$\frac{\tilde{x} p_-}{\tilde{x} p_+} = \tau = \text{const.}$$