DESY Oct. 4 91

F_L AND F_2 AT SMALL x

(STATUS)

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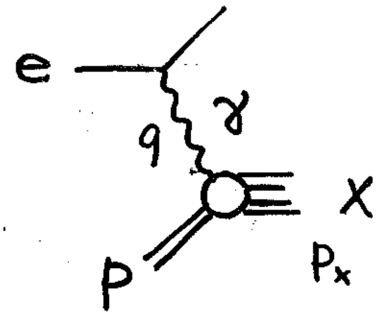
OUTLINE:

1) INTRODUCTION
2) FACTORIZATION & BASIC RELATIONS
3) $\hat{\sigma}_{F_2}$ AND $\hat{\sigma}_{F_L}$

1) INTRODUCTION



HADRONIC TENSOR:

$$W_{\mu\nu} = \frac{1}{4\pi} \sum_X (2\pi)^4 \delta^{(4)}(p+q-p_x) \langle p | J_\mu^+(0) | X \rangle \langle X | J_\nu(0) | p \rangle$$

$$= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) W_1 + \frac{1}{M_p^2}\left(p_\mu - \frac{p.q}{q^2} q_\mu\right)\left(p_\nu - \frac{p.q}{q^2} q_\nu\right) W_2$$

WITH:

$$W_1(\nu, Q^2) \longrightarrow F_1(x, Q^2) \qquad \nu = Q^2/2M_p x$$

$$\frac{\nu}{M_p} W_2(\nu, Q^2) \longrightarrow F_2(x, Q^2)$$

$$F_L = F_2 - 2x F_1$$

2 RELATIONS:

$$g_{\mu\nu} W^{\mu\nu} = \frac{1}{2x}\left[F_2 - 6x F_1\right]$$

$$p_\mu p_\nu W^{\mu\nu} = \frac{Q^2}{8x^3}\left[F_2 - 2x F_1\right] = \frac{Q^2}{8x^3} F_L$$

$$\boxed{\begin{aligned} F_2(x,Q^2) &= \frac{12x^3}{Q^2}\, P_\mu P_\nu W^{\mu\nu} - x\, g_{\mu\nu} W^{\mu\nu} \\ F_L(x,Q^2) &= \frac{8x^3}{Q^2}\, P_\mu P_\nu W^{\mu\nu} \end{aligned}}$$

- WHICH TERMS (BEYOND THE USUAL AP-LADDER) HAVE TO BE CONSIDERED AT SMALL x?

⟶ SEMIHARD RANGE

- LIPATOV EQUATION (KWIECINSKI, COLLINS, WEBBER, MARCHESINI, CATANI, CIAFALONI, HAUTMANN)
- FAN DIAGRAMS GRIBOV, LEVIN, RYSKIN
  ⟶ GLUON-EVOLUTION. KWIECINSKI, COLLINS BARTELS, JB, SCHULER MÜLLER, QIU

- ADD TO THE STANDARD EVOLUTION FOR QUARKS THE FAN-DIAGRAM TERM AND A TERM DESCRIBING GLUON RECOMBINATION INTO QUARKS (MÜLLER, QIU)

⟶ SEA QUARKS

⟶ CONVOLUTION FORMULA : F_L

KWIECINSKI, MARTIN, STIRLING, ROBERTS.

$\Longrightarrow$ IMPLICIT ASSUMPTION OF COLLINEAR POLE FACTORIZATION

$\longrightarrow$ IS NOT SUFFICIENT IN GENERAL !

(SEE : CALC. OF HEAVY FLAVOUR CROSS SECTIONS AT SMALL x (N=0)-MOMENT:

- CATANI, CIAFALONI, HAUTMANN
- LEVIN, RYSKIN, SHABELSKI, SHUVAEV
- COLLINS, R.K. ELLIS.

- REQUIRED : CORRECT FACTORIZATION

LEVIN, RYSKIN
AND ABOVE AUTHORS

1) EVALUATION OF THE GLUON INDUCED TERMS IN F_2, F_L DUE TO FAN DIAGRAMS

2) MUELLER - QIU GLUON RECOMBINATION INTO QUARKS.

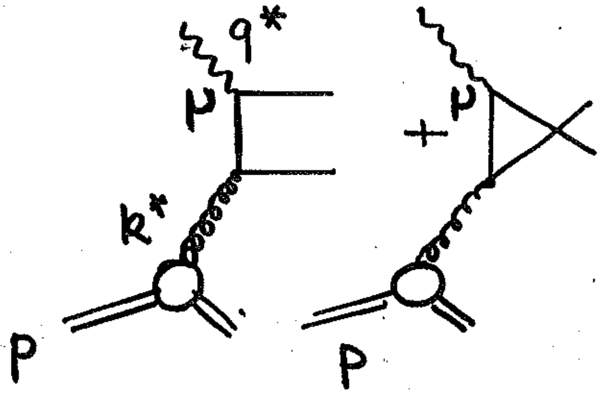
$\longrightarrow$ DERIVE THE FULL CONTRIBUTION

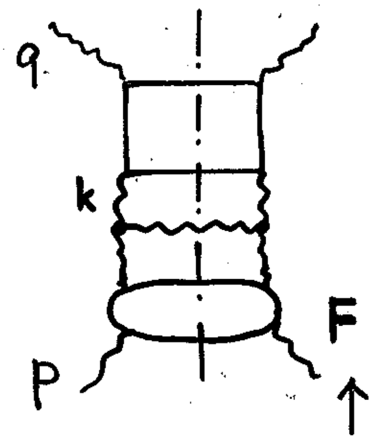
- LOW x DOMINANT TERMS : · LEVIN, RYSKIN
  · BARTELS, LOEWE, DE ROECK

- CONVOLUTIONS REQUIRE THE $x \sim 1$ BEHAVIOUR OF THE KERNELS ALSO.

2) FACTORIZATION & BASIC RELATIONS

DIAGRAMS:





+ ....

↑ CAN ALSO BE WRITTEN FOR FAN DIAGRAMS IN THIS FORM (ONLY GLUONS).

$$F_i(x, Q^2) \sim \int d^2\vec{k} \int \frac{dz}{z}\, \hat{\sigma}_i\left(\frac{x}{z}, \vec{q}, \vec{k}\right) \mathcal{F}(z, \vec{k}, Q_0^2)$$

$\hat{\sigma}_i$ IS THE $q\bar{q}$- CROSS SECTION FOR AN OFFSHELL PHOTON & AN OFF SHELL GLUON.

$$x\, G(x, Q^2) = \int^{Q^2} dk^2\, \mathcal{F}(x, \vec{k}, Q_0^2)$$

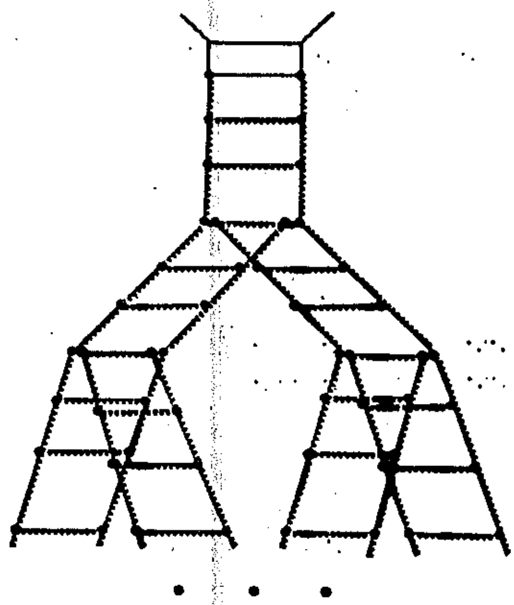
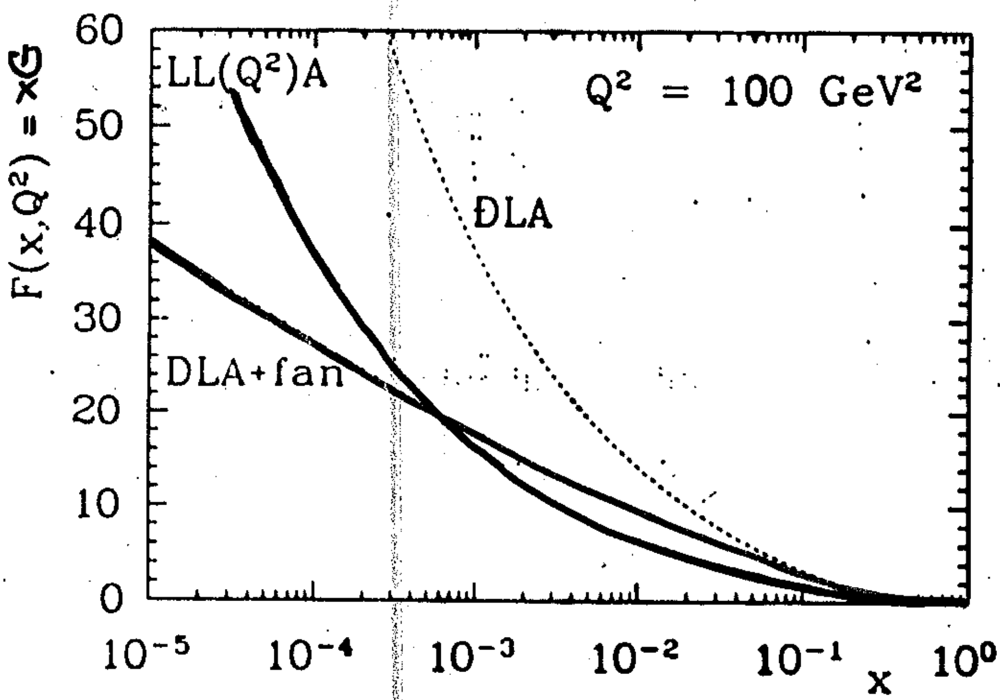


Fig. 1. Fan diagrams which are the basis for the GLR equation. The wavy lines denote gluons



HERE, $xG(x,Q^2) = \mathbb{F}(x,Q^2)$, THE SOLUTION OF THE FAN DIAGRAM EQUATION (CF. BARTELS, JB, SCHULER; COLLINS, KWIECINSKI).

$$\hat{\sigma}_{F_2} = \frac{1}{4\pi}\int dR_2\, \overline{|M_{\mu\nu\lambda g}|^2}\; \mathbb{P}^{F_2}_{\mu\nu}\, d_{\lambda g}(k)$$

$$\hat{\sigma}_{F_L} = \frac{1}{4\pi}\int dR_2\, \overline{|M_{\mu\nu\lambda g}|^2}\; \mathbb{P}^{F_L}_{\mu\nu}\, d_{\lambda g}(k)$$

$$\mathbb{P}^{F_2}_{\mu\nu} = \frac{12x^3}{Q^2} P_\mu P_\nu - x\, g_{\mu\nu}$$

$$\mathbb{P}^{F_L}_{\mu\nu} = \frac{8x^3}{Q^2} P_\mu P_\nu$$

$$d_{\lambda g}(k) = \sum_{pol} \varepsilon^*_\mu \varepsilon_\nu \Big|_{gluon}$$

! OFF SHELL QUANTITIES IN A GAUGE THEORY ARE NOT NECESSARILY GAUGE INVARIANT.

WE CHECKED, THAT: $\hat{\sigma}_{F_2}$ AND $\hat{\sigma}_{FL}$ ARE INVARIANT UNDER REPLACING:

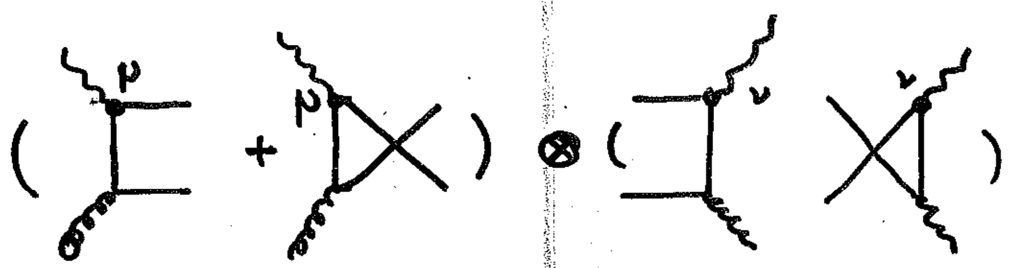
$$d_{\lambda g}(k) = -g_{\mu\nu} + \frac{1}{kc}(k_\mu c_\nu + c_\mu k_\nu) - \frac{c^2}{(kc)^2} k_\mu k_\nu\,(1-\eta)$$

$$\therefore\ \eta = \begin{cases} 1 & \text{PLANAR GAUGE} \\ 0 & \text{AXIAL GAUGE} \end{cases}$$

$$c_\nu = A\bar{q}_\nu + BP_\nu$$
$$\bar{q}_\nu = q_\nu - xP$$
$$q_\nu^2 = P_\nu^2 = 0.$$

$$d_{\lambda g}(k) = -g_{\mu\nu} + (1-\xi)\frac{k_\mu k_\nu}{k^2} \qquad (R_\xi\text{-gauge})$$

3) $\hat{\sigma}_{F_2}$ & $\hat{\sigma}_{F_L}$



$\otimes \; P_\mu P_\nu$

$\otimes \; g_{\mu\nu}$

$$\overline{|M_{\mu\nu\lambda g}|^2}\, g_{\mu\nu}\, d_{\lambda g}(k) = 2 e_q^2 g_s^2 \left\{ \left[-\frac{u}{t} - \frac{t}{u} + \frac{2sQ^2}{ut} \right] - k^2 \left[\frac{Q^2}{t^2} + \frac{Q^2}{u^2} + \frac{2s}{ut} \right] \right\} \quad {}^{*)}$$

HERE, $u+t+s = -Q^2 + k^2$. (refer to the subsystem)

THE WELL-KNOWN ON-SHELL ($k^2 \equiv 0$) RESULT is READ OFF DIRECTLY (BLACK).

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*) SET $m_q \equiv 0$ FOR BREVITY, $m_q \neq 0$ $\overline{|M|^2}$ ARE ALSO GENERATED EASILY.

$\boxed{\text{EXCURSION}}$ LIMES OF COLLINEAR-POLE FACTORIZATION

$$I_{g\mu\nu}(t) = 2e_q^2 g_s^2 \left[-\frac{u}{t} - \frac{t}{u} + \frac{2sQ^2}{ut}\right]$$

$$s+Q^2 = Q^2/x$$

$$\int_{-Q^2/x+\mu^2}^{\mu^2} dt \, I_{g\mu\nu}(t) \, \frac{x}{Q^2 \, 8\pi} = \underline{\left[x^2+(1-x)^2\right]} \, \frac{e_q^2 \alpha_s}{2\pi} \ln\left(\frac{Q^2}{\mu^2}\right) + \text{non log. terms.}$$

$\uparrow$

full P_{qg}- splitting function.

$\longrightarrow$ INTEGRATE $\overline{|M_{\mu\nu g\lambda}|^2} \, g_{\mu\nu} d_{\lambda g}(k) \; dR_2$

- USE: SUDAKOV-REP. OF $k = \alpha P + \beta \bar{q} + k_\perp$

  OF p_1', p_2'.

$$\overline{|M_{\mu\nu\lambda g}|^2} P_\mu P_\nu \, d_{\lambda g}(k) \; e_q^2 \, g_s^2 \, \frac{1}{8\pi} \, \frac{2x^3}{Q^2\pi}$$

$$= e_q^2 \frac{x^3 \alpha_s}{Q^2 \pi} \Big\{ \frac{k^2}{tu} \, 4 \big[k.P^2 + 2\, k.P \cdot q.P - 4\, k.P \cdot p_1.P + q.P^2 - 4\, q.P \cdot q_1 P + 4\, p_1.P^2 \big]$$

$$+ \frac{s}{tu} \, 4 \big[-k.P^2 - k.P \cdot q.P + 2\, k.P \cdot p_1.P + 2\, k.P \cdot p_1.P + 2\, q.P \cdot p_1.P - 2 p_1.P^2 \big]$$

$$+ \frac{1}{t} \, 4 \big[-k.P^2 - 2 k.P \cdot q.P + 2\, k.P \cdot p_1.P - q.P^2 + 3 q.P \, p_1.P - 2\, p_1.P^2 \big]$$

$$+ \frac{1}{u} \, 4 \big[-k.P^2 - k.P \, q.P + 2\, k.P \, p_1.P + q.P \, p_1.P - 2\, p_1.P^2 \big]$$

$$+ \frac{k^2}{t^2} \, 4 \big[-q.P \, p_1.P + p_1.P^2 \big]$$

$$+ \frac{k^2}{u^2} \, 4 \big[k.P^2 + k.P \, q.P - 2 k.P \, p_1.P - q.P \, p_1.P + p_1.P^2 \big] \Big\}$$

$\longrightarrow$ INTEGRATE OVER dR_2 AFTER FURTHER SIMPLIFIC.

$q.P = Q^2/2x$

$p_1.P$ & $k.P$ are got from:

$$\left.\begin{aligned} p_1 &= \alpha_1 P + \beta_1 \bar{q} + p_1^\perp \\ k &= \alpha P + \beta \bar{q} + k^\perp \end{aligned}\right\} \quad \begin{aligned} p_1 P &= \beta_1 \bar{q} P = \beta_1 q P = \beta_1 \frac{Q^2}{2x} \\ kP &= \beta q P = \beta Q^2/2x. \end{aligned}$$

THE COLLINEAR POLE RESULT IS OBTAINED BY REPLACING

$$P = \frac{1}{x} k$$

&

$$k^2 = 0$$

IN THE LAST FORMULA.

STRAIGHT-FORWARD ALGEBRA YIELDS THE WELL KNOWN RESULT:

$$\overline{|M_{\mu\nu\lambda g}|^2} K_\mu K_\nu d_{\lambda g}(k) = \frac{1}{2} e_q^2 \, 4 g_s^2 \, s$$

$$s = Q^2/x - Q^2 = Q^2 \frac{1}{x}(1-x)$$

$$\curvearrowright \quad F_L^g = \sum_q \frac{2\alpha_s}{\pi} e_q^2 \, x^2(1-x).$$

4) NEXT STEPS

- EVALUATION OF $\hat{\sigma}_{F_2}$, $\hat{\sigma}_{F_L}$
- CALCULATION OF F_2 (FAN), F_L (FAN)