# Grand Unified Theories for Pedestrians

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# 1. Introduction

- The unification of physics' most fundamental laws is an old desire and is worked out in physics whenever possible.
- Grand Unified Theories (GUTs) constitute such an attempt for the Standard Model (SM) and its present extensions.
- I will try to give an introduction to the basic theoretical structure of the most elementary theories of this kind, with emphasis on their specific construction principles and some of their experimental predictions.
- Despite being 'pedestrian', some formalism is needed for a clear understanding and solid quantification of the outcome.
- Formulae will be shown, to discuss necessary key aspects.
- Other Formulae will be shown, to provide links to experiment.
- We will not derive most of the structures, neither use extensive mathematics, but sometimes quote corresponding results.

# Very little Math's:

## Groups:

A group $\mathcal{G}$ is a set of elements with a (non-commutative) multiplication $\bullet$ with $a, b, c \in \mathcal{G} \Longrightarrow a \bullet b \in \mathcal{G}$ and $(a \bullet b) \bullet c = a \bullet (b \bullet c)$. Furthermore, there is a unique element $e$ with $a \bullet a^{-1} = e$ and $a^{-1} \in \mathcal{G}$.
You may think here of matrix multiplication, as an example - it will fully suffice.

## Sub-Groups:

A sub-group $\mathcal{B} \subseteq \mathcal{G}$ is a group contained in the covering group.

## Multiplets:

Multiplets are usually row or line vectors and sometimes matrices out of $N$ elements corresponding to so-called $N$-plets.
Examples: doublets: $\mathbf{2}$, triplets: $\mathbf{3}$, 5-plets: $\mathbf{5}$, decuplets: $\mathbf{10}$, etc.
They will frequently appear.

# Construction Principles

- We consider renormalizable gauge field field theories only in all orders in perturbation theory, i.e. those where all the parameters of which can be determined with a finite number of measurements [experiments].
- These theories have also to be anomaly free [will be explained later.]
- Unifying groups have to cover the SM and should be in a certain sense minimal.

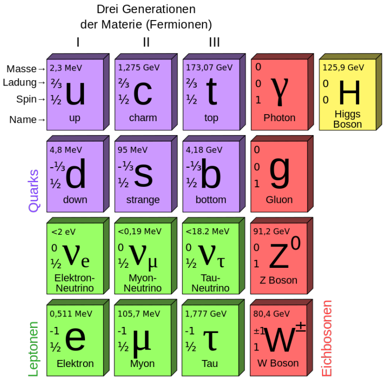
SU(5) : H. Georgi and S.L. Glashow, Unity of All Elementary Particle Forces, Phys. Rev. Lett. **32** (1974) 438-441.

SO(10) : H. Georgi, The state of the art - gauge theories (1974), AIP Conf.Proc. 23 (1975) 575-582.

SO(10) : H. Fritzsch and P. Minkowski, Unified Interactions of Leptons and Hadrons, Annals Phys. **93** (1975) 193-266.

# 2. The Standard Model

To unify the Standard Model (SM) into anything larger, we first have to commemorate essential facts of the SM.




Courtesy Wikipedia.

$$\underbrace{SU(2)_L \otimes U(1)_Y}_{\text{electro}-\text{weak}} \otimes \underbrace{SU(3)_c}_{\text{strong}} \xrightarrow{2} \underbrace{SU(3)_c}_{\text{QCD}} \otimes \underbrace{U(1)_{\text{em}}}_{\text{QED}}$$

# Occurring Matrices

$SU(2)_L$: The Pauli Matrices [rank 1]

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$SU(3)_c$: The Gell-Mann Matrices [rank 2]

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix},$$

# The Force Fields

Electro-weak fields : $SU(2)_L \otimes U(1)_Y, \quad \{B_\mu^k, \quad \mathcal{A}_\mu\} \qquad (4)$

$$\left[g' \frac{1}{2}\mathcal{A}_\mu Y + g \sum_{l=1}^{3} \tfrac{1}{2}\sigma_k B_\mu^k\right]_L, \qquad \left[g' \tfrac{1}{2}\mathcal{A}_\mu Y\right]_R$$

Gluon fields : $SU(3)_c, \quad \{A_\mu^k\} \qquad (8)$

$$g_s \mathbf{A}_\mu = g_s \sum_{k=1}^{8} \tfrac{1}{2}\lambda_k A_\mu^k$$

# The Fermion Fields

| | $(SU(2), Y, SU(3))$ |
|---|---|
| $L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$ | $(\mathbf{2}, -1, \mathbf{1})$ |
| $\nu_R$ | $(\mathbf{1}, 0, \mathbf{1})$ |
| $e_R$ | $(\mathbf{1}, -2, \mathbf{1})$ |
| $Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$ | $(\mathbf{2}, \frac{1}{3}, \mathbf{3})$ |
| $u_R$ | $(\mathbf{1}, \frac{4}{3}, \mathbf{3})$ |
| $d_R$ | $(\mathbf{1}, -\frac{2}{3}, \mathbf{3})$ |

$$Y = 2(Q_{\rm em} - I_3)$$ [Hypercharge]

Examples :

$Q(\nu_L) = -\frac{1}{2} + \frac{1}{2} = 0, \quad Q(\nu_R) = 0 + 0 = 0, \quad Q(u_R) = \frac{2}{3} + 0 = \frac{2}{3}$

# Anomaly Cancellation

At the 1-loop level graphs arise in many gauge field theories, which are just infinite, and cannot be absorbed into the parameters of the theory.

Only Solution:
The group structure has to be such that these terms exactly cancel at the end.
Any theory in which this is not the case is unrealistic and cannot fit experimental observation.

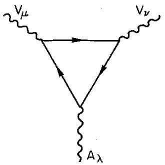



FIG. 6-23. Triangle anomaly for the axial-vector current.

Courtesy C. Quigg.

Appears in $SU(2)_L \otimes U(1)_Y$, but not in $SU(3)_c$ [gluons are vectors!]

# Anomaly Cancellation

## Condition:

$$tr\left(\{\tau_i, \tau_j\} \cdot \tau_k\right) = 0, \qquad \tau_i = \sigma_i, Y$$

The only a bit more non-trivial relation:

$$\begin{aligned} tr(Y) &= \sum_i Y_i = \sum_{\text{leptons}} Y + \sum_{\text{quarks}} Y = 0 \\ &= \{-1 - 2\}_l + N_c \left\{\tfrac{1}{3} + \tfrac{4}{3} - \tfrac{2}{3}\right\}_q = -3 + N_c \end{aligned}$$

$\Longrightarrow$ Charge Quantization Condition; family for family.

# The Higgs Field

Which representation has the Higgs field ?

Isospin $T$ - hypercharge $Y$ relation to maintain the $\rho$-parameter $= 1$ :

$$T = \frac{1}{2}\left[\sqrt{1+3Y^2} - 1\right]$$

$$\textit{doublet} \Longrightarrow \begin{array}{rcl} (T,Y) & = & \left(\frac{1}{2}, 1\right) \\ (T,Y) & = & (3,4) \\ (T,Y) & = & \left(\frac{25}{2}, 15\right) \ldots. \end{array}$$

More doublets are possible as well, with the same hypercharge.

## Higgs potential:

$V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2, \mu^2, \lambda > 0;$ $\Phi$ complex doublet

$$\langle\Phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

# The Boson and Fermion Masses

$$\mathcal{L}_Y = f^{(e)}\bar{l}_L \; \Phi \; e_R + f^{(\nu)}\bar{l}_L \; i\sigma_2 \; \Phi \; \nu_R + h.c.$$

$$m_f = \frac{f^{(f)}}{\sqrt{2}} v$$

$$\mathcal{L}^{\mathrm{vector}}_{\mathrm{mass}} = \frac{v^2}{8}\left\{ g^2 \left[ (B'^1_\mu)^2 + (B'^1_\mu)^2 \right] + (gB'^3_\mu - g'\mathcal{A}'_\mu)^2 \right\}$$

$$W^\pm_\mu = \tfrac{1}{2}\left(B'^1_\mu \mp iB'^2_\mu\right)$$

$$\frac{v^2}{8}(gB'^3_\mu - g'\mathcal{A}'_\mu)^2 = \tfrac{1}{2}(Z_\mu, A_\mu)\begin{pmatrix} M_Z^2 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}, \qquad \frac{g'}{g} = \tan\theta_W$$

$$M_W^2 = \frac{g^2}{4}v^2, \qquad M_Z^2 = \frac{g^2 + g'^2}{4}v^2, \qquad M_A^2 = 0, \quad \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \rho = 1.$$

$$m_H = \sqrt{-2\mu^2} \text{ not predicted.}$$

# Puzzles of the Standard Model

- Number of parameters: $g', g, g_s$, 4 fermion masses per family, $M_Z$, bottom quark and $\nu_k$-mixing matrices
- Why are there 3 forces of arbitrary strength ?
- Why have fermion representations this special form ?
- The nature of the Higgs field ?
- Quarks and leptons appear widely unrelated, except for anomaly cancellation
- Charge quantization: tighter embedded ?
- Why are there just 3 families ?
- Strong CP problem ?
- Size of baryon asymmetry ?

Various of these aspects will play a role in constructing GUTs of a special type.

# 3. $SU(5)$ Unification

## Goals:

- Unite the 3 sub-atomic forces
- Unite fermions (quarks and leptons)
- Give a more strict reason for charge quantization
- Reduce the number of parameters of the SM
- Design minimal extensions, whenever possible
- Try to avoid the introduction of fields without special purpose

## Principal Procedure:

Embed the SM group representations [matrices] in a certain way into (somewhat) larger ones, again represented by matrices.

# The $SU(5)$ Matrices

$N_c^2 - 1 = 24$ matrices $=$ Group Generators $\equiv$ gauge fields. [rank 4]

$$\Lambda_i|_{i=1}^{8} = \begin{pmatrix} & & & 0 & 0 \\ & \lambda_i & & 0 & 0 \\ & & & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \ldots\Lambda_{20} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & i & 0 & 0 \end{pmatrix}$$

$$\Lambda_i|_{i=20+k}^{23} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & & \\ 0 & 0 & 0 & & \sigma_k \end{pmatrix}, Y = \begin{pmatrix} -\frac{2}{3} & & & & \\ & -\frac{2}{3} & & 0 & \\ & & -\frac{2}{3} & & \\ & & & 1 & \\ & 0 & & & 1 \end{pmatrix}$$

$\frac{\sqrt{15}}{3}\Lambda_{24} = Y$

Only $SU(5)$ and $SU(3) \otimes SU(3)$ are rank 4 and have complex representations, but $SU(3) \otimes SU(3)$ cannot accommodate the necessary number of fermions.

# Charge Quantization in $SU(5)$

$$Q = T_3 + \frac{Y}{2} \equiv \tfrac{1}{2}\Lambda_{23} + \tfrac{1}{2}\,Y$$

$$Q = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} -\frac{1}{3} & & & & \\ & -\frac{1}{3} & & 0 & \\ & & -\frac{1}{3} & & \\ & & & \frac{1}{2} & \\ & 0 & & & \frac{1}{2} \end{pmatrix}$$

$$tr(Q) = 3Q_d + Q_{e^+} = 0$$

# The Gauge Field Assignments

$[SU(3), SU(2)]$

$$\begin{array}{rll} (8,1) & A_i, & \text{gluons} \\ (1,3) & W^+, W^-, Z^0 & \\ (1,1) & \mathcal{A}, & \text{hypercharge field} \\ (3,2) & X^{-4/3}, Y^{-1/3} & \\ (3^*,2) & X^{4/3}, Y^{1/3} & \end{array}$$

$$V\sqrt{2} = \begin{pmatrix} & & & X_1 & Y_1 \\ & A_i & & X_2 & Y_2 \\ & & & X_3 & Y_3 \\ X_{\bar{1}} & X_{\bar{2}} & X_{\bar{3}} & \frac{1}{\sqrt{2}}W_3 & W^+ \\ Y_{\bar{1}} & Y_{\bar{2}} & Y_{\bar{3}} & W^- & -\frac{1}{\sqrt{2}}W_3 \end{pmatrix} + \frac{\mathcal{A}}{\sqrt{30}} \begin{pmatrix} -2 & & & & \\ & -2 & & 0 & \\ & & -2 & & \\ & & & 3 & \\ & 0 & & & 3 \end{pmatrix}$$

$$gV_\mu = g\sum_{a=1}^{24} \frac{1}{2}\Lambda_a V_\mu^a$$

# Fermion Representation in $SU(5)$

The fermions, except $\nu_R$, are accommodated in a $\mathbf{5}^* \oplus \mathbf{10}$ representation :

$$5^* = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu_e \end{pmatrix}_L , \qquad 10 = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{pmatrix}_L$$

This representation is anomaly free.

$$\frac{A(5^*)}{A(10)} = \frac{tr[Q^3(\psi_i)]}{tr[Q^3(\psi_{kl})]} = \frac{3\left(\frac{1}{3}\right)^3 + (-1)^3 + 0}{3\left(-\frac{2}{3}\right)^3 + 3\left(\frac{2}{3}\right)^3 + 3\left(-\frac{1}{3}\right)^3 + 1^3} = -1$$

$$A(5^*) + A(10) = 0$$

# Symmetry Breaking of $SU(5)$

## The Higgs Potential:

$$V(H,\Phi) = V(H) + V(\Phi) + V_1(H,\Phi)$$
$$V(H) = -m_1^2 tr(H^2) + \lambda_1 (tr(H^2))^2 + \lambda_2 tr(H^4)$$
$$V(\Phi) = -m_2^2 (\Phi^\dagger \Phi) + \lambda_3 (\Phi^\dagger \Phi)^2$$

$H$ is a 5x5 matrix (with 24 elements); $\Phi$ is a complex 5-vector.

$$SU(5) \quad \xrightarrow{\text{real } \mathbf{24}\ \langle H \rangle} \quad SU(3) \otimes SU(2) \otimes U(1)$$

$$[\Lambda_i, \langle H \rangle] = 0 \quad i \in \{1, ..., 8\}$$
$$[\Lambda_i, \langle H \rangle] = 0 \quad i \in \{21, ..., 23\}$$
$$[\Lambda_{24}, \langle H \rangle] = 0 \quad \text{since} \quad \frac{1}{v_1} \langle H \rangle = \sqrt{15} \Lambda_{24}$$

Consequences:

Masses to: 12 lepto- and di-quarks $X$ and $Y$; and 12 H-Higgses.

The heavy gauge and Higgs boson masses:

$$M_X^2 = M_Y^2 = \frac{25}{2} g_5^2 v_1^2$$

$$M_{H_8}^2 = 20\lambda_2 v_2^2, \qquad M_{H_3}^2 = 80\lambda_2 v_2^2, \qquad M_{H_0}^2 = 8v_1^2(30\lambda_1 + 7\lambda_2)$$

$$SU(3) \otimes SU(2) \otimes U(1) \xrightarrow{\text{complex } \mathbf{5}\ \langle\Phi\rangle} SU(3)_c \otimes U(1)_{\rm em}$$

$$\langle \Phi_5 \rangle_0 = \frac{v_5}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Consequences: SSB similar to SM: $v_5 \simeq 246$GeV. However, 6 $h^{\pm 1/3}$ Higgses. How to make them heavy ? $\Longrightarrow$ deteriorate the lower square in $\langle H \rangle$ by $\pm\varepsilon$ through **fine tuning.**

## The Fermion Masses:

The real Higgs **24**-plet cannot participate in giving the fermions masses, since it would have to occur in the $\overline{L}R$ representations:

$$\mathbf{5}^* \otimes \mathbf{10} = \mathbf{5} \oplus \mathbf{45}$$
$$\mathbf{10} \otimes \mathbf{10} = \mathbf{5}^* \oplus \mathbf{45}^* \oplus \mathbf{50}^*,$$

which is not the case.

$$\mathcal{L}_Y = G_d \overline{\psi}^c_{j,L} \psi^{j,k}_L \Phi^\dagger_k + G_u \varepsilon_{jklmn} \overline{\psi}^{c,j,k}_L \psi^{l,km}_L \Phi^n + h.c.$$

Reduce this expression :

$$\mathcal{L}_d = -\frac{G_d v_5}{\sqrt{2}} \left( \overline{d} d + \overline{e} e \right), \quad \mathcal{L}_u = -\frac{G_d v_5}{\sqrt{2}} \overline{u} u$$

$m_d = m_e$ in SU(5); the $m_{u_i}$ are free parameters.
Only the complex Higgs **5**-plet produces the fermion masses;
here: $v_5 \simeq 246$ GeV.

# The Predictions

- Unification of forces [coupling constants].
- Running of $\sin^2\theta_W$
- The value of the GUT scale
- Relations between down-fermion masses
- Proton decay
- $N - \overline{N}$ oscillations

# The Scale Evolution of the Coupling Constants

Evolution equations: $[a_i = \alpha_i/(4\pi)]$

$$\frac{da_i}{d\ln\mu^2} = -\sum_{k=0}^{\infty} \beta_{i,k} a_i^{k+2}$$

$$\frac{1}{a_i(\mu^2)} - \frac{1}{a_i(\mu_0^2)} \approx \beta_{i,0} \ln\left(\frac{\mu^2}{\mu_0^2}\right), \quad i = 1, 2, 3.$$

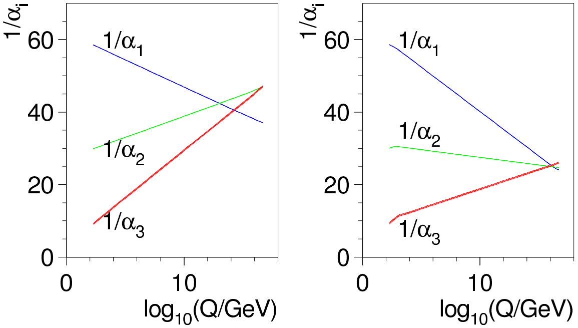
$$\beta_{SU(3),0} = 11 - \frac{2}{3} N_F \qquad \text{for} \quad \mu < M_X$$

$$\beta_{SU(2),0} = \frac{22}{3} - \frac{2}{3} N_F \qquad \text{for} \quad \mu < M_X$$

$$\beta_{U(1),0} = -\frac{2}{3} N_F \qquad \text{for} \quad \mu < M_X$$

$$\beta_{SU(5),0} = \frac{55}{3} - \frac{2}{3} N_F \qquad \text{for} \quad \mu > M_X$$

# The Scale Evolution of the Coupling Constants




The scale evolution of the coupling constants in SU(5) and MSSM SU(5);

De Boer, Sander, PL B485 (2004) 276.

One may adjust the couplings in one point within $SU(5)$ adding either a Higgs **15** plet or a Fermion **24** plet. The latter allows also to introduce the see-saw mechanism to generate $\nu$-masses.

Dosner and Fileviez Perez (2005,06); Di Luzio and Mihaila (2013).

# The Weak Mixing Angle at the GUT Scale

$$
\begin{aligned}
i g_5 \Lambda_{24} A^0_\mu &= i g' Y B_\mu \\
g_5 &\equiv g_1 = g_2 = g_s \\
g' &= \sqrt{\frac{3}{5}} g_5 \\
\sin^2 \theta_W^{\mathrm{GUT}} &= \frac{(g')^2}{g_2^2 + (g')^2} = \frac{3}{8} = 0.375.
\end{aligned}
$$

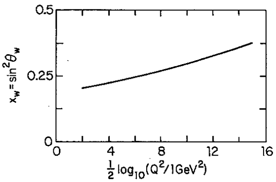



FIG. 9-4. Evolution of the $\gamma - Z^0$ mixing parameter in the $SU(5)$ model (same assumptions as for Fig. 9-3).

Courtesy C. Quigg.

$\sin^2 \theta_W^{\mathrm{GUT}}$ runs down form $M_X$ for the following reason:

# Running couplings down to accessible measurements

$$\sin^2\theta_W = \frac{1}{6} + \frac{5}{9}\frac{\alpha}{\alpha_s} \quad \Longrightarrow \quad \sin^2\theta_W \in [0.167, 0.375] \quad \sin^2\theta_W^{\mathrm{exp}} = 0.23126(5)$$

Impact of Higgs bosons on running :

$$\begin{aligned} \sin^2\theta_W &= \frac{3}{8}\left[1 - \frac{\alpha}{4\pi}\left(\frac{110}{9} - \frac{n_H}{9}\right)\ln\left(\frac{M_{\mathrm{GUT}}^2}{\mu^2}\right)\right] \\ \frac{\alpha}{\alpha_s} &= \frac{3}{8}\left[1 - \frac{\alpha}{4\pi}\left(22 + \frac{n_H}{3}\right)\ln\left(\frac{M_{\mathrm{GUT}}^2}{\mu^2}\right)\right] \end{aligned}$$

Choosing: $\mu = M_Z = 91.19\mathrm{GeV}, \alpha \approx 1/128.6, \alpha_s \approx 0.118, n_H \approx 0$ one obtains

$$M_{\mathrm{GUT}} \approx 1.28 \cdot 10^{15}\mathrm{GeV}.$$

# 5* Mass Ratios

$\nu_L$ is strictly massless due to the missing $\nu_R$.
No prediction for up particles.

$$m_d = m_e \equiv G_d^{(1)} \frac{v_5}{\sqrt{2}}, \quad m_s = m_\mu \equiv G_d^{(2)} \frac{v_5}{\sqrt{2}}, \quad m_b = m_\tau \equiv G_d^{(3)} \frac{v_5}{\sqrt{2}}$$

$$R = \frac{m_d(\mu)}{m_e(\mu)} = \left(\frac{\alpha_s(\mu)}{\alpha_s(M_{\mathrm{GUT}})}\right)^{4/(11-\frac{2}{3}N_F)} \left(\frac{\alpha(\mu)}{\alpha_s(M_{\mathrm{GUT}})}\right)^{3/(2N_F)}$$

Example: $\mu = 10$ GeV, $M_{\mathrm{GUT}} \approx 10^{14}$ GeV :

$$\begin{aligned} \frac{m_b}{m_\tau} &\approx 2.353 \quad [exp: 2.398] \\ \frac{m_s}{m_\mu} &= \sim 1[exp] \qquad [deviations] \\ \frac{m_d}{m_e} &= 9.6[exp] \qquad [deviations] \end{aligned}$$

The inclusion of an additional Higgs **45**-plet implies ...

$$\frac{m_d}{m_e} = \frac{9 m_s}{m_\mu}, \qquad 9.393 \approx 8.066$$

... and one may tune, and tune, ...

# Nucleon Decay Reactions

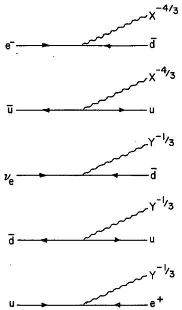



FIG. 9-1. New fermion–fermion transitions that appear in the *SU*(5) unified the

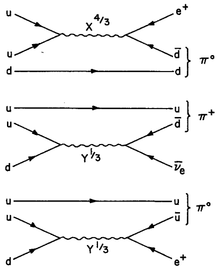



FIG. 9-2. Some mechanisms for proton decay in the *SU*(5) model of unification.

Courtesy C. Quigg

The presence of di-quarks, leptoquarks and color-triplet Higgses $h^{1/3}$ allow the decay of the lowest lying baryon, the proton (and the neutron).

Examples:

$$p \to e^+\pi^0, \quad p \to \bar{\nu}_\mu K^+, \quad n \to e + \rho^-, \ldots.$$

$$\tau_p = \tau_\mu \left(\frac{m_\mu}{m_p}\right)^5 \left(\frac{M_{\mathrm{GUT}}}{M_W}\right)^4 > 10^{34}\ yr, \quad \text{Kamioka [2009]}$$

# $N$-$\overline{N}$ Oscillation

Not in minimal $SU(5)$ SSB. Additional **15** Higgs $S_{pq}$ needed.

$$\mathcal{L}_Y = h_{15}\psi^T_{p,R}C^{-1}\psi_{q,R}S^\dagger_{pq} + \lambda_{15}M_{15}H^\dagger_p H_q S_{pq} + h.c.$$

$$N = |ddu\rangle, \qquad \overline{N} = |\bar{d}\bar{d}\bar{u}\rangle$$

$$\frac{\partial}{\partial t}\begin{pmatrix} N \\ \overline{N} \end{pmatrix} = \begin{pmatrix} E & \delta m \\ \delta m & E \end{pmatrix}\begin{pmatrix} N \\ \overline{N} \end{pmatrix}$$

Effect: $N + \overline{N} \to X$

$$A_{N\leftrightarrow\overline{N}} \lesssim 10^{-58} \quad \text{GeV}^{-5}$$

# Pro's and Con's for $SU(5)$

## Pro's

- $SU(5) \supset SU(3)_c \otimes SU(2)_{2L} \otimes U(1)_Y$
- 1 gauge coupling
- $\sin^2 \theta_W^{\mathrm{GUT}} = \frac{3}{8}$
- Correct implementation of charged current interactions
- Quantization of the electric charge [realized in the SM already by anomaly cancellation.]
- $m_b/m_\tau$ comes out about correct
- possible $p$ decay (not yet observed)

# Pro's and Con's for $SU(5)$

## Con's

- No insight into the mass and mixing patterns
- No essential reduction of the number of SM parameters
- Reducible Fermion Representations $5^* \oplus 10$
- Unifies only 3 out of 4 forces
- Desert between $M_Z$ and $M_X$
- Quite a number of new Higgs-boson emerges Growing even further in case of larger GUTs

# 4. Aspects of $SO(10)$

Why next $SO(10)$ ?

$SO(10)$ is the smallest covering group of rank 4 having complex representations [needed!]

$$\begin{aligned} SO(10) &\supset SU(5) \otimes U(1) \rightarrow Z' \\ &\supset SU(4) \otimes SU(2)_R \otimes SU(2)_L \end{aligned}$$

Possible fermion multiplets: $10 = 5 \oplus \bar{5}$, $\quad 16 = 10 \oplus \bar{5} \oplus 1$, ....

• The $\nu_R$ finds a place in a multiplet, uniting all other particles of one family.

$$m_e = m_d\,, \qquad m_u = m_\nu^{\mathrm{Dirac}}$$

A **126**-plet may introduce a Majorana mass and allow for the see-saw mechanism.

• No family unification yet.

# Symmetry Breaking of $SO(10)$

Higgs
$\downarrow$

$$
\begin{aligned}
SO(10) &\xrightarrow{16} SU(5) \otimes U(1) \\
SU(5) &\xrightarrow{45} SU(3) \otimes SU(2) \otimes U(1)
\end{aligned}
$$

$$
\begin{aligned}
SO(10) &\xrightarrow{54} SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \\
&\xrightarrow{45} SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \\
&\xrightarrow{16} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \\
&\xrightarrow{10} SU(3)_c \otimes U(1)_{\mathrm{em}}
\end{aligned}
$$

# New Phenomenology

- $SO(10)$ contains a new $Z'$.
- $\tau_{p,10} = \tau_{p,5} \left( \frac{M_5}{M_{R^+}} \right)^2$, $M_{R^+}$ is an intermediate scale
  $\tau_{p,10} > \tau_{p,5}$
- $\Delta \sin^2 \theta_W = \sin^2 \theta_{10,W} - \sin^2 \theta_{5,W} = \frac{11}{6} a(M_W) \ln \left[ \frac{M_U^2 M_5^2}{M_C^2 M_{R_+}^2} \right]$;
  $M_u \equiv M_{SO(10)}, M_C \approx M_U$
- Also modifications in the running of the other couplings.

## Even further extensions:

- $E_6 \to SO(10) \to SU(5) \to SU(3) \otimes SU(2) \otimes U(1)$
- $E_8 \to SO(16) \to SO(10) \to .....$
- SUSY extensions [of everything]
- superstring models, ....
- Theory of Everything **(NO!)**

# 5. Conclusions

- There are interesting unification scenarios, but they are not 100% convincing.
- The couplings have a tendency to cross at large scales in a small domain of scales.
- To accommodate the SM also the righthanded neutrino needs a place [not in minimal SU(5)].
- There are simple extensions of SU(5), unifying the 3 sub-atomic forces.
- Mass ratio predictions are still a problem.
- Do we need higher groups to just unify the SM ?
- Do we need supersymmetry for unification ?
- Many tunes can be performed asking for various additional large Higgs multiplets; they partly will need fine tuning as well to end up with heavy enough states.
- There is not yet any compelling experimental observation that the SM unifies into a larger gauge group (with corresponding matter representations).
- It may very well be, that the parameter reduction in the SM proceeds along quite different avenues.