Theory Perspectives: Deep-Inelastic Scattering

Johannes Blümlein
DESY

- The Major Goals
- DIS Theory Status
- Unpolarized Parton Distribution Functions
- Polarized Parton Distribution Functions
- $\Lambda_{QCD}$ and $\alpha_s(M_Z^2)$
- Advanced Technologies to Evaluate Feynman Diagrams @ 3 Loops
- Outlook
1. The Major Goals

- Precision Measurement of the Strong Coupling Constant $\alpha_s(M_Z^2)$
- Precision Measurement of the Unpolarized Parton Densities
- Precision Measurement of the Polarized Parton Densities
- Who Carries the Spin of the Proton?
- Higher Twist Effects
- Is there Saturation in DIS at small $x$? $\implies$ answered by experiment.
Theory of DIS

- Parton Model
- Light Cone Expansion [f]
- Twist 2
- Higher Twist
- Fixed Order PT: QCD
  - \( O(\alpha_s^4) \): ’73, ’74, ’80, ’97
  - \( O(\alpha_s^3) \): ’73, ’82, ’04
  - \( O(\alpha_s^3) \): ’82, ’92, ’05, ’09
- Splitting functions
- Coefficient functions
- Sum Rules
  - ’60ies - now
  - ’75, ’86
  - ’90 - ’98
- More General View
  - Non-forw. scattering
  - Angular Momentum: q, G
- Special Kinematics
  - Domain: Small x
- Diffractive Scattering
- Resummations?
- Higher Orders

New Algorithms
- Novel Mathematics

\[ O(\alpha_s^4) \]
\[ O(\alpha_s^3) \]
\[ O(\alpha_s^3) \]
Status of Highest Order Calculations

- Running $\alpha_s$: $O(\alpha_s^4)$ Larin, van Ritbergen, Vermaseren 1997
- Unpol. anomalous dimensions and Wilson coefficients: $O(\alpha_s^3)$
  Moch, Vermaseren, Vogt 2004/05
- Unpol. NS anomalous dimension 2nd Moment: $O(\alpha_s^4)$ Baikov, Chetyrkin 2006
- Pol. anomalous dimension: $O(\alpha_s^2)$; Mertig, van Neerven, 1995; Vogelsang 1995;
  $\Delta P^{qq} \Delta P_{qG}$: $O(\alpha_s^3)$ Moch, Rogal, Vermaseren, Vogt 2008
- Pol. Wilson coefficients: $O(\alpha_s^2)$; $\Delta C_{NS}^{qq}$, $\Delta C_{qG}$: van Neerven, Zijlstra 1994
- Transversity: $O(\alpha_s^2)$, some moments anom. dim.: $O(\alpha_s^3)$, Hayashigaki, Kanazawa, Koike;
  Kumano, Miyama; Vogelsang; 1997; Gracey 2006, HQ: JB, S.Klein, B. Tödtli 2008
- Unpol. Heavy Flavor Wilson Coefficients: $O(\alpha_s^2)$ Laenen, van Neerven, Riemersma, Smith, 1993
  Fast Mellin Space code: Blümlein & Alekhin, 2003
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient $F_L$: $O(\alpha_s^3)$
  Blümlein, De Freitas, van Neerven, S. Klein 2005
- $Q^2 \gg m^2$ Pol. Heavy Flavor Wilson Coefficient: $O(\alpha_s^2)$ van Neerven, Smith et al. 1996,
  Bierenbaum, Blümlein & Klein 2007
- $Q^2 \gg m^2$ Unpol. Heavy Flavor Wilson Coefficient $F_2$: $O(\alpha_s^2 \varepsilon)$: all operators
  (also polarized), Bierenbaum, Blümlein, Klein, Schneider, 2008; $O(\alpha_s^3)$: Moments 2–10(12,14)
  of the operator matrix elements, HQ Wilson coeff. Bierenbaum, Blümlein, Klein, 2008

= done at DESY (or in DESY collab.).
\[ F_j(x, Q^2) = \hat{f}_i(x, \mu^2) \otimes \sigma^i_j \left( \alpha_s \frac{Q^2}{\mu^2}, x \right) \]

\[ \uparrow \text{bare pdf} \quad \uparrow \text{sub-system cross - sect.} \]

\[ = \hat{f}_i(x, \mu^2) \otimes \Gamma^i_k \left( \alpha_s(R^2), \frac{M^2}{\mu^2}, \frac{M^2}{R^2} \right) \]

\[ \otimes C^k_j \left( \alpha_s(R^2), \frac{Q^2}{\mu^2}, \frac{M^2}{R^2}, x \right) \]

Finite pdf \( \equiv f_k \)

Finite Wilson coefficient

**Move to Mellin space:**

\[ F_j(N) = \int_0^1 dxx^{N-1}F_j(x) \]

Diagonalization of the convolutions \( \otimes \) into ordinary products.
Evolution Equations

\[
\left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} - 2\gamma_\psi(g) \right] F_i(N) = 0
\]

\[
\left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} + \gamma^K_N(g) - 2\gamma_\psi(g) \right] f_k(N) = 0
\]

\[
\left[ M \frac{\partial}{\partial M} + \beta(g) \frac{\partial}{\partial g} - \gamma^K_N(g) \right] C^j_k(N) = 0
\]

CALLAN–SYMANZIK equations for mass factorization \( \equiv \)
ALTARELLI–PARISI evolution equations

\text{x-space :}

\[
\frac{d}{d \log(\mu^2)} \left( \begin{array}{c} q^+(x, Q^2) \\ G(x, Q^2) \end{array} \right) = \frac{\alpha_s}{2\pi} P(x, \alpha_s) \otimes \left( \begin{array}{c} q^+(x, Q^2) \\ G(x, Q^2) \end{array} \right)
\]

\[
P(x, \alpha_s) = P^{(0)}(x) + \frac{\alpha_s}{2\pi} P^{(1)}(x) + \left( \frac{\alpha_s}{2\pi} \right)^2 P^{(2)}(x) + \ldots
\]
Anomalous Dimensions and Wilson Coefficients

\[ \gamma_{qq}(N) \]
\[ \gamma_{gg}(N) \]

Vermaseren, Moch, Vogt 2004

\[ \alpha_s = 0.2, N_f = 4 \]
The Basic Functions of massless QCD to $w=5$: $\equiv$ 3 Loops

Representative: $S_1(N) = \psi(N+1) + \gamma_E$ and its derivatives.

Weight $w=3$:

\[ F_1(N) = M \left[ \ln \frac{1+x}{1+x} \right] (N) \]

\[ F_2(N) = M \left[ \frac{\text{Li}_2(x)}{1+x} \right] (N), \quad F_3(N) = M \left[ \left( \frac{\text{Li}_2(x)}{1-x} \right)_+ \right] (N) \]

Yndurain et al., 1981: $F_2(N)$

Weight $w=4$:

\[ F_4(N) = M \left[ \frac{S_{1,2}(x)}{1+x} \right] (N), \quad F_5(N) := M \left[ \left( \frac{S_{1,2}(x)}{1-x} \right)_+ \right] (N) \]

$F_3(N) - F_5(N)$: J.B., 2003; J.B., V. Ravindran, 2004
Weight w=5 :

\[ F_{6,7}(N) = \mathcal{M} \left[ \left( \frac{\text{Li}_4(x)}{1 \pm x} \right)_{(+)} \right](N), \quad F_8(N) = \mathcal{M} \left[ \frac{S_{1,3}(x)}{1 + x} \right](N), \]

\[ F_{9,10}(N) = \mathcal{M} \left[ \left( \frac{S_{2,2}(x)}{1 \pm x} \right)_{(+)} \right](N), \quad F_{11}(N) = \mathcal{M} \left[ \frac{\text{Li}_2^2(x)}{1 + x} \right](N), \]

\[ F_{12,13}(N) := \mathcal{M} \left[ \left( \frac{\ln(x)S_{1,2}(-x) - \text{Li}_2^2(-x)/2}{1 \pm x} \right)_{(+)} \right](N) \]

\[ F_6(N) - F_{13}(N) : \text{J.B., S. Moch, 2004}. \]

Massless QCD to 3 Loops depends on 14 Functions.

Weight w=6 :

Complex Analysis of these Functions

- Construct exact analytic continuations to complex $N$
- The functions are meromorphic (up to soft corrections, which have a simple structure)
- Asymptotic Representation
- Recursion $z + 1 \rightarrow z$
- Solve the Evolution Equations fully analytically and form an analytic expression for the Structure functions in Mellin Space at all $Q^2$
- Include the heavy flavor Wilson coefficients in Mellin Space $\Rightarrow$ nearly accomplished to $O(a_s^3)$ I. Bierenbaum, JB, S. Klein (2009)
- Perform a single fast, numerical Mellin inversion (at high precision)

$\Rightarrow$ Fastest and most Precise Way of Analysis
3. Unpolarized Parton Distribution Functions

HERA I $e^+p$ Neutral Current Scattering - H1 and ZEUS

New ZEUS + H1 averaged $F_2(x, Q^2)$
Direct $F_L(x, Q^2)$ Measurement at HERA

H1 Preliminary $F_L(x, Q^2)$

$Q^2 = 12 \text{ GeV}^2$

$Q^2 = 15 \text{ GeV}^2$

$Q^2 = 20 \text{ GeV}^2$

$Q^2 = 25 \text{ GeV}^2$

$Q^2 = 35 \text{ GeV}^2$

$Q^2 = 45 \text{ GeV}^2$

$Q^2 = 60 \text{ GeV}^2$

$Q^2 = 90 \text{ GeV}^2$

$Q^2 = 120 \text{ GeV}^2$

$Q^2 = 150 \text{ GeV}^2$

$Q^2 = 200 \text{ GeV}^2$

$Q^2 = 250 \text{ GeV}^2$

$Q^2 = 300 \text{ GeV}^2$

$Q^2 = 400 \text{ GeV}^2$

$Q^2 = 500 \text{ GeV}^2$

$Q^2 = 650 \text{ GeV}^2$

$Q^2 = 800 \text{ GeV}^2$

$E_p = 460, 575, 920 \text{ GeV}$

$H1 \text{ (Prelim.)}$

$H1 \text{ PDF 2000}$
Direct $F_L(x, Q^2)$ Measurement at HERA (H1-prel.)
World Data Analysis: Valence Distributions

World data: NS-analysis

$W^2 > 12.5 \text{ GeV}^2, Q^2 > 4 \text{ GeV}^2$

N$^3$LO:

$\alpha_s(M_Z^2) = 0.1141^{+0.0020}_{-0.0022}$

Why an $O(\alpha_s^4)$ analysis can be performed?

assume an $\pm 100\%$ error on the Pade approximant $\rightarrow \pm 2$ MeV in $\Lambda_{QCD}$

$$\gamma_n^{approx:3} = \frac{\gamma_n^{(2)}}{\gamma_n^{(1)}}$$

Baikov & Chetyrkin, April 2006:

$$\gamma_2^{3;NS} = \frac{32}{9} a_s + \frac{9440}{243} a_s^2 + \left[ \frac{3936832}{6561} - \frac{10240}{81} \zeta_3 \right] a_s^3$$

$$+ \left[ \frac{1680283336}{1777147} - \frac{24873952}{6561} \zeta_3 + \frac{5120}{3} \zeta_4 - \frac{56969}{243} \zeta_5 \right] a_s^4$$

The results agree better than 20%.
Valence Distributions

$F_2^p(x, Q^2) \times 2^i$

- H1
- ZEUS
- BCDMS

$Q^2$, GeV$^2$

$F_2^p(x, Q^2) \times 2^i$

- BCDMS
- NMC

$Q^2$, GeV$^2$
Valence Distributions: higher twist

**PROTON**

$C_{HT}(x) \ [GeV^2]$

$4.0 \text{ GeV}^2 < W^2 < 12.5 \text{ GeV}^2$

- NLO
- NNLO
- N3LO
- N4LO

**DEUTERON**

$C_{HT}(x) \ [GeV^2]$

$4.0 \text{ GeV}^2 < W^2 < 12.5 \text{ GeV}^2$

- NLO
- NNLO
- N3LO
- N4LO

- agreement between $p$ and $d$ analysis, J.B., H. Böttcher, 2008
- LGT determination of interest
Slope of $F_2$ at low $x$

Very likely, that the $\overline{\text{MS}}$–gluon is remains positive!

J. Blümlein

Theory Perspectives : DIS

DESY, July 7th 2009

J.B., A. Guffanti 2005
Flavor distributions: light quarks (NNLO)

Current Fitting Community (NNLO): + Many NLO analyses worldwide: CTEQ, NNPDF, H1, ZEUS, ...

S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102
Correct treatment of HQ very essential: FFNS, BSMN-schemes.
full lines: ABKM error band; dashed lines: MSTW08
Flavor distributions: strangeness

FIG. 3: The strange parton distribution $xS(x)$ from the measured HERMES multiplicity for charged kaons evolved to $Q_0^2 = 2.5 \text{ GeV}^2$ assuming $\int D_S^K(z) dz = 1.27 \pm 0.13$. The solid curve is a 3-parameter fit for $S(x) = x^{-0.024} e^{-2x^{0.0404}(1-x)}$, the dashed curve gives $xS(x)$ from CTEQ6L, and the dot-dash curve is the sum of light antiquarks from CTEQ6L.

Nice HERMES measurement (hep-ex/0803.2993); still to be understood.
Heavy quarks and gluon (NNLO)

S. Alekhin, J.B., S. Klein, S. Moch, DESY 09-102
full lines: ABKM error band; dashed lines: MSTW08

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Theory Perspectives : DIS
DESY, July 7th 2009
Gluon (NNLO)

Jimenez-Delgado/Reya (2008)
High Luminosity is most important: Various precision measurements.
Polarized Parton Densities at Present

J.B., H. Böttcher (2002)
The Polarized Gluon Distribution at Present

\[ x\Delta G(x) \sim NLO \]

\[ x\Delta q(x) \]

J.B., H. Böttcher (2002)

\[ \Rightarrow \text{Currently slight move of } \Delta G \text{ towards lower values} \]

\[ \Rightarrow \text{3-loop analysis would settle theory error.} \]
Unfolding the Sea Quarks

De Florian, Sassot, Stratmann, Vogelsang, 2008
Accurate measurement highly desired.
How big is the $\tau = 3$ contribution?
Moments of PDF’s: PT + data

<table>
<thead>
<tr>
<th></th>
<th>n</th>
<th>This Fit</th>
<th>MRST04 N$^3$LO</th>
<th>A02 NNLO</th>
<th>BB, NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_v$</td>
<td>2</td>
<td>0.3006 ± 0.0031</td>
<td>0.285</td>
<td>0.304</td>
<td>0.926</td>
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<tr>
<td></td>
<td>3</td>
<td>0.0877 ± 0.0012</td>
<td>0.082</td>
<td>0.087</td>
<td>0.163 ± 0.014</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0335 ± 0.0006</td>
<td>0.032</td>
<td>0.033</td>
<td>0.055 ± 0.006</td>
</tr>
<tr>
<td>$d_v$</td>
<td>2</td>
<td>0.1252 ± 0.0027</td>
<td>0.115</td>
<td>0.120</td>
<td>-0.341</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0318 ± 0.0009</td>
<td>0.028</td>
<td>0.028</td>
<td>-0.047 ± 0.021</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0106 ± 0.0004</td>
<td>0.009</td>
<td>0.010</td>
<td>-0.015 ± 0.009</td>
</tr>
<tr>
<td>$u_v - d_v$</td>
<td>2</td>
<td>0.1754 ± 0.0041</td>
<td>0.171</td>
<td>0.184</td>
<td>1.267</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0559 ± 0.0015</td>
<td>0.055</td>
<td>0.059</td>
<td>0.210 ± 0.025</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0229 ± 0.0007</td>
<td>0.022</td>
<td>0.024</td>
<td>0.070 ± 0.011</td>
</tr>
</tbody>
</table>

Lattice Results: developing; different fermion-types studied. Low values of $m_\pi$ crucial; values approach 270 MeV now.

J.B., H. Böttcher, A. Guffanti, 2006
J.B., H. Böttcher, 2002
5. $\Lambda_{QCD}$ and $\alpha_s(M_Z^2)$

\[
\frac{\delta \alpha_{em}(0)}{\alpha_{em}(0)} \sim 3 \cdot 10^{-11} \quad \frac{\delta \alpha_{weak}}{\alpha_{weak}} \sim 7 \cdot 10^{-4} \quad \frac{\delta \alpha_s(M_Z^2)}{\alpha_s(M_Z^2)} > 2 \cdot 10^{-2}
\]

(until recently)

P. Zerwas, 2004
Overview of the Analyses

• Various NLO analyses; \( \Rightarrow \) Precision requires NNLO analysis and higher!

• Mixed S- and NS-NNLO analyses \( e(\mu)N \) world data

• S- and NS-NNLO moment analyses \( \nu N \) world data

• NS-\( N^3 \)LO analysis \( e(\mu)N \) world data

• NLO analyses polarized \( e(\mu)N \) world data

• Lattice measurements
<table>
<thead>
<tr>
<th>NLO</th>
<th>NLO ( \alpha_s(M^2_Z) )</th>
<th>expt</th>
<th>theory</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTEQ6</td>
<td>0.1165 ± 0.0065</td>
<td>[1]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MRST03</td>
<td>0.1165 ± 0.0020 ± 0.0030</td>
<td>[2]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A02</td>
<td>0.1171 ± 0.0015 ± 0.0033</td>
<td>[3]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ZEUS</td>
<td>0.1166 ± 0.0049</td>
<td>[4]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H1</td>
<td>0.1150 ± 0.0017 ± 0.0050</td>
<td>[5]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BCDMS</td>
<td>0.110 ± 0.006</td>
<td>[6]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GRS</td>
<td>0.112</td>
<td>[10]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BBG</td>
<td>0.1148 ± 0.0019</td>
<td>[9]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BB (pol)</td>
<td>0.113 ± 0.004 +0.009 −0.006</td>
<td>[7]</td>
<td></td>
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<table>
<thead>
<tr>
<th>NNLO</th>
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<th>expt</th>
<th>theory</th>
<th>Ref.</th>
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<tbody>
<tr>
<td>MRST03</td>
<td>0.1153 ± 0.0020 ± 0.0030</td>
<td>[2]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A02</td>
<td>0.1143 ± 0.0014</td>
<td>[3]</td>
<td>±0.0009</td>
<td></td>
</tr>
<tr>
<td>SY01(ep)</td>
<td>0.1166 ± 0.0013</td>
<td>[8]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SY01(\nuN)</td>
<td>0.1153 ± 0.0063</td>
<td>[8]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GRS</td>
<td>0.111</td>
<td>[10]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A06</td>
<td>0.1128 ± 0.0015</td>
<td>[11]</td>
<td></td>
<td></td>
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<tr>
<td>BBG</td>
<td>0.1134 +0.0019/ −0.0021</td>
<td>[9]</td>
<td></td>
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<table>
<thead>
<tr>
<th>N^3LO</th>
<th>N^3LO ( \alpha_s(M^2_Z) )</th>
<th>expt</th>
<th>theory</th>
<th>Ref.</th>
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<tbody>
<tr>
<td>BBG</td>
<td>0.1141 +0.0020/ −0.0022</td>
<td>[9]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NLO**

- **BBG**: \( N_f = 4 \): non-singlet data-analysis at \( O(\alpha_s^4) \): \( \Lambda = 234 \pm 26 \) MeV
- **Lattice results**: 
  - **Alpha Collab**: \( N_f = 2 \) Lattice; non-pert. renormalization \( \Lambda = 245 \pm 16 \pm 16 \) MeV
  - **QCDSF Collab**: \( N_f = 2 \) Lattice, pert. reno. \( \Lambda = 261 \pm 17 \pm 26 \) MeV
- **Lepage et al.**: Larger, but no quenched result.
\[ \delta \alpha_s(M_Z^2) \approx 1.2\% \]

(Obtained by July 1st)

<table>
<thead>
<tr>
<th></th>
<th>(\alpha_s(M_Z^2))</th>
<th>HQ:</th>
<th>dynamical approach</th>
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<tbody>
<tr>
<td>ABKM</td>
<td>0.1135 ± 0.0014</td>
<td>FFS (N_f = 3)</td>
<td></td>
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<tr>
<td>ABKM</td>
<td>0.1129 ± 0.0014</td>
<td>BSMN-approach</td>
<td></td>
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<tr>
<td>BBG (2006)</td>
<td>0.1134 (+0.0019) (-0.0021)</td>
<td></td>
<td>valence analysis, NNLO</td>
</tr>
<tr>
<td>JR (2008)</td>
<td>0.1124 ± 0.0020</td>
<td></td>
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</tr>
<tr>
<td>MSTW (2008)</td>
<td>0.1171 ± 0.0014</td>
<td></td>
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</tr>
<tr>
<td>BBG (2006)</td>
<td>0.1141 (+0.0020) (-0.0022)</td>
<td></td>
<td>valence analysis, (N^3)LO</td>
</tr>
</tbody>
</table>
\( \alpha_s(M_Z^2) \)

J.B., H. Böttcher, A. Guffanti, 2006

J. Blümlein

Theory Perspectives : DIS

DESY, July 7th 2009
• $\alpha_s(M_Z^2)$ for different data sets included are too different!
  ⇒ applies also to HERA: IS vs FS; and also DIS vs TEVATRON-jet

M. Cooper-Sarkar, 2005
6. Advanced Technologies to Evaluate Feynman Diagrams

in QED & QCD @ 3 loops and beyond

- Automatic diagram generation mandatory: QGRAF
  # 2500 - 15000 diagrams

- The ‘Only’ problem: Calculation of Feynman Parameter Integrals;
  everything else automated: FORM-codes

- Renormalization still not always trivial: $\gamma_5$, mass(es), ...

- Work with linguistic standards: Harmonic Sums, Harmonic Polylogarithms, Euler-Zagier values, etc. - Avoids the problem of Babel in analytic integration

- Generalized Hypergeometric Functions and their Generalizations are to the Heart of the Matter. M. Kalmykov et al., JB et al.

- Need: advanced Difference Equation Establishers & Solvers: Sigma

- Do not proliferate!, i.e. avoid IBP, MB, and other methods causing gigantic Zeroes.

- What remains is: Integrating the hard way.
Advanced Technologies to Evaluate Feynman Diagrams

Some Examples:

Zero-scale Problems: Euler-Zagier and Multiple Zeta Values

JB, D. Broadhurst, J. Vermaseren, DESY 09-03

find all relations: \( \Rightarrow \) Tera-Terms to be processed

alternating: all relations up to \( w = 12 \) (6-loop level);

non-alternating: all relations up to \( w = 22 \); determined.

Interesting relations: to \( w = 30 \);

Reconstructing recurrent quantities from Mellin Moments

JB, M. Kauers, S. Klein, C. Schneider DESY 09-02

Can one find the anomalous dimensions and Wilson coefficients to 3-loops just from their moments? Yes - recurrent quantities in Mellin space.

\( \leq 5114 \) Moments; difference equation fills 440 books

Complete computation: 5 CPU Months

Massive Wilson coefficients at 3 Loops

I. Bierenbaum, JB, S. Klein, DESY 09-57

first analytic massive 1-scale calculation @ 3-loops

Moments 2–10 (12/14) have been calculated for all unpolarized channels

Complete computation: 300 CPU days, partly req. 32-64 Gbyte computers
7. Outlook

Theory:
- Polarized Anomalous Dimensions & massless Wilson coefficients @ 3 Loops
- Unpolarized Heavy Flavor Wilson coefficients @ 3 Loops: general \( N \)
- Polarized Heavy Flavor Wilson coefficients @ 3 Loops
- Along with this: Development of efficient analytic calculation methods being suited for 3-Loops and higher
- \( e_p \) & \( pp \) jet cross sections at HO; progress in pdf Lattice calculations

Code:
- Creation of an Open Source Code for DIS and pp-hard scattering data for experimental precision analyzes to derive pdfs

Experiment:
- Precision Data from LHC, JLAB and EIC.

Can we get \( \delta \alpha_s \) even smaller?