The QCD Structure of the Non–Forward Compton Amplitude

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1. Introduction

Deeply Virtual Compton Scattering: A New Test Ground for QCD

- Scaling Violations: Non-Forward
- New Evolution Equations
- Generalization of the Light Cone Expansion
- New Integral Relations Specific to the Non-Forward Case

Generalization of the CALLAN–GROSS and WANDZURA–WILCZEK Relations to the Amplitude Level

Conceptional Problem: Non-Forward Light Cone Expansion & Current Conservation

How to extract the Twist–2 Contributions ?

How to resum the Spin Towers ? $\rightarrow$ Also a Problem for the Higher Twist Operators for Forward-Scattering!

2.1 Operator Structure

$$\hat{T}_{\mu\nu}(x) = iRT\left[J_\mu\left(\frac{x}{2}\right)J_\nu\left(-\frac{x}{2}\right)S\right]$$

$$\hat{T}^{\mu\nu}(x) = -e^2\frac{\tilde{x}^\lambda}{2\pi^2(x^2-i\epsilon)^2}RT\left[\overline{\psi}\left(\frac{\tilde{x}}{2}\right)\gamma^\mu\gamma^\lambda\gamma^\nu\psi\left(-\frac{\tilde{x}}{2}\right)-\overline{\psi}\left(-\frac{\tilde{x}}{2}\right)\gamma^\mu\gamma^\lambda\gamma^\nu\psi\left(\frac{\tilde{x}}{2}\right)\right.$$

$$\tilde{x} = x+\frac{\zeta}{\zeta^2}\left[\sqrt{x.\zeta^2-x^2\zeta^2}-x.\zeta\right]$$

$\tilde{x}.\tilde{x} \equiv 0.$

$$\hat{T}_{\mu\nu}(x) = -e^2\frac{\tilde{x}^\lambda}{i\pi^2(x^2-i\epsilon)^2}\left[S_{\alpha\mu\lambda\nu}O^\alpha\left(\frac{\tilde{x}}{2},-\frac{\tilde{x}}{2}\right)+i\varepsilon_{\mu\lambda\nu\sigma}O_5^\alpha\left(\frac{\tilde{x}}{2},-\frac{\tilde{x}}{2}\right)\right],$$

$$S_{\alpha\mu\lambda\nu} = g_{\alpha\mu}g_{\lambda\nu}+g_{\lambda\mu}g_{\alpha\nu}-g_{\mu\nu}g_{\lambda\alpha}\ .$$

VECTOR OPERATORS

$$O^\alpha\left(\frac{\tilde{x}}{2},-\frac{\tilde{x}}{2}\right) = \frac{i}{2}\left[\overline{\psi}\left(\frac{\tilde{x}}{2}\right)\gamma^\alpha\psi\left(-\frac{\tilde{x}}{2}\right)-\overline{\psi}\left(-\frac{\tilde{x}}{2}\right)\gamma^\alpha\psi\left(\frac{\tilde{x}}{2}\right)\right], \quad \text{UNPOL.}$$

$$O_5^\alpha\left(\frac{\tilde{x}}{2},-\frac{\tilde{x}}{2}\right) = \frac{i}{2}\left[\overline{\psi}\left(\frac{\tilde{x}}{2}\right)\gamma_5\gamma^\alpha\psi\left(-\frac{\tilde{x}}{2}\right)+\overline{\psi}\left(-\frac{\tilde{x}}{2}\right)\gamma_5\gamma^\alpha\psi\left(\frac{\tilde{x}}{2}\right)\right] \quad \text{POL.}$$

SCALAR OPERATORS

$$O\left(\frac{\tilde{x}}{2},-\frac{\tilde{x}}{2}\right) = \tilde{x}_\alpha O^\alpha\left(\frac{\tilde{x}}{2},-\frac{\tilde{x}}{2}\right)$$

$$O_5\left(\frac{\tilde{x}}{2},-\frac{\tilde{x}}{2}\right) = \tilde{x}_\alpha O_5^\alpha\left(\frac{\tilde{x}}{2},-\frac{\tilde{x}}{2}\right)$$

$$\Box O_{(5)}^{q,\,\mathrm{traceless}}(-\kappa x, \kappa x) = 0 \ .$$

HARMONIC FCT.

$$\begin{aligned} O_\sigma^{q,\,\mathrm{twist2}}(-\kappa\tilde{x}, \kappa\tilde{x}) &= \int_0^1 d\tau\, \partial_\sigma\, O^q_{\mathrm{traceless}}(-\kappa\tau x, \kappa\tau x)|_{x\to\tilde{x}} \\ &= \int_0^1 d\tau \left[\partial_\sigma + \tfrac{1}{2}(\ln\tau)x_\sigma\Box\right] O^q(-\kappa\tau x, \kappa\tau x)|_{x=\tilde{x}} \end{aligned}$$

$$\partial^\sigma O_\sigma^{q,\,\mathrm{traceless}}(-\kappa x, \kappa x) = 0 \ , \quad \Box O_\sigma^{q,\,\mathrm{traceless}}(-\kappa x, \kappa x) = 0 \ .$$

→ 3.

2.2 Operator Matrix Elements

$$e^2\left\langle p_2, S_2\left|O\left(\frac{x}{2}, -\frac{x}{2}\right)\right|p_1, S_1\right\rangle$$

$$\begin{aligned} &= i\,\overline{u}(p_2, S_2)\gamma x u(p_1, S_1)\int Dz e^{-ixp_z/2} f(z_1, z_2, p_i p_j x^2, p_i p_j, \mu_R^2) \\ &+ i\,\overline{u}(p_2, S_2)x\sigma p_- u(p_1, S_1)\int Dz e^{-ixp_z/2} g(z_1, z_2, p_i p_j x^2, p_i p_j, \mu_R^2) \end{aligned}$$

↑
'NON – FORWARD "PARTONIC" FUNCT.'

$$\Box e^2\left\langle p_2, S_2\left|O\left(\frac{x}{2}, -\frac{x}{2}\right)\right|p_1, S_1\right\rangle|_{x\to\tilde{x}} \simeq 0 \ .$$

for $p_i \cdot p_j \approx 0$.

3. Non–Forward Anomalous Dimensions

Non–local Operators ≡ Taylor Summed-up Local Operators

$$
\begin{aligned}
O^{\rm NS}(\kappa_1,\kappa_2) &= \tilde{x}^\mu\,\frac{i}{2}\left[\overline{\psi}(\kappa_1\tilde{x})\lambda_f\gamma_\mu\psi(\kappa_2\tilde{x}) - \overline{\psi}(\kappa_2\tilde{x})\lambda_f\gamma_\mu\psi(\kappa_1\tilde{x})\right]\\
O_5^{\rm NS}(\kappa_1,\kappa_2) &= \tilde{x}^\mu\,\frac{i}{2}\left[\overline{\psi}(\kappa_1\tilde{x})\lambda_f\gamma_5\gamma_\mu\psi(\kappa_2\tilde{x}) + \overline{\psi}(\kappa_2\tilde{x})\lambda_f\gamma_5\gamma_\mu\psi(\kappa_1\tilde{x})\right]\\
O^q(\kappa_1,\kappa_2) &= \tilde{x}^\mu\,\frac{i}{2}\left[\overline{\psi}(\kappa_1\tilde{x})\gamma_\mu\psi(\kappa_2\tilde{x}) - \overline{\psi}(\kappa_2\tilde{x})\gamma_\mu\psi(\kappa_1\tilde{x})\right]\\
O^G(\kappa_1,\kappa_2) &= \tilde{x}^\mu\,\tilde{x}^\nu\,\frac{1}{2}\left[F_\mu^{a\,\rho}(\kappa_1\tilde{x})F_{\nu\rho}^a(\kappa_2\tilde{x}) + F_\mu^{a\,\rho}(\kappa_2\tilde{x})F_{\nu\rho}^a(\kappa_1\tilde{x})\right]\\
O_5^q(\kappa_1,\kappa_2) &= \tilde{x}^\mu\,\frac{i}{2}\left[\overline{\psi}(\kappa_1\tilde{x})\gamma_5\gamma_\mu\psi(\kappa_2\tilde{x}) + \overline{\psi}(\kappa_2\tilde{x})\gamma_5\gamma_\mu\psi(\kappa_1\tilde{x})\right]\\
O_5^G(\kappa_1,\kappa_2) &= \tilde{x}^\mu\,\tilde{x}^\nu\,\frac{1}{2}\left[F_\mu^{a\,\rho}(\kappa_1\tilde{x})\tilde{F}_{\nu\rho}^a(\kappa_2\tilde{x}) - F_\mu^{a\,\rho}(\kappa_2\tilde{x})\tilde{F}_{\nu\rho}^a(\kappa_1\tilde{x})\right]
\end{aligned}
$$

Renormalization Group Equation:

$$
\mu^2\frac{d}{d\mu^2}O^A(\kappa_1\tilde{x},\kappa_2\tilde{x};\mu^2) = \int_{\kappa_2}^{\kappa_1} d\kappa_1' d\kappa_2' \gamma^{AB}(\kappa_1,\kappa_2,\kappa_1',\kappa_2';\mu^2)O^B(\kappa_1'\tilde{x},\kappa_2'\tilde{x};\mu^2)\ .
$$

Argument-relations of the anomalous dimension

$$
\begin{aligned}
\gamma^{AB}(\kappa_1,\kappa_2;\kappa_1',\kappa_2') &= \gamma^{AB}(\kappa_1-\kappa,\kappa_2-\kappa;\kappa_1'-\kappa,\kappa_2'-\kappa)\\
&= \lambda^{d_{AB}}\gamma^{AB}(\lambda\kappa_1,\lambda\kappa_2;\lambda\kappa_1',\lambda\kappa_2')\ ,
\end{aligned}
$$

$$
d_{AB} = 2 + d_A - d_B,
$$

$$
d_q = 1 \quad \text{and} \quad d_G = 2\ .
$$

$$
\begin{aligned}
(\kappa_2-\kappa_1)^{d_{AB}}\gamma^{AB}(\kappa_1,\kappa_2;\kappa_1',\kappa_2') &= \gamma^{AB}(0,1;\alpha_1,1-\alpha_2)\\
&\equiv \widehat{K}^{AB}(\alpha_1,\alpha_2),\\
4(\kappa_-)^{d_{AB}}\gamma^{AB}(\kappa_1,\kappa_2;\kappa_1',\kappa_2') &= 4\gamma^{AB}(-1,+1;w_1,w_2)\\
&\equiv \widetilde{K}^{AB}(w_1-w_2,w_1+w_2)
\end{aligned}
$$

$$
\alpha_1=\frac{\kappa_1'-\kappa_1}{\kappa_2-\kappa_1}, \qquad -\alpha_2=\frac{\kappa_2'-\kappa_2}{\kappa_2-\kappa_1},
$$

$$
w_1=\alpha_1-\alpha_2=\frac{\kappa'_+-\kappa_+}{\kappa_-}, \qquad w_2=1-\alpha_1-\alpha_2=\frac{\kappa'_-}{\kappa_-},
$$

Evolution Equations for Operators:

$$
\begin{aligned}
\mu^2\frac{d}{d\mu^2}O^A(\kappa_1,\kappa_2) &= \int D\alpha\,(\kappa_2-\kappa_1)^{d_B-d_A}\widehat{K}^{AB}(\alpha_1,\alpha_2)O^B(\kappa_1',\kappa_2'),\\
\mu^2\frac{d}{d\mu^2}O^A(\kappa_1,\kappa_2) &= \int Dw\,(\kappa_-)^{d_B-d_A}\widetilde{K}^{AB}(w_1,w_2)O^B(\kappa_1',\kappa_2')\\
&= \int_0^1 dw_2 \int_{-1+w_2}^{1-w_2} dw_1\,(\kappa_-)^{d_B-d_A}\widetilde{K}^{AB}_{\rm sym}(w_1,w_2)\\
&\quad \times O^B(\kappa_1',\kappa_2'),
\end{aligned}
$$

$$
\widetilde{K}^{AB}_{\rm sym}(w_1,w_2) = \frac{1}{2}\left[\widetilde{K}^{AB}_0(w_1,w_2)+(-1)^{d_B}\widetilde{K}^{AB}_0(w_1,-w_2)\right].
$$

Spin-Towers : Two-fold Moment Expansion

$$\mu^2 \frac{d}{d\mu^2} O^A_{n_1 n_2} = \sum_{n_1', n_2'} \gamma^{AB}_{n_1,n_2;n_1',n_2'} O^B_{n_1' n_2'} \ ,$$

$$\gamma^{AB}_{n_1,n_2;n_1',n_2'} = \frac{\partial^{n_1}}{\partial \kappa_1^{n_1}} \frac{\partial^{n_2}}{\partial \kappa_2^{n_2}} \int_{\kappa_2}^{\kappa_1} d\kappa_1' \int_{\kappa_2}^{\kappa_1} d\kappa_2' \frac{(\kappa_1')^{n_1'}}{n_1'!} \frac{(\kappa_2')^{n_2'}}{n_2'!} \times \gamma^{AB}(\kappa_1, \kappa_2; \kappa_1', \kappa_2')_{\kappa_1=\kappa_2=0} \quad . \qquad (2)$$

$$\gamma^{qq}_{nn'} = \binom{n}{n'} \sigma^{(-)}_{n\,n'} \int_0^1 dw_2 w_2^{n'} \left\{ 2 \int_0^{1-w_2} dw_1\, w_1^{n-n'}\, \widetilde{K}_0^{qq}(w_1, w_2) \right\},$$

$$\gamma^{qG}_{nn'} = n \binom{n-1}{n'-1} \sigma^{(-)}_{n\,n'} \int_0^1 dw_2 w_2^{n'-1} \left\{ 2 \int_0^{1-w_2} dw_1\, w_1^{n-n'}\, \widetilde{K}_0^{qG}(w_1, w_2) \right\}$$

$$\gamma^{Gq}_{nn'} = \frac{1}{n} \binom{n}{n'} \sigma^{(-)}_{n\,n'} \int_0^1 dw_2 w_2^{n'} \left\{ 2 \int_0^{1-w_2} dw_1\, w_1^{n-n'}\, \widetilde{K}_0^{Gq}(w_1, w_2) \right\},$$

$$\gamma^{GG}_{nn'} = \binom{n-1}{n'-1} \sigma^{(-)}_{n\,n'} \int_0^1 dw_2 w_2^{n'-1} \left\{ 2 \int_0^{1-w_2} dw_1\, w_1^{n-n'}\, \widetilde{K}_0^{GG}(w_1, w_2) \right\},$$

with

$$\sigma^{(\pm)}_{n\,n'} = \frac{1}{4}\left(1 + (-1)^{n-n'}\right)\left(1 \pm (-1)^{n'-2d_B}\right)$$
$$= \frac{1}{4}(1 \pm (-1)^n)(1 \pm (-1)^{n'})$$

1) Unpolarized anomalous dimensions

$$
\begin{aligned}
\widehat{K}_0^{qq}(\alpha_1,\alpha_2) &= C_F\left\{1-\delta(\alpha_1)-\delta(\alpha_2)+\delta(\alpha_1)\left[\frac{1}{\alpha_2}\right]_+ \right.\\
&\left. +\delta(\alpha_2)\left[\frac{1}{\alpha_1}\right]_+ +\frac{3}{2}\delta(\alpha_1)\delta(\alpha_2)\right\},\\
\widehat{K}_0^{qG}(\alpha_1,\alpha_2) &= -2N_fT_R\left\{1-\alpha_1-\alpha_2+4\alpha_1\alpha_2\right\},\\
\widehat{K}_0^{Gq}(\alpha_1,\alpha_2) &= -C_F\left\{\delta(\alpha_1)\delta(\alpha_2)+2\right\},\\
\widehat{K}_0^{GG}(\alpha_1,\alpha_2) &= C_A\left\{4(1-\alpha_1-\alpha_2)+12\alpha_1\alpha_2\right.\\
&+\delta(\alpha_1)\left(\left[\frac{1}{\alpha_2}\right]_+ -2+\alpha_2\right)\\
&\left.+\delta(\alpha_2)\left(\left[\frac{1}{\alpha_1}\right]_+ -2+\alpha_1\right)\right\}+\frac{1}{2}\beta_0\,\delta(\alpha_1)\delta(\alpha_2),
\end{aligned}
$$

where $C_F=(N_c^2-1)/2N_c\equiv 4/3$, $T_R=1/2$, $C_A=N_c\equiv 3$, and the β–function in leading order, $\beta_0=(11C_A-4T_RN_f)/3$.

$$
\int_0^1 dx\,[f(x,y)]_+\,\varphi(x)=\int_0^1 dx\,f(x,y)\,[\varphi(x)-\varphi(y)],
$$

if the singularity of f is of the type $\sim 1/(x-y)$. 2) Polarized anomalous dimensions

$$
\begin{aligned}
\Delta\widehat{K}_0^{qq}(\alpha_1,\alpha_2) &= \widehat{K}_0^{qq}(\alpha_1,\alpha_2),\\
\Delta\widehat{K}_0^{qG}(\alpha_1,\alpha_2) &= -2N_fT_R\left\{1-\alpha_1-\alpha_2\right\},\\
\Delta\widehat{K}_0^{Gq}(\alpha_1,\alpha_2) &= -C_F\left\{\delta(\alpha_1)\delta(\alpha_2)-2\right\},\\
\Delta\widehat{K}_0^{GG}(\alpha_1,\alpha_2) &= \widehat{K}_0^{GG}(\alpha_1,\alpha_2)-12C_A\alpha_1\alpha_2
\end{aligned}
$$

1) Unpolarized anomalous dimensions:

$$\gamma^{qq}_{nn'} = C_F\left\{\left[\frac{1}{2} - \frac{1}{(n+1)(n+2)} + 2\sum_{j=2}^{n+1}\frac{1}{j}\right]\delta_{nn'} - \left[\frac{1}{(n+1)(n+2)} + \frac{2}{n-n'}\frac{n'+1}{n+1}\right]\theta_{nn'}\right\}$$

$$\gamma^{qG}_{nn'} = -N_f T\,\frac{1}{(n+1)(n+2)(n+3)}\left[(n^2+3n+4) - (n-n')(n+1)\right]$$

$$\gamma^{Gq}_{nn'} = -C_F\frac{1}{n(n+1)(n+2)}\left[(n^2+3n+4)\delta_{nn'} + 2\theta_{nn'}\right],$$

$$\gamma^{GG}_{nn'} = C_A\left\{\left[\frac{1}{6} - \frac{2}{n(n+1)} - \frac{2}{(n+2)(n+3)} + 2\sum_{j=2}^{n+1}\frac{1}{j} + \frac{2N_f T}{3C_A}\right]\delta_{nn'} + \left[2\left(\frac{2n+1}{n(n+1)} - \frac{1}{n-n'}\right) - (n-n'+2)\left(\frac{1}{n(n+1)} + \frac{1}{(n+2)(n+3)}\right)\right]\theta_{nn'}\right\},$$

with the following notation

$$\sigma^{(\pm)}_{n\,n'} = \tfrac{1}{4}(1\pm(-1)^n)(1\pm(-1)^{n'})$$

$$\theta_{n\,n'} = \begin{cases} 1 & \text{for } n' < n, \\ 0 & \text{otherwise .}\end{cases}$$

2) Polarized local anomalous dimensions:

$$
\begin{aligned}
\Delta\gamma^{qq}_{nn'} &= \gamma^{qq}_{nn'}, \\
\Delta\gamma^{qG}_{nn'} &= -N_f T \frac{n'}{(n+1)(n+2)}, \\
\Delta\gamma^{Gq}_{nn'} &= \frac{1}{(n+1)(n+2)}\left[(n+3)\delta_{nn'} - \frac{2}{n}\theta_{nn'}\right], \\
\Delta\gamma^{GG}_{nn'} &= C_A\left\{\left[\frac{1}{6} - \frac{4}{(n+1)(n+2)} + 2\sum_{j=2}^{n+1}\frac{1}{j} + \frac{2N_f T}{3C_A}\right]\delta_{nn'}\right. \\
&+ \left[2\left(\frac{2n+1}{n(n+1)} - \frac{1}{n-n'}\right)\right. \\
&\left.\left. -(n-n'+2)\frac{2}{(n+1)(n+2)}\right]\theta_{nn'}\right\}.
\end{aligned}
$$

$$
\begin{aligned}
&e^2\left\langle p_2, S_2 \middle| O^\mu\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) \middle| p_1, S_1 \right\rangle \\
&= i\int Dz e^{-i\tilde{x}p_z/2} F(z_1, z_2)\left[\overline{u}(p_2, S_2)\gamma^\mu u(p_1, S_1) - \frac{i}{2}p_z^\mu \overline{u}(p_2, S_2)\gamma\tilde{x}u(p_1, S_1)\right] \\
&+ i\int Dz e^{-i\tilde{x}p_z/2} G(z_1, z_2)\left[\overline{u}(p_2, S_2)\sigma^{\mu\nu}p_{=\nu}u(p_1, S_1) - \frac{i}{2}p_z^\mu \overline{u}(p_2, S_2)\sigma^{\alpha\beta}\tilde{x}_\alpha p_{=\beta}u(p_1, S_1)\right]
\end{aligned}
$$

$$
\begin{aligned}
&e^2\left\langle p_2, S_2 \middle| O_5^\mu\left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2}\right) \middle| p_1, S_1 \right\rangle \\
&= i\int Dz e^{-i\tilde{x}p_z/2} F_5(z_1, z_2)\left[\overline{u}(p_2, S_2)\gamma_5\gamma^\mu u(p_1, S_1) - \frac{i}{2}p_z^\mu \overline{u}(p_2, S_2)\gamma_5\gamma\tilde{x}u(p_1, S_1)\right] \\
&+ i\int Dz e^{-i\tilde{x}p_z/2} G_5(z_1, z_2)\left[\overline{u}(p_2, S_2)\gamma_5\sigma^{\mu\nu}p_{=\nu}u(p_1, S_1) - \frac{i}{2}p_z^\mu \overline{u}(p_2, S_2)\gamma_5\sigma^{\alpha\beta}\tilde{x}_\alpha p_{=\beta}u(p_1, S_1)\right],
\end{aligned}
$$

$$H_{(5)}(z_1, z_2) = \int_0^1 \frac{d\lambda}{\lambda^2} h_{(5)} \left(\frac{z_1}{\lambda}, \frac{z_2}{\lambda} \right)$$

$$\begin{aligned} H(z_1, z_2) &= -H(-z_1, -z_2) \\ H_5(z_1, z_2) &= +H_5(-z_1, -z_2) \end{aligned}$$

$$\begin{aligned} T_{\mu\nu}(p_+, p_-, q) &= i \int d^4x \, e^{iqx} \, \langle p_2, S_2 \, | T(J_\mu(x/2) J_\nu(-x/2)) | \, p_1, S_1 \rangle \\ &= \int d^4x \, e^{iqx} \left\{ -\frac{\tilde{x}^\lambda}{i\pi^2(x^2 - i\varepsilon)^2} \left[S_{\alpha\mu\lambda\nu} \left\langle p_2 \middle| O^\alpha \left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2} \right) \middle| p_1 \right\rangle \right. \right. \\ & \qquad \left. \left. + i\varepsilon_{\mu\lambda\nu\sigma} \left\langle p_2 \middle| O_5^\alpha \left(\frac{\tilde{x}}{2}, -\frac{\tilde{x}}{2} \right) \middle| p_1 \right\rangle \right] \right\} . \end{aligned}$$

$$\begin{aligned} T_{\mu\nu}(q, p_+, p_-) &= -2 \int Dz \frac{1}{Q^2 + i\varepsilon} \left\{ \overline{u}(p_2, S_2) \Gamma^F_{\mu\nu}(q, p_+, p_-) u(p_1, S_1) \; F(z_+, z_-) \right. \\ & \qquad + \overline{u}(p_2, S_2) \Gamma^{F5}_{\mu\nu}(q, p_+, p_-) u(p_1, S_1) \; F_5(z_+, z_-) \\ & \qquad + \overline{u}(p_2, S_2) \Gamma^{G}_{\mu\nu}(q, p_+, p_-) u(p_1, S_1) \; G(z_+, z_-) \\ & \qquad \left. + \overline{u}(p_2, S_2) \Gamma^{G5}_{\mu\nu}(q, p_+, p_-) u(p_1, S_1) \; G_5(z_+, z_-) \right\} \end{aligned}$$

$$
\begin{aligned}
\Gamma^{F}_{\mu\nu}(q,p_z) &= \Big[Q_\mu\gamma_\nu + Q_\nu\gamma_\mu - g_{\mu\nu}\gamma_\alpha Q^\alpha\Big] \\
&\quad -\frac{1}{2}\Big[p_{z\mu}\gamma_\nu + p_{z\nu}\gamma_\mu - g_{\mu\nu}\gamma_\alpha p_z^\alpha\Big] \\
&\quad +\frac{1}{Q^2+i\varepsilon}\gamma_\alpha Q^\alpha\Big[Q_\nu p_{z\mu} + Q_\mu p_{z\nu} - g_{\mu\nu}Q.p_z\Big] \\
&\simeq \Big[q_\mu\gamma_\nu + q_\nu\gamma_\mu - g_{\mu\nu}\gamma_\alpha q^\alpha\Big] - [p_{z\mu}\gamma_\nu + p_{z\nu}\gamma_\mu] \\
&\quad +\frac{1}{Q^2+i\varepsilon}\gamma_\alpha q^\alpha\Big[-p_{z\nu}p_{z\mu} + q_\nu p_{z\mu} + q_\mu p_{z\nu} - g_{\mu\nu}q.p_z\Big] \\
\Gamma^{F5}_{\mu\nu}(q,p_z) &= i\,\gamma_5\varepsilon_{\mu\nu\lambda\sigma}\left[Q^\lambda\gamma^\sigma - \frac{1}{2}p_z^\sigma\gamma^\lambda + \frac{1}{Q^2+i\varepsilon}Q^\lambda p_z^\sigma\gamma_\alpha Q^\alpha\right] \\
&\simeq i\,\gamma_5\varepsilon_{\mu\nu\lambda\sigma}\left[q^\lambda\gamma^\sigma + \frac{1}{Q^2+i\varepsilon}q^\lambda p_z^\sigma\gamma_\alpha q^\alpha\right] \\
\Gamma^{G}_{\mu\nu}(q,p_z) &= \Big[Q_\mu\sigma_{\nu\alpha}p_-^\alpha + Q_\nu\sigma_{\mu\alpha}p_-^\alpha - g_{\mu\nu}\sigma_{\alpha\beta}p_-^\beta Q^\alpha\Big] \\
&\quad -\frac{1}{2}\Big[p_{z\mu}\sigma_{\nu\alpha}p_-^\alpha + p_{z\nu}\sigma_{\mu\alpha}p_-^\alpha - g_{\mu\nu}\sigma_{\beta\alpha}p_-^\alpha p_z^\beta\Big] \\
&\quad +\frac{1}{Q^2+i\varepsilon}\sigma_{\beta\alpha}p_-^\alpha Q^\beta\Big[Q_\nu p_{z\mu} + Q_\mu p_{z\nu} - g_{\mu\nu}Q.p_z\Big] \\
&\simeq \Big[q_\mu\sigma_{\nu\alpha}p_-^\alpha + q_\nu\sigma_{\mu\alpha}p_-^\alpha - g_{\mu\nu}\sigma_{\beta\alpha}p_-^\alpha q^\beta\Big] - \Big[p_{z\mu}\sigma_{\nu\alpha}p_-^\alpha + p_{z\nu}\sigma_{\mu\alpha}p_-^\alpha\Big] \\
&\quad +\frac{1}{Q^2+i\varepsilon}\sigma_{\beta\alpha}p_-^\alpha q^\beta\Big[-p_{z\mu}p_{z\nu} + q_\nu p_{z\mu} + q_\mu p_{z\nu} - g_{\mu\nu}q.p_z\Big]
\end{aligned}
$$

$$
\begin{aligned}
\Gamma^{G5}_{\mu\nu}(q,p_z) &= i\,\gamma_5\varepsilon_{\mu\nu\lambda\sigma}\left[Q^\lambda\sigma^{\sigma\alpha}p_{-\alpha} - \frac{1}{2}p_z^\sigma\sigma^{\lambda\alpha}p_{-\alpha} + \frac{1}{Q^2+i\varepsilon}Q^\lambda p_z^\sigma\sigma^{\alpha\beta}Q_\alpha p_{-\beta}\right] \\
&\simeq i\,\gamma_5\varepsilon_{\mu\nu\lambda\sigma}\left[q^\lambda\sigma^{\sigma\alpha}p_{-\alpha} + \frac{1}{Q^2+i\varepsilon}q^\lambda p_z^\sigma\sigma^{\alpha\beta}q_\alpha p_{-\beta}\right],
\end{aligned}
$$

$$
Q = q - \frac{p_z}{2}\,.
$$

2.3 Lorentz Structure

$$
\begin{aligned}
T_{\mu\nu}(q,p_+,p_-) &= -2\overline{u}(p_2,S_2)\left[\Gamma^F_{\mu\nu}(q,p_+,p_-)+\Gamma^{F5}_{\mu\nu}(q,p_+,p_-)\right.\\
&\qquad \left.+\Gamma^G_{\mu\nu}(q,p_+,p_-)+\Gamma^{G5}_{\mu\nu}(q,p_+,p_-)\right]u(p_1,S_1)\ ,
\end{aligned}
$$

$$
\begin{aligned}
\Gamma^F_{\mu\nu}(q,p_+,p_-) &= \left[q_\mu\gamma_\nu+q_\nu\gamma_\mu-g_{\mu\nu}\gamma_\alpha q^\alpha\right]F_1(\xi,\eta)\\
&\quad -\gamma_\mu F_{1,\nu}(\xi,\eta)-\gamma_\nu F_{1,\mu}(\xi,\eta)+\gamma_\alpha q^\alpha F_{2,\mu\nu}(\xi,\eta)\\
\Gamma^{F5}_{\mu\nu}(q,p_+,p_-) &= i\ \gamma_5\varepsilon_{\mu\nu\lambda\sigma}\left[q^\lambda\gamma^\sigma F_1^5(\xi,\eta)+q^\lambda\gamma_\alpha q^\alpha F_2^{\sigma,5}(\xi,\eta)\right]\\
\Gamma^G_{\mu\nu}(q,p_+,p_-) &= \left[q_\mu\sigma_{\nu\alpha}p_-^\alpha+q_\nu\sigma_{\mu\alpha}p_-^\alpha-g_{\mu\nu}\sigma_{\beta\alpha}p_-^\alpha q^\beta\right]G_1(\xi,\eta)\\
&\quad -\sigma_{\mu\alpha}p_-^\alpha G_{1,\nu}(\xi,\eta)-\sigma_{\nu\alpha}p_-^\alpha G_{1,\mu}(\xi,\eta)p_-^\alpha+\sigma_{\beta\alpha}p_-^\alpha q^\beta G_{2,\mu\nu}(\xi,\eta)\\
\Gamma^{G5}_{\mu\nu}(q,p_+,p_-) &= i\ \gamma_5\varepsilon_{\mu\nu\lambda\sigma}\left[q^\lambda\sigma^{\sigma\alpha}p_{-\alpha}G_1^5(\xi,\eta)+q^\lambda\sigma^{\alpha\beta}q_\alpha p_{-\beta}G_2^{\sigma,5}(\xi,\eta)\right]\ .
\end{aligned}
$$

$$
\begin{aligned}
H_1(\xi,\eta) &= \int Dz\frac{1}{Q^2+i\varepsilon}H(z_+,z_-)\\
H_k^\sigma(\xi,\eta) &= \int Dz\frac{p_+^\sigma z_++p_-^\sigma z_-}{(Q^2+i\varepsilon)^k}H(z_+,z_-)=\int Dz\frac{p_+^\sigma t+\pi_\sigma z_-}{(Q^2+i\varepsilon)^k}H(z_+,z_-)\\
H_{2,\mu\nu}(\xi,\eta) &= \int Dz\frac{1}{(Q^2+i\varepsilon)^2}\left[-p_{z\mu}p_{z\nu}+q_\nu p_{z\mu}+q_\mu p_{z\nu}-g_{\mu\nu}q{\cdot}p_z\right]H(z_+,z_-)\\
&= \int Dz\frac{1}{(Q^2+i\varepsilon)^2}\left[-p_{+\mu}p_{+\nu}t^2+(q_\nu p_{+\mu}+q_\mu p_{+\nu})\,t-g_{\mu\nu}q{\cdot}p_z\right.\\
&\qquad \left.-\pi_\mu\pi_\nu z_-^2+(q_\nu\pi_\mu+q_\mu\pi_\nu)\,z_-+(p_{+\nu}\pi_\mu+p_{+\mu}\pi_\nu)\,tz_-\right]\\
&\qquad \times H(z_+,z_-)\ ,
\end{aligned}
$$

$$
\begin{aligned}
t &= z_++\eta z_-\\
\pi_\sigma &= p_{-\sigma}-\eta p_{+\sigma}
\end{aligned}
$$

3 Kinematic Relations

$$
\begin{aligned}
p_+ &= p_1 + p_2 = (2E_p;\ \vec{0}) \\
-p_- &= p_1 - p_2 = (0;\ 2\vec{p}) = (0;\ 0, 0, 2p_3) \\
q &= \frac{1}{2}(q_1 + q_2) = (q_0;\ q_1, 0, q_3)\ .
\end{aligned}
$$

$$
\begin{aligned}
q_1{\cdot}q_1 &= -\nu(\xi - \eta) \\
q_2{\cdot}q_2 &= -\nu(\xi + \eta) \\
q{\cdot}p_+ &= \nu \\
q{\cdot}p_- &= \eta\nu \\
q{\cdot}q &= -\xi\nu \\
q{\cdot}p_z &= q^2 - Q^2 = (z_+ + z_-\eta)\nu \equiv t\ \nu \\
p_+^2 &\approx p_-^2 \approx p_+p_- \approx 0\ .
\end{aligned}
$$

$$
\mathsf{T}_{kl} = \varepsilon^{\mu}_{2,k} T_{\mu\nu} \varepsilon^{\nu}_{1,l}, \quad k, l \in \{0, 1, 2, 3\}
$$

$$
\begin{aligned}
n_0 &= (1;\ 0, 0, 0) \\
n_2 &= (0;\ 0, 1, 0)\ .
\end{aligned}
$$

$$
\begin{aligned}
\varepsilon^{(1)}_{0\mu} &= \frac{q_{1\mu}}{\sqrt{|q_1^2|}} = \frac{q_{1\mu}}{\nu^{1/2}}\frac{1}{\sqrt{|\xi-\eta|}} \\
\varepsilon^{(2)}_{0\mu} &= \frac{q_{2\mu}}{\sqrt{|q_2^2|}} = \frac{q_{2\mu}}{\nu^{1/2}}\frac{1}{\sqrt{|\xi+\eta|}} \\
\varepsilon^{(i)}_{1\mu} &= n_{2\mu} \\
\varepsilon^{(i)}_{2\mu} &= \frac{1}{N_{2i}}\varepsilon_{\mu\alpha\beta\gamma}n_0^\alpha n_2^\beta q_i^\gamma \\
\varepsilon^{(i)}_{3\mu} &= \frac{1}{N_{3i}}\left[q_{i\mu}q_i.n_0 - n_{0\mu}q_i.q_i\right] \ ,
\end{aligned}
$$

$$
\begin{aligned}
N_{21} &= \frac{\nu}{\mu}\sqrt{\left|1+\frac{\mu^2}{\nu}(\xi-\eta)\right|} \\
N_{22} &= \frac{\nu}{\mu}\sqrt{\left|1+\frac{\mu^2}{\nu}(\xi+\eta)\right|}
\end{aligned}
$$

$$
\begin{aligned}
N_{31} &= \frac{\nu^{3/2}}{\mu}\sqrt{|\xi-\eta|}\sqrt{\left|1+\frac{\mu^2}{\nu}(\xi-\eta)\right|} \\
N_{32} &= \frac{\nu^{3/2}}{\mu}\sqrt{|\xi+\eta|}\sqrt{\left|1+\frac{\mu^2}{\nu}(\xi+\eta)\right|}
\end{aligned}
$$

$$
\varepsilon^{(i)}_{k\mu}.\varepsilon^{(i)\mu}_{l} = s_k\delta_{kl} \ ,
$$

$$
\begin{aligned}
\varepsilon_{0\rho}^{(1(2))} &= \frac{1}{\sqrt{|\xi|}}\left[\frac{q_\rho}{\nu^{1/2}} \pm \frac{p_{-\rho}}{2\nu^{1/2}}\right]\frac{1}{\sqrt{|1 \mp \eta/\xi|}} \\
\varepsilon_{1\rho}^{(1(2))} &= n_{2\rho} \\
\varepsilon_{2\rho}^{(1(2))} &= \frac{\mu}{\nu}\left|1 - \frac{\mu^2}{2\nu}(\xi \mp \eta)\right| \varepsilon_{\rho\alpha\beta\gamma} n_0^\alpha n_2^\beta \left(q^\gamma \pm \frac{1}{2}p_-^\gamma\right) \\
\varepsilon_{3\rho}^{(1(2))} &= \frac{1}{\nu^{1/2}}\frac{1}{\sqrt{|\xi|}}\left|1 - \frac{\mu^2}{2\nu}(\xi \mp \eta)\right|\left[q_\rho \pm \frac{1}{2}p_{-\rho} + \mu n_{0\rho}(\xi \mp \eta)\right]\frac{1}{\sqrt{|1 \mp \eta/\xi|}}
\end{aligned}
$$

$$
\begin{aligned}
\varepsilon_{0\mu}^{(2)} &= \frac{1}{\sqrt{2}q_0^{(2)}} q_\mu = \frac{1}{\sqrt{2}q_0^{(2)}}(q_0,\ \vec{q}_2) \\
\varepsilon_{1\mu}^{(2)} &= n_{2\mu} \\
\varepsilon_{2\mu}^{(2)} &= \frac{1}{q_0^{(2)}} \varepsilon_{\mu\alpha\beta\gamma} n_0^\alpha n_2^\beta q_2^\gamma \ ,
\end{aligned}
$$

$$
q_0^{(2)} = \frac{\nu}{\mu} \ .
$$

$$
\tilde{\varepsilon}_{0\mu}^{(2)} = \frac{1}{\sqrt{2}q_0^{(2)}}(q_0,\ -\vec{q}_2)
$$

$$
\tilde{\varepsilon}_{0\mu}^{(2)} + \varepsilon_{0\mu}^{(2)} = \sqrt{2} n_{0\mu} \ .
$$

4 Current Conservation

$$\partial^x_\mu J^\mu(x) = 0$$

$$\begin{aligned} T_{\mu\nu}(p_+, p_-, q) &= i \int d^4 e^{-iq_2 x} \langle p_2, S_2 | RT(J_\mu(0) J_\nu(x) | p_1, S_1 \rangle \\ &= i \int d^4 e^{-iq_1 x} \langle p_2, S_2 | RT(J_\mu(-x) J_\nu(0) | p_1, S_1 \rangle \end{aligned}$$

$$q_2^\mu T_{\mu\nu} = T_{\mu\nu} q_1^\nu = 0 \ .$$

$$\begin{aligned} \overline{u}(p_2, S_2)\gamma_\mu q^\mu u(p_1, S_1) &\propto \nu \\ \varepsilon_{\alpha,\beta,\gamma,\delta}\, p_\pm^\gamma q^\delta &\propto \nu \end{aligned}$$

$$\int Dz\ H(z_+, z_-) = \int_{-1}^{+1} dz_+ \int_{-1+|z_+|}^{+1-|z_+|} H(z_+, z_-) = 0$$

$$\mathsf{T}^F_{00} = -\frac{2}{\nu}\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\frac{1}{\sqrt{|\xi^2-\eta^2|}}\int Dz\ F(z_+,z_-) \tag{4.4}$$

$$\mathsf{T}^F_{01},\mathsf{T}^F_{10},\mathsf{T}^F_{02},\mathsf{T}^F_{20} = O\left(\frac{1}{\sqrt{\nu}}\right) \tag{4.5}$$

$$\mathsf{T}^F_{03} = -\frac{2}{\nu}\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\frac{1}{|\xi-\eta|\sqrt{|\xi^2-\eta^2|}}\int Dz\ F(z_+,z_-) \tag{4.6}$$

$$\mathsf{T}^F_{30} = -\frac{2}{\nu}\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\frac{1}{|\xi+\eta|\sqrt{|\xi^2-\eta^2|}}\int Dz\ F(z_+,z_-) \tag{4.7}$$

$$\mathsf{T}^{F5}_{00} = 0 \tag{4.8}$$

$$\mathsf{T}^{F5}_{01},\mathsf{T}^{F5}_{10},\mathsf{T}^{F5}_{02},\mathsf{T}^{F5}_{20} = O\left(\frac{1}{\sqrt{\nu}}\right) \tag{4.9}$$

$$\mathsf{T}^{F5}_{03},\mathsf{T}^{F5}_{30} = O\left(\frac{1}{\nu}\right) \tag{4.10}$$

$$\mathsf{T}^G_{00} = -\frac{2}{\nu}\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\frac{1}{\sqrt{|\xi^2-\eta^2|}}\int Dz\ G(z_+,z_-) \tag{4.11}$$

$$\mathsf{T}^G_{01},\mathsf{T}^G_{10},\mathsf{T}^G_{02},\mathsf{T}^G_{20} = O\left(\frac{1}{\sqrt{\nu}}\right) \tag{4.12}$$

$$\mathsf{T}^G_{03} = -\frac{2}{\nu}\overline{u}(p_2,S_2)\sigma_{\beta\alpha}p^\alpha_- q^\beta u(p_1,S_1)\frac{1}{|\xi-\eta|\sqrt{|\xi^2-\eta^2|}}\int Dz\ G(z_+,z_-) \tag{4.13}$$

$$\mathsf{T}^G_{30} = -\frac{2}{\nu}\overline{u}(p_2,S_2)\sigma_{\beta\alpha}p^\alpha_- q^\beta u(p_1,S_1)\frac{1}{|\xi+\eta|\sqrt{|\xi^2-\eta^2|}}\int Dz\ G(z_+,z_-) \tag{4.14}$$

$$\mathsf{T}^{G5}_{00} = 0 \tag{4.15}$$

$$\mathsf{T}^{G5}_{01},\mathsf{T}^{G5}_{10},\mathsf{T}^{G5}_{02},\mathsf{T}^{G5}_{20} = O\left(\frac{1}{\sqrt{\nu}}\right) \tag{4.16}$$

$$\mathsf{T}^{G5}_{03},\mathsf{T}^{G5}_{30} = O\left(\frac{1}{\nu}\right)\ . \tag{4.17}$$

5 The Helicity Projections of the Compton Amplitude

$$\begin{aligned}
\mathsf{T}^F_{11} &= 2\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\left[F_1(\xi,\eta) + \varepsilon_1^{(2)\mu}\varepsilon_1^{(1)\nu}F_{2,\mu\nu}(\xi,\eta)\right] \\
\mathsf{T}^F_{22} &= 2\overline{u}(p_2,S_2)\gamma_\mu q^\mu u(p_1,S_1)\left[F_1(\xi,\eta) + \varepsilon_2^{(2)\mu}\varepsilon_2^{(1)\nu}F_{2,\mu\nu}(\xi,\eta)\right] \\
\mathsf{T}^F_{kl} &\propto \left(\frac{1}{\nu}\right)^{1/2+n} \qquad \text{for the other projections } \ k,l \in \{1,2,3\} \ \text{ and } \ n \geq 0\ ,
\end{aligned}$$

$$\mathsf{T}^{F5}_{12} = i\,\varepsilon^{\mu\lambda\nu\sigma}\varepsilon^{(2)}_{1\mu}\varepsilon^{(1)}_{2\nu}\int Dz\frac{q_\lambda}{Q^2+i\varepsilon}\left[S_{21,\sigma} + \frac{q.S_{21}}{Q^2+i\varepsilon}p_{z\sigma}\right]F_5(z_+,z_-)\ .$$

$$\mathsf{T}^{F5}_{kl} \propto \left(\frac{1}{\nu}\right)^{1/2+n} \qquad \text{for the other projections } \ k,l \in \{1,2,3\} \ \text{ and } \ n \geq 0\ .$$

$$\varepsilon_1^{(2)\mu}\varepsilon_1^{(1)\nu}F_{2\mu\nu}(\xi,\eta) = \varepsilon_2^{(2)\mu}\varepsilon_2^{(1)\nu}F_{2\mu\nu}(\xi,\eta) = \int Dz\frac{q{\cdot}p_z}{(Q^2+i\varepsilon)^2}F(z_+,z_-)$$

$$\mathsf{T}^F_{11} = \mathsf{T}^F_{22}\ .$$

$$\mathsf{T}^{F5}_{12} = -\mathsf{T}^{F5}_{21}$$

$$S^\sigma_{21} := -\frac{1}{2}\overline{u}(p_2,S_2)\gamma_5\gamma^\sigma u(p_1,S_1)\ .$$

$$\begin{aligned} &\propto q_\lambda S_{\sigma,21} \quad \rightarrow \quad g_1(x_B) + g_2(x_B) \\ &\propto q_\lambda p_{z\sigma} \quad \rightarrow \quad g_2(x_B) \ . \end{aligned}$$

$$\begin{aligned} \mathsf{T}^G_{11} &= 2\overline{u}(p_2, S_2)\sigma_{\alpha\beta}q^\alpha p_-^\beta u(p_1, S_1)\left[G_1(\xi,\eta) + \varepsilon_1^{(2)\mu}\varepsilon_1^{(1)\nu}G_{2,\mu\nu}(\xi,\eta)\right] \\ \mathsf{T}^G_{22} &= 2\overline{u}(p_2, S_2)\sigma_{\alpha\beta}q^\alpha p_-^\beta u(p_1, S_1)\left[G_1(\xi,\eta) + \varepsilon_2^{(2)\mu}\varepsilon_2^{(1)\nu}G_{2,\mu\nu}(\xi,\eta)\right] \\ \mathsf{T}^G_{kl} &\propto \left(\frac{1}{\nu}\right)^{1/2+n} \qquad \text{for the other projections } \ k,l \in \{1,2,3\} \ \text{ and } \ n \geq 0 \ , \end{aligned}$$

$$\mathsf{T}^{G5}_{kl} \propto \left(\frac{1}{\nu}\right)^{1/2+n} \qquad \text{for the other projections } \ k,l \in \{1,2,3\} \ \text{ and } \ n \geq 0$$

$$\mathsf{T}^{G5}_{12} = i\,\varepsilon^{\mu\lambda\nu\sigma}\varepsilon_1^{(2)\mu}\varepsilon_2^{(1)\nu}\int Dz\frac{q_\lambda}{Q^2+i\varepsilon}\left[\Sigma_{21,\sigma} + \frac{q.\Sigma_{21}}{Q^2+i\varepsilon}p_{z\sigma}\right]G_5(z_+,z_-) \ ,$$

$$\Sigma^\sigma_{21} := -\frac{1}{2}\overline{u}(p_2,S_2)\gamma_5\sigma^{\sigma\alpha}p_{-\alpha}u(p_1,S_1)$$

$$\mathsf{T}^G_{11} = \mathsf{T}^G_{22} \ .$$

$$\mathsf{T}^{G5}_{12} = -\mathsf{T}^{G5}_{21}$$

6 The Integral Relations

6.1 Unpolarized Contributions

$$\mathsf{T}_{11}^{F,G} = \mathsf{T}_{22}^{F,G}$$

$$F_2(x_B) = 2xF_1(x_B) \equiv \sum_q e_q^2 x \left[q(x_B) + \overline{q}(x_B)\right]$$

$$\begin{aligned}
\mathsf{H}_1(\xi,\eta) &= \int Dz \frac{\nu}{Q^2 + i\varepsilon} H(z_+, z_-) = -\int Dz \frac{H(z_+, z_-)}{\xi + t - i\varepsilon} \\
\mathsf{H}_2(\xi,\eta) &= \int Dz \frac{\nu\ q.p_z}{(Q^2 + i\varepsilon)^2} H(z_+, z_-) = \int Dz \frac{tH(z_+, z_-)}{(\xi + t - i\varepsilon)^2} \ .
\end{aligned}$$

$$
\begin{aligned}
A^{\mu\nu} &= -2\frac{q.P_{21}}{\nu}\left[g^{\mu\nu} - \frac{q^\mu p_+^\nu + q^\nu p_+^\mu}{q.p_+}\right]\int_{-1}^{+1} dt \frac{\mathsf{F}_1(t,\eta)}{\xi + t - i\varepsilon} \\
&+\frac{2}{\nu}\left[q^\mu\left(P_{21}^\nu - p_+^\nu \frac{q.P_{21}}{\nu}\right) + q^\nu\left(P_{21}^\mu - p_+^\mu \frac{q.P_{21}}{\nu}\right)\right]\int_{-1}^{+1} dt \frac{\hat{\mathsf{H}}(t,\eta)}{\xi + t - i\varepsilon} ,
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{q.P_{21}}{\nu^2} p_+^\mu p_+^\nu \int_{-1}^{+1} dt \frac{\mathsf{F}_2(t,\eta)}{\xi + t - i\varepsilon} \\
&-\frac{2}{\nu}\left[p_+^\mu\left(p_{21}^\nu - p_+^\nu \frac{q.P_{21}}{\nu}\right) + p_+^\nu\left(p_{21}^\mu - p_+^\mu \frac{q.P_{21}}{\nu}\right)\right]\int_{-1}^{+1} dt \frac{t\,\hat{\mathsf{H}}(t,\eta)}{\xi + t - i\varepsilon}
\end{aligned}
$$

$$
\begin{aligned}
\mathsf{F}_1(t,\eta) &= \hat{h}(t,\eta) \\
\mathsf{F}_2(t,\eta) &= 2t\,\hat{h}(t,\eta) .
\end{aligned}
$$

$$
\begin{aligned}
\hat{\mathsf{H}}(t,\eta) &= \int_t^{\mathrm{sign}(t)} \frac{dz}{z}\hat{h}(z,\eta) \\
\tilde{\mathsf{H}}_k(t,\eta) &= \int_t^{\mathrm{sign}(t)} \frac{dz}{z}\tilde{h}_k(z,t,\eta) ,
\end{aligned}
$$

$$
\tilde{h}_k(z,t,\eta) = \left(\frac{t}{z}\right)^k \int_{\rho_{\min}}^{\rho_{\max}} d\rho\,\rho^k\;h(z-\eta\rho,\rho) .
$$

$$
P_{21}^\sigma := \overline{u}(p_2,S_2)\gamma^\sigma u(p_1,S_1),
$$

$$
\Pi^\mu = P_{21}^\mu - p_+^\mu \frac{q.P_{21}}{\nu}
$$

$$
\mathsf{F}_2(t,\eta) = 2t\;\mathsf{F}_1(t,\eta)
$$

6.2 Polarized Contributions

$$\mathsf{T}_{12}^{H5} = i\,\varepsilon^{\mu\lambda\nu\sigma}\varepsilon_{1\mu}^{(2)}\varepsilon_{2\nu}^{(1)}B_{\lambda\sigma}$$

$$B_{\lambda\sigma} = \int Dz \frac{q_\lambda}{Q^2+i\varepsilon}\left[S^H_{21,\sigma} + \frac{q.S^H_{21}}{Q^2+i\varepsilon}p_{z\sigma}\right] H_5(z_+,z_-),$$

$S^H_{21} = S_{21}(\Sigma_{21})$ for $H = F(G)$. It may be rewritten as

$$B_{\lambda\sigma} = -\frac{1}{\nu}\int Dz \frac{q_\lambda}{\xi+t-i\varepsilon}\left[S^H_{21,\sigma} - \frac{1}{\nu}\frac{t\ q.S^H_{21}}{\xi+t-i\varepsilon}p_{+\sigma} + \frac{1}{\nu}\frac{q.S^H_{21}}{\xi+t-i\varepsilon}z_-\pi_\sigma\right] H_5(z_+,z_-$$

$$\begin{aligned} B_{\lambda\sigma} &= -\frac{1}{\nu}q_\lambda S^H_{21,\sigma}\int_{-1}^{+1}dt\frac{1}{\xi+t-i\varepsilon}\int_t^{\mathrm{sign}(t)}\frac{dz}{z}\hat{h}_5(z,\eta) \\ &\quad -\frac{1}{\nu^2}q_\lambda p_{+\sigma}\ q.S^H_{21}\int_{-1}^{+1}dt\frac{1}{\xi+t-i\varepsilon}\left[\hat{h}_5(t,\eta) - \int_t^{\mathrm{sign}(t)}\frac{dz}{z}\hat{h}_5(z,\eta)\right] \\ &\quad -\frac{1}{\nu^2}q_\lambda \pi_\sigma\ q.S^H_{21}\int_{-1}^{+1}dt\frac{1}{\xi+t-i\varepsilon}\int_t^{\mathrm{sign}(t)}\frac{dz}{z}\tilde{h}_5(z,t,\eta)\,. \end{aligned}$$

$$\tilde{h}_5(z,t,\eta) = \left(\frac{t}{z}\right)\int_{\rho_{\min}}^{\rho_{\max}} d\rho\ \rho h\left(z-\eta\rho,\rho\right)$$

$$\widehat{H}(t,\eta) = \int_{z_-^{\min}}^{z_-^{\max}} dz_- H(t-\eta z_-, z_-) = \int_0^1 \frac{d\lambda}{\lambda^2} \int_{z_-^{\min}}^{z_-^{\max}} dz_- h\left(\frac{t}{\lambda} - \eta\frac{z_-}{\lambda}, \frac{z_-}{\lambda}\right)$$
$$= \int_t^{\mathrm{sign}(t)} \frac{dz}{z} \hat{h}(z,\eta) ,$$

$$z_-^{\min,\max} = \frac{t \pm 1}{\eta \pm 1} ,$$

$$\hat{h}(z,\eta) = \int_{\rho_{\min}}^{\rho_{\max}} d\rho \, h\left(z - \eta\rho, \rho\right) ,$$

$$\int_{-1}^{+1} dt \frac{t}{(\xi + t - i\varepsilon)^2} \int_t^{\mathrm{sign}(t)} \frac{dz}{z} \hat{h}(z,\eta) = \int_{-1}^{+1} dt \frac{1}{\xi + t - i\varepsilon} \int_t^{\mathrm{sign}(t)} \frac{dz}{z} \hat{h}(z,\eta)$$
$$- \int_{-1}^{+1} dt \frac{1}{\xi + t - i\varepsilon} \hat{h}(t,\eta) ,$$

$$\mathsf{H}_2(\xi,\eta) = -\mathsf{H}_1(\xi,\eta) - \int_{-1}^{+1} dt \frac{\hat{h}(t,\eta)}{\xi + t - i\varepsilon} .$$

$$\mathsf{T}^H_{11(22)}(\xi,\eta) \propto - \int_{-1}^{+1} \frac{\hat{h}(t,\eta)}{\xi + t - i\varepsilon} = -\mathsf{P} \int_{-1}^{+1} dt \frac{\hat{h}(t,\eta)}{\xi + t} - i\pi \hat{h}(\xi,\eta)$$

$$B_{\lambda\sigma} = -\frac{1}{\nu} q_\lambda \int_{-1}^{+1} \frac{dt}{\xi + t - i\varepsilon}$$
$$\times \left\{ S^H_{21,\sigma} \left[\mathsf{G}_1(t,\eta) + \mathsf{G}_2(t,\eta) \right] + \frac{1}{\nu} p_{+\sigma}\; q.S^H_{21} \mathsf{G}_2(t,\eta) + \frac{1}{\nu} \pi_\sigma\; q.S^H_{21} \mathsf{G}_3(t,\eta) \right\}$$

$$\begin{aligned}
\mathsf{G}_1(t,\eta) &:= \hat{h}_5(t,\eta) \\
\mathsf{G}_2(t,\eta) &= -\mathsf{G}_1(t,\eta) + \int_t^{\mathrm{sign}(t)} \frac{dz}{z} \mathsf{G}_1(z,\eta) \\
\mathsf{G}_3(t,\eta) &:= \int_t^{\mathrm{sign}(t)} \frac{dz}{z} \tilde{h}_5(z,t,\eta) \,.
\end{aligned}$$

6.3 Forward Scattering

$$W_{\mu\nu} = \frac{1}{2\pi} \mathsf{Im}\, T_{\mu\nu} \ .$$

$$\begin{aligned} \mathsf{F}_1(t,0) &= \sum_q e_q^2 \left[q(t)\theta(t) - \overline{q}(-t)\theta(-t) \right] \\ \mathsf{G}_1(t,0) &= \sum_q e_q^2 \left[\Delta q(t)\theta(t) + \Delta\overline{q}(-t)\theta(-t) \right] \ . \end{aligned}$$

$$q_\mu A^{\mu\nu} = p^\nu \left[\int_{-1}^{+1} \frac{2\xi \mathsf{F}_1(t,0) - \mathsf{F}_2(t,0)}{\xi - t - i\varepsilon} - \int_{-1}^{+1} \frac{2\xi \mathsf{F}_1(t,0) + \mathsf{F}_2(t,0)}{\xi + t - i\varepsilon} \right] = 0 \ .$$

$$\pm 2\xi \mathsf{F}_1(\pm\xi, 0) = \mathsf{F}_2(\pm\xi, 0) \ .$$

$$\begin{aligned} F_1(x_B) &= \frac{1}{2} \left[\mathsf{F}_1(\xi,0) - \mathsf{F}_1(-\xi,0) \right] = \frac{1}{2} \sum_q e_q^2 \left[q(x_B) + \overline{q}(x_B) \right] \\ F_2(x_B) &= \mathsf{F}_2(\xi,0) + \mathsf{F}_2(-\xi,0) \ , \end{aligned}$$

$$F_2(x_B) = 2 x_B F_1(x_B)$$

$$
\begin{aligned}
B_{\lambda\sigma} &= -\frac{1}{2\nu} q_\lambda S^H_{21,\sigma} \int_{-1}^{+1} dt \left[\frac{\mathsf{G}_1(t,0)+\mathsf{G}_2(t,0)}{\xi+t-i\varepsilon} + \frac{\mathsf{G}_1(t,0)+\mathsf{G}_2(t,0)}{\xi-t-i\varepsilon} \right] \\
&\quad -\frac{1}{2\nu^2} q_\lambda p_{+\sigma}\; q.S^H_{21} \int_{-1}^{+1} dt \left[\frac{\mathsf{G}_2(t,0)}{\xi+t-i\varepsilon} + \frac{\mathsf{G}_2(t,0)}{\xi-t-i\varepsilon} \right] .
\end{aligned}
$$

$$
\begin{aligned}
g_1(x_B) &= \frac{1}{2}\left[\mathsf{G}_1(\xi,0)+\mathsf{G}_1(-\xi,0)\right] = \frac{1}{2}\sum_q e_q^2 \left[\Delta q(x_B) + \Delta\overline{q}(x_B)\right] \\
g_2(x_B) &= -g_1(x_B) + \int_{x_B}^{1} \frac{dz}{z} g_1(z) .
\end{aligned}
$$

8. Conclusions

1. The virtual Compton Amplitude for deep-inelastic Nonforward Scattering was studied in the Generalized Bjorken Region for the Twist–2 contributions.

2. There exist several equivalent methods to derive the Nonforward Evolution Kernels and anomalous dimensions, which yield the same results. The problem of Spin Towers can be solved in terms of integral representations. This is likely the solution of the Spin Tower problem arising for Higher Twist Operators, in generalized form, for Forward Scattering too.

3. The Nonforward Compton Amplitude consists of unpolarized and polarized DIRAC and PAULI –type contributions at leading twist. For the Operator-Expectaion Values an expansion in $1/\nu$ has to be performed to find the Leading Twist terms, in the spirit of the Bjorken Limit.

4. For the unpolarized terms only the Amplitude matrix elements T_{11} and T_{22} and the polarized terms the projections T_{12} and T_{21} contribute in this order.

5. In this order the Light-Cone expansion conserves the electromagnetic current. This property has to be studied twist by twist for the remaining contributions.

6. Generalizations of the CALLAN-GROSS and WANDZURA-WILCZEK relations known in the forward case for the matrix element square level for the Nonforward Case were derived at the Matrix Element Level.