The 3-Loop Heavy Flavor Corrections to Unpolarized and Polarized Deep-Inelastic Scattering

pdf4lhc meeting, CERN, Genva, December 3, 2024

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DESY AND TU DORTMUND

Recent papers:

- J. Ablinger et al., The unpolarized and polarized single-mass three-loop heavy flavor operator matrix elements $A_{gg,Q}$ and $\Delta A_{gg,Q}$, JHEP **12** (2022) 134.
- J. Ablinger et al., The first-order factorizable contributions to the three-loop massive operator matrix elements $A_{Qg}^{(3)}$ and $\Delta A_{Qg}^{(3)}$, Nucl. Phys.B 999 (2024) 116427.
- J. Ablinger et al., The non-first-order-factorizable contributions to the three-loop single-mass operator matrix elements $A_{Qg}^{(3)}$ and $\Delta A_{Qg}^{(3)}$, 2403.00513 [hep-ph].
- J. Ablinger et al., The Singe-Mass Variable Flavor Number Scheme at Three-Loop Order, DESY 24–037

The main time-line for the 3-loop corrections

- 2005 F_L [no massive 3-loop OMEs needed]
- 2010 All unpolarized N_F terms and $A_{qg,Q}^{(3)}, A_{qq,Q}^{(3).PS}$
- 2014 unpolarized logarithmic 3-loop contributions and $A_{gq,Q}^{(3)}, (\Delta)A_{qq,Q}^{(3),NS}, A_{Qq}^{(3),PS}$
- 2017 two-mass corrections $A_{gq,Q}^{(3)}, (\Delta)A_{qq,Q}^{(3),NS}, A_{Qq}^{(3),PS}$
- 2018 two-mass corrections $A_{gg,Q}^{(3)}$
- 2019 2-loop correction: $(\Delta)A_{Qq}^{(2),PS}$ whole kinematic region and $\Delta A_{Qq}^{(3),PS}$
- 2019 two-mass corrections $\Delta A_{Qq}^{(3),PS}$
- 2020 two-mass corrections $\Delta A_{gg,Q}^{(3)}$
- 2021 polarized logarithmic 3-loop contributions and $\Delta A_{gq,Q}^{(3)}, \Delta A_{qq,Q}^{(3).PS}, \Delta A_{gq}^{(3)}$
- 2022 3-loop polarized massless Wilson coefficients [JB, Marquard, Schneider, Schönwald]
- 2022 corrected the polarized 2-loop contributions
- 2022 $(\Delta)A_{gg,Q}^{(3)}$
- 2023 $(\Delta)A_{Qg}^{(3)}$: 1st order factorizing parts
- 2024 $(\Delta)A_{Qg}^{(3)}$, [two-mass corrections $(\Delta)A_{Qg}^{(3)}$]; **Project duration $\gtrsim$ 15 years**

The Wilson Coefficients at large Q^2

$$
\begin{aligned}
L_{q,(2,L)}^{\rm NS}(N_F+1) &= a_s^2\left[A_{qq,Q}^{(2),\rm NS}(N_F+1)\delta_2 + \hat{C}_{q,(2,L)}^{(2),\rm NS}(N_F)\right] + a_s^3\left[A_{qq,Q}^{(3),\rm NS}(N_F+1)\delta_2 + A_{qq,Q}^{(2),\rm NS}(N_F+1)C_{q,(2,L)}^{(1),\rm NS}(N_F+1) + \hat{C}_{q,(2,L)}^{(3),\rm NS}(N_F)\right] \\
L_{q,(2,L)}^{\rm PS}(N_F+1) &= a_s^3\left[A_{qq,Q}^{(3),\rm PS}(N_F+1)\delta_2 + N_F A_{gq,Q}^{(2),\rm NS}(N_F)\tilde{C}_{g,(2,L)}^{(1),\rm NS}(N_F+1) + N_F\hat{\tilde{C}}_{q,(2,L)}^{(3),\rm PS}(N_F)\right] \\
L_{g,(2,L)}^{\rm S}(N_F+1) &= a_s^2 A_{gg,Q}^{(1)}(N_F+1)N_F\tilde{C}_{g,(2,L)}^{(2)}(N_F+1) + a_s^3\left[A_{qg,Q}^{(3)}(N_F+1)\delta_2 + A_{gg,Q}^{(1)}(N_F+1)N_F\tilde{C}_{g,(2,L)}^{(2)}(N_F+1)\right. \\
&\quad \left. + A_{gg,Q}^{(2)}(N_F+1)N_F\tilde{C}_{g,(2,L)}^{(1)}(N_F+1) + A_{Qg}^{(1)}(N_F+1)N_F\tilde{C}_{q,(2,L)}^{(2),\rm PS}(N_F+1) + N_F\hat{\tilde{C}}_{g,(2,L)}^{(3)}(N_F)\right] \\
H_{q,(2,L)}^{\rm PS}(N_F+1) &= a_s^2\left[A_{Qq}^{(2),\rm PS}(N_F+1)\delta_2 + \tilde{C}_{q,(2,L)}^{(2),\rm PS}(N_F+1)\right] \\
&\quad + a_s^3\left[A_{Qq}^{(3),\rm PS}(N_F+1)\delta_2 + A_{gq,Q}^{(2)}(N_F+1)\tilde{C}_{g,(1,L)}^{(2)}(N_F+1) + A_{Qq}^{(2),\rm PS}(N_F+1)\tilde{C}_{q,(2,L)}^{(1),\rm NS}(N_F+1) + \tilde{C}_{q,(2,L)}^{(3),\rm PS}(N_F+1)\right] \\
H_{g,(2,L)}^{\rm S}(N_F+1) &= a_s\left[A_{Qg}^{(1)}(N_F+1)\delta_2 + \tilde{C}_{g,(2,L)}^{(1)}(N_F+1)\right] \\
&\quad + a_s^2\left[A_{Qg}^{(2)}(N_F+1)\delta_2 + A_{Qg}^{(1)}(N_F+1)\tilde{C}_{q,(2,L)}^{(1)}(N_F+1) + A_{gg,Q}^{(1)}(N_F+1)\tilde{C}_{g,(2,L)}^{(1)}(N_F+1) + \tilde{C}_{g,(2,L)}^{(2)}(N_F+1)\right] \\
&\quad + a_s^3\left[A_{Qg}^{(3)}(N_F+1)\delta_2 + A_{Qg}^{(2)}(N_F+1)\tilde{C}_{q,(2,L)}^{(1)}(N_F+1) + A_{gg,Q}^{(2)}(N_F+1)\tilde{C}_{g,(2,L)}^{(1)}(N_F+1)\right. \\
&\quad \left. + A_{Qg}^{(1)}(N_F+1)\tilde{C}_{q,(2,L)}^{(2),\rm S}(N_F+1) + A_{gg,Q}^{(1)}(N_F+1)\tilde{C}_{g,(2,L)}^{(1)}(N_F+1) + \tilde{C}_{g,(2,L)}^{(3)}(N_F+1)\right]
\end{aligned}
$$

- The case for two different masses obeys an analogous representation.
- Note the contributions of the massless Wilson coefficients.

The variable flavor number scheme

- Matching conditions for parton distribution functions:

$$f_k(N_F+2) + f_{\overline{k}}(N_F+2) = A_{qq,Q}^{\rm NS}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) \cdot \left[f_k(N_F) + f_{\overline{k}}(N_F)\right] + \frac{1}{N_F} A_{qq,Q}^{\rm PS}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) \cdot \Sigma(N_F)$$
$$+ \frac{1}{N_F} A_{qg,Q}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) \cdot G(N_F)\,,$$

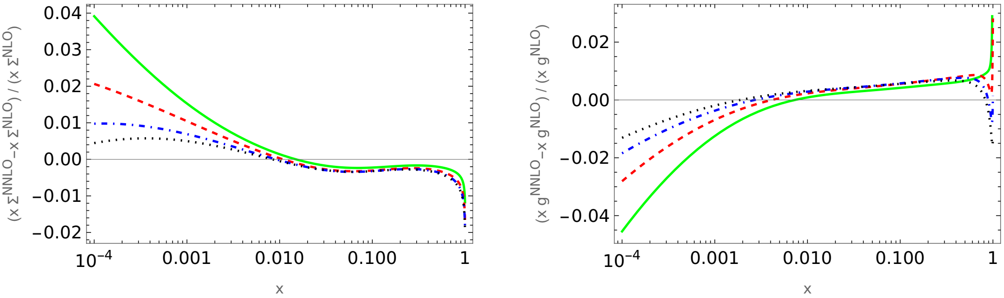
$$f_Q(N_F+2) + f_{\overline{Q}}(N_F+2) = A_{Qq}^{\rm PS}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2},\right) \cdot \Sigma(N_F) + A_{Qg}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) \cdot G(N_F)\,,$$

$$\Sigma(N_F+2) = \left[A_{qq,Q}^{\rm NS}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) + A_{qq,Q}^{\rm PS}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) + A_{Qq}^{\rm PS}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right)\right] \cdot \Sigma(N_F)$$
$$+ \left[A_{qg,Q}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) + A_{Qg}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right)\right] \cdot G(N_F)\,,$$

$$G(N_F+2) = A_{gq,Q}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) \cdot \Sigma(N_F) + A_{gg,Q}\left(N_F+2, \frac{m_1^2}{\mu^2}, \frac{m_2^2}{\mu^2}\right) \cdot G(N_F)\,.$$

The charm and bottom quark masses are not that much different.

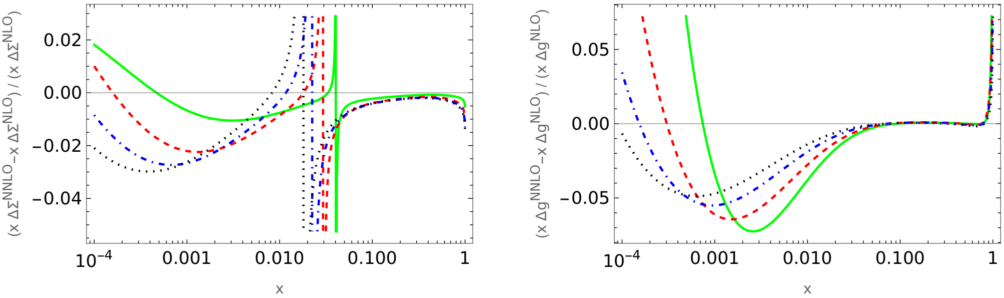
Relative effect in unpolarized NNLO evolution



$Q^2 = 10, 10^2, 10^3, 10^4$ GeV^2 dotted, dash-dotted, dashed, full lines. [M. Saragnese, 2022]

The unpolarized world deep-inelastic data have a precision of $O(1\%)$.

Relative effect in polarized NNLO evolution

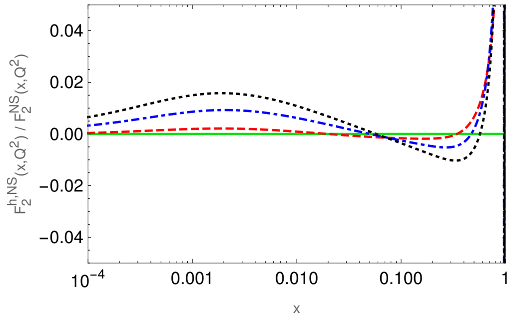


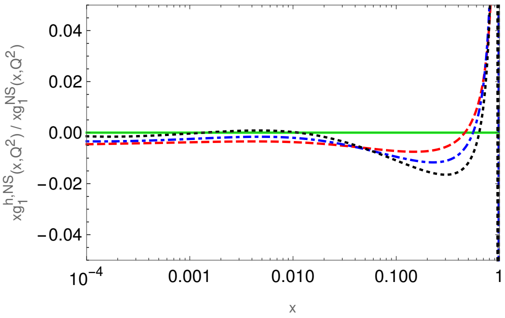
$Q^2 = 10, 10^2, 10^3, 10^4$ GeV^2 dotted, dash-dotted, dashed, full lines. [M. Saragnese, 2022]

The future polarized data at the EIC will reach a precision of $O(1\%)$.

The relative contribution of HQ to non-singlet structure functions at N^{3}LO

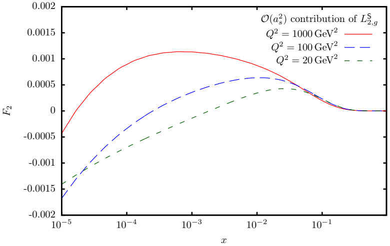
Scheme-invariant evolution



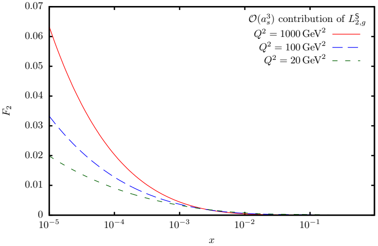


Left: The relative contribution of the heavy flavor contributions due to *c* and *b* quarks to the structure function $F_2^{\rm NS}$ at N^{3}LO; dashed lines: 100 GeV^2; dashed-dotted lines: 1000 GeV^2; dotted lines: 10000 GeV^2. Right: The same for the structure function $xg_1^{\rm NS}$ at N^{3}LO. [JB, M. Saragnese, 2021].

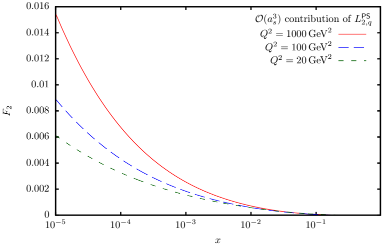
Numerical Results : $L_{g,2}^{\mathrm{S}}$ and $L_{q,2}^{\mathrm{PS}}$



$O(a_s^2)$ $L_{2,g}^{\mathrm{S}}$

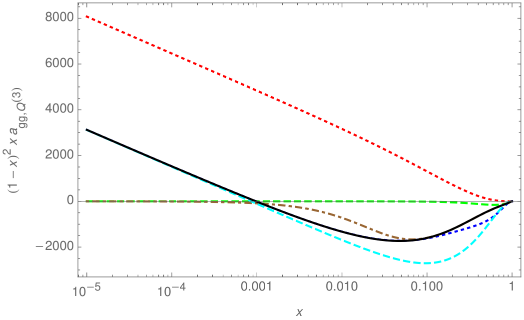


$O(a_s^3)$ $L_{2,g}^{\mathrm{S}}$

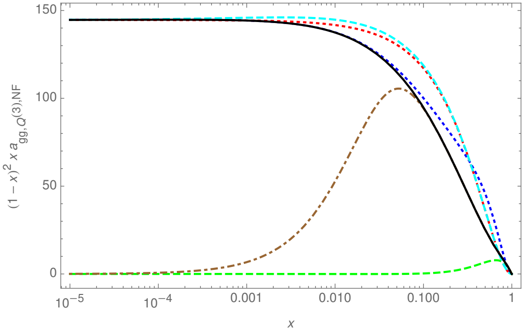


$L_{q,2}^{\mathrm{PS}}$

$a_{gg}^{(3)}$



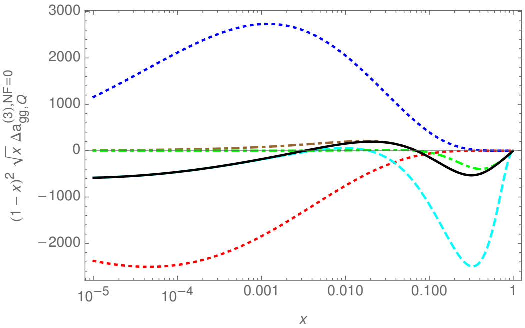
non N_F terms



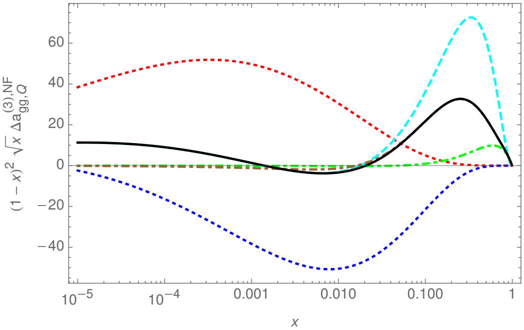
N_F terms

Left panel: The non–N_F terms of $a_{gg,Q}^{(3)}(N)$ (rescaled) as a function of x. Full line (black): complete result; upper dotted line (red): term $\propto \ln(x)/x$; lower dashed line (cyan): small x terms $\propto 1/x$; lower dotted line (blue): small x terms including all $\ln(x)$ terms up to the constant term; upper dashed line (green): large x contribution up to the constant term; dash-dotted line (brown): complete large x contribution. Right panel: the same for the N_F contribution.

$\Delta a_{gg}^{(3)}$



non N_F terms

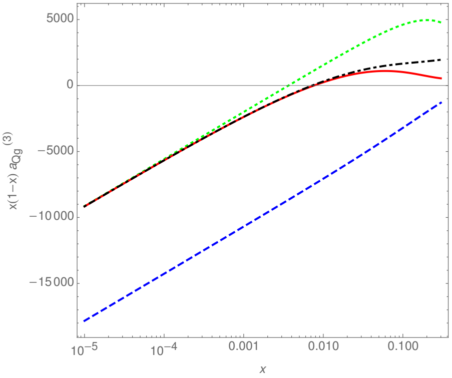


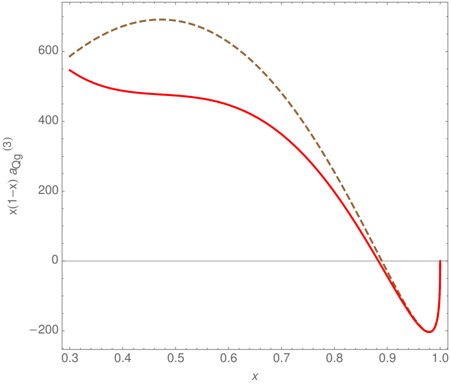
N_F terms

The non–N_F terms of $\Delta a_{gg,Q}^{(3)}(N)$ (rescaled) as a function of x. Full line (black): complete result; lower dotted line (red): term $\ln^5(x)$; upper dotted line (blue): small x terms $\propto \ln^5(x)$ and $\ln^4(x)$; upper dashed line (cyan): small x terms including all $\ln(x)$ terms up to the constant term; lower dash-dotted line (green): large x contribution up to the constant term; dash-dotted line (brown): full large x contribution. Right panel: the same for the N_F contribution.

$a_{Qg}^{(3)}$

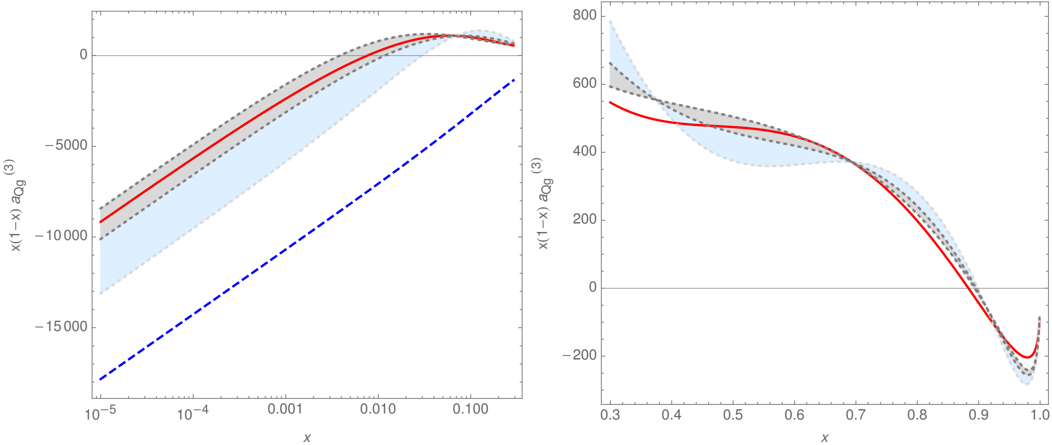
1009 of the total 1233 Feynman diagrams have first-order factorizing contributions only and are given by G-functions up to root-values letters. The letters for all constants can be rationalized.





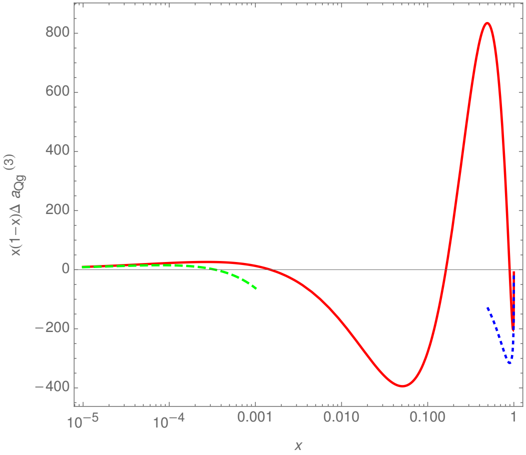
$a_{Qg}^{(3)}(x)$ as a function of x, rescaled by the factor $x(1-x)$. Left panel: smaller x region. Full line (red): $a_{Qg}^{(3)}(x)$; dashed line (blue): leading small-x term $\propto \ln(x)/x$ [Catani, Ciafaloni, Hautmann, 1990]; dotted line (green): $\ln(x)/x$ and $1/x$ term; dash-dotted line (black): all small-x terms, including also $\ln^k(x)$, $k \in \{1, \ldots, 5\}$. Right panel: larger x region. Full line (red): $a_{Qg}^{(3)}(x)$; dashed line (brown): leading large-x terms up to the terms $\propto (1-x)$, covering the logarithmic contributions of $O(\ln^k(1-x))$, $k \in \{1, 4\}$.

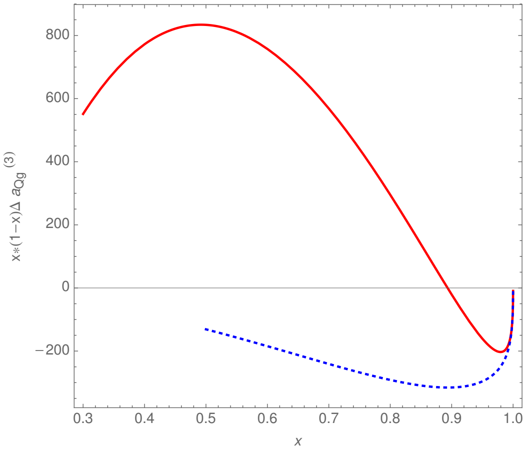
$a_{Qg}^{(3)}$



$a_{Qg}^{(3)}(x)$ as a function of x, rescaled by the factor $x(1-x)$. Left panel: smaller x region. Full line (red): $a_{Qg}^{(3)}(x)$; dashed line (blue): leading small-x term $\propto \ln(x)/x$ [Catani, Ciafaloni, Hautmann, 1990]; light blue region: estimates of [Kawamura et al., 2012]; gray region: estimates of [ABMP 2017]. Right panel: larger x region. Full line (red): $a_{Qg}^{(3)}(x)$; light blue region: estimates of [Kawamura et al., 2012] gray region: estimates of [ABMP 2017].

$\Delta a_{Qg}^{(3)}$

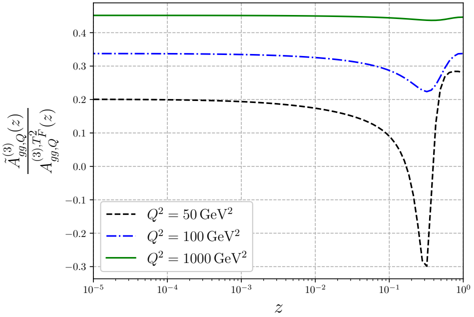




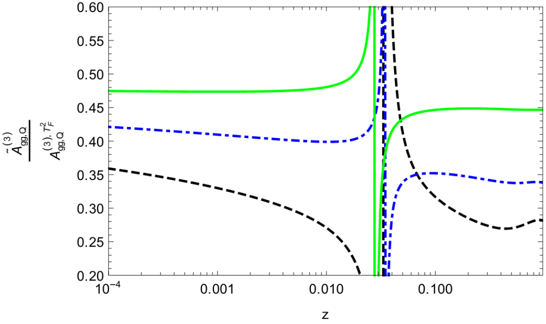
$\Delta a_{Qg}^{(3)}(x)$ as a function of x, rescaled by the factor $x(1-x)$. Left panel: full line (red): $\Delta a_{Qg}^{(3)}(x)$; dashed line (green): the small-x terms $\ln^k(x)$, $k \in \{1, \ldots, 5\}$; dotted line (blue): the large-x terms $\ln^l(1-x)$, $l \in \{1, \ldots, 4\}$. Right panel: larger x region. Full line (red): $\Delta a_{Qg}^{(3)}(x)$; dotted line (blue): the large-x terms $\ln^l(1-x)$, $l \in \{1, \ldots, 4\}$.

Two-mass Results: $\tilde{A}_{gg,Q}^{(3)}$

The two mass contributions over the whole T_F^2-contributions to the OME $\tilde{A}_{gg,Q}^{(3)}$:

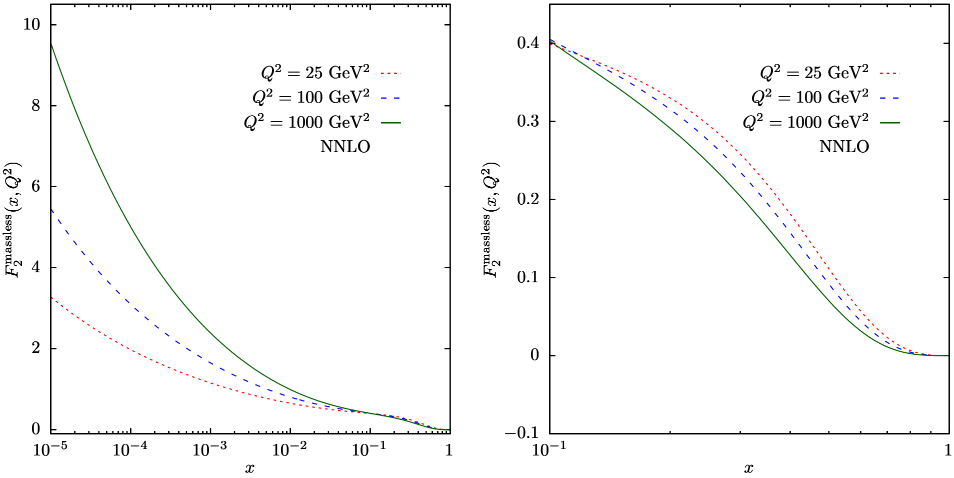


unpolarized



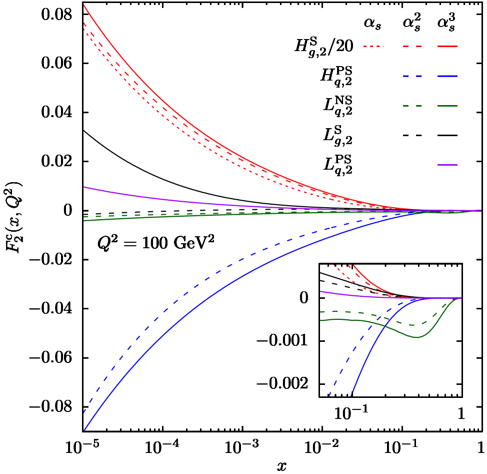
polarized

The massless contributions to F_2

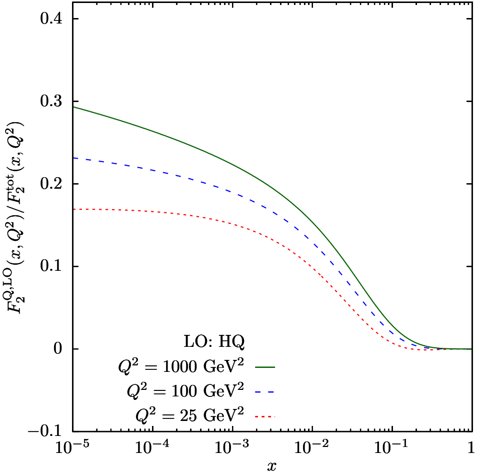


$N_F = 3$ massless quarks.

Single-mass contributions to $F_2^{c,b}$



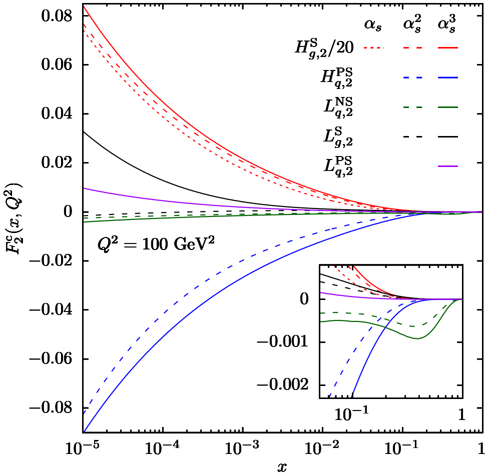
charm for $Q^2 = 100$ GeV^2.



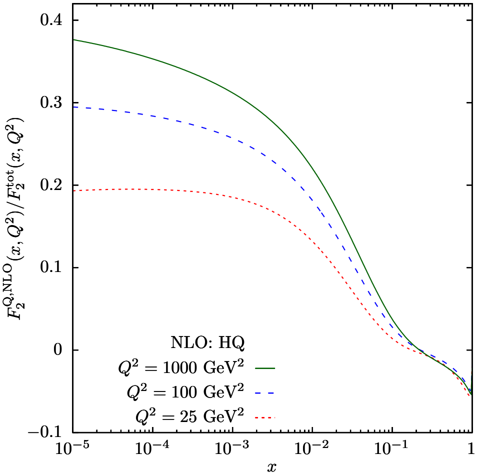
c and *b* single mass contributions

Allows to strongly reduce the current theory error on m_c.

Single-mass contributions to $F_2^{c,b}$



charm for $Q^2 = 100$ GeV^2.



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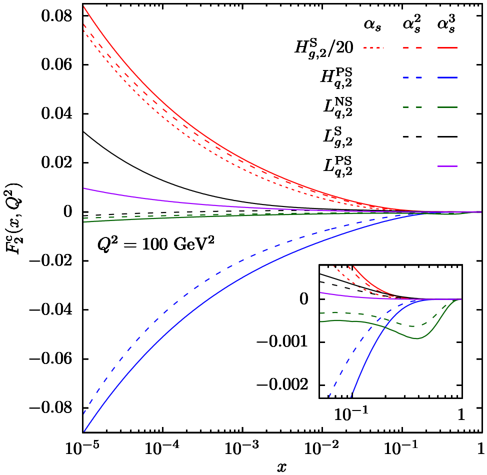
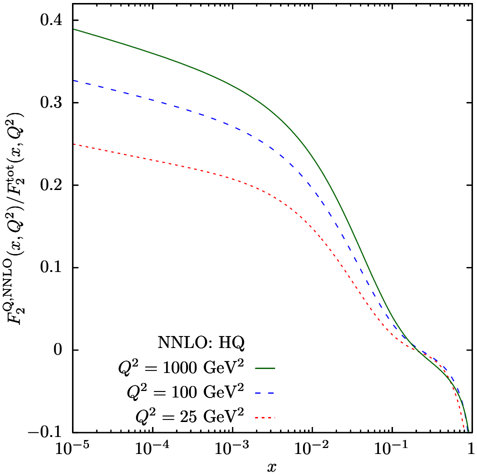
Single-mass contributions to $F_2^{c,b}$



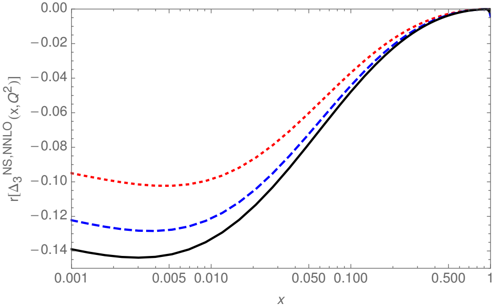
charm for $Q^2 = 100\ \mathrm{GeV}^2$.

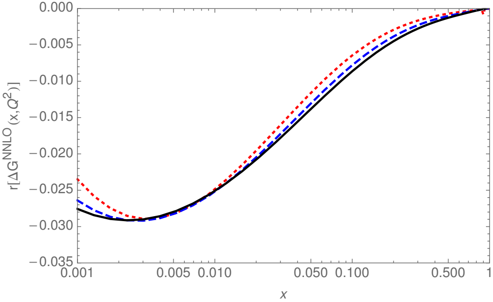


c and *b* single mass contributions

Allows to strongly reduce the current theory error on m_c.

Polarized PDF evolution in the Larin Scheme





[Dotted line: $Q^2 = 100$ GeV^2, dashed line: $Q^2 = 1000$ GeV^2, full line: $Q^2 = 10000$ GeV^2]

$$r(x, Q^2) = \frac{f^L(x, Q^2)}{f^M(x, Q^2)} - 1$$

The pdfs are necessary to match HO Larin-scheme calculations.

Conclusions

- All unpolarized and polarized single-mass OMEs and the associated massive Wilson coefficients for $Q^2 \gg m_Q^2$ have been calculated. The unpolarized and polarized massless three-loop Wilson coefficients were calculated and contribute to the present results.
- The calculation of all unpolarized and polarized two-mass OMEs, except for $(\Delta)A_{Qg}^{(3)}$, are finished and the remaining OMEs will be available very soon.
- Various new mathematical and technological methods were developed during the present project. They are available for use in further single- and two-mass calculations in other QFT projects.
- Very soon new precision analyses of the world DIS-data to measure $\alpha_s(M_Z)$ and m_c at higher precision can be carried out.
- Both the single- and two-mass VFNS at 3-loop order will be available in form of a numerical program, to be used e.g. in applications at hadron colliders.
- The results in the polarized case prepare the analysis of the precision data, which will be taken at the EIC starting at the end of this decade.
- For all sub-processes it turned out that the small x BFKL approaches fail to present the physical result due to quite a series of missing subleading terms, which substantially correct the LO behaviour. The correct description requires the full calculation.