

# LEPTOQUARK PAIR PRODUCTION IN $\gamma\gamma$ SCATTERING

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DESY

- 1) INTRODUCTION
- 2) LQ's IN  $e^+e^-$
- 3)  $\gamma\gamma \Rightarrow \phi\bar{\phi}$
- 4) QCD CORRECTIONS
- 5) CONCLUSION

# 1. Classification of Leptoquark States

B and L conserving

family-diagonal

BUCHMÜLLER,  
RÖCKL, WYLE

$SU(3)_c \times SU(2)_L \times U(1)_Y$  invariant couplings

'87

leptoquark ( $\Phi$ )	spin	$F$	colour	$T_3$	$Q_{em}$	$\lambda_L(lq)$	$\lambda_R(lq)$	$\lambda_L(\nu q)$
$S_1$	0	-2	$\bar{3}$	0	1/3	$g_{1L}$	$g_{1R}$	$-g_{1L}$
$\tilde{S}_1$	0	-2	$\bar{3}$	0	4/3	0	$\tilde{g}_{1R}$	0
$\tilde{S}_3$	0	-2	$\bar{3}$	+1	4/3	$-\sqrt{2}g_{3L}$	0	0
				0	1/3	$-g_{3L}$	0	$-g_{3L}$
$R_2$	0	0	3	1/2	5/3	$h_{2L}$	$h_{2R}$	0
				-1/2	2/3	0	$-h_{2R}$	$h_{2L}$
$\tilde{R}_2$	0	0	3	1/2	2/3	$h_{2L}$	0	0
				-1/2	-1/3	0	0	$\tilde{h}_{2L}$
$V_{2\mu}$	1	-2	$\bar{3}$	1/2	4/3	$g_{2L}$	$g_{2R}$	0
				-1/2	1/3	0	$g_{2R}$	$g_{2L}$
$\tilde{V}_{2\mu}$	1	-2	$\bar{3}$	1/2	1/3	$\tilde{g}_{2L}$	0	0
				-1/2	-2/3	0	0	$\tilde{g}_{2L}$
$U_{1\mu}$	1	0	3	0	2/3	$h_{1L}$	$h_{1R}$	$h_{1L}$
$\tilde{U}_{1\mu}$	1	0	3	0	5/3	0	$h_{1R}$	0
$\tilde{U}_{3\mu}$	1	0	3	+1	5/3	$\sqrt{2}h_{3L}$	0	0
				0	2/3	$-h_{3L}$	0	$h_{3L}$
				-1	-1/3	0	0	$\sqrt{2}h_{3L}$

$$\mathcal{L} = \mathcal{L}_{|F|=2}^f + \mathcal{L}_{F=0}^f + \mathcal{L}^{\gamma, Z, g}$$

$$\mathcal{L}^{\gamma, Z, g} = \sum_{\text{scalars}} [(D^\mu \Phi)^\dagger (D_\mu \Phi) - M^2 \Phi^\dagger \Phi] + \sum_{\text{vectors}} \left[ -\frac{1}{2} G_{\mu\nu}^a G^{\mu\nu} + M^2 \Phi^{\mu\dagger} \Phi_\mu \right]$$

$$D_\mu = \partial_\mu - ieQ^r A_\mu - ieQ^z Z_\mu - ig_s \frac{\lambda_a}{2} A_\mu^c$$

$$\begin{aligned} \mathcal{L}_{F=0}^f &= (h_{2L} \bar{u}_R l_L + h_{2R} \bar{q}_L i\tau_2 e_R) R_2 + \tilde{h}_{2L} \bar{d}_R l_L \tilde{R}_2 \\ &+ (h_{1L} \bar{q}_L \gamma^\mu l_L + h_{1R} \bar{d}_R \gamma^\mu e_R) U_{1\mu} \\ &+ \tilde{h}_{1R} \bar{u}_R \gamma^\mu e_R \tilde{U}_{1\mu} + h_{3L} \bar{q}_L \tilde{\tau} \gamma^\mu l_L \tilde{U}_{3\mu} + h.c. \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{|F|=2}^f &= (g_{1L} \bar{q}_L i\tau_2 l_L + g_{1R} \bar{u}_R^c e_R) S_1 \\ &+ \tilde{g}_{1R} \bar{d}_R^c e_R \tilde{S}_1 + g_{3L} \bar{q}_L^c i\tau_2 \tilde{\tau} l_L \tilde{S}_3 \\ &+ (g_{2L} \bar{d}_R^c \gamma^\mu l_L + g_{2R} \bar{q}_L^c \gamma^\mu e_R) V_{2\mu} \\ &+ \tilde{g}_{2L} \bar{u}_R^c \gamma^\mu l_L \tilde{V}_{2\mu} + h.c., \end{aligned}$$

## Decay Pattern for Pair Production

states	$l^+l^- + 2jets$	$l\nu + 2jets$	$\nu\bar{\nu} + 2jets$
$S_1 \quad U_1$	$\frac{4}{9} \quad 1 \quad \frac{1}{4}$	$\frac{4}{9} \quad 0 \quad \frac{1}{2}$	$\frac{1}{9} \quad 0 \quad \frac{1}{4}$
$R_2^{2/3} \quad V_2^{1/3}$	$\frac{1}{4} \quad 1 \quad 0$	$\frac{1}{2} \quad 0 \quad 0$	$\frac{1}{4} \quad 0 \quad 1$
$S_3^{1/3} \quad U_3^{2/3}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
$\tilde{S}_1 \quad S_3^{4/3} \quad R_2^{5/3} \quad \tilde{R}_2^{2/3}$ $V_2^{4/3} \quad \tilde{V}_2^{1/3} \quad \tilde{U}_1 \quad U_3^{5/3}$	1	0	0
$S_3^{-2/3} \quad \tilde{R}_2^{-1/3} \quad \tilde{V}_2^{-2/3} \quad U_3^{-1/3}$	0	0	1

Table 3: Branching ratios for final states arising from the decays of leptoquarks associated with the first ( $l = e$ ) and second ( $l = \mu$ ) family. The sequence of branching fractions given in the second and third row refers to the assumptions  $\lambda_L = \lambda_R$ ,  $\lambda_L = 0$ , and  $\lambda_R = 0$ , respectively.

# LINEAR COLLIDER:

$$\begin{array}{l}
 e^+ e^- \longrightarrow \phi \bar{\phi} \\
 \gamma \gamma \longrightarrow \phi \bar{\phi}
 \end{array}
 \left. \vphantom{\begin{array}{l} e^+ e^- \\ \gamma \gamma \end{array}} \right\}
 \begin{array}{l}
 \sigma \\
 \text{WIDELY INDEPENDENT} \\
 \text{OF } \lambda_{eq} ! \\
 \longrightarrow \text{GAUGE COUPLINGS}
 \end{array}$$

## 1) $e^+ e^-$ - ANNIHILATION:

- HEWETT/RIZZO, ZERWAS et al.
- JB, RÜCKL ; JB, BOOS, KRYUKOV.

MASS BOUNDS :  $m_\phi \lesssim \sqrt{s}/2$

- o LOSSES : BEAMSTRAHLUNG JB '93  
 QED  $O(\alpha)$  ,  $-\Delta\sigma$  : QED  $O(\alpha^2)$ .  
↑

### o ENHANCEMENT OF $\sigma$ :

QCD FS-CORR  $\frac{1}{\beta}$  JB'93

$\sqrt{s} = 500 \text{ GeV} \dots 1 \text{ TeV}$  nearly balanced,  
 i.e. losses + enhancement  $\simeq 0$ .

$\sigma_{\text{eff}} \simeq \sigma_{\text{Born}}$

## 2. $\gamma\gamma$ Scattering

$U_{em}(1)$  invariant Lagrangian:

$$\mathcal{L} = \mathcal{L}_s + \mathcal{L}_v \quad (1)$$

with

$$\mathcal{L}_s = \sum_{\text{scalars}} \left[ (D^\mu \Phi)^\dagger (D_\mu \Phi) - M_s^2 \Phi^\dagger \Phi \right] \quad (2)$$

and

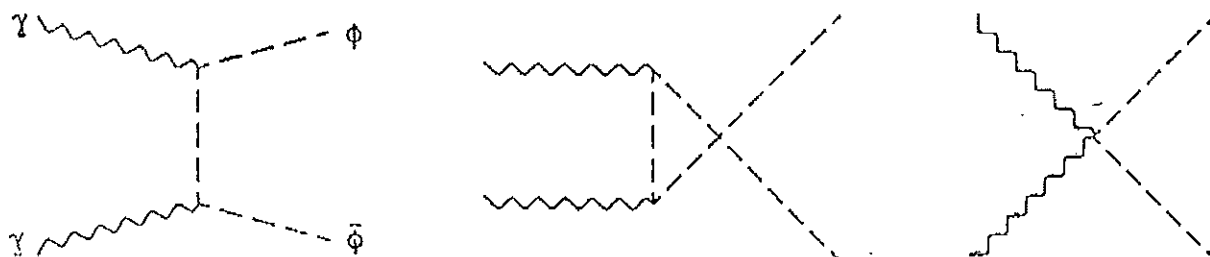
$$\mathcal{L}_v = \sum_{\text{vectors}} \left\{ -\frac{1}{2} G_{\mu\nu}^\dagger G^{\mu\nu} + M_v^2 \Phi_\mu^\dagger \Phi^\mu - ie \left[ (1 - \kappa_A) \Phi_\mu^\dagger \Phi_\nu F^{\mu\nu} + \frac{\lambda_A}{M_v^2} G_{\sigma\mu}^\dagger G_\nu^\mu F^{\nu\sigma} \right] \right\} \quad (3)$$

Field strength tensors:

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu, \\ G_{\mu\nu} &= D_\mu \Phi_\nu - D_\nu \Phi_\mu \end{aligned} \quad (4)$$

Covariant derivative:

$$D_\mu = \partial_\mu - ieQ_\gamma A_\mu \quad (5)$$



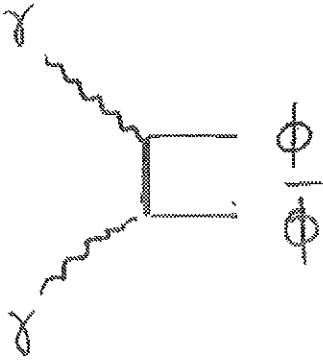
Anomalous couplings:  $\kappa_A$  and  $\lambda_A$

$\Updownarrow$

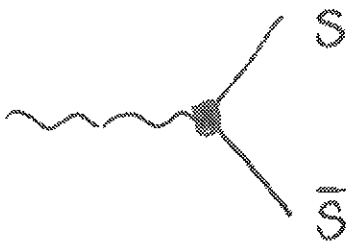
anomalous magnetic moment  $\mu_\Phi$  and electric quadrupole moment  $q_\Phi$  of leptoquarks:

$$\begin{aligned} \mu_{\Phi,A} &= \frac{eQ_\gamma}{2M_\Phi} (2 - \kappa_A + \lambda_A) \\ q_{\Phi,A} &= -\frac{eQ_\gamma}{M_\Phi^2} (1 - \kappa_A - \lambda_A) \end{aligned} \quad (6)$$

# WHY TO SEARCH FOR $\phi\bar{\phi}$ IN $\gamma\gamma$ -SCATTERING?



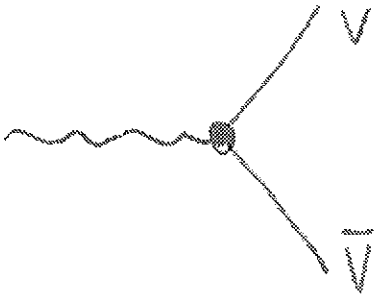
SCALARS & VECTORS



$$\sigma \sim Q_\phi^4$$

$$|Q_\phi| = \frac{1}{3} \dots \frac{5}{3} !$$

$$\sigma = O(1) \dots O(625).$$

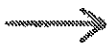


$$\sigma_V = \sigma_V(k_\gamma, \lambda_\gamma)$$

ANOMALOUS COUPLINGS.

# Sensitivity to Quantum Numbers

Process	LQ	Quantum Numbers
$e^+e^-$ Annihilation	S,V	$Q_\Phi^\gamma, Q_\Phi^Z, \lambda_{L,R}$
$\gamma\gamma$ Collider	S V	$Q_\Phi^\gamma$ $Q_\Phi^\gamma, \kappa_A, \lambda_A$
$e\gamma$ Collider	S,V	$\lambda_{L,R}, Q_\Phi^\gamma$
$ep$ Collider	single LQ pairs : S pairs : V	$\lambda_{L,R}$ $Q_\Phi^\gamma$ $Q_\Phi^\gamma, \kappa_A, \kappa_G, \lambda_A, \lambda_G$
$p\bar{p}$ Collider	V	$\kappa_G, \lambda_G$



$$\sigma \sim Q_{\Phi_{S,V}}^4$$

2. CROSS SECTIONS

BORN:

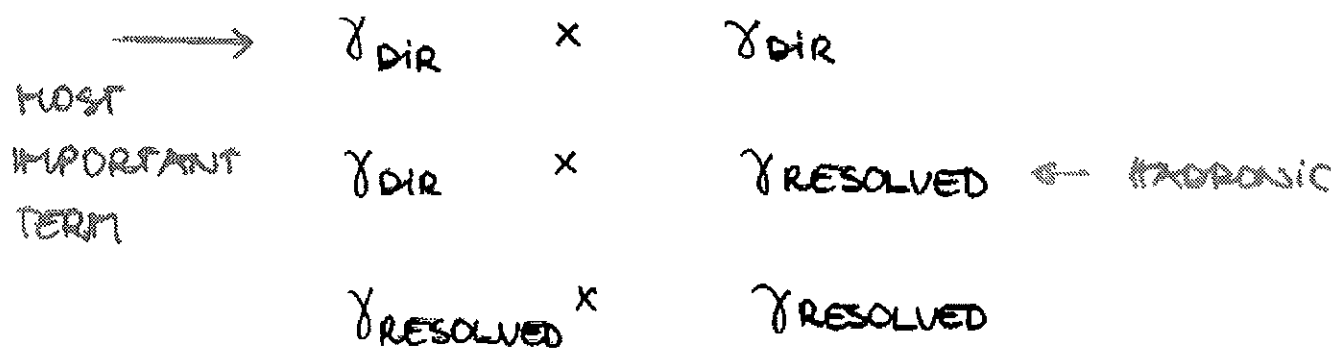
J. BLÜMUEIN, E. BOOS, NUCL. PHYS (P.S.) 37B (1994) 121

J. BLÜMUEIN, E. BOOS, A. KRYUKOV, Z. PHYS. C76 (1997) 13;

CODE: LQPAIR 1.0 : J. BLÜMUEIN, E. BOOS, A. KRYUKOV 199.

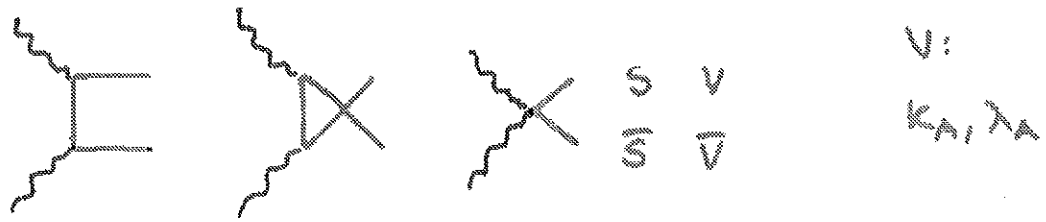
→ CONVOLUT WITH COMPTON LASER SPECTRA

THREE CONTRIBUTIONS:

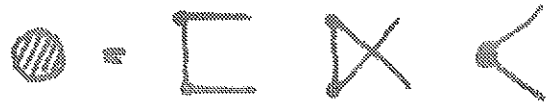
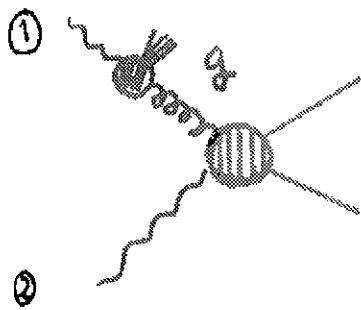




DIRECT TERMS :



DIRECT - RESOLVED CONTRIBUTIONS :

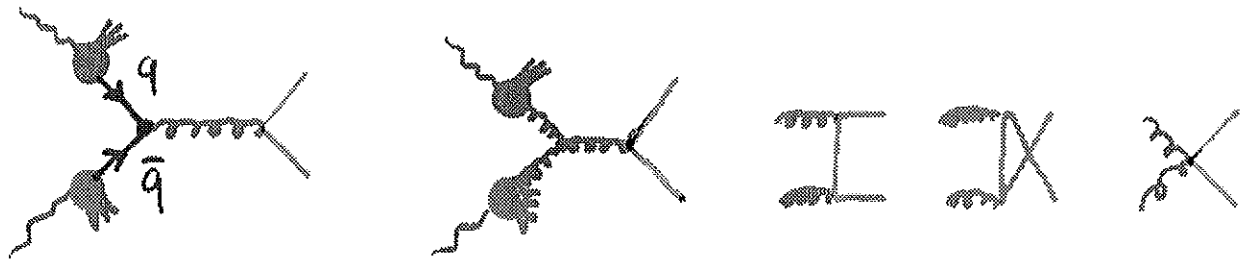


assume:  $\lambda_{qe} \ll 1$ .

+ ① ↔ ②.

V:  $\begin{matrix} \bar{1} & \bar{2} \\ K_A, K_B \\ \lambda_A, \lambda_B \end{matrix}$

RESOLVED - RESOLVED TERM :



V:  $K_B, \lambda_B$

$V$ : direct.

$$\begin{aligned}
\bar{F}_0 &= \beta \left( \frac{11}{2} - \frac{9}{4}\beta^2 + \frac{3}{4}\beta^4 \right) - \frac{3}{8} (1 - \beta^2 - \beta^4 + \beta^6) \ln \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_1 &= -8\beta - \frac{3}{2} (1 - \beta^2) \log \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_2 &= 3\beta + \frac{1}{4}\beta \frac{\hat{s}}{M_\Phi^2} + \left( \frac{7}{2} - 2\beta^2 \right) \log \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_3 &= -\frac{1}{4}\beta \frac{\hat{s}}{M_\Phi^2} + \left( -2 + \frac{3}{4}\beta^2 \right) \log \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_4 &= -\frac{1}{96}\beta + \frac{5}{48}\beta \frac{\hat{s}}{M_\Phi^2} + \frac{4 - \beta^2}{16} \log \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_5 &= -(1 - \beta^2) \log \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_6 &= -\frac{1}{6}\beta + \frac{17}{12}\beta \frac{\hat{s}}{M_\Phi^2} + \left( -3 - \frac{\beta^2}{2} + \frac{1}{2} \frac{\hat{s}}{M_\Phi^2} \right) \log \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_7 &= -\beta + \frac{11}{6}\beta \frac{\hat{s}}{M_\Phi^2} - \frac{1}{3}\beta \frac{\hat{s}^2}{M_\Phi^4} - \frac{3 + \beta^2}{4} \log \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_8 &= -\frac{1}{96}\beta + \frac{59}{80}\beta \frac{\hat{s}}{M_\Phi^2} - \frac{113}{320}\beta \frac{\hat{s}^2}{M_\Phi^4} + \frac{43}{960}\beta \frac{\hat{s}^3}{M_\Phi^6} + \left( -\frac{1}{2} - \frac{1}{16}\beta^2 + \frac{1}{8} \frac{\hat{s}}{M_\Phi^2} \right) \log \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_9 &= 2\beta + (2 + \beta^2) \log \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_{10} &= 2\beta - \frac{7}{3}\beta \frac{\hat{s}}{M_\Phi^2} + \left( 3 + \frac{5}{4}\beta^2 - \frac{1}{2} \frac{\hat{s}}{M_\Phi^2} \right) \log \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_{11} &= \frac{1}{24}\beta - \frac{59}{48}\beta \frac{\hat{s}}{M_\Phi^2} + \frac{5}{32}\beta \frac{\hat{s}^2}{M_\Phi^4} + \frac{5 + \beta^2}{4} \log \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_{12} &= -\beta + \frac{1}{2}\beta \frac{\hat{s}}{M_\Phi^2} + \left( -\frac{1}{4} - \frac{7}{4}\beta^2 \right) \log \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_{13} &= \frac{1}{24}\beta + \frac{1}{3}\beta \frac{\hat{s}}{M_\Phi^2} - \frac{1}{4} (1 - \beta^2) \log \left| \frac{1+\beta}{1-\beta} \right| \\
\bar{F}_{14} &= -\frac{1}{16}\beta + \frac{11}{96}\beta \frac{\hat{s}}{M_\Phi^2} + \frac{17}{192}\beta \frac{\hat{s}^2}{M_\Phi^4} + \left( \frac{1}{8} \frac{\hat{s}}{M_\Phi^2} - \frac{3}{4} - \frac{3}{8}\beta^2 \right) \log \left| \frac{1+\beta}{1-\beta} \right| \tag{12}
\end{aligned}$$

Tree-level unitarity:

$$\lambda_A = 0$$

$$\kappa_A^2 \left[ \left( \kappa_A - \frac{6}{5} \right)^2 + \frac{24}{25} \right] = 0. \tag{13}$$

Since  $\kappa_A, \lambda_A$  real  $\rightarrow \kappa_A \equiv \lambda_A \equiv 0$ .

# V : RESOLVED - DIRECT.

3018 J. Blumlein, E. Boos & A. Pukhov

LepLoganik Pair Production at ep Colliders 3019

$$\begin{aligned}
 F_{18} &= 2(5 - \beta^2 \cos^2 \theta) - \frac{5}{M_2^2} \frac{11 - 15\beta^2 \cos^2 \theta + 4\beta^4 \cos^4 \theta}{4} \\
 &\quad - \frac{s^2}{M_2^2} \frac{(1 - \beta^2 \cos^2 \theta)^2}{4}, \\
 F_{19} &= 3 - \beta^2 \cos^2 \theta - \frac{5}{M_2^2} \frac{7 - 8\beta^2 \cos^2 \theta + \beta^4 \cos^4 \theta}{4} \\
 &\quad + \frac{5^2}{M_2^4} \frac{11 - 13\beta^2 \cos^2 \theta + \beta^4 \cos^4 \theta + \beta^6 \cos^6 \theta}{32} \\
 &\quad + \frac{s^2}{M_2^2} \frac{5 - 7\beta^2 \cos^2 \theta - \beta^4 \cos^4 \theta + 3\beta^6 \cos^6 \theta}{128}, \\
 F_{20} &= \frac{3 - \beta^2 \cos^2 \theta}{2} + \frac{5}{M_2^2} \frac{(1 - \beta^2 \cos^2 \theta)^2}{8} \\
 &\quad + \frac{s^2}{M_2^4} \frac{11 - 23\beta^2 \cos^2 \theta + 13\beta^4 \cos^4 \theta - \beta^6 \cos^6 \theta}{64}.
 \end{aligned} \tag{A.1}$$

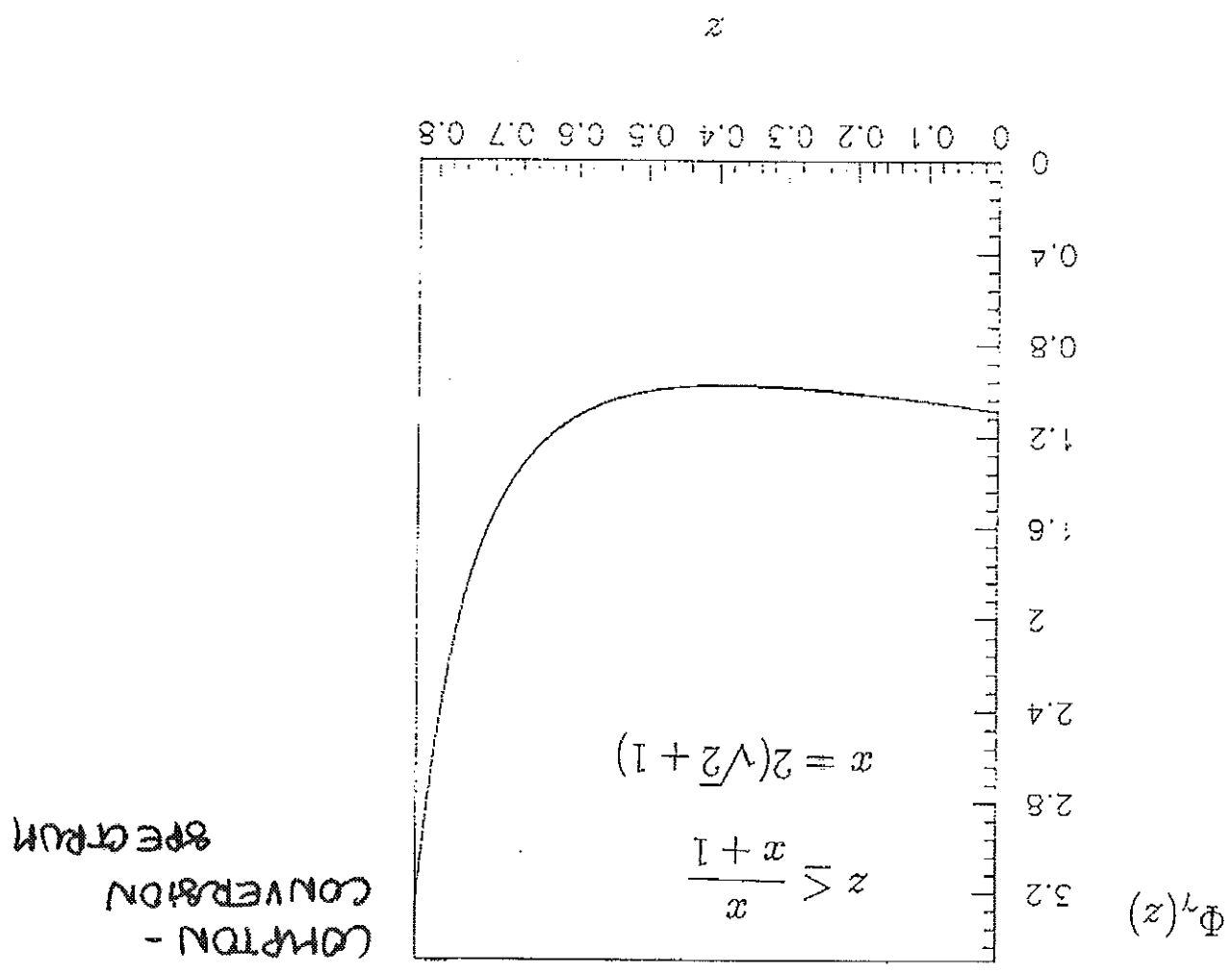
The functions  $\bar{F}_i(\beta, \beta)$ , which describe the different contributions to the integrated cross-section (13), are:

$$\begin{aligned}
 \bar{F}_0 &= \beta \left( \frac{11}{2} - \frac{9}{4} \beta^2 + \frac{3}{4} \beta^4 \right) - \frac{3}{8} (1 - \beta^2 - \beta^4 + \beta^6) \ln \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_1 &= -4\beta - \frac{3}{4} (1 - \beta^2) \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_2 &= \frac{1}{16} \beta \frac{s^2}{M_2^2} + \frac{3 - \beta^2}{4} \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_3 &= 3\beta + \frac{1}{8} \beta \frac{s^2}{M_2^2} + \left( 2 - \frac{3}{2} \beta^2 \right) \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_4 &= -\frac{1}{8} \beta \frac{s^2}{M_2^2} + \left( -1 + \frac{3}{8} \beta^2 \right) \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_5 &= -\frac{1}{96} \beta + \frac{5}{48} \beta \frac{s^2}{M_2^2} + \frac{4 - \beta^2}{16} \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_6 &= -\frac{1}{5} (1 - \beta^2) \log \left| \frac{1 + \beta}{1 - \beta} \right|.
 \end{aligned}$$

$$\begin{aligned}
 \bar{F}_7 &= \frac{7}{12} \beta \frac{s^2}{M_2^2} + \frac{1}{24} \beta \frac{s^2}{M_2^4} - \frac{5 + \beta^2}{4} \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_8 &= -\frac{1}{6} \beta + \frac{1}{4} \beta \frac{s^2}{M_2^2} - \frac{1}{12} \beta \frac{s^2}{M_2^4} + \left( -\frac{1}{2} + \frac{1}{2} \beta^2 \right) \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_9 &= -\frac{1}{2} \beta + \frac{11}{12} \beta \frac{s^2}{M_2^2} - \frac{1}{6} \beta \frac{s^2}{M_2^4} - \frac{3 + \beta^2}{8} \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_{10} &= -\frac{1}{96} \beta + \frac{59}{80} \beta \frac{s^2}{M_2^2} - \frac{113}{320} \beta \frac{s^2}{M_2^4} + \frac{43}{960} \beta \frac{s^2}{M_2^6} \\
 &\quad + \left( -\frac{1}{2} - \frac{1}{16} \beta^2 + \frac{1}{8} \beta^4 \right) \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_{11} &= \frac{1}{2} (1 + \beta^2) \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_{12} &= \beta + \frac{1}{2} \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_{13} &= \beta - \frac{5}{12} \beta \frac{s^2}{M_2^2} + \frac{1}{24} \beta \frac{s^2}{M_2^4} + \left[ -\frac{1}{4} \beta \frac{s^2}{M_2^2} + \left( \frac{3}{8} + \frac{1}{4} \beta^2 \right) \right] \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_{14} &= -\frac{11}{24} \beta \frac{s^2}{M_2^2} - \frac{1}{24} \beta \frac{s^2}{M_2^4} + \frac{9 + 3\beta^2}{8} \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_{15} &= \frac{1}{48} \beta - \frac{59}{96} \beta \frac{s^2}{M_2^2} + \frac{5}{64} \beta \frac{s^2}{M_2^4} + \frac{5 + \beta^2}{8} \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_{16} &= -\frac{1}{2} \beta - \frac{1}{8} \beta^2 \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_{17} &= -\frac{1}{96} \beta + \frac{1}{48} \beta \frac{s^2}{M_2^2} + \frac{1}{48} \beta \frac{s^2}{M_2^4} - \frac{2 + \beta^2}{16} \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_{18} &= -\frac{1}{4} \beta \frac{s^2}{M_2^2} - \frac{1 - 6\beta^2}{8} \log \left| \frac{1 + \beta}{1 - \beta} \right|, \\
 \bar{F}_{19} &= -\frac{1}{24} \beta + \frac{7}{96} \beta \frac{s^2}{M_2^2} + \frac{3}{64} \beta \frac{s^2}{M_2^4} + \left[ \frac{1}{8} \beta \frac{s^2}{M_2^2} - \frac{2 + \beta^2}{4} \right] \log \left| \frac{1 + \beta}{1 - \beta} \right|,
 \end{aligned} \tag{A.2}$$

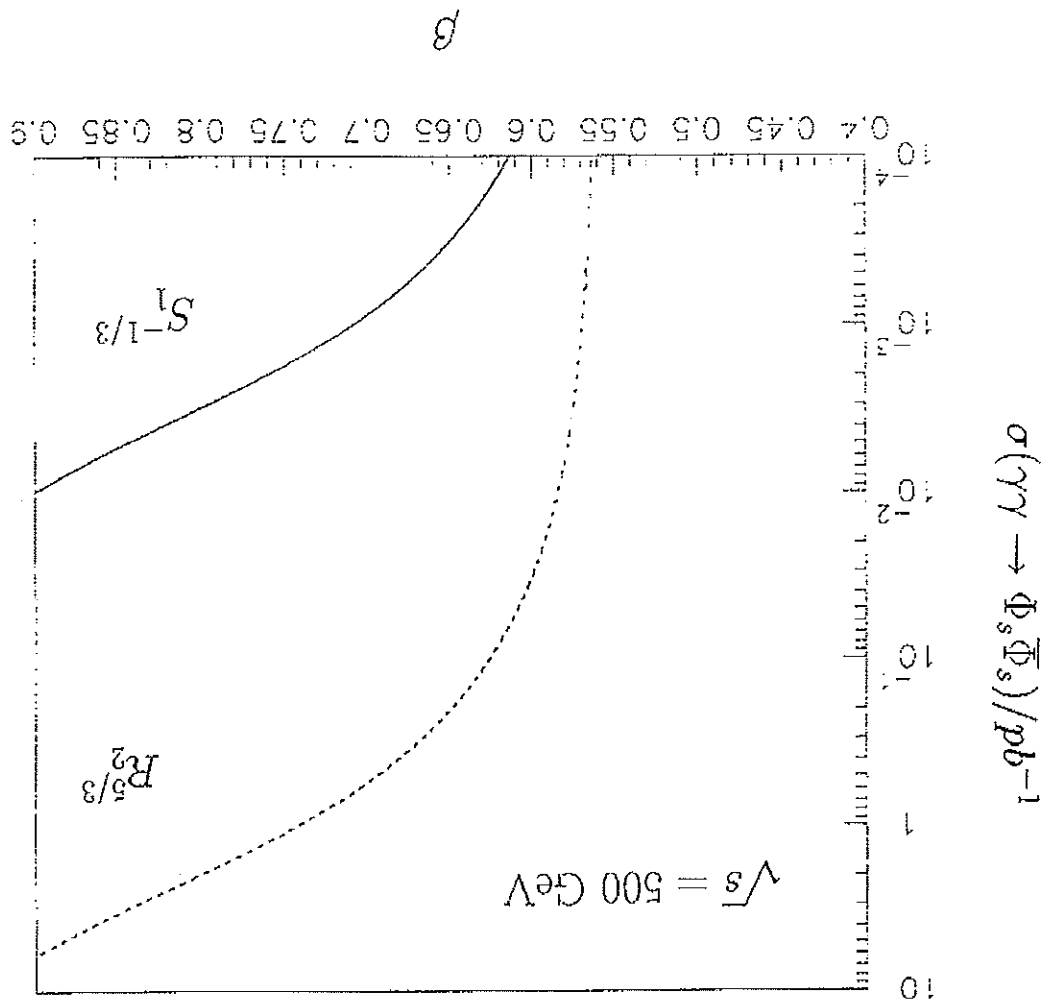
$$\Phi_\gamma(z) = \frac{1}{1-z} \left[ 1 - z + \frac{1-z}{1} - \frac{x(1-z)}{4z} + \frac{x^2(1-z)^2}{4z^2} \right] N(x)$$

$$N(x) = \frac{16 + 32x + 18x^2 + x^3}{x^2 - 4x - 8} + \frac{2x(1+x)^2}{x^2} \ln(1+x)$$



z

$\Phi_\gamma(z)$



$$\sigma = \int_0^{z_{\max}} dz_1 \int_0^{z_{\max}} dz_2 \Phi_\gamma(z_1) \Phi_\gamma(z_2) \theta(s) \theta(s - 4M_\Phi^2)$$

$$\sigma_{\text{scalar}}(s) = \frac{\pi \alpha^2}{s} Q_\Phi^4 \left\{ 2(2 - \beta^2) \beta - (1 - \beta^4) \ln \left| \frac{1 + \beta}{1 - \beta} \right| \right\}$$