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On the Resummation of small x Contributions to Unpolarized and Polarized Non–Singlet and Singlet Structure Functions

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1. Introduction

$x \to 0$:

SINGULARITIES IN THE N-PLANE

$\sim \left(\frac{\alpha}{N-1}\right)^k$ $\sim N \left(\frac{\alpha}{N^2}\right)^\ell$ ...

UNPOL. SINGLET — NON SINGLET (POL & UNPOL)
POL. SINGLET

QCD — QCD, QED

DO THEY IMPLY LARGE CORRECTIONS FOR
NS &/OR S STRUCTURE FUNCTIONS BEYOND NLO ?

$\longrightarrow$ NON-PERTURBATIVE INPUT AT Q_0^2 :
$\sim x^{\alpha_i} \ldots$

- WHAT ARE THE EFFECTS ON THE EVOLUTION ?
- FOR A SERIES OF CASES FERMION OR MOMENTUM CONSERVATION HOLDS $\longrightarrow$ $\exists$ SUBLEADING TERMS ! (HOW 'SUB'-LEADING THEY ARE ?)
- WHAT IS CHANGED BEYOND THE KNOWN NLO CONTRIBUTIONS ?
- PREDICTIONS FOR 3-LOOP SPLITTING FUNCTIONS

2. Evolution in fixed order perturbative QED and QCD

THE EVOLUTION EQS:

$$\frac{\partial q_{NS}(x,Q^2)}{\partial \log Q^2} = P^{\pm}_{NS}(x,\alpha) \otimes q_{NS}(x,Q^2)$$

$$\frac{\partial}{\partial \log Q^2} \begin{pmatrix} \Sigma(x,Q^2) \\ G(x,Q^2) \end{pmatrix} = \mathbb{P}^S(x,\alpha) \otimes \begin{pmatrix} \Sigma(x,Q^2) \\ G(x,Q^2) \end{pmatrix}$$

RGE FOR THE COUPLING CONSTANT:

$$\frac{da}{d\log Q^2} = -\sum_{k=0}^{\infty} \beta_k a^{2+k} \quad , \quad a = \frac{\alpha}{4\pi}$$

$$P^{\pm}(x,\alpha) = \sum_{\ell=0}^{\infty} a^{\ell+1} P^{\pm}_{\ell}(x) \; ; \quad \mathbb{P}^S(x,\alpha) = \sum_{\ell=0}^{\infty} a^{\ell+1} \mathbb{P}^S_{\ell}(x)$$

$$\int_0^1 dz \; P^-_{\ell}(z) = 0 \; , \; \forall \ell \quad ; \quad \int_0^1 dz \; z \sum_{p'} P^{S,\text{unp}}_{p'p;\ell}(z) = 0$$

F-number conservation $\qquad$ EM-conservation

$$F^{\pm}_i(x,Q^2) = C^{\pm}_i(x,Q^2) \otimes q^{\pm}_i(x,Q^2)$$

$$F^S_i(x,Q^2) = C^{\Sigma}_i(x,Q^2) \otimes \Sigma(x,Q^2) + C^G_i(x,Q^2) \otimes G(x,Q^2)$$

$$C_i(x,Q^2) = \delta(1-x)\,\delta_q + \sum_{\ell=1}^{\infty} a^{\ell} c_{i\ell}(x)$$

<u>NLO</u>: keep only the terms up to α^2 (α) in the splitting functions P (coefficient functions c), and β_0, β_1 in $da/d\ln Q^2$

<u>NNLO</u>: $P_2(x)$ unknown so far.

SINGULAR TERMS @ $x \to 0$:

UNPOLARIZED	NS :	$\sim N\left(\frac{a}{N^2}\right)^\ell$	$P_i^\pm$	$C_i^\pm$
UNPOLARIZED	S :	$\sim\left(\frac{a}{N-1}\right)^\ell$	$P^s, C_i^{\Sigma,G}$	
POLARIZED	NS :	$\sim N\left(\frac{a}{N^2}\right)^\ell$	$P_i^\pm$	$C_i^\pm$
POLARIZED	S :	$\sim N\left(\frac{a}{N^2}\right)^\ell$	P_i^s	$C_i^{\Sigma,G}$

SUBLEADING IN $\overline{MS}$ UP TO $O(a^2)$.

$$N\left(\frac{a}{N^2}\right)^\ell \longleftrightarrow a\frac{1}{(2\ell-2)!}\left(a\ln^2 x\right)^{\ell-1}$$

$$\left(\frac{a}{N-1}\right)^k \longleftrightarrow a\frac{1}{(k-1)!}\frac{1}{x}\left(a\ln\frac{1}{x}\right)^{k-1}$$

$\longrightarrow$ SUM RULES AS F-NUMBER & ENERGY MOMENTUM CONSERVATION ENFORCE SUBLEADING TERMS !
(ON-DIAGONAL).

3. Resummation of the dominant terms for $x \to 0$

NS RESUMMED KERNELS:

KIRSCHNER, LIPATOV

$$\Gamma^{+,QCD}_{x\to 0}(N,a) = -N\left\{1-\sqrt{1-\frac{8aC_F}{N^2}}\right\}$$

$$\Gamma^{-,QCD}_{x\to 0}(N,a) = -N\left\{1-\sqrt{1-\frac{8aC_F}{N^2}\left[1-\frac{8aC_F}{N}\frac{d}{dN}\ln\left(e^{z^2/4}D_p(z)\right)\right]}\right\}$$

$$\Gamma^{+,QED}_{x\to 0}(N,a) = -N\left\{1-\sqrt{1-\frac{8a}{N^2}}\right\}$$

JB, A. VOGT

$p = \frac{1}{2N_c^2}$

$z = N/\sqrt{2N_c a}$

$$\Gamma^{-,QED}_{x\to 0}(N,a) = -N\left\{1-\sqrt{1+\frac{8a}{N^2}\left[1-\sqrt{1-\frac{8a}{N^2}}\right]}\right\}$$

$F_2^{ep} - F_2^{en} \quad \propto \quad \Gamma^{+,QCD}$

$xF_3^{\nu N} \quad \propto \quad \Gamma^{-,QCD}$

$g_{5,NS}^{\gamma Z} \quad \propto \quad \Gamma^{+,QCD}$

$g_{1,NS}^{i} \quad \propto \quad \Gamma^{-,QCD}$

THE NLO ANOM. DIMS. AGREE WITH THE ACCORDING TERMS IN THE ABOVE RESUMMATIONS IN THEIR 'MOST SINGULAR' TERMS.

3 LOOP ANOM. DIM: $|_{\text{SING.}}$

$$P^{+,QED}_{2,x\to 0,\overline{MS}}(x,a) = \frac{2}{3}a^3\ln^4 x$$

$$P^{-,QED}_{2,x\to 0,\overline{MS}}(x,a) = -\frac{10}{3}a^3\ln^4 x \equiv K^{-,QED}_{2,x\to 0,\underline{OMS}}$$

$$P^{+,QCD}_{2,x\to 0,\overline{MS}}(x,a) = \frac{2}{3}C_F^3 a^3\ln^4 x$$

$$P^{-,QCD}_{2,x\to 0,\overline{MS}}(x,a) = \left(-\frac{10}{3}C_F^3 + 4C_F^2C_G - C_FC_G^2\right)a^3\ln^4 x$$

SINGLET RESUMMATION:

- DETAILS ARE WELL-KNOWN → UNPOLARIZED CASE — LIPATOV et al. LO; CATANI, HAUTMANN NLO$_q$
- POLARIZED CASE: BARTELS, ERMOLAEV, RYSKIN

$$\mathbb{F}_0(N,a) = 16\pi^2 \frac{a}{N}\mathbb{M}_0 - 8\frac{a}{N^2}\mathbb{F}_8(N,a)\mathbb{G}_0 + \frac{1}{8\pi^2}\frac{1}{N}\mathbb{F}_0^2(N,a)$$

$$\mathbb{F}_8(N,a) = 16\pi^2 \frac{a}{N}\mathbb{M}_8 + 2\frac{a}{N}C_A\frac{d}{dN}\mathbb{F}_8(a,N) + \frac{1}{8\pi^2 N}\mathbb{F}_8^2(N,a)$$

$$\mathbb{M}_0 = \begin{pmatrix} C_F & -2T_f N_f \\ 2C_F & 4C_A \end{pmatrix} \qquad \mathbb{M}_8 = \begin{pmatrix} C_F - C_A/2 & -T_f N_f \\ C_A & 2C_A \end{pmatrix}$$

$$\mathbb{G}_0 = \begin{pmatrix} C_F & 0 \\ 0 & C_A \end{pmatrix}.$$

- SOLVE FOR THE ANOMALOUS DIM. MATRIX.

$$\Gamma_{S,pol}(N,a) = -\frac{1}{4\pi^2}\mathbb{F}_0(N,a)$$

→ U-MATRIX IN EVOLUTION, (S).

- agrees with the 'singular' parts of $\mathbb{P}_0^{S,pol}$, $\mathbb{P}_1^{S,pol}$ $(\overline{MS})$
- again $\mathbb{P}^{S,pol}_{2,x\to 0}(x)_{\overline{MS}}$ can be derived (c_2 behaviour)

$$\mathbb{P}_S^2 = \begin{cases} P^2_{qq,x\to 0}(N) = \frac{16}{N^5}C_F\left[-5C_F^2 - 8C_F T_F N_f - 6C_A T_f N_f + 6C_A C_F - \frac{3}{2}C_A^2\right] \\ P^2_{qg,x\to 0}(N) = \frac{16}{N^5}T_f N_f\left[2C_F^2 + 8C_F T_f N_f - 6C_A C_F - 15C_A^2\right] \\ P^2_{gq,x\to 0}(N) = \frac{16}{N^5}C_F\left[-2C_F^2 - 8C_F T_f N_f + 6C_A C_F + 15C_A^2\right] \\ P^2_{gg,x\to 0}(N) = \frac{16}{\ }\left[-4C_F^2 T_f N_f - 24C_A C_F T_f + 2C_A^2 T_f + 28C_A^3\right] \end{cases}$$

4. Numerical results

- UNPOLARIZED NS · REALISTIC INPUTS — JB, A.VOGT PLB
  S · MOM. SR, F-SR. — JB, A.VOGT, S.RIEMERSMA

- POLARIZED NS · COMPARE FOR SOME INPUTS — JB, A.VOGT PLB & DESY 96-041
  S · ΔG ! — JB, A.VOGT DESY 96-050
  F-SR.

- QED : ISR (NS) @ HERA. JB, A.VOGT, S.RIEMERSMA

- HOW IMPORTANT (NOT YET KNOWN) SUBLEADING TERMS CAN BECOME ?

SUM RULES:

A : $\Gamma(N,a) \rightarrow \Gamma(N,a) - \Gamma(1,a)$

B : $\Gamma(N,a) \rightarrow \Gamma(N,a)\,(1-N)$

C : $\Gamma(N,a) \rightarrow \Gamma(N,a)\,(1-2N+N^2)$

D : $\Gamma(N,a) \rightarrow \Gamma(N,a)\,(1-2N+N^3)$

$\Gamma \equiv \Gamma^-, \Gamma^S$

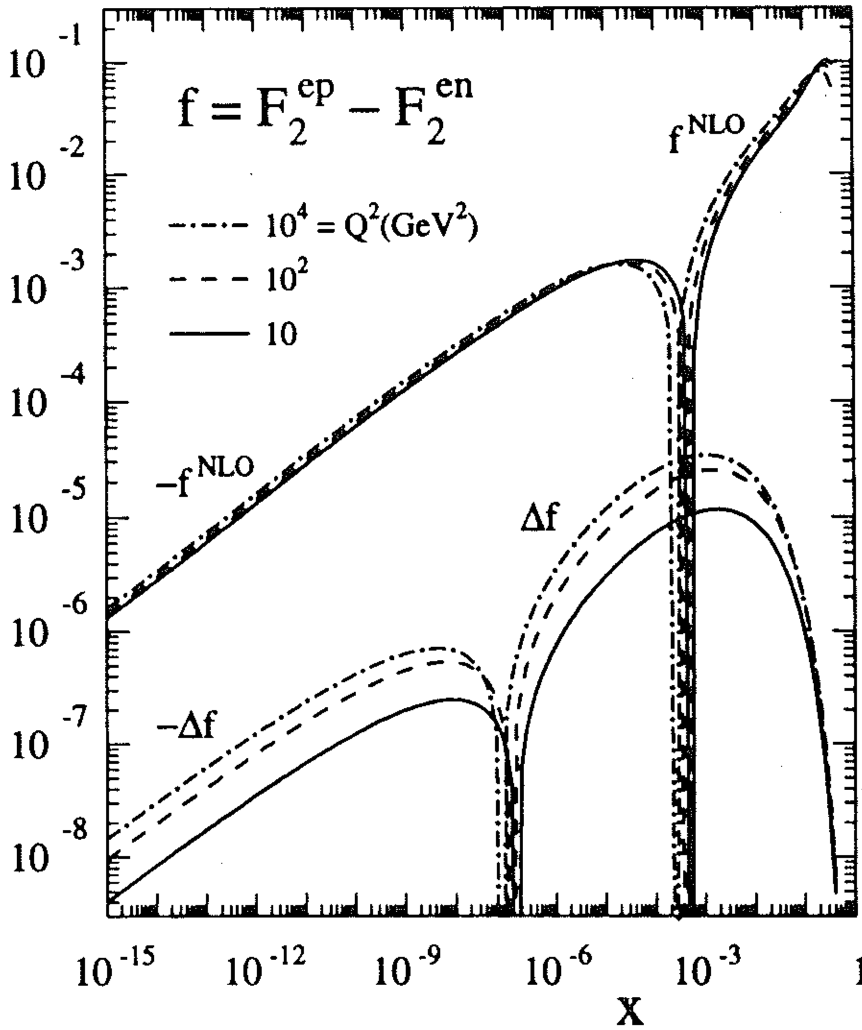
FSR, MSR

/

4.1. UNPOLARIZED NS

Γ^{+}_{QCD} :

BLÜMLEIN, VOGT
PHYS. LETT. B370 (1996) 149
& DESY 96-041



NLO

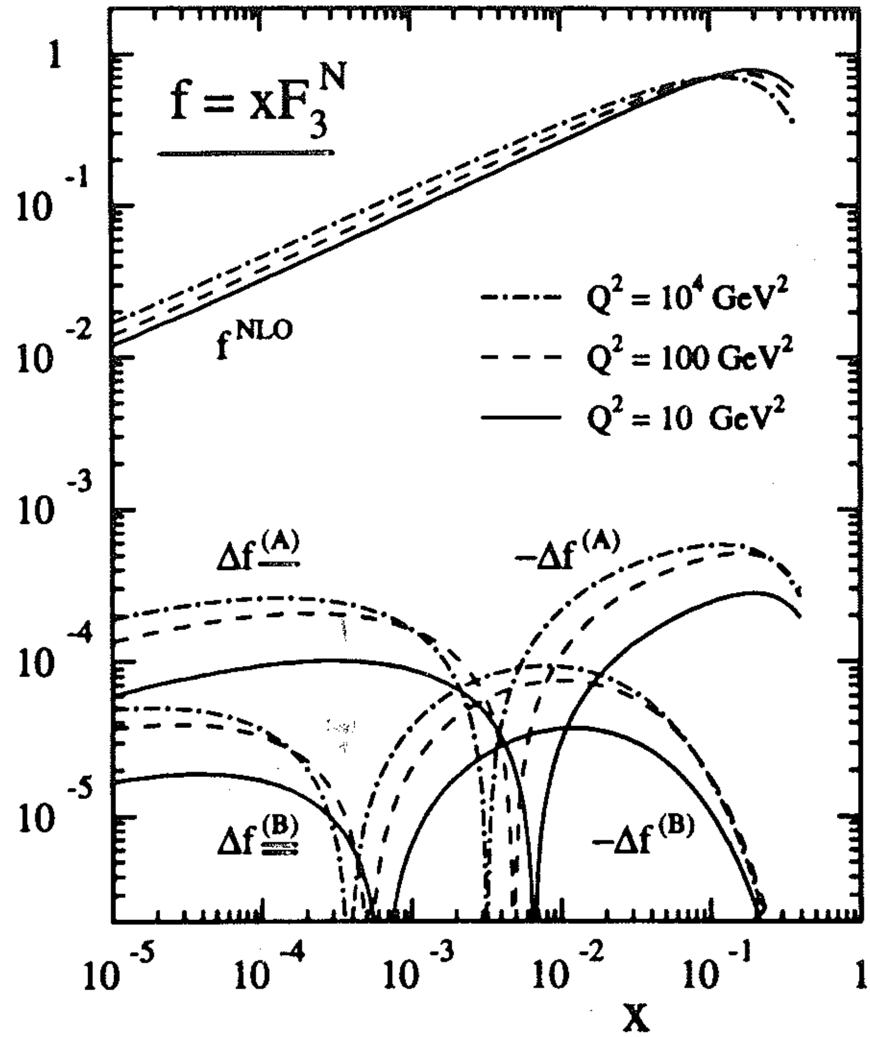
no F-number cons.

CORR: < 0 (1%)

Figure 1: The small-x Q^2–evolution of the unpolarized non–singlet structure function combination $F_2^{ep} - F_2^{en}$ in NLO and the absolute corrections to these results due to the resummed kernel derived from ref. [3]. The initial distributions at $Q_0^2 = 4$ GeV^2 have been adopted from [16].

8

Γ_{QCD}^{-} :



— TYPE

CORR.

< 0 (1%)

FACTOR 3!

F-NUMBER.

Figure 1: The small-x Q^2–evolution of the non–singlet structure function $xF_3^N \equiv \frac{1}{2}(xF_3^{\nu N} + xF_3^{\bar{\nu} N})$ for an isoscalar target N in NLO and the corrections to these results due to the resummed kernels derived from ref. [7]. 'A' and 'B' denote the two prescriptions for implementing the fermion number conservation discussed in the text.

4.2. Unpolarized Singlet

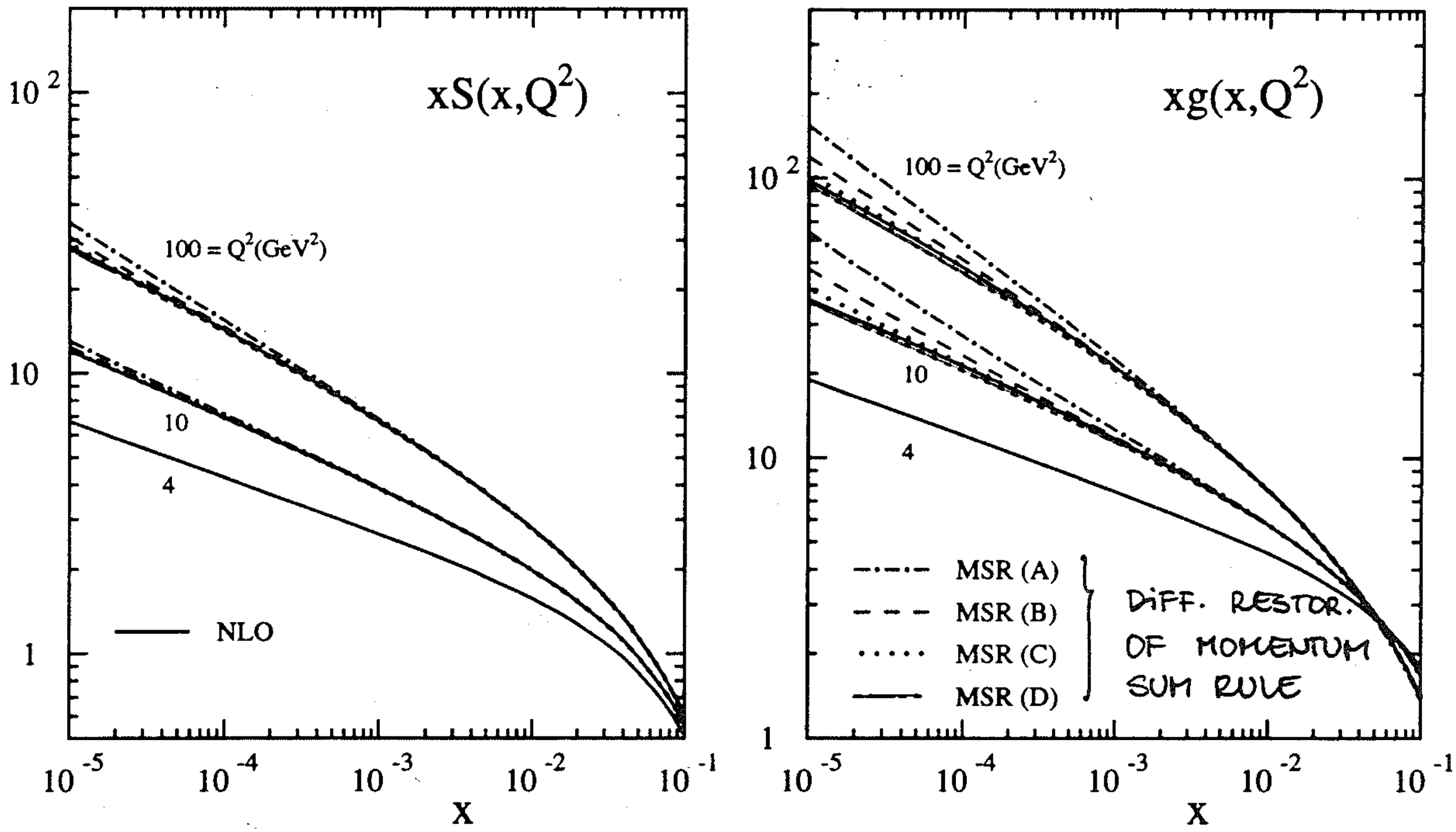
- NUMERICAL UPDATE OF EARLIER INVESTIGATIONS e.g. ELLIS, HAUTMANN, WEBBER;
  - F_2 rises @ $Q_0^2 = 4\,GeV^2$ already

$\longrightarrow$ STUDY OF SUBLEADING TERMS IN MORE DETAIL (WHAT COULD HAPPEN ?)

$\longrightarrow$ SORTING OUT OF <u>ONLY</u> SINGULAR TERMS IN HO.

NLO +
LIP. SERIES ONLY $\left(\frac{\alpha}{N-1}\right)^{\ell}$

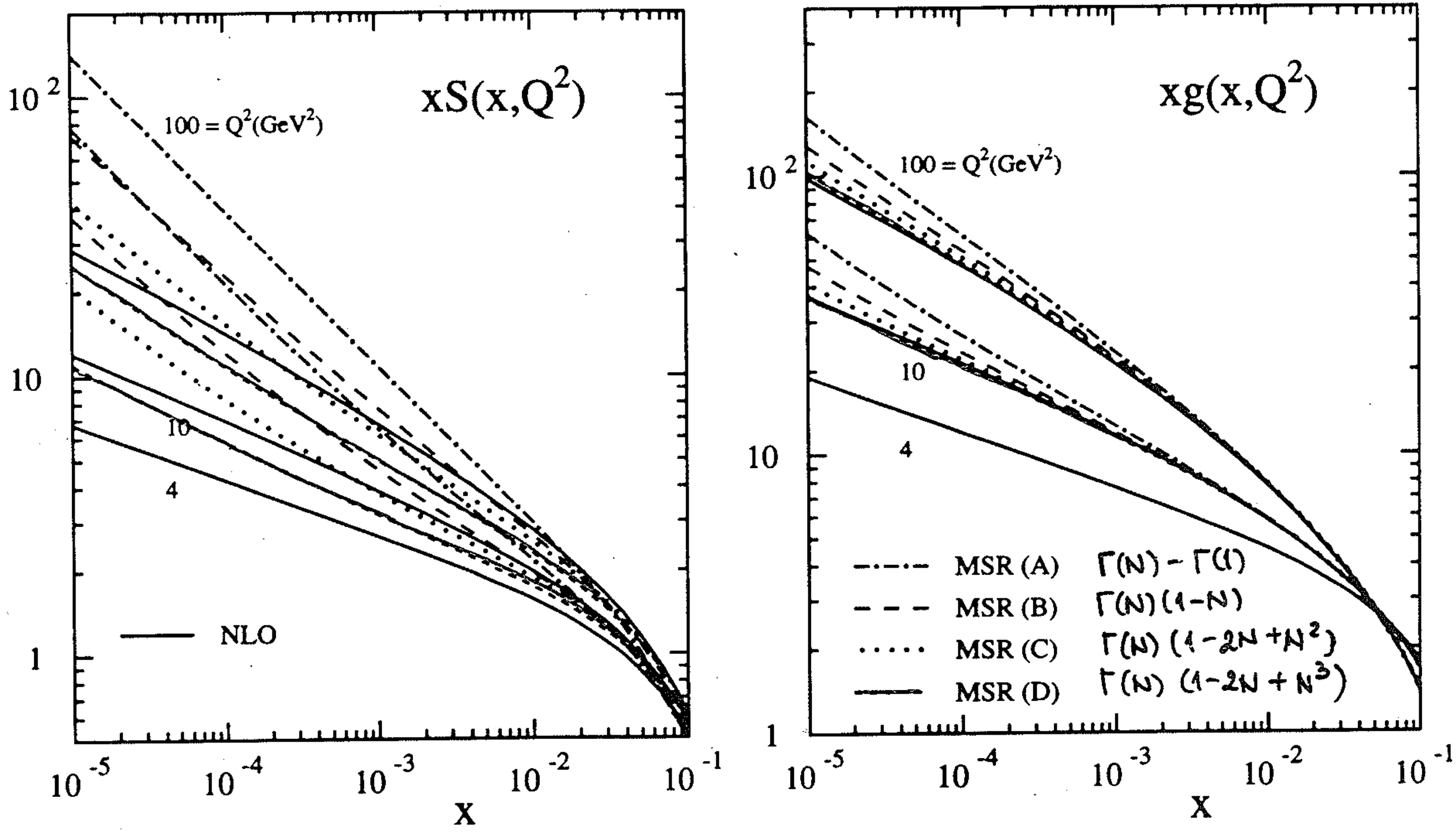
Toy input at $Q_0^2 = 4\ \mathrm{GeV}^2$, f=4, NLO (DIS) + Lx

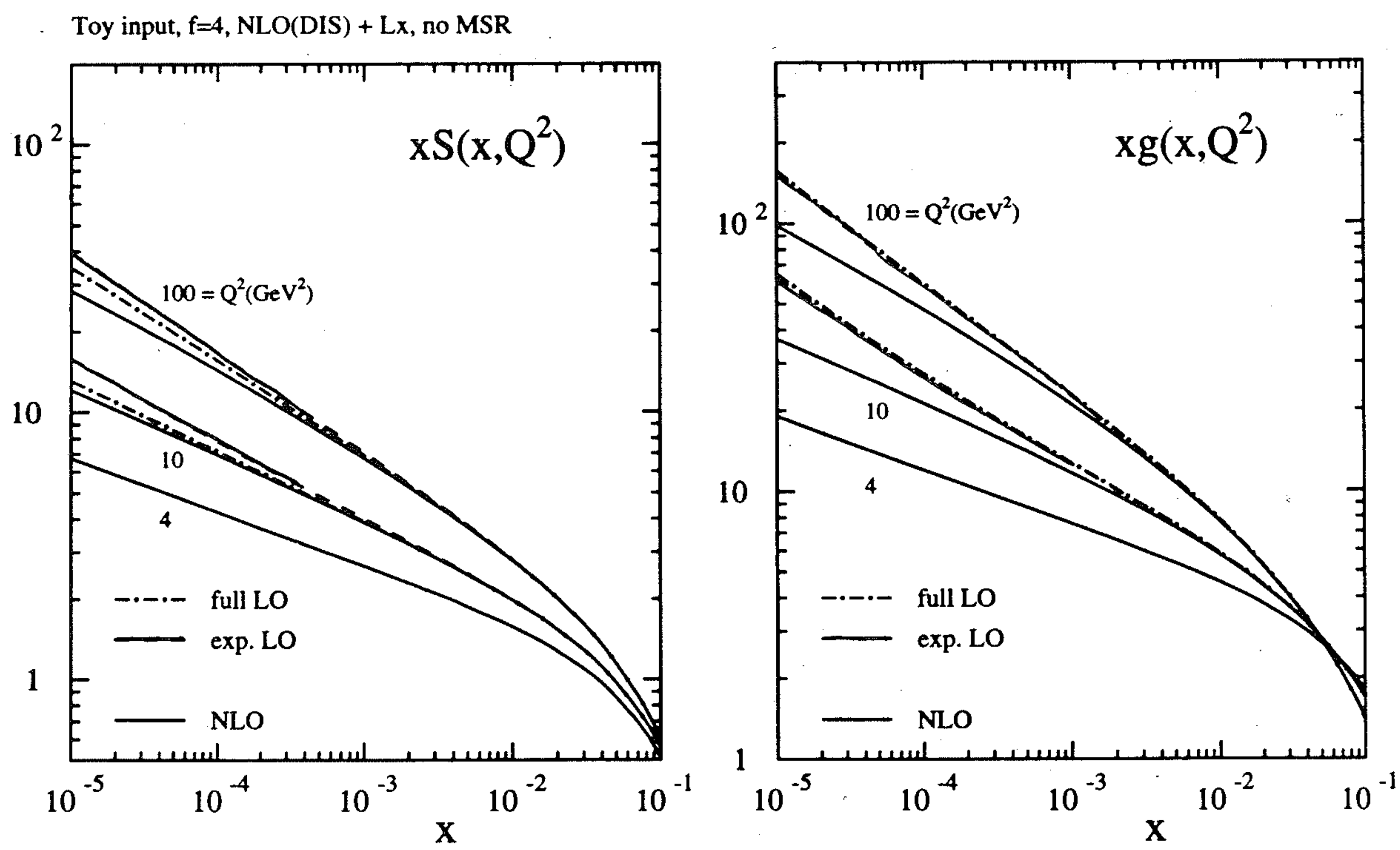


BLÜMLEIN, VOGT, RIEMERSMA

NLO + LIP + NLO SING$_q$ + $\left(\frac{\alpha}{N-1}\right)^{\ell}$ + $\alpha\left(\frac{\alpha}{N-1}\right)^{\ell}_q$

Toy input at $Q_0^2 = 4\ \mathrm{GeV}^2$, f=4, NLO (DIS) + NLx

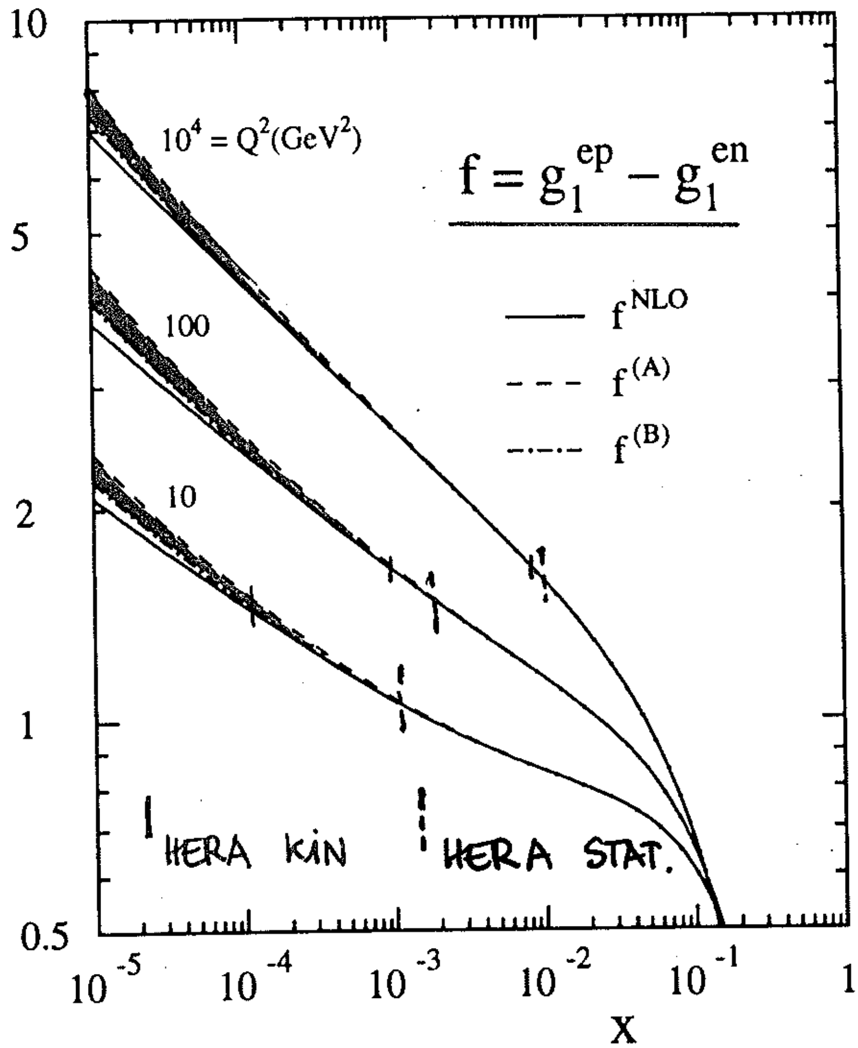




Toy input, f=4, NLO(DIS) + Lx, no MSR
xS(x,Q^2)
100 = Q^2(GeV^2)
10
4
full LO
exp. LO
NLO
10^2
10
1
10^-5
10^-4
10^-3
10^-2
10^-1
x
xg(x,Q^2)
100 = Q^2(GeV^2)
10
4
full LO
exp. LO
NLO
10^2
10
1
10^-5
10^-4
10^-3
10^-2
10^-1
x

4.3. POLARIZED NS

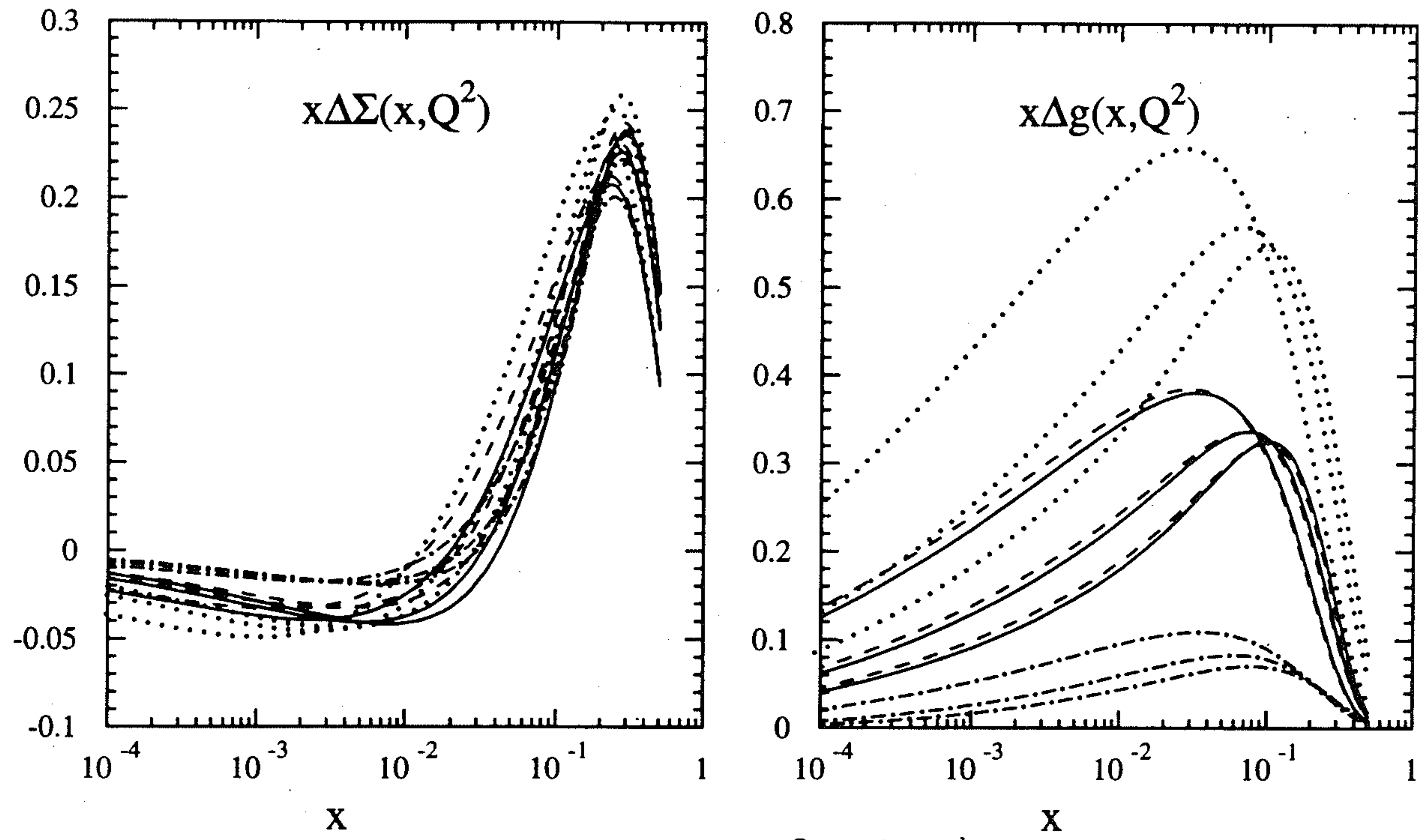
Γ^{-}) QCD



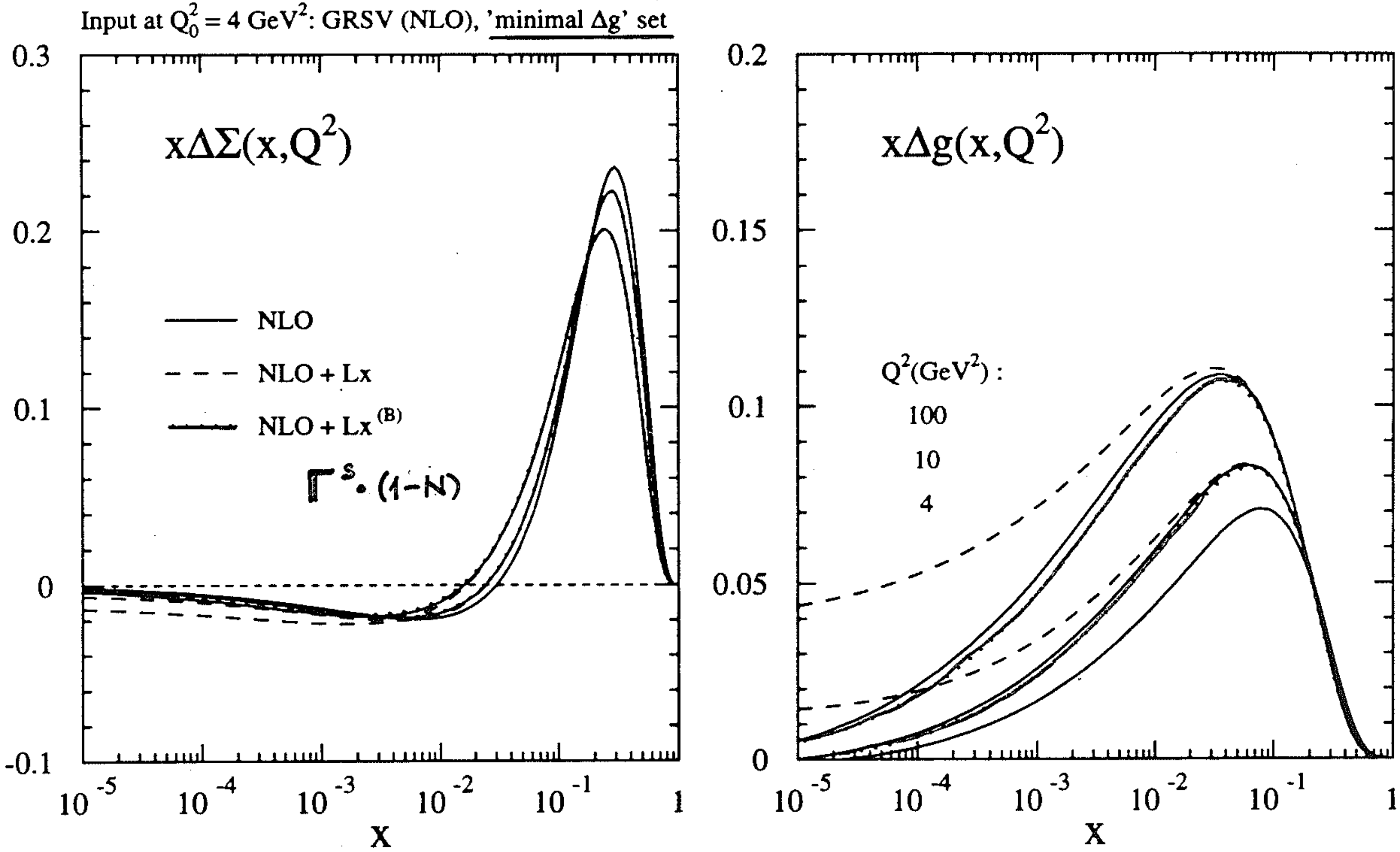
CHENG, WAI.

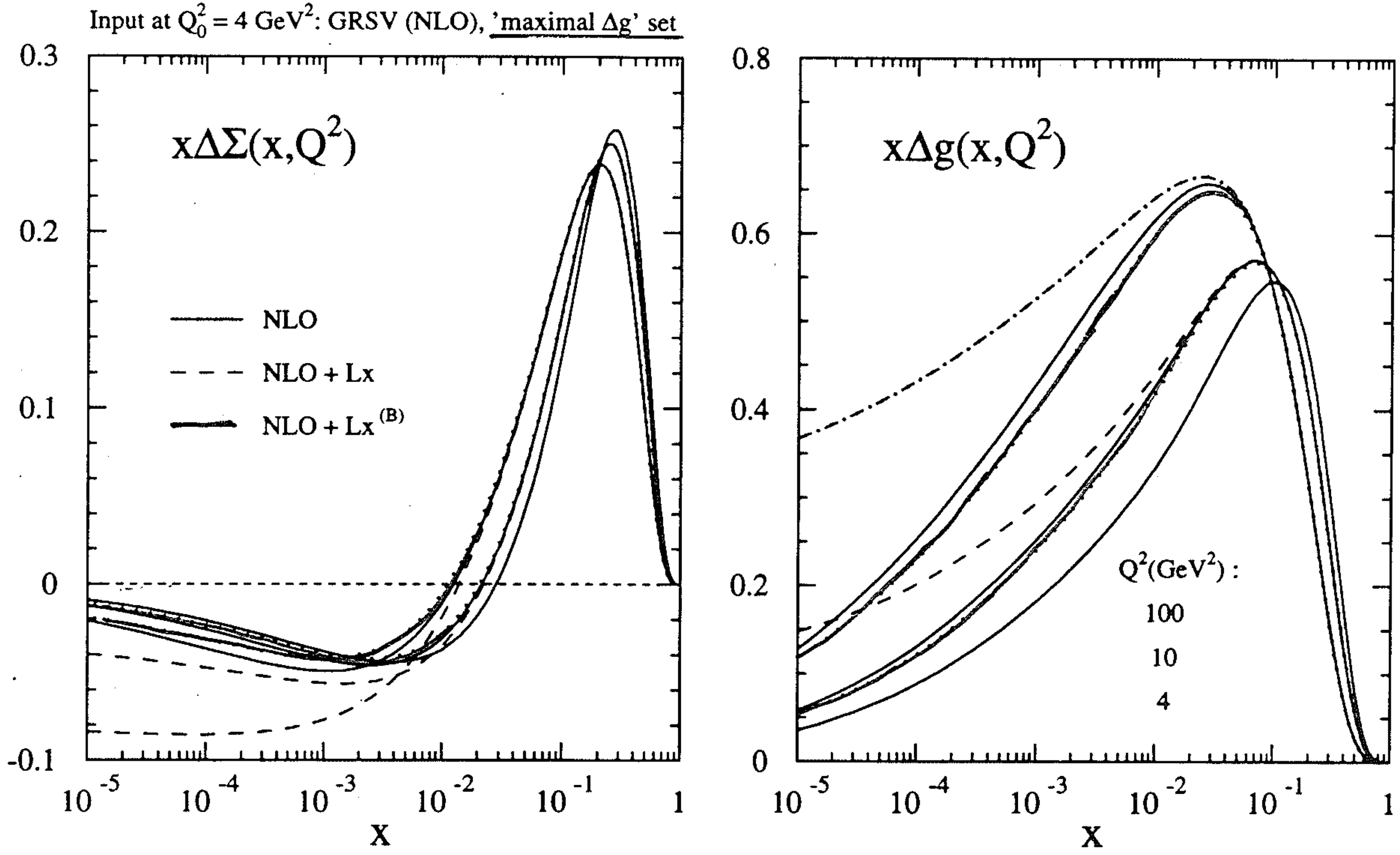
Figure 3: The small-x Q^2–evolution of the non–singlet polarized structure–function difference $g_1^{ep} - g_1^{en}$ in NLO and with the resummed kernels taken into account. Again 'A' and 'B' denote the two prescriptions for implementing the fermion number conservation discussed in the text.

4.4. POLARIZED SINGLET



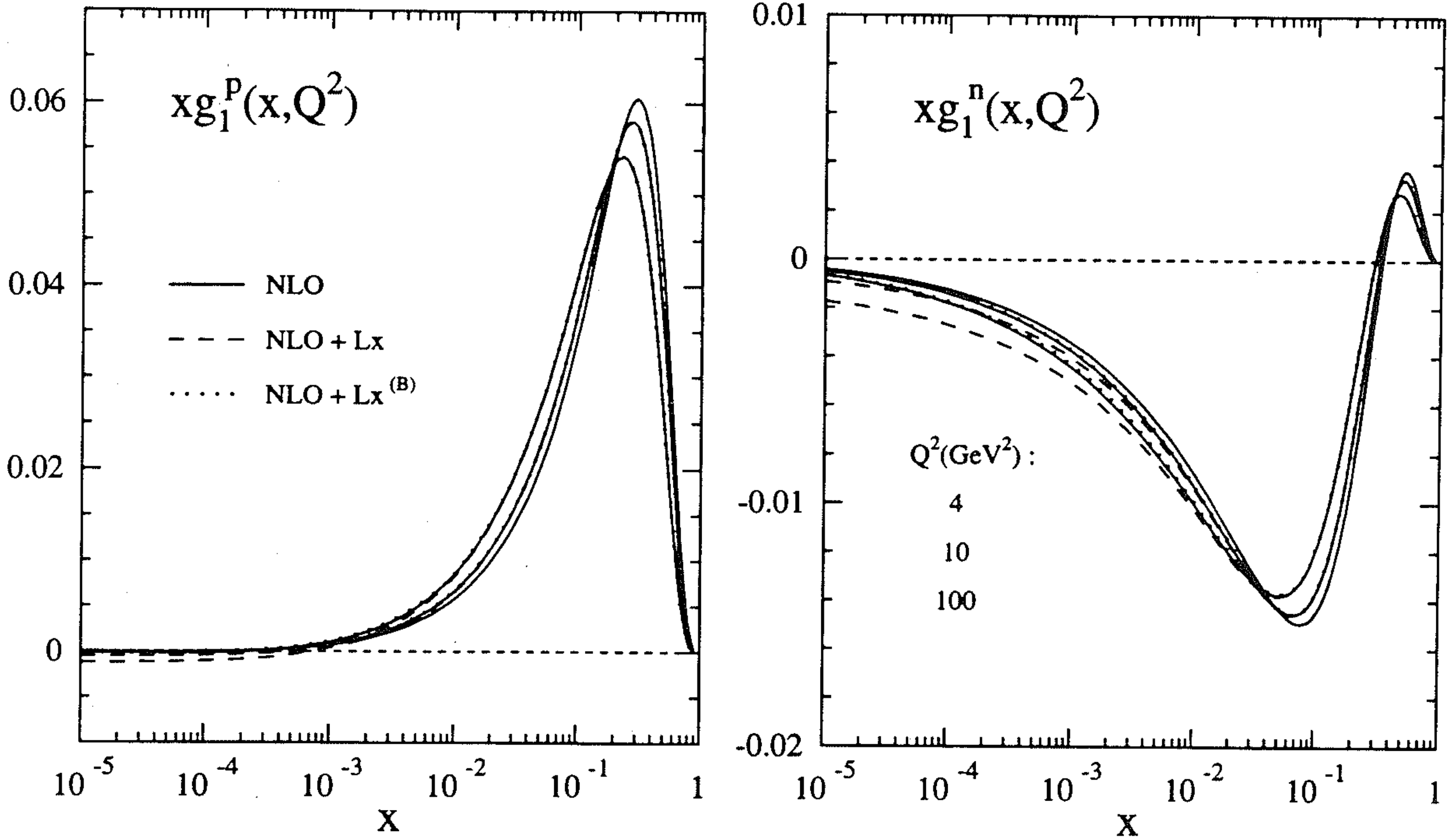
GRSV, DIFFERENT SETS.
(VAL 3 GLUONS, STD)
& THEIR SCALING VIOLATIONS

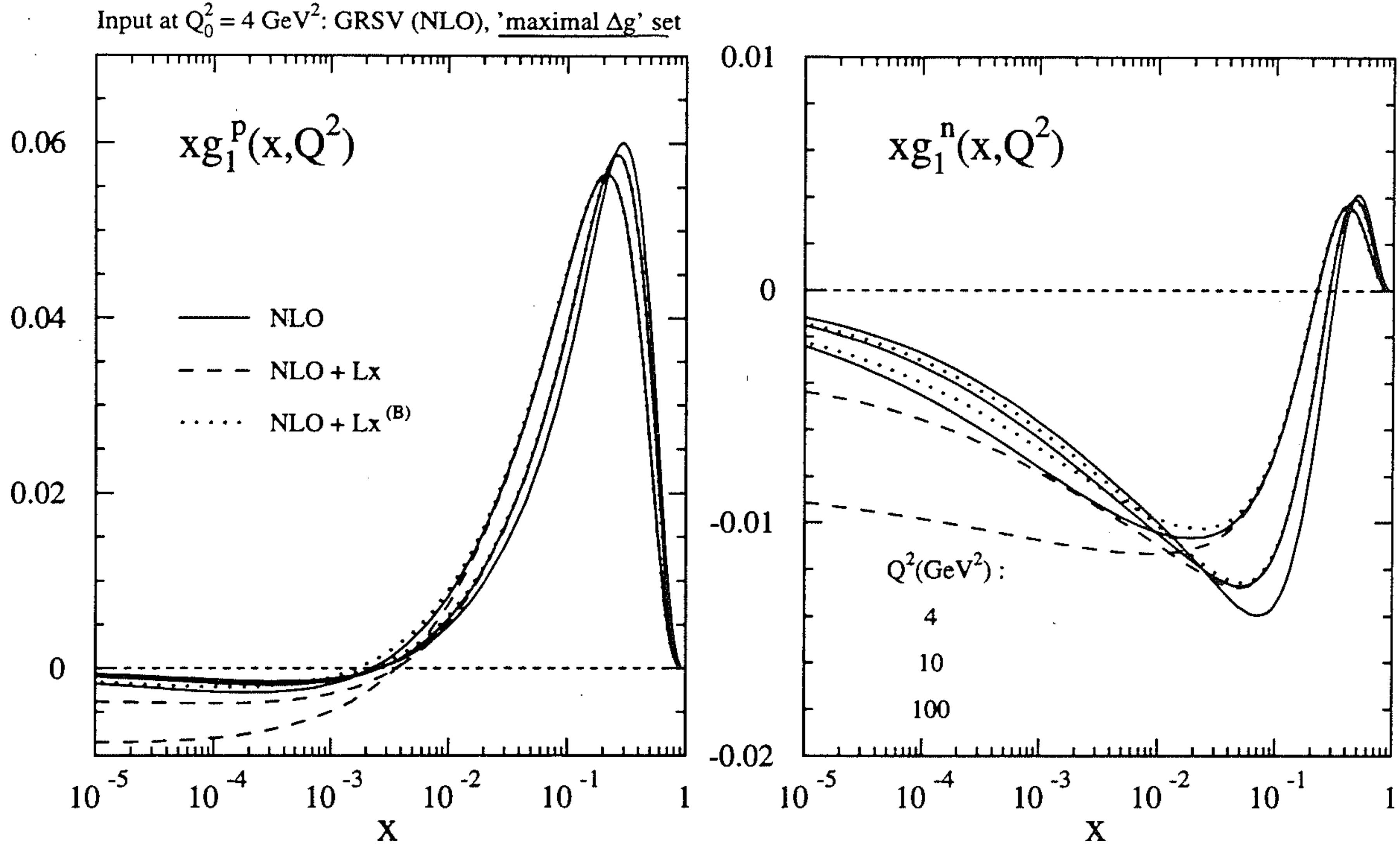

Input at $Q_0^2 = 4$ GeV^2: GRSV (NLO), 'minimal Δg' set
$x\Delta\Sigma(x,Q^2)$
NLO
NLO + Lx
NLO + Lx$^{(B)}$
$\Gamma^s \cdot (1-N)$
$x\Delta g(x,Q^2)$
Q^2(GeV^2) :
100
10
4
x
x



Input at $Q_0^2 = 4$ GeV^2: GRSV (NLO), 'maximal Δg' set
$x\Delta\Sigma(x,Q^2)$
NLO
NLO + Lx
NLO + Lx$^{(B)}$
$x\Delta g(x,Q^2)$
Q^2(GeV^2) :
100
10
4
0.3
0.2
0.1
0
-0.1
0.8
0.6
0.4
0.2
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

Input at $Q_0^2 = 4\ GeV^2$: GRSV (NLO), 'minimal Δg' set




Input at $Q_0^2 = 4$ GeV^2: GRSV (NLO), 'maximal Δg' set
$xg_1^p(x,Q^2)$
NLO
NLO + Lx
NLO + Lx$^{(B)}$
$xg_1^n(x,Q^2)$
Q^2(GeV^2) :
4
10
100
x
x

5. Conclusions

1) THE SMALL X RESUMMATIONS $\left(\frac{\alpha}{N-1}\right)^k$, $\alpha\left(\frac{\alpha}{N-1}\right)^k$, $N\left(\frac{\alpha}{N^2}\right)^k$ AGREE WITH THE ACCORDING RESULTS OF FIXED ORDER PT IN ALL KNOWN ORDERS (NLO).

2) PREDICTIONS FOR THE NNLO SPLITTING FUNCTIONS FOR THE $\alpha(\alpha \ln^2 x)^k$ TERM IN 3-LOOP ORDER CAN BE MADE DUE TO THE KNOWN BEHAVIOUR OF THE COEFFICIENT FUNCTIONS (POL. S; UNPOL. NS, QED NS) ($\overline{MS}$).

3) DUE TO THE VIOLATION OF THE GL-RELATION IN NLO NO PREDICTION CAN BE MADE FOR $q^2 > 0$.

4) THE CORRECTIONS DUE TO THE $\alpha(\alpha \ln^2 x)^k$ TERMS IS OF $< O(1\%)$ FOR ALL QCD NS STRUCTURE FCTS. (xF_3, F_2^{NS}, g_{1NS}) IN THE KIN RANGE TO BE REACHED @ HERA e.g.

5) AT HIGH y AND SMALL x A RATHER LARGE QED CORRECTION IS IMPLIED (STILL UP TO 10%), HERA RANGE.

6) FERMION NUMBER CONSERVATION (OR 4MOMENTUM CONSERVATION) MAY IMPLY DRASTIC CHANGES IN THE TERMS BEYOND NLO.

$$\Gamma \to \Gamma(N) - \Gamma(1); \quad \Gamma(N)(1-N); \quad \Gamma(N)(1-2N+N^2)$$
$$\Gamma(N)(1-2N+N^3)$$

I.E. THE $\exists$ 'SUB' LEADING TERMS ARE AS IMPORTANT. $\longrightarrow$ 3 LOOP CALCULATIONS...